Neutrosophic Knowledge, Volume 10, 2026

University of New Mexico

Conceptual HyperGraph and Conceptual SuperHyperGraph

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Received*: 03/23/2026;* Accepted*: 03/31/2026*

Abstract. A finite *hypergraph* generalizes an ordinary graph by permitting a *hyperedge* to connect any nonempty subset of vertices, thereby capturing multiway interactions. Extending this idea, a finite *SuperHyperGraph* is obtained by iterating the powerset operation, producing nested families of vertex and edge sets that model multi-layer relational structure. Separately, a *conceptual graph* is a labeled bipartite graph whose concept nodes and relation nodes encode typed relations with ordered arguments, while a *conceptual hypergraph* represents relation instances as typed hyperedges incident to an ordered tuple of vertices. In this paper, we extend these conceptual formalisms using the SuperHyperGraph framework, introducing *Conceptual SuperHyperGraphs* and examining their basic properties.

Keywords: Conceptual HyperGraph, Conceptual SuperHyperGraph, HyperGraph, SuperHyperGraph

1. Introduction

Graphs provide a standard language for modeling relational data: vertices represent entities, and edges represent pairwise interactions [1]. However, many real-world systems involve genuinely multiway relationships that cannot be faithfully captured by binary edges alone. A finite *hypergraph* addresses this limitation by allowing a *hyperedge* to connect any nonempty subset of the vertex set, thereby representing higher-order interactions [2, 3]. Hypergraphs have been extended and studied in various forms, such as fuzzy hypergraphs [4–6], directed hypergraphs [7, 8], and neutrosophic hypergraphs [9, 10].

Going beyond hypergraphs, a finite *SuperHyperGraph* is obtained by iterating the powerset operation; the resulting nested families of vertex sets naturally encode multilayer relational structures [11, 12]. In particular, SuperHyperGraphs enhance expressive power by simultaneously modeling multiway interactions and hierarchical aggregation (e.g., groups, groups of groups, and so forth), which makes them valuable for the study of layered networks, modular systems, and multiscale relational data. Such hierarchical models have been explored in a variety of applied settings, including applied science and the analysis of complex networks [13,14]. Moreover, because of the theoretical importance and strong potential applications of SuperHyperGraphs, several extended frameworks have been investigated, including *Meta SuperHyperGraphs* [15], *Plithogenic SuperHyperGraphs* [16, 17], *Topological SuperHyperGraphs* [18], and *Neutrosophic SuperHyperGraphs* [19–21]. Unless stated otherwise, the index n in $\mathcal{P}_n(\cdot)$ and the level n of an n-SuperHyperGraph are assumed to be nonnegative integers. For convenience, Table 1 presents a compact comparison of graphs, hypergraphs, and n-SuperHyperGraphs.

Aspect	**Graph**	**Hypergraph**	**SuperHyperGraph**
Objects	Vertices, edges.	Vertices, hyperedges.	n-supervertices, n-superedges.
Links connect	Two vertices.	Any nonempty vertex subset.	Nonempty supervertex subsets (with nested supervertices).
Formal model	$G = (V, E)$, $E \subseteq \binom{V}{2}$.	$H = (V, E)$, $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.	$\mathrm{SHG}^{(n)} = (V, E)$, $V \subseteq \mathcal{P}^n(V_0)$, $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.
Main emphasis	Pairwise relations.	Higher-order relations.	Higher-order *and* hierarchical (multi-level) relations.

TABLE 1. Compact comparison of graphs, hypergraphs, and n-SuperHyperGraphs.

In parallel, *conceptual graphs* and *conceptual hypergraphs* provide knowledge-representation formalisms in which relations are typed and arguments are ordered [22–24]. A *conceptual graph* is a labeled bipartite graph whose concept nodes and relation nodes encode typed relations with ordered roles, whereas a *conceptual hypergraph* represents relation instances as typed hyperedges incident to an ordered tuple of vertices. These frameworks are attractive because they are both mathematically well-structured and human-readable, supporting semantic integration and explainable reasoning in applications ranging from natural language processing to decision-support systems.

Despite the extensive development of conceptual graph and conceptual hypergraph methodology, their SuperHyperGraph counterparts have not yet been systematically investigated. This paper addresses that gap. We extend the conceptual graph and conceptual hypergraph frameworks to the SuperHyperGraph setting, introduce *Conceptual SuperHyperGraphs*, and study their basic structural properties. Table 2 summarizes the key distinctions among conceptual graphs (CG), conceptual hypergraphs (CHG), and conceptual SuperHyperGraphs (CSHG). Conceptual SuperHyperGraphs enable typed, ordered relations over nested entities, supporting hierarchical knowledge representation, explainable reasoning, and multiscale semantic integration.

Aspect	**Conceptual Graph (CG)**	**Conceptual Hypergraph (CHG)**	**Conceptual SuperHyperGraph (CSHG)**
Core structure	Bipartite graph (concept nodes / relation nodes).	Typed hypergraph (vertices / hyperedges).	SuperHyperGraph-based (n-supervertices / relation nodes) with ordered incidence.
Relation instance	A relation node linked to its argument concepts.	A typed hyperedge incident to its argument vertices.	A relation node incident to argument supervertices (forming an n-superedge).
Argument order	Position map on arcs (roles).	Ordered incidence tuple $\mathrm{inc}(a)$.	Position map (or $\mathrm{inc}(r)$) on supervertices.
Argument objects	Individual concepts.	Individual vertices.	Nested entities (supervertices built via iterated powersets).

TABLE 2. Compact comparison of CG, CHG, and CSHG.

2. Preliminaries

This section fixes the notation used throughout the paper and summarizes the background material required later. Unless stated otherwise, all sets and structures considered here are finite.

2.1. *SuperHyperGraphs*

We next recall the set-theoretic constructions underlying SuperHyperGraphs and then state the formal definition.

Definition 2.1 (Base set)**.** A *base set* S is the ambient universe of discourse associated with the context under study:

$$S = \{\, x \mid x \text{ is an admissible object in the given context} \,\}.$$

Consequently, any element of $\mathcal{P}(S)$—and, more generally, of an iterated powerset such as $\mathcal{P}_n(S)$—is a set ultimately assembled from elements of S.

Definition 2.2 (Powerset)**.** (see [25]) For a set S, the *powerset* $\mathcal{P}(S)$ is the collection of all subsets of S, namely

$$\mathcal{P}(S) \;=\; \{\, A \mid A \subseteq S \,\}.$$

In particular, $\emptyset \in \mathcal{P}(S)$ and $S \in \mathcal{P}(S)$.

Definition 2.3 (Hypergraph)**.** [26,27] A *hypergraph* is an ordered pair $H = (V, E)$ consisting of

- a finite set V of *vertices*, and
- a finite family E of nonempty subsets of V, called *hyperedges*.

Thus a hyperedge may contain more than two vertices, enabling the representation of genuinely multiway relations.

Definition 2.4 (n-th iterated powerset)**.** [28,29] For a set X, define $\mathcal{P}_1(X) := \mathcal{P}(X)$ and, for $n \geq 1$, set recursively

$$\mathcal{P}_{n+1}(X) \;:=\; \mathcal{P}\bigl(\mathcal{P}_n(X)\bigr).$$

When the empty set is excluded, we write

$$\mathcal{P}_n^{*}(X) \;:=\; \mathcal{P}_n(X) \setminus \{\emptyset\}.$$

Definition 2.5 (n–SuperHyperGraph)**.** (see [11,30]) Let V_0 be a finite, nonempty base set. Define the iterated powersets by

$$\mathcal{P}^0(V_0) := V_0, \qquad \mathcal{P}^{\,k+1}(V_0) := \mathcal{P}\bigl(\mathcal{P}^{\,k}(V_0)\bigr) \quad (k \in \mathbb{N}).$$

For $n \geq 0$, an *n–SuperHyperGraph on* V_0 is a pair

$$\mathrm{SHG}^{(n)} = (V, E)$$

satisfying

$$V \subseteq \mathcal{P}^{\,n}(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

The elements of V are called *n–supervertices*, and the elements of E are called *n–superedges*; equivalently, each n–superedge is a nonempty subset of the supervertex set V.

2.2. *Conceptual Graphs and Conceptual HyperGraphs*

A conceptual graph is a labeled bipartite graph consisting of concept nodes and relation nodes, designed to encode typed relations with ordered arguments [22–24]. In this paper, we restrict attention to relation instances whose arguments are pairwise distinct. A conceptual hypergraph is a typed hypergraph in which each hyperedge represents a relation instance together with an ordered list of argument vertices (cf. [31, 32]).

Definition 2.6 (Conceptual vocabulary)**.** A *conceptual vocabulary* is a tuple

$$\mathcal{V} = (T_C, \leq_C,\ T_R, \mathrm{ar},\ I)$$

where

- T_C is a nonempty set of *concept types* equipped with a preorder $\leq_C$ (read: "is-a"),
- T_R is a nonempty set of *relation types*,
- $\mathrm{ar} : T_R \to \mathbb{N}_{\geq 1}$ assigns to each relation type its *arity*, and
- I is a (possibly empty) set of *individual markers*.

We also adjoin a distinguished generic marker $* \notin I$, used to indicate an unspecified individual.

Definition 2.7 (Conceptual graph)**.** Fix a conceptual vocabulary $\mathcal{V} = (T_C, \leq_C, T_R, \mathrm{ar}, I)$. A *conceptual graph* over $\mathcal{V}$ is a finite structure

$$G = (C, R, \Gamma, \lambda_C, \lambda_R, \pi)$$

satisfying:

- C and R are finite disjoint sets (the *concept nodes* and *relation nodes*);
- $\Gamma \subseteq R \times C$ is a finite set of *arcs* (so the underlying graph is bipartite);
- $\lambda_C : C \to T_C \times (I \cup \{*\})$ labels each concept node c by a pair $\lambda_C(c) = (\mathrm{type}(c), \mathrm{mark}(c))$;
- $\lambda_R : R \to T_R$ labels each relation node r by its relation type $\lambda_R(r)$;
- $\pi : \Gamma \to \mathbb{N}_{\geq 1}$ assigns to each arc $(r, c) \in \Gamma$ a *position index* $\pi(r, c)$.

These data must satisfy the arity/position constraint: for every $r \in R$,

$$\{\pi(r, c)\ :\ (r, c) \in \Gamma\} = \{1, 2, \ldots, \mathrm{ar}(\lambda_R(r))\} \quad \text{and} \quad |\{c \in C : (r, c) \in \Gamma\}| = \mathrm{ar}(\lambda_R(r)).$$

Equivalently, each relation node r has exactly $\mathrm{ar}(\lambda_R(r))$ incident concept nodes, and the incident arcs are bijectively indexed by argument positions $1, \ldots, \mathrm{ar}(\lambda_R(r))$.

Definition 2.8 (Conceptual hypergraph)**.** Fix a conceptual vocabulary $\mathcal{V} = (T_C, \leq_C, T_R, \mathrm{ar}, I)$. A *conceptual hypergraph* over $\mathcal{V}$ is a finite structure

$$H = (X, \mathcal{A}, \lambda_X, \lambda_{\mathcal{A}}, \mathrm{inc})$$

such that:

- X is a finite set of *vertices* (concept vertices),
- $\mathcal{A}$ is a finite set of *hyperedges* (relation occurrences),
- $\lambda_X : X \to T_C \times (I \cup \{*\})$ assigns to each vertex $x \in X$ a label

  $$\lambda_X(x) = (\mathrm{type}(x), \mathrm{mark}(x)),$$

  where $\mathrm{type}(x) \in T_C$ and $\mathrm{mark}(x) \in I \cup \{*\}$,
- $\lambda_{\mathcal{A}} : \mathcal{A} \to T_R$ assigns to each hyperedge $a \in \mathcal{A}$ its relation type $\lambda_{\mathcal{A}}(a)$, and
- inc is an *ordered incidence map* assigning to each $a \in \mathcal{A}$ a tuple

  $$\mathrm{inc}(a) = (x_1, \dots, x_k) \in X^k, \qquad k = \mathrm{ar}(\lambda_{\mathcal{A}}(a)),$$

  such that the vertices $x_1, \dots, x_k$ are pairwise distinct.

Thus, each hyperedge a represents an ordered instance of a k-ary relation, whose arguments are precisely the vertices

$$x_1, \dots, x_k$$

listed in the order prescribed by $\mathrm{inc}(a)$.

Theorem 2.9 (Equivalence of representations)**.** *Fix a conceptual vocabulary $\mathcal{V}$. Every conceptual graph G over $\mathcal{V}$ canonically determines a conceptual hypergraph H over $\mathcal{V}$, and conversely, every conceptual hypergraph H over $\mathcal{V}$ canonically determines a conceptual graph G over $\mathcal{V}$. These constructions are mutually inverse up to isomorphism.*

Proof. (*From conceptual graphs to conceptual hypergraphs.*) Let $G = (C, R, \Gamma, \lambda_C, \lambda_R, \pi)$ be a conceptual graph. Set $X := C$, $\mathcal{A} := R$, $\lambda_X := \lambda_C$, and $\lambda_{\mathcal{A}} := \lambda_R$. For each $r \in R$, define $\mathrm{inc}(r) = (x_1, \dots, x_k)$ with $k = \mathrm{ar}(\lambda_R(r))$ by taking x_i to be the unique $c \in C$ such that $(r, c) \in \Gamma$ and $\pi(r, c) = i$. Uniqueness holds by the arity/position constraint in Definition 2.7. This yields a conceptual hypergraph H.

(*From conceptual hypergraphs to conceptual graphs.*) Let $H = (X, \mathcal{A}, \lambda_X, \lambda_{\mathcal{A}}, \mathrm{inc})$ be a conceptual hypergraph. Set $C := X$, $R := \mathcal{A}$, $\lambda_C := \lambda_X$, and $\lambda_R := \lambda_{\mathcal{A}}$. For each $a \in \mathcal{A}$ with $\mathrm{inc}(a) = (x_1, \dots, x_k)$, define arcs $(a, x_i) \in \Gamma$ for $i = 1, \dots, k$, and set $\pi(a, x_i) := i$. Then $\{\pi(a, c) : (a, c) \in \Gamma\} = \{1, \dots, k\}$ and $k = \mathrm{ar}(\lambda_R(a))$ by definition of inc, so the arity/position constraint holds and we obtain a conceptual graph G.

It is immediate from the definitions that applying the second construction after the first (and vice versa) recovers the original structure up to the evident renaming of nodes, i.e. up to isomorphism. □

3. Main Results: Conceptual Superhypergraphs

A *Conceptual SuperHyperGraph* is a labeled ordered incidence structure whose supervertices carry concepts, whose superedges represent relation instances, and whose underlying shadow is a SuperHyperGraph. We assume the conceptual vocabulary $\mathcal{V} = (T_C, \leq_C, T_R, \mathrm{ar}, I)$ from Definition 2.6. Throughout, V_0 is a finite nonempty base set and $n \geq 0$.

Definition 3.1 (Conceptual n–SuperHyperGraph)**.** A *conceptual n–SuperHyperGraph* over $\mathcal{V}$ on V_0 is a finite structure

$$\mathfrak{C} \;=\; (V, R, \Gamma, \lambda_V, \lambda_R, \pi)$$

such that:

- V and R are finite disjoint sets (the *n-supervertices* and *relation nodes*);
- $V \subseteq \mathcal{P}^n(V_0)$;
- $\Gamma \subseteq R \times V$ is a finite set of *arcs* (so the underlying graph is bipartite);
- $\lambda_V : V \to T_C \times (I \cup \{*\})$ labels each supervertex v by a pair $\lambda_V(v) = (\mathrm{type}(v), \mathrm{mark}(v))$;
- $\lambda_R : R \to T_R$ labels each relation node r by its relation type $\lambda_R(r)$;
- $\pi : \Gamma \to \mathbb{N}_{\geq 1}$ assigns to each arc $(r, v) \in \Gamma$ a *position index* $\pi(r, v)$,

and these data satisfy the arity/position constraint: for every $r \in R$, letting $k = \mathrm{ar}(\lambda_R(r))$,

$$\{\pi(r, v) \;:\; (r, v) \in \Gamma\} = \{1, 2, \ldots, k\} \quad \text{and} \quad |\{v \in V : (r, v) \in \Gamma\}| = k.$$

Example 3.2 (A conceptual 2–SuperHyperGraph)**.** Let the conceptual vocabulary be $\mathcal{V} = (T_C, \leq_C, T_R, \mathrm{ar}, I)$ with

$$T_C = \{\mathrm{Person}, \mathrm{City}\}, \qquad T_R = \{\mathrm{LivesIn}\}, \qquad \mathrm{ar}(\mathrm{LivesIn}) = 2, \qquad I = \{\mathrm{Alice}, \mathrm{Tokyo}\}.$$

Let the base set be $V_0 = \{a, b\}$ and take $n = 2$, so

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}(V_0)\big).$$

Define four 2-supervertices (i.e., subsets of $\mathcal{P}(V_0)$) by

$$u_1 := \big\{\{a\}\big\}, \qquad u_2 := \big\{\{b\}\big\}, \qquad u_3 := \big\{\{a, b\}\big\}, \qquad u_4 := \big\{\{a\}, \{a, b\}\big\}.$$

Then

$$V = \{u_1, u_2, u_3, u_4\} \;\subseteq\; \mathcal{P}^2(V_0).$$

Let $R = \{r\}$ be a singleton set of relation nodes, and define the arc set

$$\Gamma = \{(r, u_4), (r, u_2)\} \;\subseteq\; R \times V.$$

Assign labels to 2-supervertices by

$$\lambda_V(u_4) = (\mathrm{Person}, \mathrm{Alice}), \qquad \lambda_V(u_2) = (\mathrm{City}, \mathrm{Tokyo}),$$

$$\lambda_V(u_1) = (\mathrm{City}, *), \qquad \lambda_V(u_3) = (\mathrm{Person}, *),$$

label the relation node by

$$\lambda_R(r) = \text{LivesIn},$$

and assign argument positions by

$$\pi(r, u_4) = 1, \qquad \pi(r, u_2) = 2.$$

Then $k = \mathrm{ar}(\lambda_R(r)) = 2$, and indeed

$$\{\pi(r, v) : (r, v) \in \Gamma\} = \{1, 2\}, \qquad |\{v \in V : (r, v) \in \Gamma\}| = 2,$$

so $\mathfrak{C} = (V, R, \Gamma, \lambda_V, \lambda_R, \pi)$ is a well-defined conceptual 2–SuperHyperGraph. Intuitively, the supervertex $u_4 = \{\{a\}, \{a, b\}\}$ encodes a *nested* entity (a set of subsets of V_0), so the statement "Alice lives in Tokyo" is represented with arguments that already carry a two-level grouping structure.

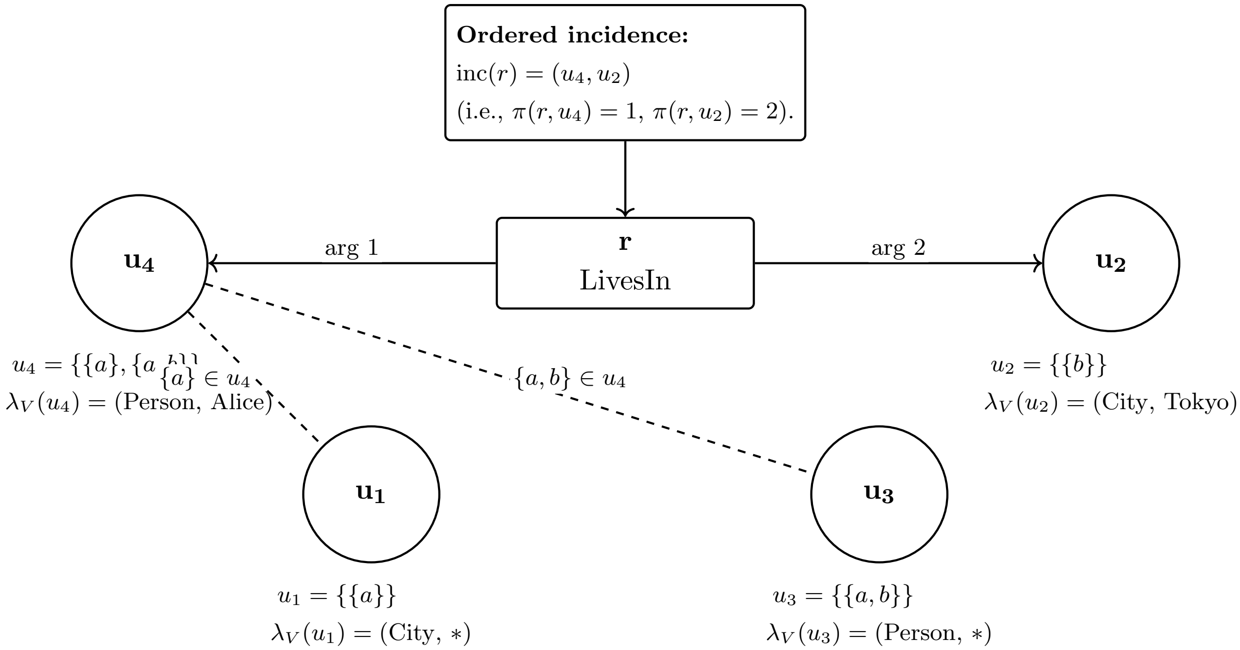

FIGURE 1. A clearer depiction of a conceptual 2–SuperHyperGraph. The relation node **r** (type LivesIn) has two ordered arguments: arg 1 is the 2-supervertex $u_4 = \{\{a\}, \{a, b\}\}$ labeled (Person, Alice), and arg 2 is $u_2 = \{\{b\}\}$ labeled (City, Tokyo). Dashed lines indicate that $\{a\}$ and $\{a, b\}$ are elements of the nested set u_4.

Example 3.3 (Another conceptual 2–SuperHyperGraph)**.** We present a second, self-contained example of a conceptual 2–SuperHyperGraph, this time with a *ternary* relation.

Vocabulary. Let the conceptual vocabulary be $\mathcal{V} = (T_C, \leq_C,\ T_R, \mathrm{ar},\ I)$, where

$$T_C = \{\text{Person}, \text{City}\}, \qquad \leq_C \text{ is the identity preorder on } T_C,$$

$$T_R = \{\text{Travel}\}, \qquad \text{ar}(\text{Travel}) = 3, \qquad I = \{\text{Alice}, \text{Osaka}, \text{Tokyo}\}.$$

Intuitively, $\text{Travel}(p, c_1, c_2)$ means: "person p travels from city c_1 to city c_2."

Base set and level. Let $V_0 = \{a, b, c\}$ and take $n = 2$. Hence

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}(V_0)\big).$$

Supervertices. Define the following 2-supervertices (each is a subset of $\mathcal{P}(V_0)$):

$$u_1 := \big\{\{a\}, \{a, b\}\big\}, \qquad u_2 := \big\{\{b\}\big\}, \qquad u_3 := \big\{\{c\}\big\}, \qquad u_4 := \big\{\{b, c\}\big\}.$$

Let

$$V = \{u_1, u_2, u_3, u_4\} \ \subseteq\ \mathcal{P}^2(V_0).$$

Relation node and arcs. Let $R = \{r\}$ be a singleton set of relation nodes and define

$$\Gamma = \{(r, u_1), (r, u_2), (r, u_3)\} \ \subseteq\ R \times V.$$

Labels and argument order. Assign labels to 2-supervertices by

$$\lambda_V(u_1) = (\text{Person}, \text{Alice}), \qquad \lambda_V(u_2) = (\text{City}, \text{Osaka}),$$

$$\lambda_V(u_3) = (\text{City}, \text{Tokyo}), \qquad \lambda_V(u_4) = (\text{City}, *).$$

Label the relation node by

$$\lambda_R(r) = \text{Travel},$$

and define the argument-position map $\pi : \Gamma \to \mathbb{N}_{\geq 1}$ by

$$\pi(r, u_1) = 1, \qquad \pi(r, u_2) = 2, \qquad \pi(r, u_3) = 3.$$

Then $k = \text{ar}(\lambda_R(r)) = 3$ and

$$\{\pi(r, v) : (r, v) \in \Gamma\} = \{1, 2, 3\}, \qquad |\{v \in V : (r, v) \in \Gamma\}| = 3,$$

so $\mathfrak{C} = (V, R, \Gamma, \lambda_V, \lambda_R, \pi)$ is a well-defined conceptual 2–SuperHyperGraph. It encodes the statement: "Alice travels from Osaka to Tokyo," where the first argument $u_1 = \{\{a\}, \{a, b\}\}$ is a genuinely nested (two-level) entity.

Definition 3.4 (Incidence set and ordered incidence)**.** Let $\mathfrak{C} = (V, R, \Gamma, \lambda_V, \lambda_R, \pi)$ be a conceptual n–SuperHyperGraph. For each $r \in R$ define its (crisp) *incidence set*

$$\text{Inc}(r) \ :=\ \{\, v \in V :\ (r, v) \in \Gamma \,\} \ \subseteq\ V.$$

Also define its *ordered incidence tuple* $\text{inc}(r) = (v_1, \dots, v_k) \in V^k$, where $k = \text{ar}(\lambda_R(r))$ and v_i is the unique supervertex such that $(r, v_i) \in \Gamma$ and $\pi(r, v_i) = i$.

Lemma 3.5 (Well-definedness of $\text{inc}(r)$)**.** *In Definition 3.4, for each $r \in R$ the tuple $\text{inc}(r)$ is well-defined and unique. Moreover, $\text{Inc}(r) \neq \emptyset$ for every $r \in R$.*

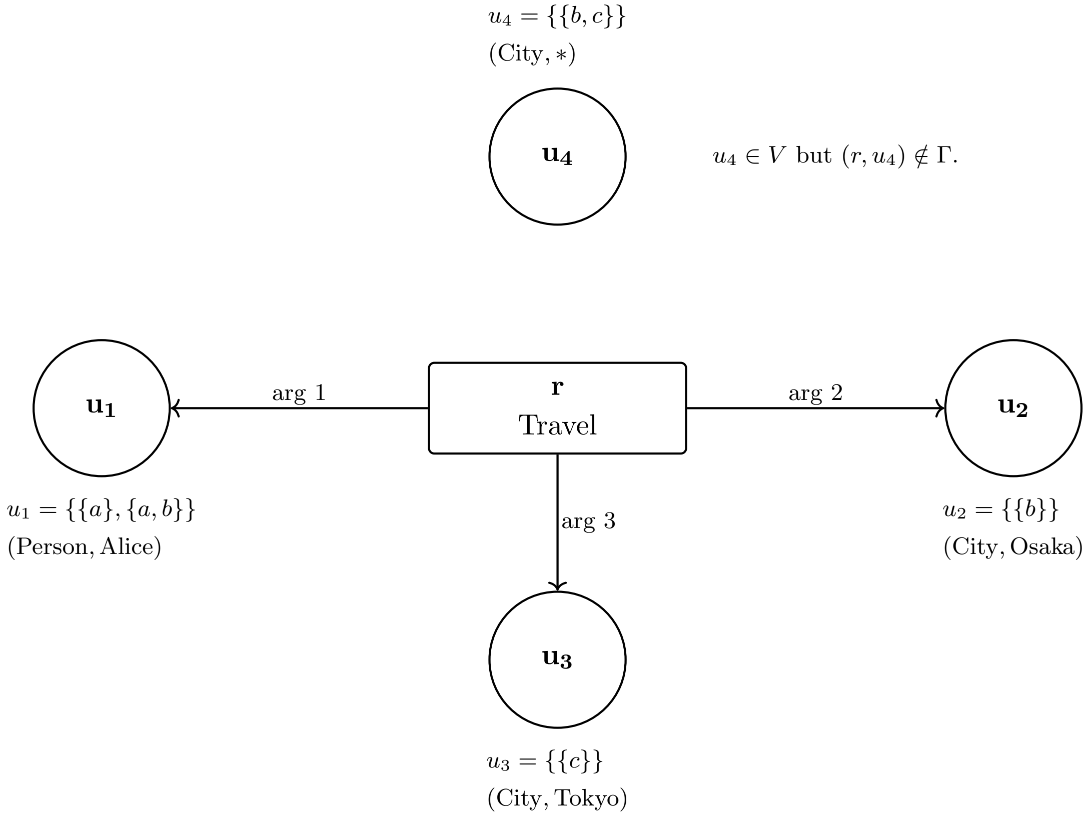

FIGURE 2. A conceptual 2–SuperHyperGraph with one ternary relation node Travel and ordered arguments (u_1, u_2, u_3).

Proof. Fix $r \in R$ and set $k = \mathrm{ar}(\lambda_R(r)) \geq 1$. By Definition 3.1, the set of position indices $\{\pi(r, v) : (r, v) \in \Gamma\}$ equals $\{1, \ldots, k\}$, so for each $i \in \{1, \ldots, k\}$ there exists at least one $v \in V$ with $(r, v) \in \Gamma$ and $\pi(r, v) = i$. The second constraint $|\{v : (r, v) \in \Gamma\}| = k$ forces this v to be unique for each i, hence $\mathrm{inc}(r)$ is well-defined and unique. Finally, $k \geq 1$ implies $\mathrm{Inc}(r)$ contains at least one vertex, hence $\mathrm{Inc}(r) \neq \emptyset$. □

Theorem 3.6 (Underlying n–SuperHyperGraph structure). *Let $\mathfrak{C} = (V, R, \Gamma, \lambda_V, \lambda_R, \pi)$ be a conceptual n–SuperHyperGraph on V_0. Define*

$$E := \{\mathrm{Inc}(r) \subseteq V : r \in R\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Then (V, E) is an n–SuperHyperGraph on V_0 in the sense of Definition 2.5.

Proof. By Definition 3.1, we have $V \subseteq \mathcal{P}^n(V_0)$. For each $r \in R$, $\mathrm{Inc}(r) \subseteq V$ by definition and $\mathrm{Inc}(r) \neq \emptyset$ by Lemma 3.5. Hence $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, and therefore (V, E) is an n–SuperHyperGraph on V_0. □

Remark 3.7 (Forgetting concept/relation labels and argument order)**.** Theorem 3.6 formalizes the fact that every conceptual n-SuperHyperGraph determines an underlying n-SuperHyperGraph. This underlying structure is obtained by forgetting the concept and relation labels λ_V, λ_R as well as the argument-order information π, and retaining only the supervertex set

$$V \subseteq \mathcal{P}^n(V_0)$$

together with the superedge family

$$E = \{\mathrm{Inc}(r) \mid r \in R\}.$$

Hence, the shadow records only which supervertices are incident with each relation node, but it does not retain labels, marks, or positional information. In particular, distinct relation nodes having the same incidence set give rise to the same superedge in the underlying n-SuperHyperGraph.

Theorem 3.8 (Conceptual graphs embed as conceptual 0–SuperHyperGraphs)**.** *Let* $G = (C, R, \Gamma, \lambda_C, \lambda_R, \pi)$ *be a conceptual graph over* $\mathcal{V}$ *(Definition 2.7). Let* $V_0 := C$ *and* $n := 0$*. Then* $C = \mathcal{P}^0(V_0)$*, and*

$$\mathfrak{C}_G \;:=\; (V, R, \Gamma, \lambda_V, \lambda_R, \pi) \quad \textit{with} \quad V := C,\ \lambda_V := \lambda_C$$

is a conceptual 0*–SuperHyperGraph on* V_0*. Moreover, the underlying* 0*–SuperHyperGraph of* $\mathfrak{C}_G$ *(Theorem 3.6) has supervertex set* C *and superedges* $\{\mathrm{Inc}(r) : r \in R\}$*.*

Proof. Since $n = 0$, we have $\mathcal{P}^0(V_0) = V_0 = C$, hence $V = C \subseteq \mathcal{P}^0(V_0)$. All other axioms of Definition 3.1 are exactly those of Definition 2.7. The last statement follows directly from Theorem 3.6. □

Theorem 3.9 (Conceptual hypergraphs embed as conceptual 0–SuperHyperGraphs)**.** *Let* $H = (X, \mathcal{A}, \lambda_X, \lambda_{\mathcal{A}}, \mathrm{inc})$ *be a conceptual hypergraph over* $\mathcal{V}$ *(Definition 2.8). Let* $V_0 := X$ *and* $n := 0$*. Define*

$$V := X, \qquad R := \mathcal{A}, \qquad \lambda_V := \lambda_X, \qquad \lambda_R := \lambda_{\mathcal{A}}.$$

For each $a \in \mathcal{A}$ *with* $\mathrm{inc}(a) = (x_1, \dots, x_k)$ *(where* $k = \mathrm{ar}(\lambda_{\mathcal{A}}(a))$*), set*

$$\Gamma \;:=\; \{(a, x_i) : a \in \mathcal{A},\ 1 \le i \le k\}, \qquad \pi(a, x_i) \;:=\; i.$$

Then $\mathfrak{C}_H = (V, R, \Gamma, \lambda_V, \lambda_R, \pi)$ *is a conceptual* 0*–SuperHyperGraph on* V_0*. Furthermore, the ordered incidence tuple of each* $a \in R$ *in* $\mathfrak{C}_H$ *equals the original* $\mathrm{inc}(a)$ *in* H*.*

Proof. Because $n = 0$ and $V_0 = X$, we have $V = X \subseteq \mathcal{P}^0(V_0) = V_0$. Fix $a \in R = \mathcal{A}$ and write $\mathrm{inc}(a) = (x_1, \ldots, x_k)$ with $k = \mathrm{ar}(\lambda_R(a))$. By construction, the arcs incident to a are exactly (a, x_i) for $i = 1, \ldots, k$, and their position indices are $\pi(a, x_i) = i$. Hence

$$\{\pi(a, v) : (a, v) \in \Gamma\} = \{1, \ldots, k\} \quad \text{and} \quad |\{v : (a, v) \in \Gamma\}| = k,$$

so the arity/position constraint of Definition 3.1 holds. Thus $\mathfrak{C}_H$ is a conceptual 0–SuperHyperGraph. Finally, the ordered incidence $\mathrm{inc}(a)$ recovered from π is $(x_1, \ldots, x_k)$, which is exactly the original tuple in H. □

4. **Conclusion**

In this paper, we extended classical conceptual formalisms within the SuperHyperGraph framework by introducing *Conceptual SuperHyperGraphs* and investigating their basic structural properties, including their hierarchical organization and their relationships with conceptual graphs and conceptual hypergraphs. We expect that future research will further develop this framework in several directions, such as the incorporation of Neutrosophic Sets [33, 34], Double-Valued Neutrosophic Sets [35,36], Plithogenic Sets [37,38], and Uncertain Sets [39,40]. Such extensions may provide richer ways to represent indeterminacy, contradiction, attribute interaction, and incomplete information in higher-order conceptual knowledge structures.

Funding

This study was conducted without any financial support from external organizations or grants.

Acknowledgments

We would like to express our sincere gratitude to everyone who provided valuable insights, support, and encouragement throughout this research. We also extend our thanks to the readers for their interest and to the authors of the referenced works, whose scholarly contributions have greatly influenced this study. Lastly, we are deeply grateful to the publishers and reviewers who facilitated the dissemination of this work.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and we do not employ them in any way that violates ethical standards.

Supplementary Information

No supplementary materials accompany this paper.

Disclaimer

The ideas presented here are theoretical and have not yet been validated through empirical testing. While we have strived for accuracy and proper citation, inadvertent errors may remain. Readers should verify any referenced material independently. The opinions expressed are those of the authors and do not necessarily reflect the views of their institutions.

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