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# Cartesian Product of Complex Neutrosophic Soft Graphs

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**Abstract**. The Cartesian product of complex neutrosophic soft graphs is defined and its basic structural aspects are examined in this work. Under the product operation, the membership functions of vertices and edges are examined. The work shows how complex neutrosophic soft graphs can be methodically built from simpler components and provides preservation findings.

**Keywords:** Soft Graph; Neutrosophic Soft Graph; Complex Neutrosophic Soft Graph.

## 1. Introduction

Zadeh [1] developed fuzzy set theory to deal with uncertainty. Since then, many researchers have studied fuzzy logic and fuzzy set theory in an attempt to solve a range of real-world problems involving ambiguous circumstances. According to Smarandache [2], neutrosophic sets (NSs) are a useful mathematical tool for dealing with confusing, inconsistent, and incomplete real-world data. Neutrosophic sets are distinguished by a truth-membership function ($T$), an indeterminacy-membership function ($I$), and a falsity-membership function ($F$) that can all have values in the standard or non-standard interval of 0 to 1. Because the concept of a Neutrosophic Set is important, several extended notions of Neutrosophic Sets have also been introduced [20–22].

Akram [3–6] distinguishes between two soft computing methods for conveying ambiguity and uncertainty: fuzzy sets and soft sets. They used soft computing models to examine graph ambiguity and uncertainty. Several regular fuzzy soft graph properties were investigated. He then discussed intuitionistic fuzzy soft graphs, neutrosophic soft graphs, and their uses. In a soft graph, each vertex and edge has a set of parameters. According to Satham Hussain [7], fuzzy sets and neutrosophic soft sets are effective ways to describe data with information. This method was used to create neutrosophic soft graphs. Decision-makers can then customize their findings to fit their particular fields of expertise.

Satham Hussain [8] introduced the ideas of *neutrosophic graphs* and *strong neutrosophic graphs*, which served as a basis for more in-depth research. Neutrosophic graphs and graph components are combined in this new theoretical framework. The total membership values for truth, indeterminacy, and falsity in this case fall between 0 and 2 as truth and falsity are considered dependent variables. Applications in operations research and issues in social networks are the focus of this theory's development. In the latter industry, the rise of phony or false profiles is an especially serious issue. A new solution to these problems is provided by the neutrosophic graph model. Examples were provided to demonstrate the suggested concepts, and strong, complete, and self-complementary neutrosophic graphs were examined.

The complex neutrosophic soft set model, which blends neutrosophic and soft sets, was proposed by Said Broumi [9, 10]. It presents and illustrates the fundamental operations of set theory along with other concepts related to this model structure. The use of this paradigm is demonstrated using a decision-making challenge including subjective and ambiguous information. This illustration shows how well the approach manages ambiguity and encourages well-informed decision-making. It also introduced fermatean neutrosophic graphs and provides opportunities for further research to enhance the methodology and take into account its ramifications in other domains, such as artificial intelligence and decision science.

The concept of complex neutrosophic soft graphs (CNSGs), which provide a sophisticated framework for simulating indeterminacy, ambiguity, and uncertainty in complex systems, is introduced and developed by Suriya [11–14]. High degrees of neutrosophic uncertainty and complex-valued memberships for both edges and vertices under soft set parameters define a strong complex neutrosophic soft graph (SCNSG). We present a novel measure that considers the cumulative strength and uncertainty of connections within graph characteristics, as well as the complements of a complex neutrosophic soft graph and a strong complex neutrosophic soft graph.

The Cartesian product of complex neutrosophic soft graphs is defined and its basic structural characteristics are examined in this work. The Cartesian product procedure is used to analyze vertex and edge membership functions. A number of preservation outcomes concerning graph structure and connectivity are established. The suggested structure shows how smaller graphs can be methodically transformed into complex neutrosophic soft graphs. Additionally, possible uses of complex neutrosophic soft graphs for complex network systems and uncertainty modeling are examined.

## 2. Preliminaries

This section provides a basic description and example to support the key findings.

**Definition 2.1.** *[10] Assume $U \neq \varphi$, a complex fuzzy set $A$ is an entity with the following structure:*

$$A = \{(x, \rho_A(x)) : x \in U\} = \{x, \mathfrak{X}_A(x)e^{iw_A(x)}\}$$

*where $i = \sqrt{-1}, \mathfrak{X}_A(x) \in [0,1]$ and $0 \leq w_A(x) \leq 2\pi$.*

**Definition 2.2.** *[1] Assume $U \neq \varphi$. A complex intuitionistic fuzzy set $A$ is defined as follows:*

$$A = \{(x, \mathfrak{X}_A(x), \mathfrak{Z}_A(x)) : x \in U\}$$

$$A = \{(x, \mathfrak{X}_A(x)e^{i\alpha_A(x)}, \mathfrak{Z}_A(x)e^{i\gamma_A(x)}) : x \in U\}$$

*where $i = \sqrt{-1}, \mathfrak{X}_A(x), \mathfrak{Z}_A(x) \in [0,1], \alpha_A(x), \gamma_X(x) \in [0, 2\pi]$ and $0^- \leq \mathfrak{X}_A(x) + \mathfrak{Z}_A(x) \leq 1^+$.*

**Definition 2.3.** *[1] Assume $U \neq \varphi$. An object with the shape of a complex neutrosophic set $A$ is*

$$A = \{(x, \mathfrak{X}_A(x), \mathfrak{Y}_A(x), \mathfrak{Z}_F(x)) : x \in U\}$$

$$A = \{(x, \mathfrak{X}_A(x)e^{i\alpha_A(x)}, \mathfrak{Y}_A(x)e^{i\beta_A(x)}, \mathfrak{Z}_A(x)e^{I\gamma_A(x)}) : x \in U\}$$

*where $i = \sqrt{-1}, \mathfrak{X}_A(x), \mathfrak{Y}_A(x), \mathfrak{Z}_A(x) \in [0,1], \alpha_A(x), \beta_A(x), \gamma_X(x) \in [0, 2\pi]$ and $0^- \leq \mathfrak{X}_A(x) + \mathfrak{Y}_A(x) + \mathfrak{Z}_A(x) \leq 3^+$.*

**Definition 2.4.** *( [4]) A fuzzy soft graph $G = (G^*, M, N, R)$ is an ordered quadruple that meets the following conditions:*

- *$G^* = (V, E)$ is a simple graph,*
- *$R$ is a non-empty set of parameters,*
- *$(M, R)$ is a fuzzy set defined over the vertex set $V$,*

- *$(N, R)$ is a fuzzy set defined over the edge set $E$,*
- *For each $a \in R$, the pair $(M(a), N(a))$ forms a fuzzy graph associated with $G^*$, such that:*

$$N(a)(xy) \leq \min\{M(a)(x), M(a)(y)\}$$

*holds for all $x, y \in V$.*

*$\mathfrak{H}(a)$ represents the fuzzy graph $(M(a), N(a))$. A fuzzy soft graph can be thought of as a family of fuzzy graphs parameterized by $R$. $M(G^*)$ denotes the class of all fuzzy soft graphs over $G^*$.*

**Definition 2.5.** *[3] An intuitionistic fuzzy soft graph $G = (G^*, F, K, Q)$ is an ordered four tuple if it satisfies the following conditions:*

- *$G^* = (V, E)$ is a simple graph,*
- *$Q$ is a non-empty set of parameters,*
- *$(F, Q)$ is an intuitionistic fuzzy set over $V$,*
- *$(K, Q)$ is an intuitionistic fuzzy set over $E$,*
- *$(F(a), K(a))$ is an intuitionistic fuzzy graph of $G^*$ for all $a \in Q$. That is*

$$\mathfrak{X}_{K(a)}(ab) \leq \min\{\mathfrak{X}_{F(a)}(a), \mathfrak{X}_{F(a)}(b)\}$$

$$\mathfrak{Z}_{K(a)}(ab) \geq \max\{\mathfrak{Z}_{F(a)}(a), \mathfrak{Z}_{F(a)}(b)\}$$

*for all $a \in Q$ and $x, y \in V$. The intuitionistic fuzzy graph $(F(a), K(a))$ is denoted by $H(a)$ for convenience. An intuitionistic fuzzy soft graph is a parameterized family of intuitionistic fuzzy graphs.*

**Definition 2.6.** *[7] A neutrosophic soft graph $G = (G^*, M, N, R)$ is defined as a quadruple satisfying the following conditions:*

- *$G^* = (V, E)$ represents a simple (i.e., undirected and without loops or multiple edges) graph,*
- *$R$ is a non-empty collection of parameters,*
- *$(M, R)$ denotes a neutrosophic set defined over the vertex set $V$,*
- *$(N, R)$ denotes a neutrosophic set defined over the edge set $E$,*
- *For every $a \in R$, the pair $(M(a), N(a))$ forms a neutrosophic subgraph of $G^*$. Specifically, the following conditions hold:*

$$\mathfrak{X}_{N(a)}(xy) \leq \min\{\mathfrak{X}_{M(a)}(x), \mathfrak{X}_{M(a)}(y)\},$$

$$\mathfrak{Y}_{N(a)}(xy) \leq \min\{\mathfrak{Y}_{M(a)}(x), \mathfrak{Y}_{M(a)}(y)\},$$

$$\mathfrak{Z}_{N(a)}(xy) \geq \max\{\mathfrak{Z}_{M(a)}(x), \mathfrak{Z}_{M(a)}(y)\},$$

*For ease of use, $\mathfrak{H}(a)$ is used to represent the neutrosophic fuzzy graph $(M(a), N(a))$. A neutrosophic fuzzy soft graph may therefore be thought of as a family of neutrosophic fuzzy* *graphs that are parameterized by $R$. $M(G^*)$ represents the class of all neutrosophic fuzzy soft graphs over $G^*$, for all $a \in R$ and $x, y \in V$.*

## 3. Cartesian Product of Complex Neutrosophic Soft Graphs.

We explain the Cartesian product of complex neutrosophic soft graphs.

**Definition 3.1.** *A **Complex Neutrosophic Soft Graph (CNSG)** is defined as an ordered quadruple $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$ that satisfies the following conditions:*

- *$G^* = (V, E)$ is a simple graph.*
- *$L$ is a non-empty set of parameters.*
- *$(\mathfrak{S}, L)$ constitutes a complex neutrosophic soft (CNS) set over the vertex set $V$.*
- *$(\mathfrak{K}, L)$ represents a CNS set over the edge set $E$.*
- *For each parameter $l \in L$, the pair $(\mathfrak{S}(l), \mathfrak{K}(l))$ forms a complex neutrosophic soft graph (CNSG) on $G^*$.*

*1. Let $V = \{l_1, l_2, \cdots, l_n\}$, and consider the functions:*

$$\mathfrak{X}_{\mathfrak{S}(l)} : V \to [0,1], \quad \mathfrak{Y}_{\mathfrak{S}(l)} : V \to [0,1], \quad \mathfrak{Z}_{\mathfrak{S}(l)} : V \to [0,1]$$

*The degrees of truth-membership, indeterminacy-membership and falsity-membership for each vertex $p_i \in V$ under the parameter $l \in L$ are represented by these functions, respectively. Furthermore, in $[0, 2\pi]$, let $A_{\mathfrak{S}(l)}, B_{\mathfrak{S}(l)}$, and $C_{\mathfrak{S}(l)}$ represent the corresponding argument functions for every membership function. Afterward, for every vertex $p_i$:*

$$0 \leq \mathfrak{X}_{\mathfrak{S}(l)}(p_i)e^{iA_{\mathfrak{S}(l)}(p_i)} + \mathfrak{Y}_{\mathfrak{S}(l)}(p_i)e^{iB_{\mathfrak{S}(l)}(p_i)} + \mathfrak{Z}_{\mathfrak{S}(l)}(p_i)e^{iC_{\mathfrak{S}(l)}(p_i)} \leq 2$$

*2. For the edge set $E \subseteq V \bigotimes V$, the corresponding functions are defined as:*

$$\mathfrak{X}_{\mathfrak{K}(l)} : E \to [0,1], \quad \mathfrak{Y}_{\mathfrak{K}(l)} : E \to [0,1], \quad \mathfrak{Z}_{\mathfrak{K}(l)} : E \to [0,1]$$

*With regard to parameter $l$, these functions indicate the degrees of truth-membership, indeterminacy-membership and falsity-membership for every edge $p_ip_j \in E$. The functions $A_{\mathfrak{K}(l)}, B_{\mathfrak{K}(l)}$, and $C_{\mathfrak{K}(l)} \in [0, 2\pi]$ are argument functions. These also correspond to the associated membership functions.*

*For each edge $p_ip_j \in E$, the following conditions must be satisfied:*

$$\mathfrak{X}_{\mathfrak{K}(l)}(p_ip_j)e^{iA_{\mathfrak{K}(l)}(p_ip_j)} \leq [\mathfrak{X}_{\mathfrak{S}(l)}(p_i) \wedge \mathfrak{X}_{\mathfrak{S}(l)}(p_j)]e^{i\{A_{\mathfrak{S}(l)}(p_i) \wedge A_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{Y}_{\mathfrak{K}(l)}(p_ip_j)e^{iB_{\mathfrak{K}(l)}(p_ip_j)} \leq [\mathfrak{Y}_{\mathfrak{S}(l)}(p_i) \wedge \mathfrak{Y}_{\mathfrak{S}(l)}(p_j)]e^{i\{B_{\mathfrak{S}(l)}(p_i) \wedge B_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{Z}_{\mathfrak{K}(l)}(p_ip_j)e^{iC_{\mathfrak{K}(l)}(p_ip_j)} \geq [\mathfrak{Z}_{\mathfrak{S}(l)}(p_i) \vee \mathfrak{Z}_{\mathfrak{S}(l)}(p_j)]e^{i\{C_{\mathfrak{S}(l)}(p_i) \vee C_{\mathfrak{S}(l)}(p_j)\}}$$

*Furthermore, each edge satisfies the bounded magnitude condition:*

$$0 \leq \mathfrak{X}_{\mathfrak{K}(l)}(p_ip_j)e^{iA_{\mathfrak{K}(l)}(p_ip_j)} + \mathfrak{Y}_{\mathfrak{K}(l)}(p_ip_j)e^{iB_{\mathfrak{K}(l)}(p_ip_j)} + \mathfrak{Z}_{\mathfrak{K}(l)}(p_ip_j)e^{iC_{\mathfrak{K}(l)}(p_ip_j)} \leq 2$$

*For simplicity, the pair* $(\mathfrak{S}(l), \mathfrak{K}(l))$ *is known as the complex neutrosophic graph* $\mathbb{G}$*. Thus, a CNSG is a parameterized structure that expands on the idea of a complex neutrosophic graph.*

**Definition 3.2** (Cartesian product of complex neutrosophic soft graphs)**.** *Let*

$$G_1 = (G_1^*, \mathfrak{S}_1, \mathfrak{K}_1, L_1) \qquad \text{and} \qquad G_2 = (G_2^*, \mathfrak{S}_2, \mathfrak{K}_2, L_2)$$

*be two complex neutrosophic soft graphs (CNSGs), where*

$$G_1^* = (V_1, E_1), \qquad G_2^* = (V_2, E_2).$$

*The* Cartesian product *of* $G_1$ *and* $G_2$*, denoted by*

$$G_1 \bigotimes G_2,$$

*is the CNSG*

$$G_1 \bigotimes G_2 = \big(G^*, \mathfrak{S}, \mathfrak{K}, L_1 \times L_2\big),$$

*where the underlying crisp graph is*

$$G^* = (V_1 \times V_2, E),$$

*with*

$$E = \big\{(p, q_1)(p, q_2) \mid p \in V_1,\ q_1q_2 \in E_2\big\} \cup \big\{(p_1, q)(p_2, q) \mid p_1p_2 \in E_1,\ q \in V_2\big\}.$$

*For each* $(l, k) \in L_1 \times L_2$*, the vertex-membership values are defined by*

$$\mathfrak{X}_{\mathfrak{S}_1(l)\otimes\mathfrak{S}_2(k)}(p, q)\, e^{iA_{\mathfrak{S}_1(l)\otimes\mathfrak{S}_2(k)}(p,q)} = \big[\mathfrak{X}_{\mathfrak{S}_1(l)}(p) \wedge \mathfrak{X}_{\mathfrak{S}_2(k)}(q)\big] e^{i\left(A_{\mathfrak{S}_1(l)}(p)\wedge A_{\mathfrak{S}_2(k)}(q)\right)},$$

$$\mathfrak{Y}_{\mathfrak{S}_1(l)\otimes\mathfrak{S}_2(k)}(p, q)\, e^{iB_{\mathfrak{S}_1(l)\otimes\mathfrak{S}_2(k)}(p,q)} = \big[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p) \wedge \mathfrak{Y}_{\mathfrak{S}_2(k)}(q)\big] e^{i\left(B_{\mathfrak{S}_1(l)}(p)\wedge B_{\mathfrak{S}_2(k)}(q)\right)},$$

$$\mathfrak{Z}_{\mathfrak{S}_1(l)\otimes\mathfrak{S}_2(k)}(p, q)\, e^{iC_{\mathfrak{S}_1(l)\otimes\mathfrak{S}_2(k)}(p,q)} = \big[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p) \vee \mathfrak{Z}_{\mathfrak{S}_2(k)}(q)\big] e^{i\left(C_{\mathfrak{S}_1(l)}(p)\vee C_{\mathfrak{S}_2(k)}(q)\right)},$$

*for all* $(p, q) \in V_1 \times V_2$*.*
*The edge-membership values are defined as follows.*

***(1)*** *If* $(p, q_1)(p, q_2) \in E$*, where* $p \in V_1$ *and* $q_1q_2 \in E_2$*, then*

$$\mathfrak{X}_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}\big((p, q_1)(p, q_2)\big)\, e^{iA_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}((p,q_1)(p,q_2))} = \big[\mathfrak{X}_{\mathfrak{S}_1(l)}(p) \wedge \mathfrak{X}_{\mathfrak{K}_2(k)}(q_1q_2)\big] \cdot e^{i\left(A_{\mathfrak{S}_1(l)}(p)\wedge A_{\mathfrak{K}_2(k)}(q_1q_2)\right)},$$

$$\mathfrak{Y}_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}\big((p, q_1)(p, q_2)\big)\, e^{iB_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}((p,q_1)(p,q_2))} = \big[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p) \wedge \mathfrak{Y}_{\mathfrak{K}_2(k)}(q_1q_2)\big] \cdot e^{i\left(B_{\mathfrak{S}_1(l)}(p)\wedge B_{\mathfrak{K}_2(k)}(q_1q_2)\right)},$$

$$\mathfrak{Z}_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}\big((p, q_1)(p, q_2)\big)\, e^{iC_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}((p,q_1)(p,q_2))} = \big[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p) \vee \mathfrak{Z}_{\mathfrak{K}_2(k)}(q_1q_2)\big] \cdot e^{i\left(C_{\mathfrak{S}_1(l)}(p)\vee C_{\mathfrak{K}_2(k)}(q_1q_2)\right)}.$$

***(2)*** *If* $(p_1,q)(p_2,q)\in E$*, where* $p_1p_2\in E_1$ *and* $q\in V_2$*, then*

$$\mathfrak{X}_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}\big((p_1,q)(p_2,q)\big)\,e^{iA_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}((p_1,q)(p_2,q))}=\big[\mathfrak{X}_{\mathfrak{K}_1(l)}(p_1p_2)\wedge\mathfrak{X}_{\mathfrak{S}_2(k)}(q)\big]\cdot e^{i\left(A_{\mathfrak{K}_1(l)}(p_1p_2)\wedge A_{\mathfrak{S}_2(k)}(q)\right)},$$

$$\mathfrak{Y}_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}\big((p_1,q)(p_2,q)\big)\,e^{iB_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}((p_1,q)(p_2,q))}=\big[\mathfrak{Y}_{\mathfrak{K}_1(l)}(p_1p_2)\wedge\mathfrak{Y}_{\mathfrak{S}_2(k)}(q)\big]\cdot e^{i\left(B_{\mathfrak{K}_1(l)}(p_1p_2)\wedge B_{\mathfrak{S}_2(k)}(q)\right)},$$

$$\mathfrak{Z}_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}\big((p_1,q)(p_2,q)\big)\,e^{iC_{\mathfrak{K}_1(l)\otimes\mathfrak{K}_2(k)}((p_1,q)(p_2,q))}=\big[\mathfrak{Z}_{\mathfrak{K}_1(l)}(p_1p_2)\vee\mathfrak{Z}_{\mathfrak{S}_2(k)}(q)\big]\cdot e^{i\left(C_{\mathfrak{K}_1(l)}(p_1p_2)\vee C_{\mathfrak{S}_2(k)}(q)\right)}.$$

**Theorem 3.3.** *Cartesian product* $G_1\bigotimes G_2=(V_1\bigotimes V_2,E_1\bigotimes E_2)$ *of two CNSGs,* $G_1$ *and* $G_2$*, is also the CNSG of* $G_1\bigotimes G_2$*.*

***Proof:*** *Two cases are examined. Case 1: For* $p\in V_1$*,* $q_1q_2\in E_2$

$$\mathfrak{X}_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((pq_1)(pq_2))e^{iA_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((pq_1)(pq_2))}=[\mathfrak{X}_{\mathfrak{S}_1(l)}(p)\wedge\mathfrak{X}_{\mathfrak{K}_2(k)}(q_1q_2)]e^{i\{A_{\mathfrak{S}_1(l)}(p)\wedge A_{\mathfrak{K}_2(k)}(q_1q_2)\}}$$

$$\leq[\mathfrak{X}_{\mathfrak{S}_1(l)}(p)\wedge[\mathfrak{X}_{\mathfrak{S}_2(k)}(q_1)\wedge\mathfrak{X}_{\mathfrak{S}_2(k)}(q_2)]]e^{i\{A_{\mathfrak{S}_1(l)}(p)\wedge\{A_{\mathfrak{S}_2(k)}(q_1)\wedge A_{\mathfrak{S}_2(k)}(q_2)\}\}}$$

$$=[[\mathfrak{X}_{\mathfrak{S}_1(l)}(p)\wedge\mathfrak{X}_{\mathfrak{S}_2(k)}(q_1)]\wedge[\mathfrak{X}_{\mathfrak{S}_1(l)}(p)\wedge\mathfrak{X}_{\mathfrak{S}_2(k)}(q_2)]]e^{i\{A_{\mathfrak{S}_1(l)}(p)\wedge A_{\mathfrak{S}_2(k)}(q_1)\}\wedge\{(A_{\mathfrak{S}_1(l)}(p)\wedge A_{\mathfrak{S}_2(k)}(q_2)\}}$$

$$=[\mathfrak{X}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_1)\wedge\mathfrak{X}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_2)]e^{i\{A_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_1)\wedge A_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_2)\}}$$

$$\mathfrak{Y}_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((pq_1)(pq_2))e^{iB_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((pq_1)(pq_2))}=[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p)\wedge\mathfrak{Y}_{\mathfrak{K}_2(k)}(q_1q_2)]e^{i\{B_{\mathfrak{S}_1(l)}(p)\wedge B_{\mathfrak{K}_2(k)}(q_1q_2)\}}$$

$$\leq[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p)\wedge[\mathfrak{Y}_{\mathfrak{S}_2(k)}(q_1)\wedge\mathfrak{Y}_{\mathfrak{S}_2(k)}(q_2)]]e^{i\{B_{\mathfrak{S}_1(l)}(p)\wedge\{B_{\mathfrak{S}_2(k)}(q_1)\wedge B_{\mathfrak{S}_2(k)}(q_2)\}\}}$$

$$=[[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p)\wedge\mathfrak{Y}_{\mathfrak{S}_2(k)}(q_1)]\wedge[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p)\wedge\mathfrak{Y}_{\mathfrak{S}_2(k)}(q_2)]]e^{i\{B_{\mathfrak{S}_1(l)}(p)\wedge B_{\mathfrak{S}_2(k)}(q_1)\}\wedge\{(B_{\mathfrak{S}_1(l)}(p)\wedge B_{\mathfrak{S}_2(k)}(q_2)\}}$$

$$=[\mathfrak{Y}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_1)\wedge\mathfrak{Y}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_2)]e^{i\{B_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_1)\wedge B_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_2)\}}$$

$$\mathfrak{Z}_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((pq_1)(pq_2))e^{iC_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((pq_1)(pq_2))}=[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p)\vee\mathfrak{Z}_{\mathfrak{K}_2(k)}(q_1q_2)]e^{i\{C_{\mathfrak{S}_1(l)}(p)\vee C_{\mathfrak{K}_2(k)}(q_1q_2)\}}$$

$$\leq[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p)\vee[\mathfrak{Z}_{\mathfrak{S}_2(k)}(q_1)\vee\mathfrak{Z}_{\mathfrak{S}_2(k)}(q_2)]]e^{i\{C_{\mathfrak{S}_1(l)}(p)\vee\{C_{\mathfrak{S}_2(k)}(q_1)\vee C_{\mathfrak{S}_2(k)}(q_2)\}\}}$$

$$=[[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p)\vee\mathfrak{Z}_{\mathfrak{S}_2(k)}(q_1)]\vee[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p)\vee\mathfrak{Z}_{\mathfrak{S}_2(k)}(q_2)]]e^{i\{C_{\mathfrak{S}_1(l)}(p)\vee C_{\mathfrak{S}_2(k)}(q_1)\}\vee\{(C_{\mathfrak{S}_1(l)}(p)\vee C_{\mathfrak{S}_2(k)}(q_2)\}}$$

$$=[\mathfrak{Z}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_1)\vee\mathfrak{Z}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_2)]e^{i\{C_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_1)\vee C_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(rq_2)\}}$$

*for all* $pq_1,pq_2\in G_1\bigotimes G_2$

*Case 2: For* $q\in V_2$*,* $p_1p_2\in E_1$

$\mathfrak{X}_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((p_1q)(p_2q))e^{iA_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((p_1q)(p_2q))} =$
$[\mathfrak{X}_{\mathfrak{S}_2(k)}(q)\wedge\mathfrak{X}_{\mathfrak{K}_1(l)}(p_1p_2)]e^{i\{A_{\mathfrak{S}_2(k)}(q)\wedge A_{\mathfrak{K}_1(l)}(p_1p_2)\}}$

$$\leq [\mathfrak{X}_{\mathfrak{S}_2(k)}(q)\wedge[\mathfrak{X}_{\mathfrak{S}_1(l)}(p_1)\wedge\mathfrak{X}_{\mathfrak{S}_1(l)}(p_2)]]e^{i\{A_{\mathfrak{S}_2(k)}(q)\wedge\{A_{\mathfrak{S}_1(l)}(p_1)\wedge A_{\mathfrak{S}_1(l)}(p_2)\}\}}$$
$$= [[\mathfrak{X}_{\mathfrak{S}_2(k)}(q)\wedge\mathfrak{X}_{\mathfrak{S}_1(l)}(p_1)]\wedge[\mathfrak{X}_{\mathfrak{S}_2(k)}(q)\wedge\mathfrak{X}_{\mathfrak{S}_1(l)}(p_2)]]e^{i\{A_{\mathfrak{S}_2(k)}(q)\wedge A_{\mathfrak{S}_1(l)}(p_1)\}\wedge\{A_{\mathfrak{S}_2(k)}(q)\wedge A_{\mathfrak{S}_1(l)}(p_2)\}}$$
$$= [\mathfrak{X}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_1q)\wedge\mathfrak{X}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_2q)]e^{i\{A_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_1q)\wedge A_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_2q)\}}$$

$\mathfrak{Y}_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((p_1q)(p_2q))e^{iB_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((p_1q)(p_2q))} =$
$[\mathfrak{Y}_{\mathfrak{S}_2(k)}(q)\wedge\mathfrak{Y}_{\mathfrak{K}_1(l)}(p_1p_2)]e^{i\{B_{\mathfrak{S}_2(k)}(q)\wedge B_{\mathfrak{K}_1(l)}(p_1p_2)\}}$

$$\leq [\mathfrak{Y}_{\mathfrak{S}_2(k)}(q)\wedge[\mathfrak{Y}_{\mathfrak{S}_1(l)}(p_1)\wedge\mathfrak{Y}_{\mathfrak{S}_1(l)}(p_2)]]e^{i\{B_{\mathfrak{S}_2(k)}(q)\wedge\{B_{\mathfrak{S}_1(l)}(p_1)\wedge B_{\mathfrak{S}_1(l)}(p_2)\}\}}$$
$$= [[\mathfrak{Y}_{\mathfrak{S}_2(k)}(q)\wedge\mathfrak{Y}_{\mathfrak{S}_1(l)}(p_1)]\wedge[\mathfrak{Y}_{\mathfrak{S}_2(k)}(q)\wedge\mathfrak{Y}_{\mathfrak{S}_1(l)}(p_2)]]e^{i\{B_{\mathfrak{S}_2(k)}(q)\wedge B_{\mathfrak{S}_1(l)}(p_1)\}\wedge\{B_{\mathfrak{S}_2(k)}(q)\wedge B_{\mathfrak{S}_1(l)}(p_2)\}}$$
$$= [\mathfrak{Y}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_1q)\wedge\mathfrak{Y}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_2q)]e^{i\{B_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_1q)\wedge B_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_2q)\}}$$

$\mathfrak{Z}_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((p_1q)(p_2q))e^{iC_{\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k)}((p_1q)(p_2q))} =$
$[\mathfrak{Z}_{\mathfrak{S}_2(k)}(q)\vee\mathfrak{Z}_{\mathfrak{K}_1(l)}(p_1p_2)]e^{i\{C_{\mathfrak{S}_2(k)}(q)\vee C_{\mathfrak{K}_1(l)}(p_1p_2)\}}$

$$\leq [\mathfrak{Z}_{\mathfrak{S}_2(k)}(q)\vee[\mathfrak{Z}_{\mathfrak{S}_1(l)}(p_1)\vee\mathfrak{Z}_{\mathfrak{S}_1(l)}(p_2)]]e^{i\{C_{\mathfrak{S}_2(k)}(q)\vee\{C_{\mathfrak{S}_1(l)}(p_1)\vee C_{\mathfrak{S}_1(l)}(p_2)\}\}}$$
$$= [[\mathfrak{Z}_{\mathfrak{S}_2(k)}(q)\vee\mathfrak{Z}_{\mathfrak{S}_1(l)}(p_1)]\vee[\mathfrak{Z}_{\mathfrak{S}_2(k)}(q)\vee\mathfrak{Z}_{\mathfrak{S}_1(l)}(p_2)]]e^{i\{C_{\mathfrak{S}_2(k)}(q)\vee C_{\mathfrak{S}_1(l)}(p_1)\}\vee\{C_{\mathfrak{S}_2(k)}(q)\vee C_{\mathfrak{S}_1(l)}(p_2)\}}$$
$$= [\mathfrak{Z}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_1q)\vee\mathfrak{Z}_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_2q)]e^{i\{C_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_1q)\vee C_{\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k)}(p_2q)\}}$$

*for all* $p_1q, p_2q \in G_1 \bigotimes G_2$.

**Example 3.4.** *Examine two CNSGs* $G_1 = (V_1, E_1)$ *and* $G_2 = (V_2, E_2)$ *of* $G = (V, E)$*, as shown in figure 1. From this, we obtain the cartesian product* $(G_1 \bigotimes G_2)$ *as shown in figure 2. Let* $(\mathfrak{S}, L)$ *be a CNSS over* $V$ *and let* $L = (l)$ *be a set of parameters. The complex neutrosophic approximation function* $\mathfrak{S} : L \to L(V)$ *is defined by*

$$\mathfrak{S}(l_1) = \begin{cases} \{p_1, (.3e^{i\pi(1.6)}, .7e^{i\pi(1.8)}, .6e^{i\pi(1.4)})\} \\ \{p_2, (.2e^{i\pi(1.7)}, .6e^{i\pi(1.7)}, .7e^{i\pi(1.8)})\} \\ \{p_3, (.5e^{i\pi(1.4)}, .8e^{i\pi(1.6)}, .4e^{i\pi(1.6)})\} \end{cases}$$

*Let* $(\mathfrak{S}, K)$ *be a CNSS over* $V$ *and let* $K = (k_1, k_2)$ *be a set of parameters. The complex neutrosophic approximation function* $\mathfrak{S} : K \to K(V)$ *is defined by*

$$\mathfrak{S}(l_1) = \begin{cases} \{q_1, (.4e^{i\pi(1.5)}, .5e^{i\pi(1.4)}, .6e^{i\pi(1.5)})\} \\ \{q_2, (.6e^{i\pi(1.3)}, .7e^{i\pi(1.5)}, .4e^{i\pi(1.8)})\} \end{cases}$$
$$\mathfrak{S}(l_2) = \begin{cases} \{q_1, (.3e^{i\pi(1.2)}, .7e^{i\pi(1.4)}, .6e^{i\pi(1.7)})\} \\ \{q_2, (.2e^{i\pi(1.6)}, .6e^{i\pi(1.5)}, .8e^{i\pi(1.5)})\} \end{cases}$$

*Let $(\mathfrak{K}, L)$ be a CNSS over $E$ and let $L = (l)$ be a set of parameters. The complex neutrosophic approximation function $\mathfrak{K} : L \to L(E)$ is defined by*

$$\mathfrak{K}(l_1) = \begin{cases} \{p_1p_2, (.1e^{i\pi(1.5)}, .5e^{i\pi(1.6)}, .8e^{i\pi(1.9)})\} \\ \{p_2p_3, (.2e^{i\pi(1.3)}, .6e^{i\pi(1.5)}, .7e^{i\pi(1.8)})\} \end{cases}$$

*Let $(\mathfrak{K}, K)$ be a CNSS over $E$ and let $K = (k_1, k_2)$ be a set of parameters. The complex neutrosophic approximation function $\mathfrak{K} : K \to K(E)$ is defined by*

$$\mathfrak{K}(k_1) = \Big\{\{q_1q_2, (.3e^{i\pi(1.2)}, .4e^{i\pi(1.3)}, .7e^{i\pi(1.8)})\}$$
$$\mathfrak{K}(k_2) = \Big\{\{q_1, q_2(.2e^{i\pi(1.2)}, .6e^{i\pi(1.4)}, .8e^{i\pi(1.7)})\}$$

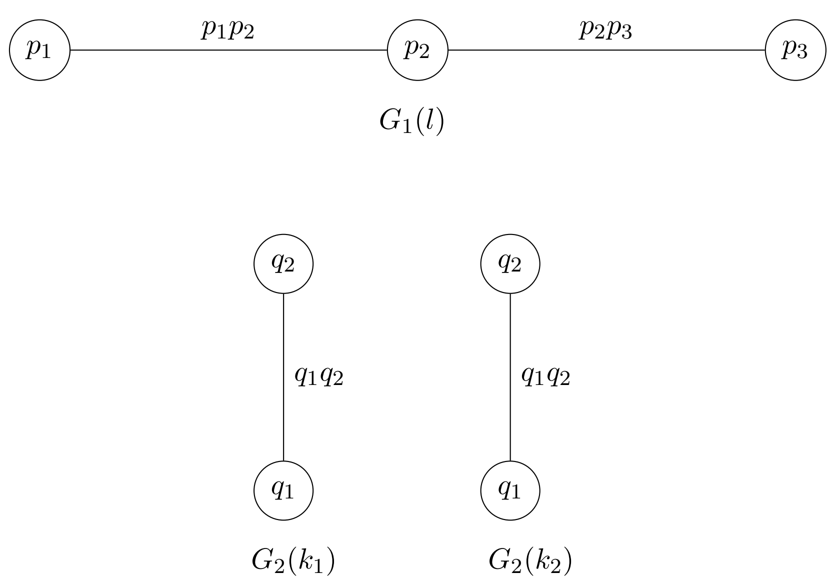


***Figure 1: Complex Neutrosophic Soft Graphs.***

*Vertices of CNSG $G_1 \bigotimes G_2$*

$$\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k_1)=\begin{cases}\{p_1q_1,(.3e^{i\pi(1.5)},.5e^{i\pi(1.4)},.6e^{i\pi(1.5)})\}\\\{p_2q_1,(.2e^{i\pi(1.5)},.5e^{i\pi(1.4)},.7e^{i\pi(1.8)})\}\\\{p_3q_1,(.4e^{i\pi(1.4)},.5e^{i\pi(1.4)},.6e^{i\pi(1.6)})\}\\\{p_1q_2,(.3e^{i\pi(1.3)},.7e^{i\pi(1.5)},.6e^{i\pi(1.8)})\}\\\{p_2q_2,(.2e^{i\pi(1.3)},.6e^{i\pi(1.5)},.7e^{i\pi(1.8)})\}\\\{p_3q_2,(.5e^{i\pi(1.3)},.7e^{i\pi(1.5)},.4e^{i\pi(1.8)})\}\end{cases}$$

$$\mathfrak{S}_1(l)\bigotimes\mathfrak{S}_2(k_2)=\begin{cases}\{p_1q_1,(.3e^{i\pi(1.2)},.7e^{i\pi(1.4)},.6e^{i\pi(1.7)})\}\\\{p_2q_1,(.2e^{i\pi(1.2)},.6e^{i\pi(1.4)},.7e^{i\pi(1.8)})\}\\\{p_3q_1,(.3e^{i\pi(1.2)},.7e^{i\pi(1.4)},.6e^{i\pi(1.7)})\}\\\{p_1q_2,(.2e^{i\pi(1.6)},.6e^{i\pi(1.5)},.8e^{i\pi(1.5)})\}\\\{p_2q_2,(.2e^{i\pi(1.6)},.6e^{i\pi(1.5)},.8e^{i\pi(1.8)})\}\\\{p_3q_2,(.2e^{i\pi(1.4)},.6e^{i\pi(1.5)},.8e^{i\pi(1.8)})\}\end{cases}$$

*Edges of CNSG* $G_1\bigotimes G_2$

$$\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k_1)=\begin{cases}\{p_1(q_1q_2),(.3e^{i\pi(1.2)},.4e^{i\pi(1.3)},.7e^{i\pi(1.8)})\}\\\{p_2(q_1q_2),(.2e^{i\pi(1.2)},.4e^{i\pi(1.3)},.7e^{i\pi(1.8)})\}\\\{p_3(q_1q_2),(.3e^{i\pi(1.2)},.4e^{i\pi(1.3)},.7e^{i\pi(1.8)})\}\\\{(p_1p_2)q_1,(.1e^{i\pi(1.5)},.5e^{i\pi(1.4)},.8e^{i\pi(1.9)})\}\\\{(p_1p_2)q_2,(.1e^{i\pi(1.3)},.6e^{i\pi(1.5)},.8e^{i\pi(1.9)})\}\\\{(p_2p_3)q_1,(.2e^{i\pi(1.3)},.5e^{i\pi(1.4)},.7e^{i\pi(1.8)})\}\\\{(p_2p_3)q_2,(.2e^{i\pi(1.3)},.6e^{i\pi(1.5)},.7e^{i\pi(1.8)})\}\end{cases}$$

$$\mathfrak{K}_1(l)\bigotimes\mathfrak{K}_2(k_2)=\begin{cases}\{p_1(q_1q_2),(.2e^{i\pi(1.2)},.6e^{i\pi(1.4)},.8e^{i\pi(1.7)})\}\\\{p_2(q_1q_2),(.2e^{i\pi(1.2)},.6e^{i\pi(1.4)},.8e^{i\pi(1.8)})\}\\\{p_3(q_1q_2),(.2e^{i\pi(1.2)},.6e^{i\pi(1.4)},.8e^{i\pi(1.7)})\}\\\{(p_1p_2)q_1,(.1e^{i\pi(1.2)},.5e^{i\pi(1.4)},.8e^{i\pi(1.9)})\}\\\{(p_1p_2)q_2,(.1e^{i\pi(1.5)},.5e^{i\pi(1.5)},.8e^{i\pi(1.9)})\}\\\{(p_2p_3)q_1,(.2e^{i\pi(1.2)},.6e^{i\pi(1.4)},.7e^{i\pi(1.8)})\}\\\{(p_2p_3)q_2,(.2e^{i\pi(1.3)},.6e^{i\pi(1.5)},.7e^{i\pi(1.8)})\}\end{cases}$$

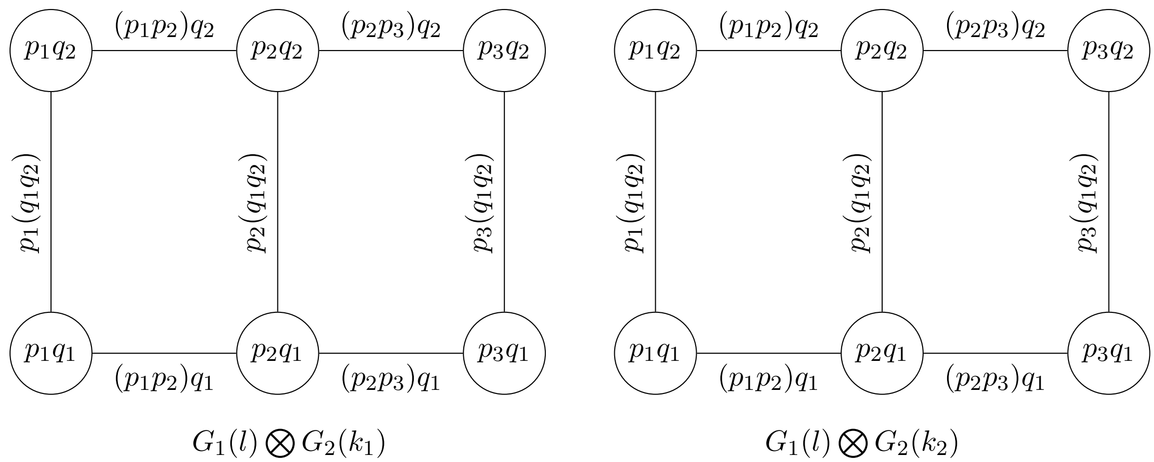


***Figure 2: Cartesian product of CNSG.***

## 4. Application: Decision-Making in Wireless Communication Networks using CNSG

A wireless sensor network (WSN) monitors system or environmental conditions by connecting several sensor nodes and exchanging data. However, uncertainties like noise, interference, and propagation delay, which result in phase changes in sent signals, often impede communication. Additionally, incomplete or missing data from particular sensors results in ambiguity, while transmission errors or packet losses lead to inaccurate information. Because of these features, it is difficult to assess communication link reliability using conventional deterministic or even fuzzy models. Implementing a more comprehensive framework that can simultaneously describe and analyze multiple types of uncertainty is therefore crucial. The Complex Neutrosophic soft Graph model generates such a structure by integrating truth, indeterminacy, and falsity degrees with complex (phase) and soft computing techniques. The goal of this technique is to determine the most reliable communication link between sensor nodes while accounting for all of the entangled uncertainty.

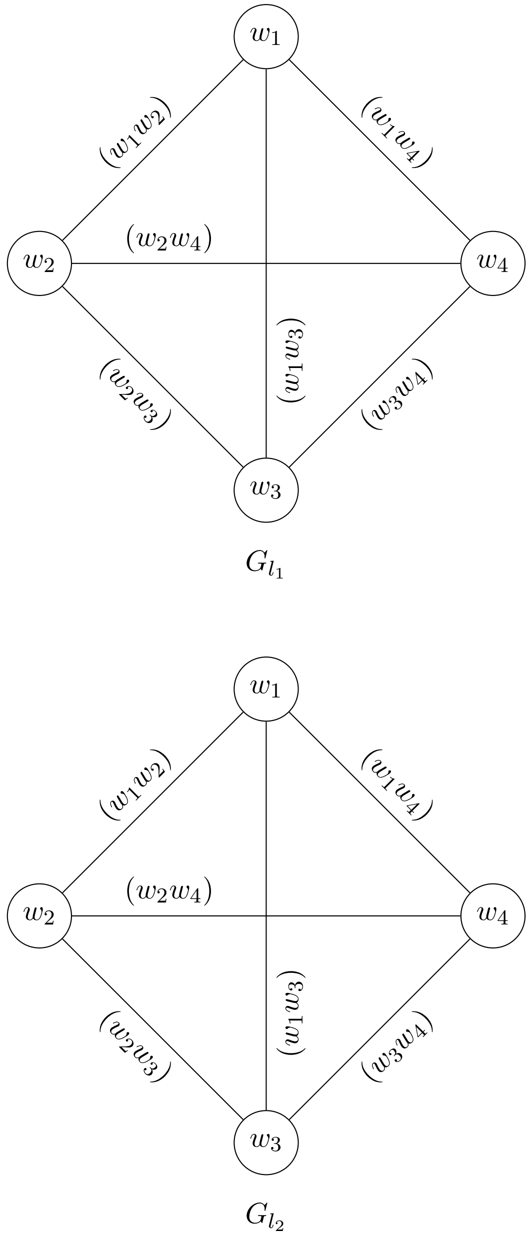


**Figure 3: Wireless Communication Networks.**

Sensor nodes:

$$\mathfrak{S}(l_1) = \begin{cases} \{u_1, (.5e^{i1.8\pi}, .4e^{i1.6\pi}, .4e^{i1.3\pi})\} \\ \{u_2, (.7e^{i1.6\pi}, .7e^{i1.3\pi}, .3e^{i1.5\pi})\} \\ \{u_3, (.5e^{i1.7\pi}, .6e^{i1.4\pi}, .5e^{i1.8\pi})\} \\ \{u_4, (.4e^{i1.9\pi}, .5e^{i1.2\pi}, .5e^{i1.7\pi})\} \end{cases}$$

$$\mathfrak{S}(l_2) = \begin{cases} \{u_1, (.6e^{i1.7\pi}, .3e^{i1.4\pi}, .3e^{i1.7\pi})\} \\ \{u_2, (.8e^{i1.6\pi}, .4e^{i1.3\pi}, .2e^{i1.5\pi})\} \\ \{u_3, (.6e^{i1.8\pi}, .6e^{i1.7\pi}, .4e^{i1.3\pi})\} \\ \{u_4, (.7e^{i1.9\pi}, .5e^{i1.8\pi}, .3e^{i1.4\pi})\} \end{cases}$$

Communication path:

$$\mathfrak{K}(l_1) = \begin{cases} \{u_1u_2, (.4e^{i1.6\pi}, .3e^{i1.2\pi}, .4e^{i1.5\pi})\} \\ \{u_2u_3, (.5e^{i1.5\pi}, .6e^{i1.3\pi}, .6e^{i1.6\pi})\} \\ \{u_3u_4, (.3e^{i1.7\pi}, .5e^{i1.2\pi}, .5e^{i1.8\pi})\} \\ \{u_1u_4, (.4e^{i1.6\pi}, .4e^{i1.1\pi}, .7e^{i1.6\pi})\} \\ \{u_1u_3, (.5e^{i1.6\pi}, .4e^{i1.3\pi}, .6e^{i1.7\pi})\} \\ \{u_2u_4, (.3e^{i1.5\pi}, .5e^{i1.2\pi}, .5e^{i1.6\pi})\} \end{cases}$$

$$\mathfrak{K}(l_2) = \begin{cases} \{u_1u_2, (.6e^{i1.5\pi}, .3e^{i1.2\pi}, .3e^{i1.6\pi})\} \\ \{u_2u_3, (.5e^{i1.4\pi}, .4e^{i1.3\pi}, .5e^{i1.4\pi})\} \\ \{u_3u_4, (.6e^{i1.7\pi}, .5e^{i1.6\pi}, .4e^{i1.3\pi})\} \\ \{u_1u_4, (.5e^{i1.6\pi}, .3e^{i1.5\pi}, .3e^{i1.2\pi})\} \\ \{u_1u_3, (.4e^{i1.6\pi}, .3e^{i1.4\pi}, .6e^{i1.5\pi})\} \\ \{u_2u_4, (.6e^{i1.5\pi}, .4e^{i1.2\pi}, .3e^{i1.4\pi})\} \end{cases}$$

Each edge in the Complex Neutrosophic Soft Graph is assigned a composite reliability score ($\mathfrak{R}$), which measures the overall reliability of each communication channel. This score takes into account the results of the complex neutrosophic representation's truth, indeterminacy, and falsity components. The equation for calculating the dependability of an edge is

$$\mathfrak{R}(w_iw_j) = \sum \left| \mathfrak{X}_{\mathfrak{K}(l)}(w_iw_j) - \frac{[\mathfrak{Y}_{\mathfrak{K}(l)}(w_iw_j) + \mathfrak{Z}_{\mathfrak{K}(l)}(w_iw_j)]}{2} \right| e^{i\sum\left|A_{\mathfrak{K}(l)}(w_iw_j) - \frac{[B_{\mathfrak{K}(l)}(w_iw_j)+C_{\mathfrak{K}(l)}(w_iw_j)]}{2}\right|}$$

Then,

$$\mathfrak{R}(w_1w_2) = .35e^{i0.35\pi}$$

$$\mathfrak{R}(w_2w_3) = .15e^{i0.10\pi}$$

$$\mathfrak{R}(w_3w_4) = .35e^{i0.55\pi}$$

$$\mathfrak{R}(w_1w_4) = .35e^{i0.50\pi}$$

$$\mathfrak{R}(w_1w_3) = .05e^{i0.25\pi}$$

$$\mathfrak{R}(w_2w_4) = .45e^{i0.30\pi}$$

The most reliable path is between $w_2$ and $w_4$ with $R(w_2w_4) = .45e^{i0.30\pi}$.

The least reliable path is $w_1 - w_3$ with $R(w_1 w_3) = 0.05e^{i0.25\pi}$.

As a result, CNSG suggests the communication link $(w_2 w_4)$ as the most reliable path for data transfer.

## 5. Conclusion

This paper presented a formal definition and analysis of the Cartesian product of complex neutrosophic soft graphs. Vertex and edge membership functions were employed to investigate the fundamental structural properties of the resulting graphs. By establishing preservation results under the Cartesian product operation and examining its applications in CNSGs, it was shown that complex neutrosophic soft graphs can be systematically constructed from simpler ones. The proposed framework strengthens the theoretical foundations of complex neutrosophic soft graph theory and opens promising directions for further research and applications in complex network analysis and uncertainty modeling. It is also hoped that future studies will explore extensions based on Plithogenic Graphs [15, 16], HyperGraphs [17], and SuperHyperGraphs [18, 19, 23, 24].

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### Use of Generative AI and AI-Assisted Tools

Generative AI and AI-assisted tools were used for tasks such as English grammar checking, and they were not used in any way that violates ethical standards.

### Supplementary Information

No supplementary materials accompany this paper.

### Disclaimer

The ideas presented here are theoretical and have not yet been validated through empirical testing. While we have strived for accuracy and proper citation, inadvertent errors may remain. Readers should verify any referenced material independently. The opinions expressed are those of the authors and do not necessarily reflect the views of their institutions.

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