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Neutrosophic Quadratic Equations

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Abstract: This study develops a comprehensive framework for solving neutrosophic quadratic equation (NQE). These equations are classified into three distinct types based on the coefficients of the x^2 term, with systematic solution methods derived for each type. For NQE Type 1, we establish conditions for the existence of distinct real solutions by analyzing the discriminants of the real and indeterminate components. Explicit solution formulations are derived, and the algebraic properties of their summation and product are rigorously examined. The methodologies are extended to solve NQE Types 2 and 3. To validate the theoretical findings, illustrative examples are provided for each equation type, demonstrating the practical implementation of the proposed methods.

Keywords: Neutrosophic real number; Neutrosophic complex number; Neutrosophic limit; Neutrosophic algebraic equation

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1. Introduction

Neutrosophic Logic, introduced by Smarandache, serves as a modern alternative to classical logical frameworks, offering a mathematical model to handle uncertainty, vagueness, and inconsistency in data. Rooted in his philosophical concept of neutrosophy, this approach reconceptualizes the mathematical representation of indeterminate information [1, 2, 3, 4]. Subsequent advancements have further enriched this framework. For instance, Al-Tahan extended neutrosophic logic to single-valued neutrosophic (weak) polygroups, broadening its algebraic applications [23]. Additionally, Edalatpanah introduced an algorithm for neutrosophic linear programming utilizing triangular neutrosophic numbers for variables and constraints [24], and Chakraborty applied pentagonal neutrosophic numbers to network optimization, addressing shortest-path problems [25, 26]. Further, Abdel-Basset leveraged neutrosophic numbers in group decision-making using the TOPSIS technique for supplier selection [27], and Alvaracín Jarrín et al. applied neutrosophic statistics to model ambiguity in social science data [19].

In foundational work on Neutrosophic Real Numbers (NRNs), Smarandache established conditions for division and root-taking of both real and complex neutrosophic numbers, defining a standard form for Neutrosophic Complex Numbers (NCNs) [6, 7, 18]. His contributions further extend to Neutrosophic Probability, Statistics, and a unique neutrosophic calculus encompassing concepts such as the mereo-limit, mereo-derivative, and mereo-integral [5, 8].

Recent studies have increasingly explored the applications of neutrosophic numbers in advanced mathematical contexts. Alhasan's contributions to neutrosophic complex numbers in exponential form [9, 10] and his development of neutrosophic integrals [17, 14] have provided a foundation for further research in this area. Neutrosophic mathematics has demonstrated its relevance across diverse domains. For instance, Abobala and Hatip (2021) introduced neutrosophic Euclidean geometry as a generalization of classical Euclidean geometry [11]. Similarly, Çeven and Sekmen (2023) proposed the concept of neutrosophic square matrices and developed methods for solving systems of neutrosophic linear equations [21].

Alhasan's extensive work has established a robust framework for neutrosophic differential calculus, linear equations, and integration, providing valuable tools for analyzing uncertainty in mathematical settings [17, 16]. His studies on neutrosophic straight lines and circles (2023) have enhanced the visualization of uncertain data in two-dimensional spaces [15], while his formulation of double neutrosophic integrals has enabled multidimensional analyses [20]. Additionally, Alhasan and Musa (2023) introduced the concept of neutrosophic limits [12]. More recently, Narzary and Basumatary (2025) investigated the n-th derivative of neutrosophic functions, advancing the theory of differential calculus under uncertainty [13].

Quadratic equations are foundational to numerous areas of mathematics. Abobala (2020) examined linear and quadratic equations in neutrosophic fields [22]. However, his work does not provide a systematic method for determining the roots of neutrosophic quadratic equations, nor does it address the behavior of their solutions. This paper aims to fill this gap by developing a structured approach to solving neutrosophic quadratic equations of the form: $(p_1 + p_2 I)x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I$, where I is a neutrosophic indeterminate unit satisfying $I^2 = I$.

2. Preliminaries

Definition 2.1 *[7] Let N be a neutrosophic real number, which takes the standard form $N = a + bI$, where $a, b \in \mathbb{R}$ and I represents an indeterminate unit such that $I \cdot 0 = 0$ and $I^n = I$ for all $n \in \mathbb{Z}$.*

Definition 2.2 *[7] Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two neutrosophic numbers, The operations between these neutrosophic numbers are defined as follows:*

1. *Addition:*

$$N_1 + N_2 = (a_1 + a_2) + (b_1 + b_2)I$$

2. Subtraction:

$$N_1 - N_2 = (a_1 - a_2) + (b_1 - b_2)I$$

3. Multiplication:

$$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$$

4. Division:

$$\frac{N_1}{N_2} = \frac{a_1}{a_2} + \frac{b_1 a_2 - a_1 b_2}{a_2(a_2 + b_2)} I \quad a_2 \neq 0 \quad and \quad a_2 \neq -b_2.$$

We denote neutrosophic zero as $0_I = 0 + 0I$.

3. General Neutrosophic Quadratic Equation

Consider the general form of the neutrosophic quadratic equation:

$$(p_1 + p_2 I)x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I, \quad (1)$$

where p_1, p_2, q_1, q_2, r_1, and r_2 are real numbers. Here, we note that there are six parameters involved. To simplify the problem, we divide the entire equation by $p_1 + p_2 I$. This division leads to three distinct cases, with each case corresponding to a specific type of neutrosophic quadratic equation (NQE). The following are the cases:

Case 1: $\boldsymbol{p_1 \neq 0}$ **and** $\boldsymbol{p_1 \neq -p_2}$

If $p_1 \neq 0$ and $p_1 \neq -p_2$, the original equation can be normalized as follows:

$$x^2 + \left(\frac{q_1+q_2 I}{p_1+p_2 I}\right)x + \left(\frac{r_1+r_2 I}{p_1+p_2 I}\right) = 0_I.$$

Introducing simplified coefficients, we write:

$$\boldsymbol{x^2 + (\bar{q}_1 + \bar{q}_2 I)x + (\bar{r}_1 + \bar{r}_2 I) = 0_I},$$

where

$$\bar{q}_1 + \bar{q}_2 I = \frac{q_1+q_2 I}{p_1+p_2 I}, \qquad \bar{r}_1 + \bar{r}_2 I = \frac{r_1+r_2 I}{p_1+p_2 I}.$$

This form is referred to as a Neutrosophic Quadratic Equation Type 1 (NQE Type 1).

Case 2: $\boldsymbol{p_1 = 0}$

If $p_1 = 0$, the equation reduces to:

$$\boldsymbol{p_2 I x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I}.$$

This form is identified as a Neutrosophic Quadratic Equation Type 2 (NQE Type 2).

Case 3: $\boldsymbol{p_1 = -p_2}$

If $p_1 = -p_2$, the equation transforms into:

$$\boldsymbol{p_2(-1 + I)x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I}.$$

This variation is termed a Neutrosophic Quadratic Equation Type 3 (NQE Type 3).

In the following three sections, we will find the solution for each of the cases outlined above, providing an example for each case to illustrate the approach.

4. Solution of NQE Type 1

In this section, we develop a method to solve a NQE Type 1 of the form:

$$x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I \quad (2)$$

To determine the solution, we substitute $x = a + bI$ into equation (2):

$$(a + bI)^2 + (q_1 + q_2 I)(a + bI) + (r_1 + r_2 I) = 0_I. \quad (3)$$

Expanding $(a + bI)^2$:

$$(a + bI)^2 = a^2 + 2abI + b^2 I^2 = a^2 + (2ab + b^2)I, \text{ since } I^2 = I.$$

Next, we expand $(q_1 + q_2 I)(a + bI)$:

$$(q_1 + q_2 I)(a + bI) = q_1 a + q_1 bI + q_2 aI + q_2 bI^2 = q_1 a + (q_1 b + q_2 a + q_2 b)I.$$

Substituting these expanded expressions into equation (3) yields:

$$a^2 + (2ab + b^2)I + q_1 a + (q_1 b + q_2 a + q_2 b)I + r_1 + r_2 I = 0_I.$$

Separating the real and indeterminate parts, we obtain:

$$a^2 + q_1 a + r_1 + (2ab + b^2 + q_1 b + q_2 a + q_2 b + r_2)I = 0_I.$$

Equating the real and indeterminate parts separately, we have:

$$a^2 + q_1 a + r_1 = 0. \quad (4)$$

$$2ab + b^2 + q_1 b + q_2 a + q_2 b + r_2 = 0,$$

which can be rearranged as:

$$b^2 + (2a + q_1 + q_2)b + (q_2 a + r_2) = 0. \quad (5)$$

To solve for a, consider the quadratic equation (4): $a^2 + q_1 a + r_1 = 0$.

The discriminant of this equation is $\Delta_1 = q_1^2 - 4r_1$, yielding the solutions for a:

$$a_1 = \frac{-q_1 + \sqrt{\Delta_1}}{2}, \qquad a_2 = \frac{-q_1 - \sqrt{\Delta_1}}{2}.$$

Solution for b, For each $a = a_1$, Equation (5) becomes:

$$b^2 + (2a_1 + q_1 + q_2)b + (q_2 a_1 + r_2) = 0.$$

Using the quadratic formula, the solutions for b are:

$$b_1 = \frac{-(2a_1 + q_1 + q_2) + \sqrt{(2a_1 + q_1 + q_2)^2 - 4(q_2 a_1 + r_2)}}{2},$$

$$b_2 = \frac{-(2a_1 + q_1 + q_2) - \sqrt{(2a_1 + q_1 + q_2)^2 - 4(q_2 a_1 + r_2)}}{2}.$$

For $a = a_2$, Equation (5) becomes:

$$b^2 + (2a_2 + q_1 + q_2)b + (q_2a_2 + r_2) = 0.$$

The solutions for b are:

$$b_3 = \frac{-(2a_2 + q_1 + q_2) + \sqrt{(2a_2 + q_1 + q_2)^2 - 4(q_2a_2 + r_2)}}{2},$$

$$b_4 = \frac{-(2a_2 + q_1 + q_2) - \sqrt{(2a_2 + q_1 + q_2)^2 - 4(q_2a_2 + r_2)}}{2}.$$

Thus, the solutions for a and b are obtained from the two quadratic equations (4) and (5). Using these values, we have the following four solutions:

$$x_1 = a_1 + b_1I, x_2 = a_1 + b_2I, x_3 = a_2 + b_3I, x_4 = a_2 + b_4I.$$

Substituting $2a_1 + q_1 = \sqrt{\Delta_1}$ and $2a_2 + q_1 = -\sqrt{\Delta_1}$, we obtain:

$$x_1 = a_1 + b_1I = \frac{-q_1 + \sqrt{\Delta_1}}{2} + \left(\frac{-(\sqrt{\Delta_1} + q_2) + \sqrt{(\sqrt{\Delta_1} + q_2)^2 - 4(q_2a_1 + r_2)}}{2}\right)I.$$

$$x_2 = a_1 + b_2I = \frac{-q_1 + \sqrt{\Delta_1}}{2} + \left(\frac{-(\sqrt{\Delta_1} + q_2) - \sqrt{(\sqrt{\Delta_1} + q_2)^2 - 4(q_2a_1 + r_2)}}{2}\right)I.$$

$$x_3 = a_2 + b_3I = \frac{-q_1 - \sqrt{\Delta_1}}{2} + \left(\frac{-(-\sqrt{\Delta_1} + q_2) + \sqrt{(-\sqrt{\Delta_1} + q_2)^2 - 4(q_2a_2 + r_2)}}{2}\right)I.$$

$$x_4 = a_2 + b_4I = \frac{-q_1 - \sqrt{\Delta_1}}{2} + \left(\frac{-(-\sqrt{\Delta_1} + q_2) - \sqrt{(-\sqrt{\Delta_1} + q_2)^2 - 4(q_2a_2 + r_2)}}{2}\right)I.$$

Theorem 4.2 *Let* $x^2 + (q_1 + q_2I)x + (r_1 + r_2I) = 0_I$ *be a NQE of Type 1, where* $x = a + bI$ *and* a *satisfies the real part equation* $a^2 + q_1a + r_1 = 0$. *If* $\Delta_1 = q_1^2 - 4r_1 > 0$, *yielding two distinct real solutions* a_1 *and* a_2, *then the discriminants* Δ_{a_1} *and* Δ_{a_2} *of the equations*

$$b^2 + (2a_1 + q_1 + q_2)b + (q_2a_1 + r_2) = 0$$

and

$$b^2 + (2a_2 + q_1 + q_2)b + (q_2a_2 + r_2) = 0$$

are equal, i.e., $\Delta_{a_1} = \Delta_{a_2}$.

Proof. Since $\Delta_1 = q_1^2 - 4r_1 > 0$, the equation $a^2 + q_1a + r_1 = 0$ has two distinct real solutions, given by

$$a_1 = \frac{-q_1 + \sqrt{\Delta_1}}{2}, a_2 = \frac{-q_1 - \sqrt{\Delta_1}}{2}.$$

Now, the discriminant Δ_{a_1} of the quadratic equation: $b^2 + (2a_1 + q_1 + q_2)b + (q_2a_1 + r_2) = 0$ is given by

$$\begin{aligned}\Delta_{a_1} &= (2a_1 + q_1 + q_2)^2 - 4(q_2 a_1 + r_2)\\ &= \left(2 \cdot \tfrac{-q_1+\sqrt{\Delta_1}}{2} + q_1 + q_2\right)^2 - 4\left(q_2 \cdot \tfrac{-q_1+\sqrt{\Delta_1}}{2} + r_2\right)\\ &= (\sqrt{\Delta_1} + q_2)^2 - 4\left(\tfrac{-q_1q_2+q_2\sqrt{\Delta_1}}{2} + r_2\right)\\ &= \Delta_1 + q_2^2 + 2q_1q_2 - 4r_2\\ &= (q_1+q_2)^2 - 4(r_1+r_2), \qquad \textit{since}\ \ \Delta_1 = q_1^2 - 4r_1.\end{aligned}$$

Similarly, the discriminant Δ_{a_2} of the quadratic equation $b^2 + (2a_2 + q_1 + q_2)b + (q_2a_2 + r_2) = 0$ is given by

$$\begin{aligned}\Delta_{a_2} &= (2a_2 + q_1 + q_2)^2 - 4(q_2 a_2 + r_2)\\ &= \left(2 \cdot \tfrac{-q_1-\sqrt{\Delta_1}}{2} + q_1 + q_2\right)^2 - 4\left(q_2 \cdot \tfrac{-q_1-\sqrt{\Delta_1}}{2} + r_2\right)\\ &= (-\sqrt{\Delta_1} + q_2)^2 - 4\left(\tfrac{-q_1q_2-q_2\sqrt{\Delta_1}}{2} + r_2\right)\\ &= \Delta_1 + q_2^2 + 2q_1q_2 - 4r_2\\ &= (q_1+q_2)^2 - 4(r_1+r_2), \qquad \textit{since}\ \ \Delta_1 = q_1^2 - 4r_1.\end{aligned}$$

Thus, $\Delta_{a_1} = \Delta_{a_2}$ as required.

4.3 Solutions Formula for NQE Type 1

Using the above theorem, we can simplify the solutions for the NQE Type 1. The roots of this equation are derived as follows:

$$\begin{aligned}x_1 = a_1 + b_1 I &= \frac{-q_1+\sqrt{\Delta_1}}{2} + \frac{-q_2 - (\sqrt{\Delta_1} - \sqrt{\Delta_2})}{2} I,\\ x_2 = a_1 + b_2 I &= \frac{-q_1+\sqrt{\Delta_1}}{2} + \frac{-q_2 - (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2} I,\\ x_3 = a_2 + b_3 I &= \frac{-q_1-\sqrt{\Delta_1}}{2} + \frac{-q_2 + (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2} I,\\ x_4 = a_2 + b_4 I &= \frac{-q_1-\sqrt{\Delta_1}}{2} + \frac{-q_2 + (\sqrt{\Delta_1} - \sqrt{\Delta_2})}{2} I,\end{aligned}$$

where the discriminants are defined as $\Delta_1 = q_1^2 - 4r_1$ and $\Delta_2 = (q_1+q_2)^2 - 4(r_1+r_2)$.

4.4 Count Number of Distinct Real Solutions for NQE Type 1

To determine the conditions under which the NQE Type 1 has distinct real solutions, we expand and separate the equation into real and indeterminate components. This yields the following equations:

1. **Real Part:** $a^2 + q_1 a + r_1 = 0$.
2. **Indeterminate Part:** $b^2 + (2a + q_1 + q_2)b + (q_2 a + r_2) = 0$.

The discriminant for the real part is given by $\Delta_1 = q_1^2 - 4r_1$.
The number of real solutions for a depends on the value of Δ_1:

If $\Delta_1 > 0$: There are two distinct real solutions for a, denoted by a_1 and a_2. For each a_i, the discriminant of the indeterminate part, given by $\Delta_2 = (q_1 + q_2)^2 - 4(r_1 + r_2)$,
determines the number of real solutions for x:

- If $\Delta_2 > 0$, there are four distinct real solutions for x.
- If $\Delta_2 = 0$, there are two distinct real solutions for x.
- If $\Delta_2 < 0$, there are no real solutions for x.

If $\Delta_1 = 0$: There is a single (repeated) real solution for a. In this case, the discriminant

$$\Delta_2 = (q_1 + q_2)^2 - 4(r_1 + r_2)$$

dictates the number of solutions for x:

- If $\Delta_2 > 0$, there are two distinct real solutions for x.
- If $\Delta_2 = 0$, there is one distinct real solution for x.
- If $\Delta_2 < 0$, there are no real solutions for x.

If $\Delta_1 < 0$: There are no real solutions for a, hence there are no real solutions for x.

Condition on Δ_1	**Condition on Δ_2**	**Number of Distinct Real Solutions for x**
$\Delta_1 > 0$	$\Delta_2 > 0$	4
$\Delta_1 > 0$	$\Delta_2 = 0$	2
$\Delta_1 > 0$	$\Delta_2 < 0$	0
$\Delta_1 = 0$	$\Delta_2 > 0$	2
$\Delta_1 = 0$	$\Delta_2 = 0$	1
$\Delta_1 = 0$	$\Delta_2 < 0$	0
$\Delta_1 < 0$	-	0

Table 1: Count of Distinct Real Solutions of NQE Type 1

From the above discussion we conclude next theorem as

Theorem 4.5 *The NQE Type 1 has all solutions are neutrosophic real numbers if and only if $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$.*

Theorem 4.6 *Let $x_1 = a_1 + b_1 I$, $x_2 = a_1 + b_2 I$, $x_3 = a_2 + b_3 I$, and $x_4 = a_2 + b_4 I$ be the four solutions of the neutrosophic quadratic equation $x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I$. Then the following holds:*

a) $x_1 + x_4 = x_2 + x_3 = -q_1 - q_2 I$
b) $x_1 + x_2 + x_3 + x_4 = -2q_1 - 2q_2 I$.

c) $x_1x_4 = x_2x_3 = r_1 + r_2I$
d) $x_1x_2x_3x_4 = (r_1 + r_2I)^2$.

Proof. (a) From section 4.3 adding x_1 and x_4, we get:

$$x_1 + x_4 = \frac{-q_1 + \sqrt{\Delta_1} - q_1 - \sqrt{\Delta_1}}{2} + \frac{-q_2 - (\sqrt{\Delta_1} - \sqrt{\Delta_2}) - q_2 + (\sqrt{\Delta_1} - \sqrt{\Delta_2})}{2} I = -q_1 - q_2I.$$

Similarly, adding x_2 and x_3:

$$x_2 + x_3 = \frac{-q_1 + \sqrt{\Delta_1} - q_1 - \sqrt{\Delta_1}}{2} + \frac{-q_2 - (\sqrt{\Delta_1} + \sqrt{\Delta_2}) - q_2 + (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2} I = -q_1 - q_2I.$$

Thus,

$$x_1 + x_4 = x_2 + x_3 = -q_1 - q_2I.$$

(b) Summing all four roots:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= (x_1 + x_4) + (x_2 + x_3) \\ &= -q_1 - q_2I + -q_1 - q_2I \\ &= -2q_1 - 2q_2I. \end{aligned}$$

Thus,

$$x_1 + x_2 + x_3 + x_4 = -2q_1 - 2q_2I.$$

(c) Let's calculate the product of x_2 and x_3. We are given:

$$x_2 = a_1 + b_2I, x_3 = a_2 + b_3I,$$

where

$$a_1 = \frac{-q_1 + \sqrt{\Delta_1}}{2}, b_2 = \frac{-q_2 - (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2},$$

$$a_2 = \frac{-q_1 - \sqrt{\Delta_1}}{2}, b_3 = \frac{-q_2 + (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2}.$$

Thus, $x_2x_3 = a_1a_2 + (a_1b_3 + b_2a_2 + b_2b_3)I$.

Calculate the expressions for $a_1a_2a_1b_3$, b_2a_2, and b_2b_3:

$$a_1a_2 = \left(\frac{-q_1+\sqrt{\Delta_1}}{2}\right)\left(\frac{-q_1-\sqrt{\Delta_1}}{2}\right) = \frac{q_1^2-(\sqrt{\Delta_1})^2}{4} = \frac{q_1^2-\Delta_1}{4} = \frac{4r_1}{4} = r_1.$$

$$a_1b_3 = \left(\frac{-q_1+\sqrt{\Delta_1}}{2}\right)\left(\frac{-q_2+\sqrt{\Delta_1}+\sqrt{\Delta_2}}{2}\right) = \frac{q_1q_2-q_1(\sqrt{\Delta_1}+\sqrt{\Delta_2})-q_2\sqrt{\Delta_1}+\Delta_1+\sqrt{\Delta_1}\sqrt{\Delta_2}}{4}$$

.

$$b_2a_2 = \left(\frac{-q_2-\sqrt{\Delta_1}-\sqrt{\Delta_2}}{2}\right)\left(\frac{-q_1-\sqrt{\Delta_1}}{2}\right) = \frac{q_1q_2+q_1(\sqrt{\Delta_1}+\sqrt{\Delta_2})+q_2\sqrt{\Delta_1}+\Delta_1+\sqrt{\Delta_1}\sqrt{\Delta_2}}{4}$$

$$b_2b_3 = \left(\frac{-q_2-\sqrt{\Delta_1}-\sqrt{\Delta_2}}{2}\right)\left(\frac{-q_2+\sqrt{\Delta_1}+\sqrt{\Delta_2}}{2}\right) = \frac{q_2^2-(\sqrt{\Delta_1}+\sqrt{\Delta_2})^2}{4} = \frac{q_2^2-(\Delta_1+\Delta_2+2\sqrt{\Delta_1}\sqrt{\Delta_2})}{4}$$

Now, we add the expressions for a_1b_3, b_2a_2, and b_2b_3:

$$a_1b_3 + b_2a_2 + b_2b_3 = \frac{q_1q_2 - q_1(\sqrt{\Delta_1} + \sqrt{\Delta_2}) - q_2\sqrt{\Delta_1} + \Delta_1 + \sqrt{\Delta_1}\sqrt{\Delta_2}}{4}$$

$$+\frac{q_1q_2 + q_1(\sqrt{\Delta_1} + \sqrt{\Delta_2}) + q_2\sqrt{\Delta_1} + \Delta_1 + \sqrt{\Delta_1}\sqrt{\Delta_2}}{4} + \frac{q_2^2 - (\Delta_1 + \Delta_2 + 2\sqrt{\Delta_1}\sqrt{\Delta_2})}{4}.$$

After combining like terms, we get:

$$a_1b_3 + b_2a_2 + b_2b_3 = \frac{2q_1q_2 + q_2^2 + \Delta_1 - \Delta_2}{4}$$

$$= \frac{2q_1q_2 + q_2^2 + (q_1^2 - 4r_1) - [(q_1 + q_2)^2 - 4(r_1 + r_2)]}{4}$$

$$= \frac{2q_1q_2 + q_2^2 + q_1^2 - 4r_1 - (q_1^2 + 2q_1q_2 + q_2^2) + 4r_1 + 4r_2}{4} = \frac{4r_2}{4} = r_2.$$

Thus, we have shown that:

$$a_1b_3 + b_2a_2 + b_2b_3 = r_2.$$

After performing all the calculations and simplifying, we conclude:

$$x_2x_3 = a_1a_2 + (a_1b_3 + b_2a_2 + b_2b_3)I = r_1 + r_2I.$$

Similarly we can prove that

$$x_1x_4 = a_2a_2 + (a_1b_4 + b_1a_2 + b_1b_4)I = r_1 + r_2I.$$

(d) Now product of all solutions

$$\begin{aligned} x_1x_2x_3x_4 &= (x_1x_4)(x_2x_3) \\ &= (r_1 + r_2I)(r_1 + r_2I) \\ &= (r_1 + r_2I)^2. \end{aligned}$$

This completes the proof.

Example 4.7 *Consider the neutrosophic quadratic equation Type 1*

$$x^2 + (4 + 2I)x + (3 + 2I) = 0_I.$$

Here, we identify the coefficients $q_1 = 4,\ q_2 = 2,\ r_1 = 3,$ and $r_2 = 2$. The roots are determined as follows. The discriminants are given by: $\Delta_1 = q_1^2 - 4r_1 = 4^2 - 4 \cdot 3 = 16 - 12 = 4,$

$$\Delta_2 = (q_1 + q_2)^2 - 4(r_1 + r_2) = (4 + 2)^2 - 4(3 + 2) = 6^2 - 4 \cdot 5 = 36 - 20 = 16.$$

The square roots of the discriminants are:

$$\sqrt{\Delta_1} = \sqrt{4} = 2, \qquad \sqrt{\Delta_2} = \sqrt{16} = 4.$$

Using the formulas for the roots, we substitute $q_1 = 4,\ \sqrt{\Delta_1} = 2,\ q_2 = 2,$ and $\sqrt{\Delta_2} = 4$. The roots x_1, x_2, x_3, x_4 are computed as follows:

$$x_1 = \frac{-q_1 + \sqrt{\Delta_1}}{2} + \frac{-q_2 - (\sqrt{\Delta_1} - \sqrt{\Delta_2})}{2}I = \frac{-4 + 2}{2} + \frac{-2 - (2 - 4)}{2}I = -1$$

$$x_2 = \frac{-q_1 + \sqrt{\Delta_1}}{2} + \frac{-q_2 - (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2} I == \frac{-4+2}{2} + \frac{-2-(2+4)}{2} I = -1 - 4I$$

$$x_3 = \frac{-q_1 - \sqrt{\Delta_1}}{2} + \frac{-q_2 + (\sqrt{\Delta_1} + \sqrt{\Delta_2})}{2} I == \frac{-4-2}{2} + \frac{-2+(2+4)}{2} I = -3 + 2I$$

$$x_4 = \frac{-q_1 - \sqrt{\Delta_1}}{2} + \frac{-q_2 + (\sqrt{\Delta_1} - \sqrt{\Delta_2})}{2} I == \frac{-4-2}{2} + \frac{-2+(2-4)}{2} I == -3 - 2I.$$

The roots of the equation $x^2 + (4+2I)x + (3+2I) = 0_I$ are:

$$x_1 = -1,\ x_2 = -1 - 4I, x_3 = -3 + 2I,\ x_4 = -3 - 2I.$$

5. Solution of NQE Type 2

The neutrosophic quadratic equation of Type 2 is expressed as:

$$pIx^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I, \tag{6}$$

where p, q_1, q_2, r_1, r_2 are real constants, and I denotes the indeterminate unit.

To solve this equation, we substitute $x = a + bI$ into Equation (6):

$$pI(a+bI)^2 + (q_1 + q_2 I)(a + bI) + (r_1 + r_2 I) = 0_I. \tag{7}$$

First, expand $pI(a+bI)^2$:

$$pI(a+bI)^2 = pI(a^2 + 2abI + b^2 I^2) = (pa^2 + 2pab + pb^2)I, (\text{since } \mathrm{I}^2 = \mathrm{I}).$$

Next, expand $(q_1 + q_2 I)(a + bI)$:

$$(q_1 + q_2 I)(a + bI) = q_1 a + q_1 bI + q_2 aI + q_2 bI^2 = q_1 a + (q_1 b + q_2 a + q_2 b)I, (\text{since } \mathrm{I}^2 = \mathrm{I}).$$

Substitute these expansions back into Equation (7):

$$(pa^2 + 2pab + pb^2)I + q_1 a + (q_1 b + q_2 a + q_2 b)I + (r_1 + r_2 I) = 0_I.$$

Separate the real and indeterminate components:

$$(q_1 a + r_1) + (pa^2 + 2pab + pb^2 + q_1 b + q_2 a + q_2 b + r_2)I = 0_I.$$

Equating the coefficients of the real and indeterminate parts to zero gives:

$$q_1 a + r_1 = 0, \tag{8}$$

$$pb^2 + (2pa + q_1 + q_2)b + (pa^2 + q_2 a + r_2) = 0. \tag{9}$$

Solving for a and b From Equation (8), solve for a:

$$a = -\frac{r_1}{q_1}, \text{where } q_1 \neq 0.$$

Substitute $a = -\frac{r_1}{q_1}$ into Equation (9). The resulting quadratic equation in b has solutions b_1 and b_2. Therefore, the complete solution to the neutrosophic quadratic equation of Type 2 is:

$$x = a + b_1 I \quad \text{and} \quad x = a + b_2 I.$$

Example 5.1 *Consider the NQE of Type 2:*

$$2Ix^2 + (3 + 4I)x + (-6 + 8I) = 0_I.$$

Comparing this equation with Equation (6), we identify:

$$p = 2, q_1 = 3, q_2 = 4, r_1 = -6, r_2 = 8.$$

From Equation (8):

$$q_1 a + r_1 = 03a - 6 = 0 \qquad a = \frac{6}{3} = 2.$$

Next, substitute $a = 2$ into Equation (9):

$$pb^2 + (2pa + q_1 + q_2)b + (pa^2 + q_2 a + r_2) = 0.$$

Substituting the known values:

$$2b^2 + 15b + 24 = 0.$$

Solve this quadratic equation using the quadratic formula:

$$b_{1,2} = \frac{-15 \pm \sqrt{15^2 - 4 \cdot 2 \cdot 24}}{2 \cdot 2} = \frac{-15 \pm \sqrt{33}}{4}.$$

Thus, the two possible solutions for b are:

$$b_1 = \frac{-15 + \sqrt{33}}{4}, b_2 = \frac{-15 - \sqrt{33}}{4}.$$

Final Solutions The solutions for x are:

$$x_1 = 2 + \frac{-15 + \sqrt{33}}{4} I, \qquad x_2 = 2 + \frac{-15 - \sqrt{33}}{4} I.$$

6. Solution of NQE Type 3

The neutrosophic quadratic equation of Type 3 is expressed as:

$$p(-1 + I)x^2 + (q_1 + q_2 I)x + (r_1 + r_2 I) = 0_I. \tag{10}$$

To solve this equation, substitute $x = a + bI$ into Equation (10):

$$p(-1 + I)(a + bI)^2 + (q_1 + q_2 I)(a + bI) + (r_1 + r_2 I) = 0_I.$$

Expand the first term:

$$\begin{aligned} p(-1 + I)(a + bI)^2 &= p(-1 + I)(a^2 + (b^2 + 2ab)I) \\ &= -pa^2 + pa^2 I - p(b^2 + 2ab)I + p(b^2 + 2ab)I \\ &= -pa^2 + pa^2 I. \end{aligned}$$

Expand the second term:

$$(q_1 + q_2I)(a + bI) = q_1a + q_1bI + q_2aI + q_2bI^2$$
$$= q_1a + (q_1b + q_2a + q_2b)I, \text{ (since } \mathrm{I}^2 = \mathrm{I}).$$

Substituting the expansions into the equation:

$$(-pa^2 + pa^2I) + (q_1a + (q_1b + q_2a + q_2b)I) + (r_1 + r_2I) = 0_I.$$

Separation of Components Group the real and I-components:

$$-pa^2 + q_1a + r_1 + (pa^2 + q_1b + q_2a + q_2b + r_2)I = 0_I.$$

Equating the coefficients of the real and indeterminate (I) parts to zero:

$$-pa^2 + q_1a + r_1 = 0, \tag{11}$$

$$pa^2 + q_1b + q_2a + q_2b + r_2 = 0. \tag{12}$$

Equation (11) is a quadratic equation in a. Solving using the quadratic formula:

$$a = \frac{-q_1 \pm \sqrt{q_1^2 - 4(-p)(r_1)}}{2(-p)}.$$

Simplifying further:

$$a_1 = \frac{q_1 + \sqrt{q_1^2 + 4pr_1}}{2p}, a_2 = \frac{q_1 - \sqrt{q_1^2 + 4pr_1}}{2p}.$$

Solving for b Substitute $a = a_1$ and $a = a_2$ into Equation (12):

$$pa^2 + q_1b + q_2a + q_2b + r_2 = 0.$$

Grouping terms for b:

$$(q_1 + q_2)b = -pa^2 - q_2a - r_2.$$

Solve for b:

$$b_1 = \frac{-pa_1^2 - q_2a_1 - r_2}{q_1 + q_2}, b_2 = \frac{-pa_2^2 - q_2a_2 - r_2}{q_1 + q_2}.$$

The complete solutions to the neutrosophic quadratic equation are:

$$x_1 = a_1 + b_1I = a_1 + \frac{-pa_1^2 - q_2a_1 - r_2}{q_1 + q_2}I,$$

$$x_2 = a_2 + b_2I = a_2 + \frac{-pa_2^2 - q_2a_2 - r_2}{q_1 + q_2}I.$$

Example 6.1 *Consider the neutrosophic quadratic equation of Type 3:*

$$-(-1 + I)x^2 + (-5 + 1I)x + (6 + 2I) = 0_I.$$

Here:

$$p = -1, q_1 = -5, q_2 = 1, r_1 = 6, r_2 = 2.$$

Solving for a From Equation (11):

$$-pa^2 + q_1a + r_1 = 0.$$

Substitute the values:

$$a^2 - 5a + 6 = 0.$$

Using the quadratic formula:

$$a = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm \sqrt{1}}{2}.$$

This gives:

$$a_1 = \frac{5+1}{2} = 3, a_2 = \frac{5-1}{2} = 2.$$

Solving for b Using Equation (12):

$$pa^2 + q_1 b + q_2 a + q_2 b + r_2 = 0.$$

Substitute the values:

$$b = \frac{-pa^2 - q_2 a - r_2}{q_1 + q_2} = \frac{a^2 - a - 2}{-4}.$$

For $a_1 = 3$:

$$b_1 = \frac{3^2 - 3 - 2}{-4} = -1.$$

For $a_2 = 2$:

$$b_2 = \frac{2^2 - 2 - 2}{-4} = 0.$$

The solutions for x are:

$$x_1 = 3 - I,\ x_2 = 2.$$

7. Conclusions

This work advances the study of neutrosophic quadratic equations by addressing key gaps in the literature, particularly the determination of roots and their behavior under uncertainty. The proposed framework enables systematic analysis and classification, contributing to the broader understanding of neutrosophic mathematics. Further research may extend this approach to higher-order neutrosophic polynomials. Additionally, applications of neutrosophic solutions in complex systems modeling, uncertain decision-making processes, and artificial intelligence are promising avenues for investigation. Studying the stability and behavior of these solutions in dynamic and probabilistic contexts could also reveal new properties and potential applications in systems with inherent indeterminacy.

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