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Article

Indeterminacy Fuzzy Set coincides with the Picture Fuzzy Set, and both are particular cases of the Refined Neutrosophic Set

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Abstract: In this short note, we analyze the relationship between C. Ardil's Indeterminacy Fuzzy Set (IDFS) and Bui Cong Cuong's Picture Fuzzy Set (PFS). We show that Ardil's "epistemic freedom residual" mathematically coincides with Cuong's "refusal degree," and that the two formalisms satisfy the same admissible simplex constraint. Consequently, IDFS and PFS are mathematically equivalent representations under different notations and interpretations. Furthermore, we show that both structures can be regarded as particular cases of the n-refined neutrosophic set and its generalized extensions.

Keywords: Indeterminacy Fuzzy Set, Picture Fuzzy Set, n-Refined Neutrosophic Set, Nonstandard Refined Neutrosophic Set, Refined Neutrosophic Over/Under/Off-Set, Nonstandard Refined Neutrosophic Over/Under/Off-Set.

1. Introduction

We present, for comparison, the definitions of Indeterminacy Fuzzy Set [1] and Picture Fuzzy Set [2] below, and it is clearly seeing that they are the same. Since the Picture Fuzzy Set was introduced in 2014 by Cuong [2], it may represent an earlier formulation of the same mathematical structure.

The other definitions below (n-Refined Neutrosophic Set, Nonstandard Refined Neutrosophic Set, Refined Neutrosophic Over/Under/Off-Set, Nonstandard Refined Neutrosophic Over/Under/Off-Set) are displayed because they are generalizations of the Indeterminacy Fuzzy Set (Picture Fuzzy Set).

2. Definition 1.

Indeterminacy Fuzzy Set. Let X be a nonempty universe. An IDFS A on X is

$$A = \{\langle x, TA(x), IA(x), FA(x)\rangle \mid x \in X\},$$

where $TA, IA, FA : X \to [0, 1]$ satisfy the simplex constraint $0 \le TA(x) + IA(x) + FA(x) \le 1$ ($\forall x \in X$).

(1) The quantity $\varrho A(x) = 1 - TA(x) - IA(x) - FA(x) \ge 0$ is the <u>epistemic freedom residual</u>.

The admissible state space is the closed simplex

$$\Delta* = \{(T, I, F) \in [0, 1]^3 \mid T + I + F \le 1\}.$$

(2) The boundary $T + I + F = 1$ (i.e., $\varrho = 0$) contains all IFS states embedded by

$$\Phi(\mu, \nu) = (\mu, 1 - \mu - \nu, \nu).$$

Interior points ($\varrho > 0$) are exclusive to IDFS.

The maximum entropy IDFS state (1/4, 1/4, 1/4) with ϱ = 1/4 achieves entropy ln4, strictly exceeding the IFS maximum ln3. [1]

3. Definition 2.

Picture Fuzzy Set A on a universe X is an object in the form of

$$A = \{(x, \mu A(x), \eta A(x), \nu A(x)) \mid x \in X\}$$

where μA (x) ∈ [0, 1] is called the degree of positive membership of x in A, ηA(x) ∈ [0, 1] is called the degree of neutral membership of x in A and νA(x) ∈ [0, 1] is called the degree of negative membership of x in A, and where μA, ηA and νA satisfy the following condition:

$$(\forall x \in X)\ (\mu A(x) + \eta A(x) + \nu A(x) \leq 1).$$

Now (1 − (μA(x) + ηA(x) + νA(x)) could be called the degree of refusal membership of x in A.

Let PFS(X) denote the set of all the picture fuzzy sets on a universe X. Basically, picture fuzzy sets based models may be adequate in situations when we face human opinions involving more answers of types: yes, abstain, no, refusal. [2]

4. Theorem 1 (Equivalence of IDFS and PFS)

Let A be an Indeterminacy Fuzzy Set (IDFS) on universe X:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

such that

$$T_A(x) + I_A(x) + F_A(x) \leq 1.$$

Let B be a Picture Fuzzy Set (PFS):

$$B = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\}$$

with

$$\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1.$$

Then IDFS and PFS are mathematically equivalent under the bijective mapping:

$$T_A(x) \leftrightarrow \mu_A(x),$$
$$I_A(x) \leftrightarrow \eta_A(x),$$
$$F_A(x) \leftrightarrow \nu_A(x).$$

Moreover,

$$\rho_A(x) = 1 - T_A(x) - I_A(x) - F_A(x)$$

coincides with the Picture Fuzzy Set refusal degree:

$$\pi_A(x) = 1 - \mu_A(x) - \eta_A(x) - \nu_A(x).$$

Therefore,

$$\rho_A(x) = \pi_A(x),$$

which proves the equivalence of the two formalisms.

Proof.

Under the above correspondence between membership components, the admissible state spaces are identical since both satisfy the same simplex constraint. The residual quantity in IDFS equals the refusal degree in PFS through direct substitution. Hence every IDFS induces a unique PFS and vice versa, establishing a one-to-one correspondence. ■

5. Definition 3

Single-Valued n-Refined Neutrosophic Set/Logic/Probability [3], where there are many refinements $T_1, T_2, \ldots T_p$; $I_1, I_2, \ldots I_r$; $F_1, F_2, \ldots, F_s$, with integers p, r, s ≥ 1, and p + r + s = n, and all numerical $T_1, T_2, \ldots T_p$, $I_1, I_2, \ldots I_r$, $F_1, F_2, \ldots, F_s \in [0, 1]$.

And Set-Valued n-Refined Neutrosophic Set/Logic/Probability, where similarly there are many refinements $T_1, T_2, \ldots T_p$; $I_1, I_2, \ldots I_r$; $F_1, F_2, \ldots, F_s$, with integers p, r, s ≥ 1 and p + r + s = n, and all subsets $T_1, T_2, \ldots T_p$, $I_1, I_2, \ldots I_r$, $F_1, F_2, \ldots, F_s \subseteq [0, 1]$.

6. Definition 4

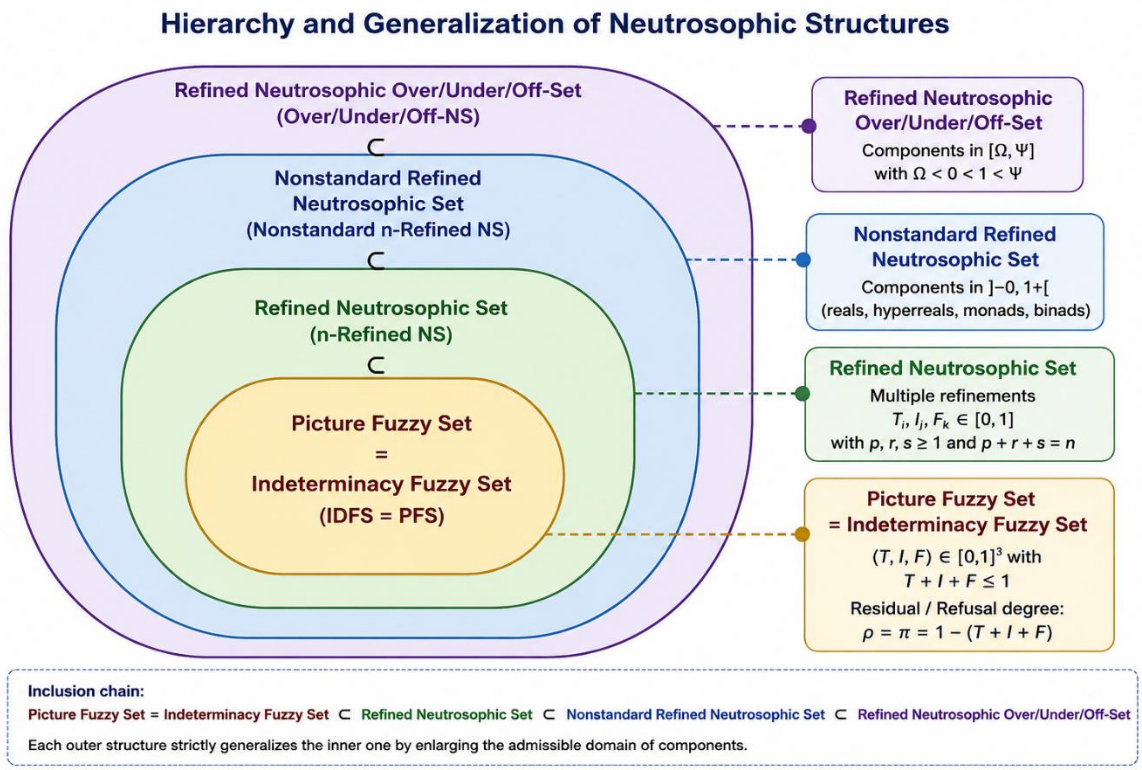
More generalization there is the Nonstandard Refined Neutrosophic Set [4] (within the Extended Nonstandard Analysis), where $T_1, T_2, \ldots T_p, \ I_1, I_2, \ldots I_r, F_1, F_2, \ldots, F_s \in]^-0, 1^+[$ and they may be reals, hyperreals, monads, or binads.

7. Definition 5

Further on, one has the **Refined Neutrosophic Over/Under/Off-Set/Logic/Probability** [3, 4], where all subcomponents $T_1, T_2, \ldots T_p, \ I_1, I_2, \ldots I_r, F_1, F_2, \ldots, F_s \in [\Omega, \Psi]$, with $\Omega < 0 < 1 < \Psi$.

8. Definition 6

Also, **Nonstandard Refined Neutrosophic Over/Under/Off-Set/Logic/Probability**, where all subcomponents $T_1, T_2, \ldots T_p, \ I_1, I_2, \ldots I_r, F_1, F_2, \ldots, F_s \in [^-\Omega, \Psi^+]$, with $\Omega < 0 < 1 < \Psi$, and they may be reals, hyperreals, monads, or binads (within Extended NonStandard Analysis). [4, 5]



9. Conclusion

Definitions 1 and 2 coincide, but using different notations. While the mathematical structure of the Indeterminacy Fuzzy Set coincides with that of the Picture Fuzzy Set, one may still distinguish the semantic interpretation of the residual component. In the IDFS framework, the quantity ρ is interpreted as an *epistemic freedom residual*, whereas in the PFS framework it is interpreted as a *refusal degree*. Therefore, the present note does not deny possible semantic reinterpretations, but rather establishes that the underlying mathematical formalism is equivalent.

References

1. Ardil, C. (2026). Quantum logic framework for autonomous loitering munition systems selection: An indeterminacy fuzzy sets approach. *International Journal of Electrical and Computer Engineering, 20*(5).
2. Cuong, B. C. (2014). Picture fuzzy sets. *Journal of Computer Science and Cybernetics, 30*(4), 409–420. https://doi.org/10.15625/1813-9663/30/4/5032
3. Florentin Smarandache (2013). n-valued refined neutrosophic logic and its applications to physics. *Progress in Physics, 4*, 143–146. https://fs.unm.edu/RefinedNeutrosophicSet.pdf
4. Florentin Smarandache. (n.d.). *Improved definition of nonstandard neutrosophic logic and introduction to neutrosophic hyperreals* (3rd version). arXiv. https://fs.unm.edu/NonStandardAnalysis-Imamura-proven-wrong.pdf
5. Florentin Smarandache. (2016). *Neutrosophic overset, neutrosophic underset, and neutrosophic offset similarly for neutrosophic over-/under-/off-logic, probability, and statistics*. Pons Editions. https://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf, https://fs.unm.edu/Over-Under-Off.htm