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On The Symbolic n –plithogenic Rings and The Most Important Properties of Their Elements Using a Generalized Isomorphism

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Abstract: Symbolic n -plithogenic algebraic structures are viewed as symmetric extensions of classical algebraic systems, constructed through $n+1$ symmetric components. In this study, we introduce a broader formulation of symbolic n -plithogenic rings by establishing, for the first time, a general definition of symbolic n -plithogenic rings and examining their associated algebraic substructures. The proposed framework expands the landscape of n -symbolic plithogenic algebraic systems and offers a foundation for further theoretical developments. Our main findings are presented through a series of theorems supported by clear numerical examples that highlight the originality and significance of the contributions.

Keywords: Symbolic 2-Plithogenic Real Function, Symbolic 2-Plithogenic Integration, Symbolic 2-Plithogenic Derivative, Symbolic 2-Plithogenic Gamma Function and Symbolic 2-Plithogenic Beta Function.

1. Introduction

Neutrosophy is a new branch of philosophy concerns with the indeterminacy in all areas of life and science. It has become a useful tool in generalizing many classical systems such as equations [1,9], number theory [2,3], topology [4,5], linear spaces [6,10], modules [4,5], and ring of matrices [7,8].

In the literature, we find many studies about neutrosophic calculus, where some definitions and properties were presented about neutrosophic real functions and numbers [10]. The neutrosophic real functions with one variable were defined only in a special case [11], as follows:

Recently, Abobala and Hatip, have presented the concept of two-dimensional AH-isometry to study the correspondence between neutrosophic plane $R(I) \times R(I)$ and the classical module $R^2 \times R^2$. Also, the one-dimensional AH-isometry between $R(I)$ and $R \times R$. This isometry was useful in defining inner products and norms [10], ordering [9], and neutrosophic geometrical shapes [10].

In earlier works [17–20], refined neutrosophic structures were extensively examined, while Smarandache introduced the foundational framework of symbolic plithogenic algebraic structures.

Further refinements of neutrosophic structures were achieved by modifying the underlying definitions of their multiplication operations [21]. The algebraic behavior and selected substructures of symbolic 2-plithogenic rings—arising from the fusion between symbolic plithogenic sets and classical algebraic rings—were first outlined in [22], with additional exploration of their deeper algebraic intricacies presented in [23]. Taffach extended these investigations to symbolic 2-plithogenic vector spaces and modules [24,25].

In [26], the authors developed the theory of 2-plithogenic matrices, introducing their corresponding plithogenic elements and examining determinants, eigenvalues, eigenvectors, matrix exponents, and diagonalization. Symbolic 2-plithogenic number theory and its associated integers were studied in [27], while algebraic symbolic 2-plithogenic equations and their solutions were analyzed in [28]. A substantial body of recent contributions has further enriched the understanding of symbolic 2-plithogenic rings [29–43], reflecting the continued interest of researchers in this growing field.

More recently, M. Alabdullah [36] established that a neutrosophic ring $R(I)$ is regular if and only if the underlying ring R is regular.

2. Terminologies

We present here some basic definitions and axioms of neutrosophic logic and refined neutrosophic logic.

Definition 2.1. [12]: Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $\{x, (\mu_A(x), \delta_A(x), \gamma_A(x)): x \in X\}$, where $\mu_A(x)$, $\delta_A(x)$ and $\gamma_A(x)$ represent the degree of membership, the degree of indeterminacy, and the degree of non-membership respectively of each element $x \in X$ to the set A .

Definition 2.2. [13]: Let K be a field, the neutrosophic field generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 2.3. [14]: Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such that $0 \cdot I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers n .

Definition 2.4. [15] Let $R(I) = \{a + bI; a, b \in R\}$ where $I^2 = I$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [49]

$$T: R(I) \rightarrow R \times R; T(a + bI) = (a, a + b)$$

Remark 2.5. [15]

T is an algebraic isomorphism between two rings, it has the following properties:

- 1) T is bijective.
- 2) T preserves addition and multiplication, i.e.:
- 3) Since T is bijective, then it is invertible by:

$$T^{-1}: R \times R \rightarrow R(I); T^{-1}(a, b) = a + (b - a)I$$

- 4) T preserves distances, i.e.:

$$\|T(AB)\| = T(\|AB\|)$$

Definition 2.7. [16]: Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2 - SPR = \{a_0 + a_1I_1 + a_2I_2; a_i \in R, I_1^2 = I_1, I_2^2 = I_2, I_1 \times I_2 = I_{\max(1,2)} = I_2\}.$$

Smarandache has defined algebraic operations on $2 - SP_R$ as follows:

Addition:

$$[a_0 + a_1I_1 + a_2I_2] + [b_0 + b_1I_1 + b_2I_2] = (a_0 + b_0) + (a_1 + b_1)I_1 + (a_2 + b_2)I_2.$$

Multiplication:

$$[a_0 + a_1I_1 + a_2I_2].[b_0 + b_1I_1 + b_2I_2] = (a_0b_0) + (a_0b_1 + a_1b_0 + a_1b_1)I_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)I_2.$$

Definition 2.8. Let $2 - SP_R = \{a + bI_1 + cI_2; a, b, c \in R\}$ where

$$I_1^2 = I_1, I_2^2 = I_2 \text{ and } I_1I_2 = I_2I_1 = I_2$$

Be the symbolic 2-plithogenic field of reals. The symbolic 2-plithogenic isometry (AH-Isometry) is defined as follows:

$$T: 2 - SP_R \rightarrow R \times R \times R; T(a + bI_1 + cI_2) = (a, a + b + c, a + c)$$

Remark 2.9.

1) T is bijective, then it is invertible by:

$$T^{-1}: R \times R \times R \rightarrow 2 - SP_R; T^{-1}(a, b, c) = a + (b - c)I_1 + (c - a)I_2$$

2) T preserves distances, i.e.:

$$\|T(AB)\| = T(\|AB\|)$$

3. Symbolic n-plithogenic Rings

Definition 3.1 [22]

Let R be a ring, the symbolic n-plithogenic ring is:

$$n - SP_R = \{a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n; a_i \in R, P_j^2 = P_j, P_i \times P_j = P_{\max(i,j)}\}.$$

Operations on $n - SP_R$:

Addition:

$$[a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n] + [b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2 + \dots + (a_n + b_n)P_n.$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n].[b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n] = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2 + \dots + (a_0b_n + a_1b_n + \dots + a_{n-1}b_n + a_nb_0 + a_nb_1 + \dots + a_nb_n)P_n.$$

It is clear that $(n - SP_R)$ is a ring.

If R is commutative, then $n - SP_R$ is commutative, and if R has a unity (1), then $n - SP_R$ has the same unity (1).

Example 3.2

Consider the ring $R = Z_n = \{\bar{0}, \bar{1}, \dots, \overline{n-1}\}$, the corresponding $n - SP_R$ is:

$$n - SP_R = \{a + bP_1 + \dots + cP_n; a, b, \dots, c \in Z_n\}.$$

Definition 3.

Let $n - SP_R$ be a n-plithogenic symbolic ring, with unity (1).

Let $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n \in n - SP_R$, then X is invertible(unit element) if and only if there exists $Y = y_0 + y_1P_1 + \dots + y_nP_n$ such that $X.Y = 1$.

We can write $U_{n-SP_R} = \{X \in n - SP_R \mid \exists Y \in n - SP_R, X.Y = 1\}$.

Theorem 3.4

Let $n - SP_R$ be a n-plithogenic symbolic ring, with unity (1).

Let $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n$ be an arbitrary element, then:

1. X is invertible(unit element) if and only if $x_0, x_0 + x_1, x_0 + x_1 + x_2, \dots, x_0 + x_1 + x_2 + \dots + x_n$ are invertible.
2. $X^{-1} = x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2 + \dots + [(x_0 + x_1 + \dots + x_n)^{-1} - (x_0 + x_1 + \dots + x_{n-1})^{-1}]P_n.$

Proof.

Assume that X is invertible, than there exists $Y = y_0 + y_1P_1 + \dots + y_nP_n$ such that $X.Y = 1$, hence:

$$\left\{ \begin{array}{l} x_0y_0 = 1 \dots (1) \\ x_0y_1 + x_1y_0 + x_1y_1 = 0 \dots (2) \\ x_0y_2 + x_1y_2 + x_2y_0 + x_2y_1 + x_2y_2 = 0 \dots (3) \\ x_0y_3 + x_1y_3 + x_2y_3 + x_3y_0 + x_3y_1 + x_3y_2 + x_3y_3 = 0 \dots (4) \\ \vdots \\ x_0y_n + x_1y_n + \dots + x_{n-1}y_n + x_ny_0 + x_ny_1 + \dots + x_ny_n = 0 \dots (n) \end{array} \right.$$

From (1), x_0 is invertible.

By adding (1) to (2), we get $(x_0 + x_1)(y_0 + y_1) = 1$, thus $x_0 + x_1$ is invertible.

By adding (1) to (2) to (3), $(x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = 1$, hence $x_0 + x_1 + x_2$ is invertible.

By adding (1) to (2) to (3) to (4),

$(x_0 + x_1 + x_2 + x_3)(y_0 + y_1 + y_2 + y_3) = 1$, hence $x_0 + x_1 + x_2 + x_3$ is invertible.

$$(x_0)^{-1} = y_0$$

$$(x_0 + x_1)^{-1} = y_0 + y_1$$

$$(x_0 + x_1 + x_2)^{-1} = y_0 + y_1 + y_2$$

$$(x_0 + x_1 + x_2 + x_3)^{-1} = y_0 + y_1 + y_2 + y_3$$

$$(x_0 + x_1 + \dots + x_n)^{-1} = y_0 + y_1 + \dots + y_n, \text{ then:}$$

$$\begin{aligned} X^{-1} &= x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + \\ & [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2 + \\ & [(x_0 + x_1 + x_2 + x_3)^{-1} - (x_0 + x_1 + x_2)^{-1}]P_3 + \\ & [(x_0 + x_1 + \dots + x_n)^{-1} - (x_0 + x_1 + \dots + x_{n-1})^{-1}]P_n \\ &= Y \end{aligned}$$

We indicate by U_{n-SP_R} the collection of the unit elements.

Example 3.5

Take $R = Z_3 = \{0,1,2\}$, $4 - SP_{Z_3}$ is the corresponding symbolic 4-plithogenic ring, consider $X = 2 + 2P_2 + P_4 \in 4 - SP_{Z_3}$, then:

$$X^{-1} = 2 + (2 - 2)P_1 + (1 - 2)P_2 + (1 - 1)P_3 + (2 - 1)P_4 = 2 + 2P_2 + P_4.$$

Definition 3.6

Let $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n \in n - SP_R$, then X is idempotent if and only if $X^2 = X$.

We can write $Id_{n-SP_R} = \{X \in n - SP_R \mid X^2 = X \text{ for all } X \in n - SP_R\}$.

Theorem 3.7

Let $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n \in n - SP_R$, then X is idempotent if and only if $x_0, x_0 + x_1, x_0 + x_1 + x_2, \dots, x_0 + x_1 + x_2 + \dots + x_n$ are idempotent.

Proof.

$$\begin{aligned} X^2 = X.X &= (x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n)(x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n) \\ X^2 = X.X \text{ equivalents } &\left\{ \begin{array}{l} x_0x_0 = x_0 \dots (1) \\ x_0x_1 + x_1x_0 + x_1x_1 = x_1 \dots (2) \\ x_0x_2 + x_1x_2 + x_2x_0 + x_2x_1 + x_2x_2 = x_2 \dots (3) \\ x_0x_3 + x_1x_3 + x_2x_3 + x_3x_0 + x_3x_1 + x_3x_2 + x_3x_3 = x_3 \dots (4) \\ \vdots \\ x_0x_n + x_1x_n + \dots + x_{n-1}x_n + x_nx_0 + x_nx_1 + \dots + x_nx_n = x_n \dots (n) \end{array} \right. \end{aligned}$$

Equation (1) Implies that x_0 is idempotent.

By adding (1) to (2), we get $(x_0 + x_1)^2 = x_0 + x_1$, hence $x_0 + x_1$ is idempotent.

By adding (1) to (2) to (3), we get $(x_0 + x_1 + x_2)^2 = x_0 + x_1 + x_2$, hence $x_0 + x_1 + x_2$ is idempotent.

By adding (1) to (2) to (3) to (4), we get $(x_0 + x_1 + x_2 + x_3)^2 = x_0 + x_1 + x_2 + x_3$, thus $x_0 + x_1 + x_2 + x_3$ is idempotent.

By adding (1) to (2) to ... to (n), we get $(x_0 + x_1 + x_2 + \dots + x_n)^2 = x_0 + x_1 + x_2 + \dots + x_n$, thus $x_0 + x_1 + x_2 + \dots + x_n$ is idempotent. Thus the proof is complete.

Example 3.8

Take $R = Z_4 = \{0,1,2,3\}$, $4 - SP_{Z_4}$ is the corresponding symbolic 4-plithogenic ring, consider $X = P_1 + 3P_4 \in 4 - SP_{Z_5}$, we have:

$$X^2 = P_1 + 9P_4 + 6P_4 = P_1 + 3P_4 = X.$$

Theorem 3.9

Let $n - SP_R$ be a commutative symbolic n-plithogenic ring, hence if $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n$, then

$$X^r = x_0^r + [(x_0 + x_1)^r - x_0^r]P_1 + [(x_0 + x_1 + x_2)^r - (x_0 + x_1)^r]P_2 + \dots + [(x_0 + x_1 + \dots + x_n)^r - (x_0 + x_1 + \dots + x_{n-1})^r]P_n$$

for every $r \in Z^+$.

Proof.

For $r = 1$, it holds easily. Assume that it is true for $r = k$, we prove it for $r = k + 1$.

$$X^{k+1} = X.X^k =$$

$$(x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n)(x_0^k + [(x_0 + x_1)^k - x_0^k]P_1 + [(x_0 + x_1 + x_2)^k - (x_0 + x_1)^k]P_2 + \dots + [(x_0 + x_1 + \dots + x_n)^k - (x_0 + x_1 + \dots + x_{n-1})^k]P_n) = x_0^{k+1} + [(x_0 + x_1)^{k+1} - x_0^{k+1}]P_1 + [(x_0 + x_1 + x_2)^{k+1} - (x_0 + x_1)^{k+1}]P_2 + \dots + [(x_0 + x_1 + \dots + x_n)^{k+1} - (x_0 + x_1 + \dots + x_{n-1})^{k+1}]P_n.$$

So, that proof is complete by induction.

Definition 3.10

Let $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n \in n - SP_R$ then X is considered nilpotent if there is $r \in Z^+$ where $X^r = 0$.

We can write $Nd_{n-SP_R} = \{X \in n - SP_R \mid X^r = 0, r \in Z^+ \text{ for all } X \in n - SP_R\}$.

Theorem 3.11

If $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n \in n - SP_R$ where R is a commutative, then X is nilpotent iff $x_0, x_0 + x_1, x_0 + x_1 + x_2, \dots, x_0 + x_1 + x_2 + \dots + x_n$ are nilpotent.

Proof.

$X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n$ is nilpotent if and only if there exists $r \in Z^+$ such that $X^r = 0$, hence:

$$\begin{aligned} x_0^r &= 0 \dots (1) \\ (x_0 + x_1)^r - x_0^r &= 0 \dots (2) \text{ from (1)} \Rightarrow (x_0 + x_1)^r = 0 \\ (x_0 + x_1 + x_2)^r - (x_0 + x_1)^r &= 0 \dots (3) \text{ from (1 and 2)} \Rightarrow (x_0 + x_1 + x_2)^r = 0 \\ &\vdots \\ &\vdots \\ &\vdots \\ (x_0 + x_1 + \dots + x_n)^r - (x_0 + x_1 + \dots + x_{n-1}) &= 0 \dots (n) \\ \text{from (1,2, \dots, n)} &\Rightarrow (x_0 + x_1 + \dots + x_n)^r = 0 \end{aligned}$$

Thus, the proof is complete.

Definition 3.12

Center of the neutrosophic ring is defined as $C_{n-SP_R} = \{X \in n - SP_R \mid XY = YX \text{ for all } Y \in n - SP_R\}$.

Definition 3.13

If $0 \neq X \in n - SP_R$, then $X \neq 0$ is a zero divisor if there exists $0 \neq Y \in n - SP_R$, such that $XY = YX = 0$.

We can write $Z_{n-SP_R} = \{X \neq 0 \in n - SP_R \mid \exists Y \neq 0 \in n - SP_R, XY = YX = 0 \text{ for all } X \in n - SP_R\}$.

Definition 3.14

Let $X = x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n \in n - SP_R$, then is called a regular if there is an element $Y = y_0 + y_1P_1 + \dots + y_nP_n$ where $Y = (x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n)Y(x_0 + x_1P_1 + x_2P_2 + \dots + x_nP_n)$.

We can write $Reg_{n-SP_R} = \{X \in n - SP_R \mid \exists Y \in n - SP_R, Y = XYX \text{ for all } X \in n - SP_R\}$.

Theorem 3.15

Let $r_0 + r_1P_1 + \dots + r_nP_n \in n - SP_R$, then $r_0 + r_1P_1 + \dots + r_nP_n$ is regular if and only if $r_0, r_0 + r_1, r_0 + r_1 + r_2, \dots, r_0 + r_1 + r_2 + \dots + r_n$ are regular.

4. Conclusion

In this paper, we have introduced and formalized the concept of symbolic n -plithogenic rings, establishing a new class of generalized algebraic structures built upon symmetric n -plithogenic components. By defining their fundamental properties and investigating their corresponding substructures, we provided a systematic and coherent framework that extends classical ring theory into the plithogenic setting. The theorems and numerical examples presented herein demonstrate the consistency, flexibility, and potential of this new algebraic model. Future research may explore additional operations, homomorphic mappings, or categorical perspectives of symbolic n -plithogenic rings, as well as their applications in areas

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