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DOUBLE-VALUED NEUTROSOPHIC HYPERGRAPHS

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Abstract. Double-valued neutrosophic sets provide an effective framework for representing uncertainty, indeterminacy, and inconsistency by incorporating truth, falsity, and two distinct indeterminacy components. Hypergraphs generalize classical graphs by allowing a hyperedge to connect more than two vertices, making them suitable for modeling higher-order relationships in complex systems.

In this paper, we extend the concept of double-valued neutrosophic sets to hypergraph structures and introduce the notion of a double-valued neutrosophic hypergraph (DVNHG). We develop the fundamental theory of DVNHGs by defining their height, level hypergraphs, lower and upper truncations, transition levels, and core hypergraph structures. Furthermore, we investigate ordered and sectionally elementary DVNHGs and study properties of double-valued neutrosophic transversals and minimal transversals.

In addition, we introduce the concept of T-related double-valued neutrosophic hypergraphs and establish several fundamental results concerning their transversal structures. The proposed framework generalizes existing neutrosophic hypergraph models and provides a richer mathematical tool for representing complex uncertainty in higher-order relational systems.

Keywords: Neutrosophic hypergraph, Single-valued neutrosophic hypergraph, Double-valued neutrosophic hypergraph.

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1. **Introduction**

Fuzzy set theory was introduced by Zadeh [1] as a mathematical framework for representing vague and imprecise information. Later, Smarandache introduced neutrosophic sets (NSs) as a more general tool for handling ambiguous, incomplete, indeterminate, and inconsistent information [2, 3]. A neutrosophic set is characterized by three independent components: a truth-membership function (T), an indeterminacy-membership function (I), and a falsity-membership function (F), whose values are usually taken from the standard or nonstandard real unit interval. To facilitate the application of neutrosophic sets in practical problems, Wang et al. [4] proposed the notion of a single-valued neutrosophic set (SVNS), in which the three membership degrees are restricted to the standard interval $[0, 1]$.

Neutrosophic sets generalize fuzzy sets and intuitionistic fuzzy sets by treating truth, indeterminacy, and falsity as independent information components. This feature makes neutrosophic models more flexible for describing uncertainty than classical, fuzzy, or intuitionistic fuzzy models. In graph theory, neutrosophic information has been incorporated into vertices and edges, giving rise to several types of neutrosophic graphs (cf. [35,36]). Broumi et al. [5] studied various classes of single-valued neutrosophic graphs, including complete, constant, and strong single-valued neutrosophic graphs. Akram et al. [6, 7] further investigated operations and structural properties of complex and single-valued neutrosophic graphs. Moreover, plithogenic sets are also known as an extended concept of neutrosophic sets [26, 27].

Double-valued neutrosophic sets, together with their applications to minimal spanning trees and clustering techniques, were first proposed by Ilanthenral Kandasamy [8]. Double-valued neutrosophic graphs were subsequently introduced by Takaaki Fujita [9]. In the double-valued neutrosophic framework, the indeterminacy component is divided into two distinct parts: indeterminacy leaning toward truth and indeterminacy leaning toward falsity. This separation enables a more refined representation of ambiguous and mixed information than the single-valued neutrosophic setting. More recently, triple-valued neutrosophic sets were proposed as an extension of double-valued neutrosophic sets [10, 11, 25].

Hypergraphs provide a natural extension of classical graphs by allowing a hyperedge to connect more than two vertices. In uncertain environments, this higher-order structure has been combined with fuzzy and neutrosophic frameworks. Kaufmann introduced the notion of fuzzy hypergraphs, and Chen [12] later introduced interval-valued fuzzy hypergraphs. Lee-Kwang [13] generalized the concept of fuzzy hypergraphs and reformulated it for the fuzzy partition of a system. Akram and Dudek [14] investigated several properties of intuitionistic fuzzy hypergraphs and presented their applications. Akram [15] further introduced various concepts related to single-valued neutrosophic hypergraphs, including line graphs, dual hypergraphs, and transversal single-valued neutrosophic hypergraphs. Because of the importance of studying neutrosophic hypergraphs, neutrosophic hypergraphs have been investigated in works such as [31–34]. Table 1 provides a concise comparison between single-valued neutrosophic hypergraphs and double-valued neutrosophic hypergraphs.

Many related concepts have also been developed in the theory of neutrosophic graphs. For example, neutrosophic soft graphs combine soft-set parameters with neutrosophic graph structures, thereby modeling uncertain relationships through truth, indeterminacy, and falsity degrees assigned to vertices and edges [21–24]. In addition, complex neutrosophic soft graphs (CNSGs) have been developed by SuriyaKumar et al. [16–20] as an advanced framework for representing ambiguity, uncertainty, and indeterminacy in complex systems. In particular, strong complex neutrosophic soft graphs (SCNSGs) incorporate soft-set parameters together with complex-valued membership degrees for both vertices and edges, providing a flexible representation of neutrosophic uncertainty. Related studies have investigated the complements of complex neutrosophic soft graphs and strong complex neutrosophic soft graphs, proposed measures reflecting the cumulative strength and uncertainty of graph relations, and investigated structural properties of strong double-valued complex neutrosophic graphs, including their subgraphs and complements.

The advantages of DVNHG over SVNHG are DVNHGs provide a more detailed representation of uncertainty by separating indeterminacy into truth-oriented and falsity-oriented components. DVNHGs can effectively handle inconsistent, incomplete, and conflicting information. DVNHGs possess greater descriptive power for modelling complex systems. DVNHGs offer more accurate representations of real-world scenarios involving multiple information sources. DVNHGs enhance decision-making processes under uncertain environments. The relationship between SVNHG and DVNHG is an SVNHG can be regarded as a special case of a DVNHG when the truth-oriented and falsity-oriented indeterminacy components are combined into a single indeterminacy degree. Therefore,

$$\text{SVNHG} \subseteq \text{DVNHG}.$$

Hence, DVNHGs constitute a more generalized and expressive framework for modelling uncertainty in hypergraph structures.

Table 1. Comparison between SVNHGs and DVNHGs

Feature	**SVNHG**	**DVNHG**
Basic Structure	Based on Single-Valued Neutrosophic Sets (SVNSs)	Based on Double-Valued Neutrosophic Sets (DVNSs)
Membership Components	Truth ($\mathfrak{t}$), Indeterminacy ($\mathfrak{i}$), Falsity ($\mathfrak{f}$)	Truth ($\mathfrak{t}$), Truth-oriented Indeterminacy ($\mathfrak{i}^{\mathfrak{t}}$), Falsity-oriented Indeterminacy ($\mathfrak{i}^{\mathfrak{f}}$), Falsity ($\mathfrak{f}$)
Vertex Representation	Each vertex has three degrees: $(\mathfrak{t}_S, \mathfrak{i}_S, \mathfrak{f}_S) \in [0,1]^3$.	Each vertex has four degrees: $(\mathfrak{t}_S, \mathfrak{i}^{\mathfrak{t}}_S, \mathfrak{i}^{\mathfrak{f}}_S, \mathfrak{f}_S) \in [0,1]^4$.
Hyperedge Representation	Each hyperedge has three degrees: $(\mathfrak{t}_K, \mathfrak{i}_K, \mathfrak{f}_K) \in [0,1]^3$.	Each hyperedge has four degrees: $(\mathfrak{t}_K, \mathfrak{i}^{\mathfrak{t}}_K, \mathfrak{i}^{\mathfrak{f}}_K, \mathfrak{f}_K) \in [0,1]^4$.
Constraint	$0 \leq \mathfrak{t} + \mathfrak{i} + \mathfrak{f} \leq 3$	$0 \leq \mathfrak{t} + \mathfrak{i}^{\mathfrak{t}} + \mathfrak{i}^{\mathfrak{f}} + \mathfrak{f} \leq 4$
Height of Hypergraph	$(\max \mathfrak{t}, \max \mathfrak{i}, \min \mathfrak{f})$	$(\max \mathfrak{t}, \max \mathfrak{i}^{\mathfrak{t}}, \max \mathfrak{i}^{\mathfrak{f}}, \min \mathfrak{f})$
Treatment of Indeterminacy	Single indeterminacy degree	Two independent indeterminacy degrees
Information Richness	Moderate	High
Ability to Model Uncertainty	Good	Better
Ability to Capture Conflicting Information	Limited	More effective
Expressiveness	Lower	Higher
Computational Complexity	Lower	Higher
Main advantage	Simpler and easier to handle.	More expressive for ambiguous and mixed evidence.

Motivated by these developments, this paper introduces double-valued neutrosophic hypergraphs. Specifically, we define double-valued neutrosophic hypergraphs, level hypergraphs, lower truncations, upper truncations, and transition levels. Furthermore, we introduce T-related double-valued neutrosophic hypergraphs and discuss several properties of their minimal transversals.

2. Preliminaries

In this section, we recall the basic definitions and notation needed in the sequel. In particular, we review single-valued neutrosophic sets, single-valued neutrosophic hypergraphs, single-valued neutrosophic graphs, and double-valued neutrosophic graphs. These notions serve as the foundation for the construction of double-valued neutrosophic hypergraphs developed in the next section.

Definition 2.1. *The support of a single-valued neutrosophic set* $A = \{(x, \mathfrak{t}_A(x), \mathfrak{i}_A(x), \mathfrak{f}_A(x)) : x \in X\}$ *is denoted by supp(A), defined by* $supp(A) = \{x \mid \mathfrak{t}_A(x) \neq 0; \mathfrak{i}_A(x) \neq 0; \mathfrak{f}_A(x) \neq 0\}$*. The support of a single-valued neutrosophic set is a crisp set.*

Definition 2.2. *The height of a single-valued neutrosophic set* $A = \{(x, \mathfrak{t}_A(x), \mathfrak{i}_A(x), \mathfrak{f}_A(x)) : x \in X\}$ *is defined as* $h(A) = (sup_{x\in X}\mathfrak{t}_A(x); sup_{x\in X}\mathfrak{i}_A(x); inf_{x\in X}\mathfrak{f}_A(x))$*. A single-valued neutrosophic set A is called normal if there exists at least one element* $x \in X$ *such that* $\mathfrak{t}_A(x) = 1, \mathfrak{i}_A(x) = 1, \mathfrak{f}_A(x) = 0$*.*

Definition 2.3. *Let* $A = \{(x, \mathfrak{t}_A(x), \mathfrak{i}_A(x), \mathfrak{f}_A(x)) : x \in X\}$ *be a single-valued neutrosophic set on X and let* $a, b, c \in [0, 1]$ *such that* $a + b + c \leq 3$*. Then the set* $A_{(a,b,c)} = \{(x; \mathfrak{t}_A(x) \geq a, \mathfrak{i}_A(x) \geq b, \mathfrak{f}_A(x) \leq c)\}$ *is called* (a, b, c) *-level subset of A.* (a, b, c) *-level set is a crisp set.*

Definition 2.4. *The height of a single-valued neutrosophic hypergraph* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$*, is denoted by* $h(\mathfrak{H})$*, is defined by* $h(\mathfrak{H}) = \bigvee_i \{h(\mathfrak{E}_i) \mid \mathfrak{E}_i \in \mathfrak{E}\}$

Definition 2.5. *Let* $G^* = (V, E)$ *be a graph. A SVNG on* G^* *or an SVNG where* $S = (\mathfrak{t}_S, \mathfrak{i}_S, \mathfrak{f}_S)$ *is a single-valued neutrosophic set on V and* $K = (\mathfrak{t}_K, \mathfrak{i}_K, \mathfrak{f}_K)$ *is a single-valued neutrosophic set on E*

1. $V = \{u_1, u_2, ..., u_n\}$ *such that* $\mathfrak{t}_S : V \to [0, 1]$, $\mathfrak{i}_S : V \to [0, 1]$ *and* $\mathfrak{f}_S : V \to [0, 1]$*, indicates the degree of truth, indeterminacy and falsity membership function of the element* $u_i \in V$

$$0 \leq \mathfrak{t}_S(u_i) + \mathfrak{i}_S(u_i) + \mathfrak{f}_S(u_i) \leq 3$$

2. $E \subseteq V \times V$ *where* $\mathfrak{t}_K : V \times V \to [0, 1]$, $\mathfrak{i}_K : V \times V \to [0, 1]$ *and* $\mathfrak{f}_K : V \times V \to [0, 1]$*. indicates the degree of truth, indeterminacy and falsity membership of the element* $u_iu_j \in E$*, respectively and such that,*

$$0 \leq \mathfrak{t}_K(u_iu_j) + \mathfrak{i}_K(u_iu_j) + \mathfrak{f}_K(u_iu_j) \leq 3$$

such that

$$\mathfrak{t}_K(u_iu_j) \leq \min\{\mathfrak{t}_S(u_i), \mathfrak{t}_S(u_j)\}$$

$$\mathfrak{i}_K(u_iu_j) \leq \min\{\mathfrak{i}_S(u_i), \mathfrak{i}_S(u_j)\}$$

$$\mathfrak{f}_K(u_iu_j) \geq \max\{\mathfrak{f}_S(u_i), \mathfrak{f}_S(u_j)\}$$

Definition 2.6. *Let us examine the graph $G^* = (V, E)$. A double-valued neutrosophic graph (DVNG) on G^* or a DVNG where $S = (\mathfrak{t}_S, \mathfrak{i}_S^{\mathfrak{t}}, \mathfrak{i}_S^{\mathfrak{f}}, \mathfrak{f}_S)$ is a double-valued neutrosophic set on V and $K = (\mathfrak{t}_K, \mathfrak{i}_K^{\mathfrak{t}}, \mathfrak{i}_K^{\mathfrak{f}}, \mathfrak{f}_K)$ is a double-valued neutrosophic set $E \subseteq V \times V$ where,*

1. $V = \{u_1, u_2, ..., u_n\}$ such that $\mathfrak{t}_S : V \to [0, 1]$, $\mathfrak{i}_S^{\mathfrak{t}} : V \to [0, 1]$, $\mathfrak{i}_S^{\mathfrak{f}} : V \to [0, 1]$ and $\mathfrak{f}_S : V \to [0, 1]$, indicates the degree of truth membership function, indeterminacy leaning towards truth, indeterminacy leaning towards falsity and falsity membership function of the element $u_i \in V$

$$0 \leq \mathfrak{t}_S(u_i) + \mathfrak{i}_S^{\mathfrak{t}}(u_i) + \mathfrak{i}_S^{\mathfrak{f}}(u_i) + \mathfrak{f}_S(u_i) \leq 4$$

2. $E \subseteq V \times V$ where $\mathfrak{t}_K : V \times V \to [0, 1]$, $\mathfrak{i}_K^{\mathfrak{t}} : V \times V \to [0, 1]$, $\mathfrak{i}_K^{\mathfrak{f}} : V \times V \to [0, 1]$ and $\mathfrak{f}_K : V \times V \to [0, 1]$. indicates the degree of truth membership function, indeterminacy membership and falsehood membership of the element $u_i u_j \in E$, respectively and such that,

$$0 \leq \mathfrak{t}_K(u_i u_j) + \mathfrak{i}_K^{\mathfrak{t}}(u_i u_j) + \mathfrak{i}_K^{\mathfrak{f}}(u_i u_j) + \mathfrak{f}_K(u_i u_j) \leq 4$$

such that

$$\mathfrak{t}_K(u_i u_j) \leq \min\{\mathfrak{t}_S(u_i), \mathfrak{t}_S(u_j)\}$$
$$\mathfrak{i}_K^{\mathfrak{t}}(u_i u_j) \leq \min\{\mathfrak{i}_S^{\mathfrak{t}}(u_i), \mathfrak{i}_S^{\mathfrak{t}}(u_j)\}$$
$$\mathfrak{i}_K^{\mathfrak{f}}(u_i u_j) \leq \min\{\mathfrak{i}_S^{\mathfrak{f}}(u_i), \mathfrak{i}_S^{\mathfrak{f}}(u_j)\}$$
$$\mathfrak{f}_K(u_i u_j) \geq \max\{\mathfrak{f}_S(u_i), \mathfrak{f}_S(u_j)\}$$

Definition 2.7. *Let S be a finite nonempty set of vertices. A fuzzy hypergraph is an ordered pair $G = (S, K)$, where $K = \{K_1, K_2, \cdots, K_m\}$ is a family of fuzzy subsets of S. Each hyperedge S_i is defined by a membership function $\mathfrak{t}_{K_i} : S \to [0, 1]$, where $\mathfrak{t}_{K_i}(u)$ represents the degree of membership of the vertex u in the hyperedge K_i. Hence, a fuzzy hypergraph may be written as*

$$G = (S, \{\mathfrak{t}_{K_1}, \mathfrak{t}_{K_2}, \cdots, \mathfrak{t}_{K_m}\}).$$

Definition 2.8. *Let S be a finite nonempty set of vertices. An intuitionistic fuzzy hypergraph is an ordered pair $G = (S, K)$, where $K = \{K_1, K_2, \cdots, K_m\}$ is a family of intuitionistic fuzzy subsets of S. For each hyperedge K_i, there exist two functions $\mathfrak{t}_{K_i} : S \to [0, 1]$ and $\mathfrak{f}_{K_i} : S \to [0, 1]$, representing the membership degree and non-membership degree of a vertex in K_i, respectively, satisfying*

$$0 \leq \mathfrak{t}_{K_i}(u) + \mathfrak{f}_{K_i}(u) \leq 1, \qquad \forall u \in S.$$

The hesitation degree is defined by

$$\rho_{K_i}(u) = 1 - \mathfrak{t}_{K_i}(u) - \mathfrak{f}_{K_i}(u).$$

Therefore, an intuitionistic fuzzy hypergraph can be represented as

$$G = (S, \{(\mathfrak{t}_{K_1}, \mathfrak{f}_{K_1}), (\mathfrak{t}_{K_2}, \mathfrak{f}_{K_2}), \cdots, (\mathfrak{t}_{K_m}, \mathfrak{f}_{K_m})\}).$$

Definition 2.9. *Let S be a finite nonempty set of vertices. A neutrosophic hypergraph is an ordered pair $G = (S, K)$, where $K = \{K_1, K_2, \cdots, K_m\}$ is a family of neutrosophic subsets of S. For each hyperedge K_i, there exist three functions $\mathfrak{t}_{K_i} : S \to [0,1]$, $\quad \mathfrak{i}_{K_i} : S \to [0,1]$, $\quad \mathfrak{f}_{K_i} : S \to [0,1]$, where $\mathfrak{t}_{K_i}(u)$, $\mathfrak{i}_{K_i}(u)$, and $\mathfrak{f}_{K_i}(u)$ denote the truth-membership, indeterminacy-membership, and falsity-membership degrees of the vertex u in the hyperedge K_i, respectively. These functions satisfy*

$$0 \leq \mathfrak{t}_{K_i}(u) + \mathfrak{i}_{K_i}(u) + \mathfrak{f}_{K_i}(u) \leq 3, \qquad \forall u \in S.$$

Therefore, a neutrosophic hypergraph can be represented as

$$G = \left(S, \{(\mathfrak{t}_{K_1}, \mathfrak{i}_{K_1}, \mathfrak{f}_{K_1}), (\mathfrak{t}_{K_2}, \mathfrak{i}_{K_2}, \mathfrak{f}_{K_2}), \cdots, (\mathfrak{t}_{K_m}, \mathfrak{i}_{K_m}, \mathfrak{f}_{K_m})\}\right).$$

Definition 2.10. *A Neutrosophic Super-Hypergraph is a generalization of a neutrosophic hypergraph in which both vertices and hyperedges may themselves be neutrosophic sets. This model is capable of representing higher-order and hierarchical relationships under uncertainty. Let $S = \{S_1, S_2, \cdots, S_n\}$ be a finite family of neutrosophic sets called super-vertices. A* Neutrosophic Super-Hypergraph *is an ordered pair $\mathcal{G} = (S, K)$, where $K = \{K_1, K_2, \cdots, K_m\}$ is a family of neutrosophic super-hyperedges joining one or more super-vertices. Each super-vertex S_i is associated with three membership functions $\mathfrak{t}_S(S_i)$, $\quad \mathfrak{i}_S(S_i)$, $\quad \mathfrak{f}_S(S_i) : S \to [0,1]$, representing the truth-membership, indeterminacy-membership, and falsity-membership degrees, respectively. Similarly, each super-hyperedge K_j is characterized by $\mathfrak{t}_K(K_j)$, $\quad \mathfrak{i}_K(K_j)$, $\quad \mathfrak{f}_K(K_j) : K \to [0,1]$, such that*

$$0 \leq \mathfrak{t}_S(S_i) + \mathfrak{i}_S(S_i) + \mathfrak{f}_S(S_i) \leq 3,$$

and

$$0 \leq \mathfrak{t}_K(K_j) + \mathfrak{i}_K(K_j) + \mathfrak{f}_K(K_j) \leq 3.$$

Therefore, a Neutrosophic Super-Hypergraph is represented as

$$\mathcal{G} = \left(S, K, \mathfrak{t}_S, \mathfrak{i}_S, \mathfrak{f}_S, \mathfrak{t}_K, \mathfrak{i}_K, \mathfrak{f}_K\right).$$

3. DOUBLE-VALUED NEUTROSOPHIC HYPERGRAPHS (DVNHGs)

Double-valued neutrosophic hypergraphs model higher-order relations by assigning truth, falsity, and two indeterminacy degrees to vertices and hyperedges under compatibility constraints, enabling richer information representation.

Definition 3.1. *A double-valued neutrosophic hypergraph on $\mathfrak{G}$ is an ordered pair*

$$\mathfrak{H} = (\mathfrak{V}, \mathfrak{E}),$$

where

$$\mathfrak{V} = \{V_1, V_2, \ldots, V_n\}$$

is a finite family of double-valued neutrosophic sets on $\mathfrak{G}$, and $\mathfrak{E}$ is a double-valued neutrosophic relation on $\mathfrak{V}$ satisfying the following conditions.

(1) For every crisp hyperedge

$$e = \{u_1, u_2, \ldots, u_r\} \subseteq \mathfrak{G},$$

the following inequalities hold:

$$\mathfrak{t}_{\mathfrak{E}}(e) \leq \min\{\mathfrak{t}_{V_i}(u_1), \mathfrak{t}_{V_i}(u_2), \ldots, \mathfrak{t}_{V_i}(u_r)\},$$

$$\mathfrak{i}^{\mathfrak{t}}_{\mathfrak{E}}(e) \leq \min\{\mathfrak{i}^{\mathfrak{t}}_{V_i}(u_1), \mathfrak{i}^{\mathfrak{t}}_{V_i}(u_2), \ldots, \mathfrak{i}^{\mathfrak{t}}_{V_i}(u_r)\},$$

$$\mathfrak{i}^{\mathfrak{f}}_{\mathfrak{E}}(e) \leq \min\{\mathfrak{i}^{\mathfrak{f}}_{V_i}(u_1), \mathfrak{i}^{\mathfrak{f}}_{V_i}(u_2), \ldots, \mathfrak{i}^{\mathfrak{f}}_{V_i}(u_r)\},$$

and

$$\mathfrak{f}_{\mathfrak{E}}(e) \geq \max\{\mathfrak{f}_{V_i}(u_1), \mathfrak{f}_{V_i}(u_2), \ldots, \mathfrak{f}_{V_i}(u_r)\}.$$

Moreover,

$$0 \leq \mathfrak{t}_{\mathfrak{E}}(e) + \mathfrak{i}^{\mathfrak{t}}_{\mathfrak{E}}(e) + \mathfrak{i}^{\mathfrak{f}}_{\mathfrak{E}}(e) + \mathfrak{f}_{\mathfrak{E}}(e) \leq 4.$$

(2) The supports of the double-valued neutrosophic sets in $\mathfrak{V}$ cover $\mathfrak{G}$, that is,

$$\bigcup_{i=1}^{n} \operatorname{supp}(V_i) = \mathfrak{G}.$$

The set $e = \{u_1, u_2, \ldots, u_r\}$ is called a crisp hyperedge of $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$.

Definition 3.2. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a DVNHG on $\mathfrak{G}$. The height of $\mathfrak{H}$, denoted by $h(\mathfrak{H})$, is defined by*

$$h(\mathfrak{H}) = \big(h_{\mathfrak{t}}(\mathfrak{H}), h_{\mathfrak{i}^{\mathfrak{t}}}(\mathfrak{H}), h_{\mathfrak{i}^{\mathfrak{f}}}(\mathfrak{H}), h_{\mathfrak{f}}(\mathfrak{H})\big),$$

where

$$h_{\mathfrak{t}}(\mathfrak{H}) = \max_{\delta_i \in \mathfrak{E},\, v \in \mathfrak{G}} \mathfrak{t}_{\delta_i}(v),$$

$$h_{\mathfrak{i}^{\mathfrak{t}}}(\mathfrak{H}) = \max_{\delta_i \in \mathfrak{E},\, v \in \mathfrak{G}} \mathfrak{i}^{\mathfrak{t}}_{\delta_i}(v),$$

$$h_{\mathfrak{i}^{\mathfrak{f}}}(\mathfrak{H}) = \max_{\delta_i \in \mathfrak{E},\, v \in \mathfrak{G}} \mathfrak{i}^{\mathfrak{f}}_{\delta_i}(v),$$

and

$$h_{\mathfrak{f}}(\mathfrak{H}) = \min_{\delta_i \in \mathfrak{E},\, v \in \mathfrak{G}} \mathfrak{f}_{\delta_i}(v).$$

Here, $\mathfrak{t}_{\delta_i}(v)$*,* $\mathfrak{i}^{\mathfrak{t}}_{\delta_i}(v)$*,* $\mathfrak{i}^{\mathfrak{f}}_{\delta_i}(v)$*, and* $\mathfrak{f}_{\delta_i}(v)$ *denote, respectively, the truth-membership degree, the truth-oriented indeterminacy degree, the falsity-oriented indeterminacy degree, and the falsity-membership degree of the vertex* v *with respect to the hyperedge* δ_i*.*

Definition 3.3. *Assume that* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *is a DVNHG on* $\mathfrak{G}$*. Let* $A, B^A, B^C, C \in [0,1]$ *satisfy*

$$0 \le A + B^A + B^C + C \le 4.$$

The (A, B^A, B^C, C)*-level hypergraph of* $\mathfrak{H}$ *is the crisp hypergraph*

$$\mathfrak{H}^{(A,B^A,B^C,C)} = \big(\mathfrak{V}^{(A,B^A,B^C,C)}, \mathfrak{E}^{(A,B^A,B^C,C)}\big),$$

where, for each $\delta_i \in \mathfrak{E}$*,*

$$\mathfrak{E}_i^{(A,B^A,B^C,C)} = \left\{ x \in \mathfrak{G} \;\middle|\; \mathfrak{t}_{\delta_i}(x) \ge A,\ \mathfrak{i}^{\mathfrak{t}}_{\delta_i}(x) \ge B^A,\ \mathfrak{i}^{\mathfrak{f}}_{\delta_i}(x) \ge B^C,\ \mathfrak{f}_{\delta_i}(x) \le C \right\}.$$

The hyperedge set is defined by

$$\mathfrak{E}^{(A,B^A,B^C,C)} = \left\{ \mathfrak{E}_i^{(A,B^A,B^C,C)} \;\middle|\; \delta_i \in \mathfrak{E},\ \mathfrak{E}_i^{(A,B^A,B^C,C)} \ne \emptyset \right\},$$

and the vertex set is defined by

$$\mathfrak{V}^{(A,B^A,B^C,C)} = \bigcup_{\delta_i \in \mathfrak{E}} \mathfrak{E}_i^{(A,B^A,B^C,C)}.$$

Thus, $\mathfrak{H}^{(A,B^A,B^C,C)}$ *is a crisp hypergraph.*

Definition 3.4. *Assume that* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *is a DVNHG on* $\mathfrak{G}$*, and write*

$$h(\mathfrak{H}) = \big(h_{\mathfrak{t}}, h_{\mathfrak{i}^{\mathfrak{t}}}, h_{\mathfrak{i}^{\mathfrak{f}}}, h_{\mathfrak{f}}\big).$$

For parameters satisfying

$$0 < A \le h_{\mathfrak{t}}, \qquad 0 < B^A \le h_{\mathfrak{i}^{\mathfrak{t}}}, \qquad 0 < B^C \le h_{\mathfrak{i}^{\mathfrak{f}}}, \qquad h_{\mathfrak{f}} \le C \le 1,$$

let

$$\mathfrak{H}^{(A,B^A,B^C,C)} = \big(\mathfrak{V}^{(A,B^A,B^C,C)}, \mathfrak{E}^{(A,B^A,B^C,C)}\big)$$

be the (A, B^A, B^C, C)*-level hypergraph of* $\mathfrak{H}$*.*

A finite sequence

$$\big\{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), \ldots, (A_m, B_m^A, B_m^C, C_m)\big\}$$

is called a fundamental sequence of $\mathfrak{H}$ *if the following conditions hold:*

$$h_{\mathfrak{t}} = A_1 > A_2 > \cdots > A_m > 0,$$

$$h_{\mathfrak{i}^{\mathfrak{t}}} = B_1^A > B_2^A > \cdots > B_m^A > 0,$$

$$h_{\mathfrak{if}} = B_1^C > B_2^C > \cdots > B_m^C > 0,$$

and

$$h_{\mathfrak{f}} = C_1 < C_2 < \cdots < C_m \leq 1.$$

Moreover, for each $k \in \{1, 2, \ldots, m-1\}$*, the following two conditions are satisfied:*

(i) *If*

$$A_{k+1} < A' \leq A_k, \qquad B_{k+1}^A < (B^A)' \leq B_k^A, \qquad B_{k+1}^C < (B^C)' \leq B_k^C,$$

and

$$C_k \leq C' < C_{k+1},$$

then

$$\mathfrak{E}^{(A',(B^A)',(B^C)',C')} = \mathfrak{E}^{(A_k,B_k^A,B_k^C,C_k)}.$$

(ii) *The level hyperedge sets are strictly nested as*

$$\mathfrak{E}^{(A_k,B_k^A,B_k^C,C_k)} \subset \mathfrak{E}^{(A_{k+1},B_{k+1}^A,B_{k+1}^C,C_{k+1})}.$$

The fundamental sequence of $\mathfrak{H}$ *is denoted by* $\mathfrak{F}_s(\mathfrak{H})$*.*

The set of level hypergraphs

$$\left\{\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}, \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}, \ldots, \mathfrak{H}^{(A_m,B_m^A,B_m^C,C_m)}\right\}$$

is called the set of core hypergraphs, or the core set, of $\mathfrak{H}$ *and is denoted by* $c(\mathfrak{H})$*.*

Example 3.5. *Assume a DVNHG* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *on*

$$\mathfrak{G} = \{u_1, u_2, u_3, u_4, u_5, u_6\}$$

, where $u_1 = (0.8, 0.6, 0.7, 0.5)$*,* $u_2 = (0.5, 0.5, 0.4, 0.3)$*,* $u_3 = (0.6, 0.4, 0.5, 0.4)$*,* $u_4 = (0.2, 0.3, 0.2, 0.1)$*,* $u_5 = (0.7, 0.6, 0.8, 0.5)$*,* $u_6 = (0.4, 0.5, 0.4, 0.3)$*. The DVNR* $\mathfrak{E}$ *is given as,* $\mathfrak{E}(\{u_1, u_2, u_3\}) = (0.5, 0.4, 0.4, 0.3)$*,* $\mathfrak{E}(\{u_4, u_5, u_6\}) = (0.2, 0.3, 0.2, 0.1)$*,* $\mathfrak{E}(\{u_1, u_6\}) = (0.4, 0.5, 0.4, 0.4)$*,* $\mathfrak{E}(\{u_2, u_5\}) = (0.5, 0.5, 0.4, 0.4)$ *and* $\mathfrak{E}(\{u_3, u_4\}) = (0.2, 0.3, 0.2, 0.1)$*. The corresponding DVNHG is given in Figure 1.*

$$(A_1, B_1^A, B_1^C, C_1) = (0.8, 0.6, 0.7, 0.5)$$

$$(A_2, B_2^A, B_2^C, C_2) = (0.5, 0.4, 0.4, 0.3)$$

$$(A_3, B_3^A, B_3^C, C_3) = (0.4, 0.4, 0.4, 0.3)$$

$$(A_4, B_4^A, B_4^C, C_4) = (0.2, 0.3, 0.2, 0.1)$$

Note that the sequence

$$\{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), (A_3, B_3^A, B_3^C, C_3), (A_4, B_4^A, B_4^C, C_4)\}$$

satisfies all the conditions of definition 3.4. Thus, it is a fundamental sequence of $\mathfrak{H}$*. The corresponding* (A_j, B_j^A, B_j^C, C_j) *-level hypergraphs are shown in Figures 2-5.*

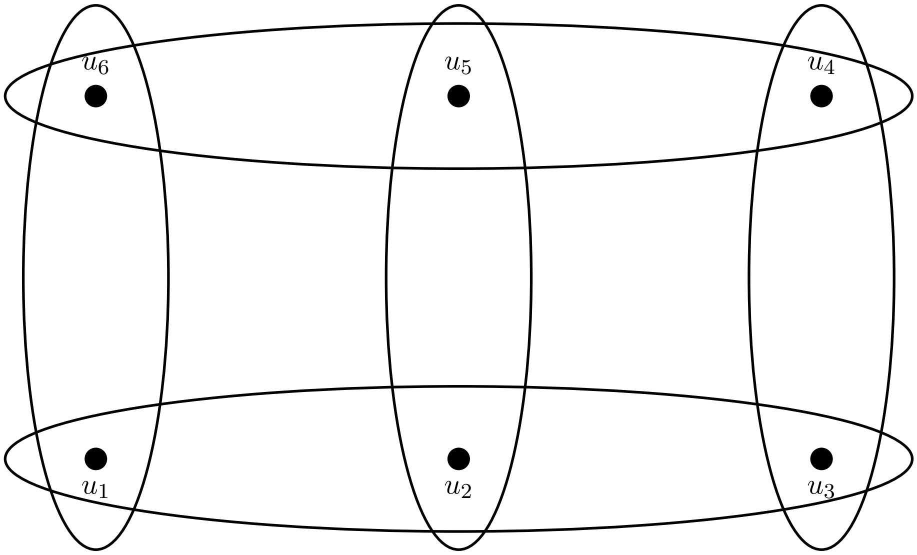

FIGURE 1. double-valued neutrosophic hypergraph $\mathfrak{H}$

FIGURE 2. $\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}$ -level hypergraph

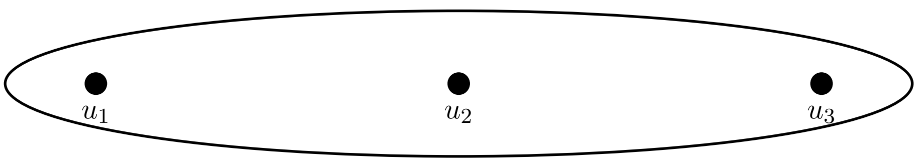

FIGURE 3. $\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}$ -level hypergraph

Definition 3.6. *A DVNHG $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is called ordered if its core set $c(\mathfrak{H})$ is ordered. More precisely, if*

$$c(\mathfrak{H}) = \left\{\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}, \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}, \ldots, \mathfrak{H}^{(A_m,B_m^A,B_m^C,C_m)}\right\},$$

then

$$\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)} \subseteq \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)} \subseteq \cdots \subseteq \mathfrak{H}^{(A_m,B_m^A,B_m^C,C_m)}.$$

A DVNHG $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is called simply ordered if $c(\mathfrak{H})$ is simply ordered. That is, writing

$$\mathfrak{H}^{(A_i,B_i^A,B_i^C,C_i)} = (\mathfrak{G}_i, \mathfrak{E}_i), \qquad i = 1, 2, \ldots, m,$$

if

$$e \in \mathfrak{E}_{i+1} \setminus \mathfrak{E}_i,$$

then

$$e \nsubseteq \mathfrak{G}_i.$$

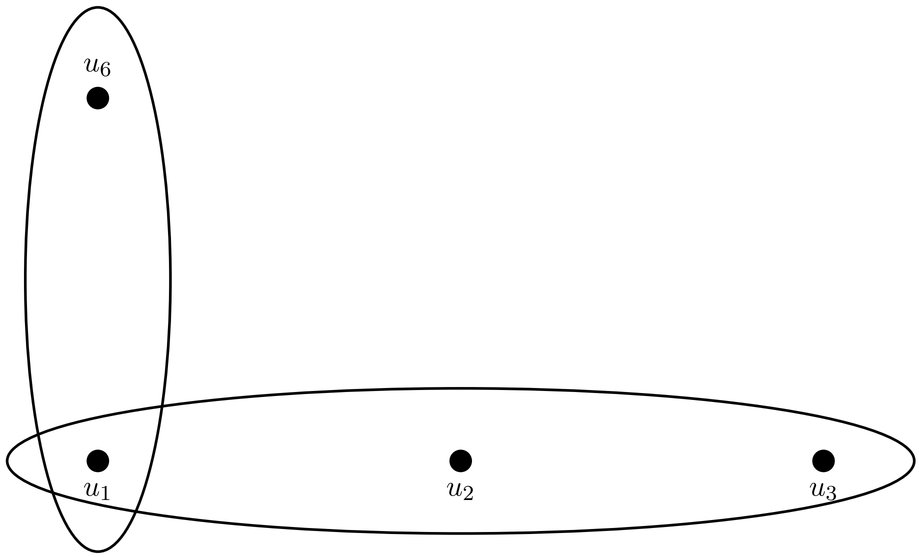

FIGURE 4. $\mathfrak{H}^{(A_3,B_3^A,B_3^C,C_3)}$ -level hypergraph

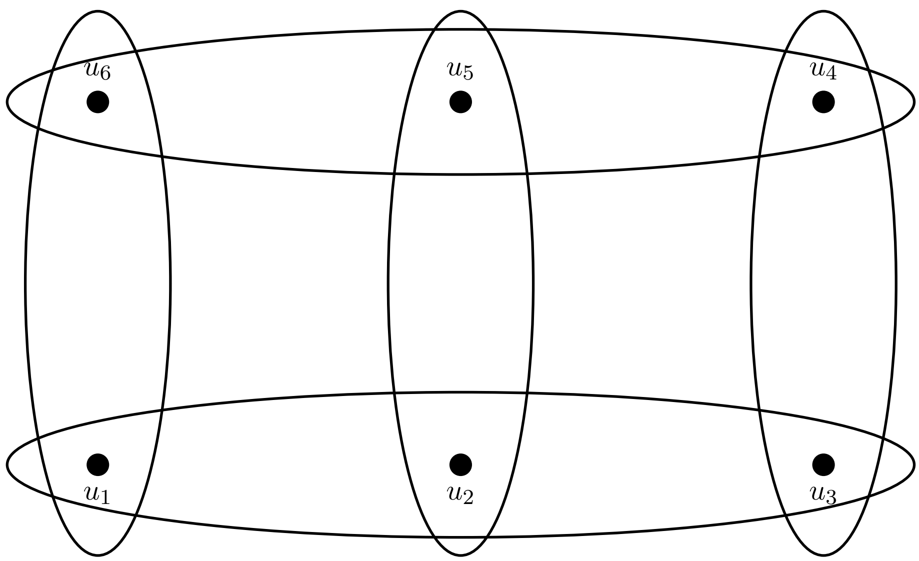

FIGURE 5. $\mathfrak{H}^{(A_4,B_4^A,B_4^C,C_4)}$ -level hypergraph

Example 3.7. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a DVNHG as shown in Figure 1. The set of core hypergraphs is given by*

$$c(\mathfrak{H}) = \left\{\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}, \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}, \mathfrak{H}^{(A_3,B_3^A,B_3^C,C_3)}, \mathfrak{H}^{(A_4,B_4^A,B_4^C,C_4)}\right\},$$

where

$$\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)} = (\mathfrak{G}_1, \mathfrak{E}_1), \qquad \mathfrak{G}_1 = \{u_1\}, \qquad \mathfrak{E}_1 = \emptyset,$$

$$\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)} = (\mathfrak{G}_2, \mathfrak{E}_2), \qquad \mathfrak{G}_2 = \{u_1, u_2, u_3\}, \qquad \mathfrak{E}_2 = \{\{u_1, u_2, u_3\}\},$$

$$\mathfrak{H}^{(A_3,B_3^A,B_3^C,C_3)} = (\mathfrak{G}_3, \mathfrak{E}_3), \qquad \mathfrak{G}_3 = \{u_1, u_2, u_3, u_6\},$$

$$\mathfrak{E}_3 = \{\{u_1, u_2, u_3\}, \{u_1, u_6\}\},$$

and

$$\mathfrak{H}^{(A_4,B_4^A,B_4^C,C_4)} = (\mathfrak{G}_4,\mathfrak{E}_4), \qquad \mathfrak{G}_4 = \{u_1,u_2,u_3,u_4,u_5,u_6\},$$

$$\mathfrak{E}_4 = \{\{u_1,u_2,u_3\},\{u_1,u_6\},\{u_4,u_5,u_6\},\{u_2,u_5\},\{u_3,u_4\}\}.$$

Note that

$$\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)} \subseteq \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)} \subseteq \mathfrak{H}^{(A_3,B_3^A,B_3^C,C_3)} \subseteq \mathfrak{H}^{(A_4,B_4^A,B_4^C,C_4)}.$$

Hence, $\mathfrak{H} = (\mathfrak{V},\mathfrak{E})$ *is an ordered DVNHG. Also,* $\mathfrak{H} = (\mathfrak{V},\mathfrak{E})$ *is simply ordered.*

Definition 3.8. *Let* $\mathfrak{H} = (\mathfrak{V},\mathfrak{E})$ *be a DVNHG with*

$$\mathfrak{J}_s(\mathfrak{H}) = \left\{(A_1,B_1^A,B_1^C,C_1),(A_2,B_2^A,B_2^C,C_2),\ldots,(A_m,B_m^A,B_m^C,C_m)\right\}.$$

Then $\mathfrak{H}$ *is called sectionally elementary if, for every* $k \in \{1,2,\ldots,m-1\}$*, the equality*

$$\mathfrak{E}^{(A',(B^A)',(B^C)',C')} = \mathfrak{E}^{(A_k,B_k^A,B_k^C,C_k)}$$

holds for all parameters satisfying

$$A_{k+1} < A' \le A_k, \qquad B_{k+1}^A < (B^A)' \le B_k^A, \qquad B_{k+1}^C < (B^C)' \le B_k^C,$$

and

$$C_k \le C' < C_{k+1}.$$

Definition 3.9. *Assume that* V *is a DVNS on* $\mathfrak{J}$*. For* $0 < A, B^A, B^C, C < 1$*, define the level set*

$$V^{(A,B^A,B^C,C)} = \left\{u \in \mathfrak{J} \;\middle|\; \mathfrak{t}_V(u) \ge A,\ \mathfrak{i}_V^{\mathfrak{t}}(u) \ge B^A,\ \mathfrak{i}_V^{\mathfrak{f}}(u) \ge B^C,\ \mathfrak{f}_V(u) \le C\right\}.$$

The lower truncation of V *at level* (A,B^A,B^C,C) *is the DVNS* $V_{[(A,B^A,B^C,C)]}$ *defined by*

$$\mathfrak{t}_{V_{[(A,B^A,B^C,C)]}}(u) = \begin{cases} \mathfrak{t}_V(u), & \text{if } u \in V^{(A,B^A,B^C,C)},\\ 0, & \text{otherwise,}\end{cases}$$

$$\mathfrak{i}^{\mathfrak{t}}_{V_{[(A,B^A,B^C,C)]}}(u) = \begin{cases} \mathfrak{i}^{\mathfrak{t}}_V(u), & \text{if } u \in V^{(A,B^A,B^C,C)},\\ 0, & \text{otherwise,}\end{cases}$$

$$\mathfrak{i}^{\mathfrak{f}}_{V_{[(A,B^A,B^C,C)]}}(u) = \begin{cases} \mathfrak{i}^{\mathfrak{f}}_V(u), & \text{if } u \in V^{(A,B^A,B^C,C)},\\ 0, & \text{otherwise,}\end{cases}$$

and

$$\mathfrak{f}_{V_{[(A,B^A,B^C,C)]}}(u) = \begin{cases} \mathfrak{f}_V(u), & \text{if } u \in V^{(A,B^A,B^C,C)},\\ 0, & \text{otherwise.}\end{cases}$$

Definition 3.10. *Assume that V is a DVNS on $\mathfrak{J}$. For $0 < A, B^A, B^C, C < 1$, define*

$$V^{(A,B^A,B^C,C)} = \left\{ u \in \mathfrak{J} \;\middle|\; \mathfrak{t}_V(u) \geq A,\ \mathfrak{i}^{\mathfrak{t}}_V(u) \geq B^A,\ \mathfrak{i}^{\mathfrak{f}}_V(u) \geq B^C,\ \mathfrak{f}_V(u) \leq C \right\}.$$

The upper truncation of V at level (A, B^A, B^C, C) is the DVNS $V^{[(A,B^A,B^C,C)]}$ defined by

$$\mathfrak{t}_{V^{[(A,B^A,B^C,C)]}}(u) = \begin{cases} A, & \text{if } u \in V^{(A,B^A,B^C,C)}, \\ \mathfrak{t}_V(u), & \text{otherwise}, \end{cases}$$

$$\mathfrak{i}^{\mathfrak{t}}_{V^{[(A,B^A,B^C,C)]}}(u) = \begin{cases} B^A, & \text{if } u \in V^{(A,B^A,B^C,C)}, \\ \mathfrak{i}^{\mathfrak{t}}_V(u), & \text{otherwise}, \end{cases}$$

$$\mathfrak{i}^{\mathfrak{f}}_{V^{[(A,B^A,B^C,C)]}}(u) = \begin{cases} B^C, & \text{if } u \in V^{(A,B^A,B^C,C)}, \\ \mathfrak{i}^{\mathfrak{f}}_V(u), & \text{otherwise}, \end{cases}$$

and

$$\mathfrak{f}_{V^{[(A,B^A,B^C,C)]}}(u) = \begin{cases} C, & \text{if } u \in V^{(A,B^A,B^C,C)}, \\ \mathfrak{f}_V(u), & \text{otherwise}. \end{cases}$$

Definition 3.11. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a DVNHG. The lower truncation of $\mathfrak{H}$ at level (A, B^A, B^C, C) is denoted by*

$$\mathfrak{H}_{[(A,B^A,B^C,C)]} = \left(\mathfrak{V}_{[(A,B^A,B^C,C)]}, \mathfrak{E}_{[(A,B^A,B^C,C)]} \right),$$

where

$$\mathfrak{V}_{[(A,B^A,B^C,C)]} = \left\{ V_{[(A,B^A,B^C,C)]} \;\middle|\; V \in \mathfrak{V} \right\},$$

and $\mathfrak{E}_{[(A,B^A,B^C,C)]}$ is obtained by applying the same lower truncation to each member of $\mathfrak{E}$. The upper truncation of $\mathfrak{H}$ at level (A, B^A, B^C, C) is denoted by

$$\mathfrak{H}^{[(A,B^A,B^C,C)]} = \left(\mathfrak{V}^{[(A,B^A,B^C,C)]}, \mathfrak{E}^{[(A,B^A,B^C,C)]} \right),$$

where

$$\mathfrak{V}^{[(A,B^A,B^C,C)]} = \left\{ V^{[(A,B^A,B^C,C)]} \;\middle|\; V \in \mathfrak{V} \right\},$$

and $\mathfrak{E}^{[(A,B^A,B^C,C)]}$ is obtained by applying the same upper truncation to each member of $\mathfrak{E}$.

Definition 3.12. *Assume that V is a DVNS on $\mathfrak{G}$. A quadruple (A, B^A, B^C, C) is called a transition level of V if*

$$A \in (0, \mathfrak{t}(h(V))), \qquad B^A \in (0, \mathfrak{i}^{\mathfrak{t}}(h(V))), \qquad B^C \in (0, \mathfrak{i}^{\mathfrak{f}}(h(V))),$$

and

$$C \in (\mathfrak{f}(h(V)), 1],$$

and the level cut of V changes at (A, B^A, B^C, C).

Example 3.13. *Assume that* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *is a DVNHG as shown in Figure 1. The* $(0.4, 0.4, 0.4, 0.3)$*-level hypergraph of* $\mathfrak{H}$ *is shown in Figure 4. Then the lower truncation*

$$\mathfrak{H}_{[(0.4,0.4,0.4,0.3)]} = \left(\mathfrak{V}_{[(0.4,0.4,0.4,0.3)]}, \mathfrak{E}_{[(0.4,0.4,0.4,0.3)]}\right)$$

of $\mathfrak{H}$ *is a DVNHG on*

$$\mathfrak{G}_1 = \{u_1, u_2, u_3, u_6\}$$

as given in Figure 6.

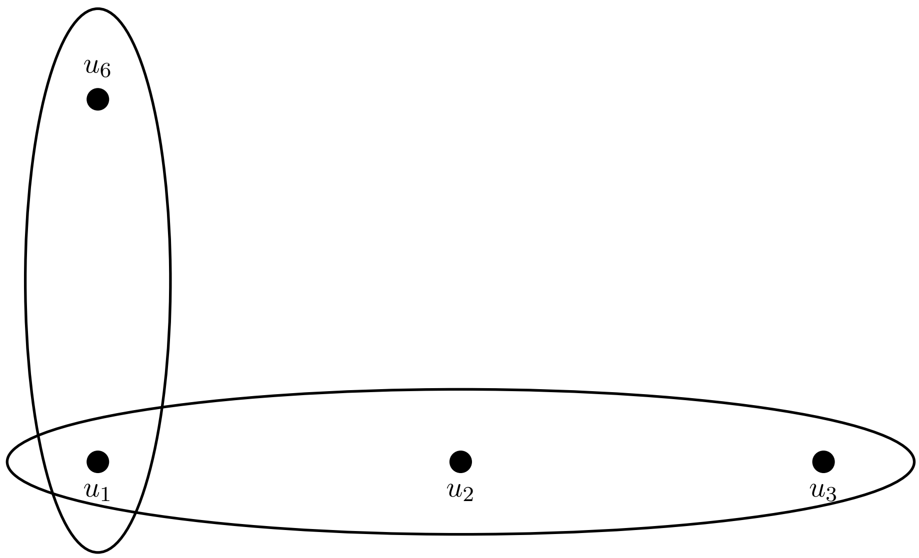

FIGURE 6. Lower truncation of $\mathfrak{H}$

Note that

$$\mathfrak{G}_1 = \bigcup_{V \in \mathfrak{V}} \text{supp}\left(V_{[(0.4,0.4,0.4,0.3)]}\right).$$

The upper truncation

$$\mathfrak{H}^{[(0.4,0.4,0.4,0.3)]} = \left(\mathfrak{V}^{[(0.4,0.4,0.4,0.3)]}, \mathfrak{E}^{[(0.4,0.4,0.4,0.3)]}\right)$$

of $\mathfrak{H}$ *is a DVNHG on*

$$\mathfrak{G} = \{u_1, u_2, u_3, u_4, u_5, u_6\}$$

as given in Figure 7.

Definition 3.14. *Assume that* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *is a DVNHG. A double-valued neutrosophic transversal (DVNT)* ρ *is a DVNS of* $\mathfrak{G}$ *satisfying the condition* $\delta^{h(\delta)} \cap \rho^{h(\delta)} \neq \emptyset$*, for all* $\delta \in E$*, where* $h(\delta)$*, is the height of* δ*.*
A minimal double-valued neutrosophic transversal ρ_1 *is the DVNT of* $\mathfrak{H}$ *with the property that if* $\rho \subset \rho_1$*, then* ρ *is not a DVNT of* $\mathfrak{H}$*.*
Let us denote the family of minimal DVNTs of $\mathfrak{H}$ *by* $T_r(\mathfrak{H})$*.*

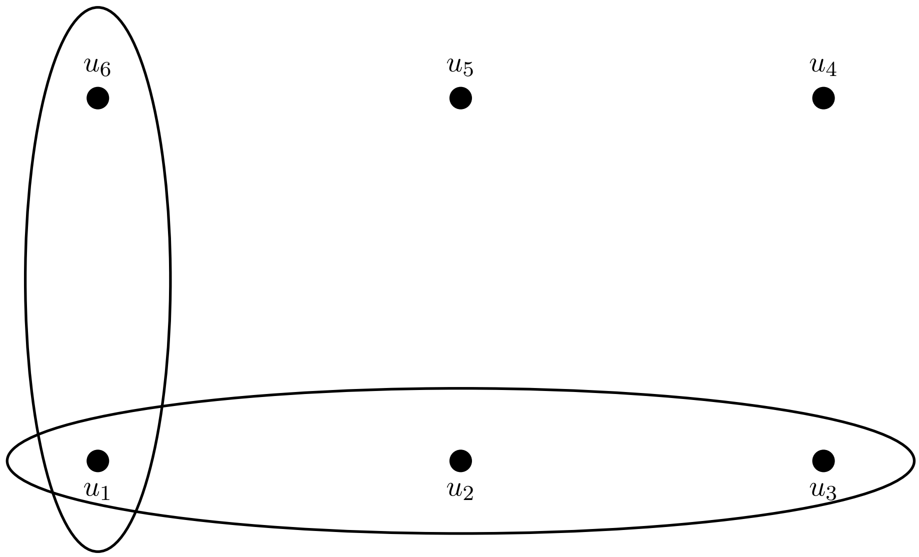

FIGURE 7. Upper truncation of $\mathfrak{H}$

Definition 3.15. *A DVNT ρ with the property that $\rho^{(A,B^A,B^C,C)} \in T_r(\mathfrak{H}^{(A,B^A,B^C,C)})$, for all$(A, B^A, B^C, C) \in [0,1]$, is called the locally minimal DVNT of $\mathfrak{H}$. The collection of all locally minimal DVNTs of $\mathfrak{H}$ is represented by $T_r^*(\mathfrak{H})$.*
Note that $T_r^*(\mathfrak{H}) \subseteq T_r(\mathfrak{H})$, but the converse is not generally true.

Definition 3.16. *Assume that V is a DVNS on $\mathfrak{G}$. Then, the basic sequence of V determined by V, denoted by $D_s(V)$, is defined as $\{(A_1, B_1^A, B_1^C, C_1)^V, (A_2, B_2^A, B_2^C, C_2)^V, \cdots, (A_n, B_n^A, B_n^C, C_n)^V\}$, where*

- $A_1 > A_2 > \cdots > A_n$,$B_1^A > B_2^A > \cdots > B_n^A$, $B_1^C > B_2^C > \cdots > B_n^C$, $C_1 < C_2 < \cdots < C_n$
- $(A_1, B_1^A, B_1^C, C_1) = h(V)$
- $\{(A_2, B_2^A, B_2^C, C_2)^V, \cdots, (A_n, B_n^A, B_n^C, C_n)^V\}$ *are the transition levels of V*

Definition 3.17. *Assume that $D_s(V) = \{(A_1, B_1^A, B_1^C, C_1)^V, (A_2, B_2^A, B_2^C, C_2)^V, \cdots, (A_n, B_n^A, B_n^C, C_n)^V\}$ is the basic sequence of V. Then the set of basic cuts $D_s(V)$ is defined as,*

$$D_c(V) = \{V^{(A,B^A,B^C,C)} | (A, B^A, B^C, C) \in D_s(V)\}$$

.

Lemma 3.18. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a DVNHG with*
$\mathfrak{I}_s(\mathfrak{H}) = \{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), \cdots, (A_n, B_n^A, B_n^C, C_n)\}$. *Then*

- *If (A, B^A, B^C, C) is a transition level of $\rho \in T_r(\mathfrak{H})$, then there exists an $\epsilon > 0$ such that for all $A_1 \in (A, A+\epsilon]$, $B_1^A \in (B^A, B^A+\epsilon]$, $B_1^C \in (B^C, B^C+\epsilon]$, $C_1 \in (C, C+\epsilon]$ is* *minimal* $\mathfrak{H}^{(A,B^A,B^C,C)}$ *transversal extension of* $\rho^{(A,B^A,B^C,C)}$*, i.e.,* $\rho^{(A_1,B_1^A,B_1^C,C_1)} \subseteq J \subset \rho^{(A,B^A,B^C,C)}$*, then* J *is not a transversal of* $\mathfrak{H}^{(A,B^A,B^C,C)}$*.*

- $T_r(\mathfrak{H})$*, i.e., the collection of minimal transversals of* $\mathfrak{H}$ *is sectionally elementary.*
- $\mathfrak{F}_s(T_r(\mathfrak{H}))$ *is properly contained in* $\mathfrak{F}_s(\mathfrak{H})$*.*
- $\rho^{(A,B^A,B^C,C)} \in T_r(\mathfrak{H}^{(A,B^A,B^C,C)})$*, for all* $\rho \in T_r(\mathfrak{H})$ *and for every* $A_2 < A \leq A_1$*,* $B_2^A < B^A \leq B_1^A$*,* $B_2^C < B^C \leq B_1^C$*,* $C_2 > C \geq C_1$*.*

4. **T-Related DVNHGs**

T-related DVNHGs are double-valued neutrosophic hypergraphs whose consecutive core hypergraphs preserve minimal transversals compatibly, ensuring global and locally minimal transversal structures coincide across transition levels.

Definition 4.1. *A DVNHG* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *is N-tempered DVNHG of* $\mathbb{H} = (\mathbb{V}, \mathbb{E})$ *if there exists* $\mathbb{H} = (\mathbb{V}, \mathbb{E})$*, a crisp hypergraph and a DVNS* N *such that* $\mathfrak{E} = \{\xi_e | e \in \mathbb{E}\}$*, where*

$$\mathfrak{t}_\xi(x) = \begin{cases} \min\{\mathfrak{t}_N(x) | x \in e\}, & if \;\; x \in e \\ 0, & otherwise. \end{cases}$$

$$\mathfrak{i^t}_\xi(x) = \begin{cases} \min\{\mathfrak{i^t}_N(x) | x \in e\}, & if \;\; x \in e \\ 0, & otherwise. \end{cases}$$

$$\mathfrak{i^f}_\xi(x) = \begin{cases} \min\{\mathfrak{i^f}_N(x) | x \in e\}, & if \;\; x \in e \\ 0, & otherwise. \end{cases}$$

$$\mathfrak{f}_\xi(x) = \begin{cases} \max\{\mathfrak{f}_N(x) | x \in e\}, & if \;\; x \in e \\ 0, & otherwise. \end{cases}$$

An N-tempered DVNHG $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *determined by* $\mathbb{H}$ *and DVNS* N *is denoted by* $N \otimes \mathbb{H}$*.*

Definition 4.2. *A pair* $(\mathfrak{G}, \mathfrak{J})$ *of crisp hypergraphs is T-related if whenever* g *is a minimal transversal of* $\mathfrak{G}$*,* k *is any transversal of* $\mathfrak{J}$ *and* $g \subseteq k$*, then there exists a minimal transversal* l *of* $\mathfrak{J}$ *such that* $g \subseteq l \subseteq k$*.*

Definition 4.3. *Assume that* $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ *is a DVNHG with* $\mathfrak{F}_s(\mathfrak{H}) = \{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), \cdots, (A_n, B_n^A, B_n^C, C_n)\}$*. Then,* $\mathfrak{H}$ *is T-related if from the core set*

$$c(\mathfrak{H}) = \{\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}, \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}, \cdots, \mathfrak{H}^{(A_m,B_m^A,B_m^C,C_m)}\}$$

of $\mathfrak{H}$*, every successive ordered pair* $(\mathfrak{H}^{(A_j,B_j^A,B_j^C,C_j)}, \mathfrak{H}^{(A_{j-1},B_{j-1}^A,B_{j-1}^C,C_{j-1})})$ *is T-related.*
If $\mathfrak{F}_s(\mathfrak{H})$ *contains only one element,* $\mathfrak{H}$ *is considered to be trivially T-related.*

Theorem 4.4. *If $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a T-related DVNHG, then*

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H}).$$

Proof. Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a T-related DVNHG with

$$\mathfrak{F}_s(\mathfrak{H}) = \left\{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), \ldots, (A_n, B_n^A, B_n^C, C_n)\right\}.$$

We prove that every minimal DVNT of $\mathfrak{H}$ is locally minimal. Since

$$T_r^*(\mathfrak{H}) \subseteq T_r(\mathfrak{H}),$$

it is enough to show that

$$T_r(\mathfrak{H}) \subseteq T_r^*(\mathfrak{H}).$$

Let $\delta \in T_r(\mathfrak{H})$. We show that, for every admissible level (A, B^A, B^C, C),

$$\delta^{(A,B^A,B^C,C)} \in T_r\big(\mathfrak{H}^{(A,B^A,B^C,C)}\big).$$

First, suppose that $\mathfrak{F}_s(\mathfrak{H})$ consists of only one element, say

$$\mathfrak{F}_s(\mathfrak{H}) = \left\{(A_1, B_1^A, B_1^C, C_1)\right\}.$$

Then Lemma 3.18 implies that, for each $\delta \in T_r(\mathfrak{H})$,

$$\delta^{(A,B^A,B^C,C)} \in T_r\big(\mathfrak{H}^{(A,B^A,B^C,C)}\big)$$

for all admissible levels satisfying

$$0 < A \le \mathfrak{t}\big(h(\mathfrak{H})\big), \qquad 0 < B^A \le \mathfrak{i}^{\mathfrak{t}}\big(h(\mathfrak{H})\big),$$
$$0 < B^C \le \mathfrak{i}^{\mathfrak{f}}\big(h(\mathfrak{H})\big), \qquad 0 < \mathfrak{f}\big(h(\mathfrak{H})\big) \le C.$$

Hence,

$$\delta \in T_r^*(\mathfrak{H}).$$

Therefore,

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H})$$

in this case.

Next, suppose that $\mathfrak{F}_s(\mathfrak{H})$ contains at least two elements. Let

$$\delta^{(A_1,B_1^A,B_1^C,C_1)} \subseteq \delta^{(A_2,B_2^A,B_2^C,C_2)} \subseteq \cdots \subseteq \delta^{(A_n,B_n^A,B_n^C,C_n)}$$

be the corresponding sequence of level cuts of δ.

We first prove that

$$\delta^{(A_2,B_2^A,B_2^C,C_2)} \in T_r\big(\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}\big).$$

Suppose, to the contrary, that

$$\delta^{(A_2,B_2^A,B_2^C,C_2)} \notin T_r\big(\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}\big).$$

Since

$$\delta^{(A_1,B_1^A,B_1^C,C_1)} \in T_r\big(\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}\big)$$

and the ordered pair

$$\Big(\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}, \mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}\Big)$$

is T-related, there exists a minimal transversal ρ_2 of $\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}$ such that

$$\delta^{(A_1,B_1^A,B_1^C,C_1)} \subseteq \rho_2 \subset \delta^{(A_2,B_2^A,B_2^C,C_2)}.$$

Let

$$\rho_1 = \delta^{(A_1,B_1^A,B_1^C,C_1)}.$$

Construct a DVNS ξ from δ by replacing the cut $\delta^{(A_2,B_2^A,B_2^C,C_2)}$ with ρ_2, while preserving the remaining lower-level structure. Equivalently, one may write

$$\xi = \delta^{(A_3,B_3^A,B_3^C,C_3)} \cup (\tau_2 \cap \tau_1),$$

where τ_k is an elementary DVNS with support ρ_k and height

$$(A_k, B_k^A, B_k^C, C_k), \qquad k = 1, 2.$$

Then ξ is a DVNT of $\mathfrak{H}$ and

$$\xi \subset \delta,$$

which contradicts the minimality of δ in $T_r(\mathfrak{H})$. Therefore,

$$\delta^{(A_2,B_2^A,B_2^C,C_2)} \in T_r\big(\mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}\big).$$

Repeating the same argument successively for all adjacent pairs in the fundamental sequence, we obtain

$$\delta^{(A_k,B_k^A,B_k^C,C_k)} \in T_r\big(\mathfrak{H}^{(A_k,B_k^A,B_k^C,C_k)}\big)$$

for every $k = 1, 2, \ldots, n$. By Lemma 3.18, this also holds for every admissible level lying between two consecutive fundamental levels. Hence

$$\delta^{(A,B^A,B^C,C)} \in T_r\big(\mathfrak{H}^{(A,B^A,B^C,C)}\big)$$

for every admissible level (A, B^A, B^C, C). Thus,

$$\delta \in T_r^*(\mathfrak{H}).$$

Since $\delta \in T_r(\mathfrak{H})$ was arbitrary, we obtain

$$T_r(\mathfrak{H}) \subseteq T_r^*(\mathfrak{H}).$$

Together with $T_r^*(\mathfrak{H}) \subseteq T_r(\mathfrak{H})$, this proves

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H}).$$

□

Example 4.5. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is a DVNHG represented by the incidence matrix in Table 2. Note that*

$$\mathfrak{E}^{(0.8,0.8,0.8,0.8)} = \{\{u_1, u_2\}, \{u_2, u_3\}, \{u_1, u_3\}\},$$
$$\mathfrak{E}^{(0.4,0.4,0.4,0.4)} = \{\{u_1, u_2, u_4\}, \{u_2, u_3, u_5\}, \{u_1, u_3, u_4\}\},$$
$$\mathfrak{E}^{(0.2,0.2,0.2,0.2)} = \{\{u_1, u_2, u_4, u_5\}, \{u_2, u_3, u_4, u_5\}, \{u_1, u_3, u_4, u_5\}\}.$$

Clearly,

$$\mathfrak{F}_s(\mathfrak{H}) = \{(0.8, 0.8, 0.8, 0.8), (0.4, 0.4, 0.4, 0.4), (0.2, 0.2, 0.2, 0.2)\}.$$

Also,

$$T_r(\mathfrak{H}) = \{\rho_1, \rho_2, \rho_3\} = T_r^*(\mathfrak{H}),$$

where

$$\rho_1 = \{(u_1, 0.8, 0.8, 0.8, 0.8), (u_2, 0.8, 0.8, 0.8, 0.8)\},$$

$$\rho_2 = \{(u_2, 0.8, 0.8, 0.8, 0.8), (u_3, 0.8, 0.8, 0.8, 0.8)\},$$

and

$$\rho_3 = \{(u_1, 0.8, 0.8, 0.8, 0.8), (u_3, 0.8, 0.8, 0.8, 0.8)\}.$$

Table 2. Incidence matrix of the DVNHG $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$

	$\mathfrak{E}_1$	$\mathfrak{E}_2$	$\mathfrak{E}_3$
u_1	$(0.8, 0.8, 0.8, 0.8)$	$(0, 0, 0, 1)$	$(0.8, 0.8, 0.8, 0.8)$
u_2	$(0.8, 0.8, 0.8, 0.8)$	$(0.8, 0.8, 0.8, 0.8)$	$(0, 0, 0, 1)$
u_3	$(0, 0, 0, 1)$	$(0.8, 0.8, 0.8, 0.8)$	$(0.8, 0.8, 0.8, 0.8)$
u_4	$(0.4, 0.4, 0.4, 0.4)$	$(0.4, 0.4, 0.4, 0.4)$	$(0.2, 0.2, 0.2, 0.2)$
u_5	$(0.2, 0.2, 0.2, 0.2)$	$(0.2, 0.2, 0.2, 0.2)$	$(0.4, 0.4, 0.4, 0.4)$

Since

$$\{u_4, u_5\} \in T_r\big(\mathfrak{H}^{(0.4,0.4,0.4,0.4)}\big)$$

and

$$\{u_4\} \in T_r\big(\mathfrak{H}^{(0.2,0.2,0.2,0.2)}\big),$$

no minimal transversal of $\mathfrak{H}^{(0.2,0.2,0.2,0.2)}$ contains $\{u_4, u_5\}$. Hence the ordered pair

$$\Big(\mathfrak{H}^{(0.4,0.4,0.4,0.4)}, \mathfrak{H}^{(0.2,0.2,0.2,0.2)}\Big)$$

is not T-related. Therefore, $\mathfrak{H}$ is not T-related.

Theorem 4.6. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is an ordered DVNHG. Then*

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H}) \iff \mathfrak{H} \text{ is T-related.}$$

Proof. By Theorem 4.4, if $\mathfrak{H}$ is T-related, then

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H}).$$

It remains to prove the converse.
Assume that

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H})$$

and suppose, for a contradiction, that $\mathfrak{H}$ is not T-related. Let

$$\mathfrak{F}_s(\mathfrak{H}) = \left\{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), \ldots, (A_n, B_n^A, B_n^C, C_n)\right\},$$

and let

$$c(\mathfrak{H}) = \left\{\mathfrak{H}^{(A_1,B_1^A,B_1^C,C_1)}, \mathfrak{H}^{(A_2,B_2^A,B_2^C,C_2)}, \ldots, \mathfrak{H}^{(A_n,B_n^A,B_n^C,C_n)}\right\}.$$

Since $\mathfrak{H}$ is not T-related, there exists an index $j \in \{1, 2, \ldots, n-1\}$ such that the ordered pair

$$\left(\mathfrak{H}^{(A_j,B_j^A,B_j^C,C_j)}, \mathfrak{H}^{(A_{j+1},B_{j+1}^A,B_{j+1}^C,C_{j+1})}\right)$$

is not T-related.
By the definition of T-relatedness, there exist a minimal transversal

$$\rho_j \in T_r\left(\mathfrak{H}^{(A_j,B_j^A,B_j^C,C_j)}\right)$$

and a transversal ρ_{j+1} of

$$\mathfrak{H}^{(A_{j+1},B_{j+1}^A,B_{j+1}^C,C_{j+1})}$$

such that

$$\rho_j \subseteq \rho_{j+1},$$

but no minimal transversal N of $\mathfrak{H}^{(A_{j+1},B_{j+1}^A,B_{j+1}^C,C_{j+1})}$ satisfies

$$\rho_j \subseteq N \subseteq \rho_{j+1}.$$

Since $\mathfrak{H}$ is ordered, we have

$$\mathfrak{H}^{(A_j,B_j^A,B_j^C,C_j)} \subseteq \mathfrak{H}^{(A_{j+1},B_{j+1}^A,B_{j+1}^C,C_{j+1})}.$$

Therefore, ρ_j is not a transversal of $\mathfrak{H}^{(A_{j+1},B_{j+1}^A,B_{j+1}^C,C_{j+1})}$; otherwise, it would contradict the above choice of ρ_j and ρ_{j+1}.
Choose a minimal transversal ξ of $\mathfrak{H}^{(A_{j+1},B_{j+1}^A,B_{j+1}^C,C_{j+1})}$ such that

$$\xi \subseteq \rho_{j+1}.$$

By the non-T-relatedness assumption, ξ cannot contain ρ_j. Hence

$$\rho_j \nsubseteq \xi.$$

Using ρ_j as the cut at level (A_j, B_j^A, B_j^C, C_j) and ξ as the cut at level $(A_{j+1}, B_{j+1}^A, B_{j+1}^C, C_{j+1})$, one can construct a minimal DVNT δ of $\mathfrak{H}$ such that

$$\delta \in T_r(\mathfrak{H})$$

but

$$\delta^{(A_{j+1}, B_{j+1}^A, B_{j+1}^C, C_{j+1})} \notin T_r\big(\mathfrak{H}^{(A_{j+1}, B_{j+1}^A, B_{j+1}^C, C_{j+1})}\big).$$

Thus,

$$\delta \notin T_r^*(\mathfrak{H}).$$

This gives

$$\delta \in T_r(\mathfrak{H}) \setminus T_r^*(\mathfrak{H}),$$

which contradicts the assumption

$$T_r(\mathfrak{H}) = T_r^*(\mathfrak{H}).$$

Therefore, $\mathfrak{H}$ must be T-related. □

Corollary 4.7. *Assume that $\mathfrak{H} = (\mathfrak{V}, \mathfrak{E})$ is an ordered DVNHG with*

$$\mathfrak{F}_s(\mathfrak{H}) = \big\{(A_1, B_1^A, B_1^C, C_1), (A_2, B_2^A, B_2^C, C_2), \ldots, (A_n, B_n^A, B_n^C, C_n)\big\}$$

and

$$c(\mathfrak{H}) = \Big\{\mathfrak{H}^{(A_1, B_1^A, B_1^C, C_1)}, \mathfrak{H}^{(A_2, B_2^A, B_2^C, C_2)}, \ldots, \mathfrak{H}^{(A_n, B_n^A, B_n^C, C_n)}\Big\}.$$

If an ordered pair

$$\Big(\mathfrak{H}^{(A_j, B_j^A, B_j^C, C_j)}, \mathfrak{H}^{(A_{j+1}, B_{j+1}^A, B_{j+1}^C, C_{j+1})}\Big)$$

is not T-related, then:

- $$(A_{j+1}, B_{j+1}^A, B_{j+1}^C, C_{j+1}) \in \mathfrak{F}_s\big(T_r(\mathfrak{H})\big).$$

- $$(A_{j+1}, B_{j+1}^A, B_{j+1}^C, C_{j+1})$$

  is a transition level for some

$$\delta \in T_r(\mathfrak{H}) \setminus T_r^*(\mathfrak{H}).$$

5. Conclusions

In this study, we integrated the concept of double-valued neutrosophic sets into hypergraph theory in order to handle uncertainty and indeterminacy situations more effectively. The proposed double-valued neutrosophic hypergraph model provides a flexible framework for representing uncertainty through truth, indeterminacy, and falsity membership degrees. We investigated several fundamental properties of double-valued neutrosophic hypergraphs, including transition levels, lower truncations, and upper truncations. Moreover, we studied the properties of minimal transversals and their relationship with T-related double-valued neutrosophic hypergraphs. The proposed framework shows its potential for describing double-valued uncertain systems and identifying hidden relationships among interconnected structures. Future research may extend this framework to interval-valued (hyperfuzzy) [39, 40], bipolar [37, 38], and other generalized neutrosophic hypergraphs. We also hope that further extensions using HyperSoft graphs [28], SuperHyperGraphs [29, 30], and related higher-order structures will be investigated in future studies.

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