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Article

Neutrosophic Split Quaternions and Their Matrix Forms

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Abstract: In this study, we developed a new approach to split quaternions using neutrosophic sets, which have recently gained attention in mathematics, and examined some properties of neutrosophic split quaternions. We also provided the matrix form of these new quaternions.

Keywords: neutrosophic numbers; neutrosophic split quaternions; neutrosophic split quaternion matrices.

1. Introduction

Quaternions were introduced by the Irish mathematician Sir William Rowan Hamilton in 1843 and hold a significant place in mathematics, [1]. Quaternions, also known as four-dimensional hypercomplex numbers, are widely utilized in various mathematical fields such as commutative ring theory, number theory, group theory, geometric topology, spectral theory of Riemannian manifolds, algebraic geometry, as well as in physics and engineering.

After the work of Hamilton, in 1849, James Cockle introduced the set of split quaternions, [2]. Later, Alagöz, Oral and Yüce gave the matrix form of split quaternions and studied their properties, [3]. Split quaternions can be represented as

$$H_S = \{q \mid q = a_0 + a_1 i + a_2 j + a_3 k, a_n \in \mathbb{R}, n = 0,1,2,3\},$$

where the split quaternion bases $i,\ j$ and k satisfy the following identities are known as

$$i^2 = -1, j^2 = k^2 = 1, ijk = 1.$$

For any $q = a_0 + a_1 i + a_2 j + a_3 k \in H_S$, the real part of q defined by $Re(q) = a_0$ and the imaginary part of q defined by $Im(q) = a_1 i + a_2 j + a_3 k$. The conjugate of a split quaternion number is defined as $\bar{q} = a_0 - a_1 i - a_2 j - a_3 k$. The split quaternion multiplication is not commutative.

Neutrosophic numbers, a concept with roots in neutrosophy, were first introduced by Florentin Smarandache in the early 21st century, [4]. The set of real neutrosophic real numbers is defined as

$$N(\mathbb{R}) = \{q_I = a + bI \mid a, b \in \mathbb{R}\},$$

where $I^n = I,\ 0.I = 0,\ I$ represents indeterminacy.

More detailed information about neutrosophic numbers can be found in [4]. Also, using this definition, Alhasan defined the general exponential form of a neutrosophic complex number, [5]. A neutrosophic complex number is represented as

$$(x_1 + y_1 I) + (x_2 + y_2 I)i,$$

where $x_1, x_2, y_1, y_2 \in \mathbb{R}$, $i^2 = -1$ and I is the indeterminacy element.

Neutrosophic numbers provide an extended mathematical framework by incorporating indeterminacy into classical number systems. Within this framework, uncertain information can be modeled more effectively by taking into account the components of truth, indeterminacy and falsity together. This approach is particularly advantageous for the analysis of complex and uncertain data encountered in fields such as artificial intelligence, decision-making and quantum mechanics.

In recent years, Dertli and Tetik have combined the recently popular concepts of non-Newtonian calculus and neutrosophy to define and examine some of the properties of non-Newtonian neutrosophic numbers and non-Newtonian neutrosophic complex numbers, [6]. They also presented the non-Newtonian neutrosophic triangle inequality, frequently used in analysis and geometry and some properties of the non-Newtonian neutrosophic norm. This provides a broader perspective compared to existing studies, encompassing fields such as quantum mechanics, artificial intelligence, analysis, geometry and medicine.

Drawing on the works of Smarandache, Alhasan et al. introduced the notion of neutrosophic quaternion numbers in 2024, [7]. The set of neutrosophic quaternion numbers are defined as

$$H_N = \left\{ \begin{matrix} q_I = a_0 + b_0 I + v_I \mid v_I = (a_1 + b_1 I)i + (a_2 + b_2 I)j + (a_3 + b_3 I)k, \\ a_s, b_s \in \mathbb{R}, s = 0,1,2,3 \end{matrix} \right\},$$

where $I^n = I,\ 0.I = 0,\ I$ represents the indeterminacy and the quaternion bases $i,\ j$ and k satisfy the following identities

$$i^2 = j^2 = k^2 = ijk = -1, ij = -ji = k, jk = -kj = i, ki = -ik = j.$$

The neutrosophic quaternion number $q_I = a_0 + b_0 I + v_I$ consist of two parts. The part $a_0 + b_0 I$ is called the neutrosophic real (scalar) part; the part $v_I = (a_1 + b_1 I)i + (a_2 + b_2 I)j + (a_3 + b_3 I)k$ is called the neutrosophic vector part. The conjugate of a neutrosophic quaternion number is defined as $\overline{q_I} = a_0 + b_0 I - v_I = a_0 + b_0 I - (a_1 + b_1 I)i - (a_2 + b_2 I)j - (a_3 + b_3 I)k$. The neutrosophic quaternion multiplication is not commutative.

Based on these definitions, in this study we define neutrosophic split quaternions and their matrix representations. Moreover, we examine some of their properties.

2. The Neutrosophic Split Quaternions

Definition 2.1. The set of neutrosophic split quaternion numbers is defined as

$$H_{S_N} = \left\{ \begin{matrix} q_N = a_0 + b_0 I + v_I \mid v_I = (a_1 + b_1 I)i + (a_2 + b_2 I)j + (a_3 + b_3 I)k, \\ a_s, b_s \in \mathbb{R}, s = 0,1,2,3 \end{matrix} \right\},$$

where $I^n = I,\ 0.I = 0,\ I$ represents the indeterminacy and the split quaternion bases $i,\ j$ and k satisfy the following identities

$$i^2 = -1, j^2 = k^2 = 1, ij = -ji = k, jk = -kj = -i, ki = -ik = j.$$

Definition 2.2. The operations of addition and multiplication for neutrosophic split quaternions are given by

$$\begin{aligned} +: H_{S_N} \times H_{S_N} &\rightarrow H_{S_N} \\ (q_N, p_N) &\rightarrow q_N + p_N = (a_0 + b_0 I + v_I) + (c_0 + d_0 I + u_I) \\ &= (a_0 + c_0) + (b_0 + d_0)I \\ &+ \big((a_1 + c_1) + (b_1 + d_1)I\big)i \\ &+ \big((a_2 + c_2) + (b_2 + d_2)I\big)j \\ &+ \big((a_3 + c_3) + (b_3 + d_3)I\big)k \end{aligned}$$

and

$$\begin{aligned} \cdot : H_{S_N} \times H_{S_N} &\rightarrow H_{S_N} \\ (q_N, p_N) &\rightarrow q_N \cdot p_N = (a_0 + b_0 I)(c_0 + d_0 I) - (a_1 + b_1 I)(c_1 + d_1 I) \\ &+ (a_2 + b_2 I)(c_2 + d_2 I) + (a_3 + b_3 I)(c_3 + d_3 I) \\ &+ xi + yj + zk, \end{aligned}$$

where

$$\begin{aligned} x &= (a_0+b_0I)(c_1+d_1I)+(a_1+b_1I)(c_0+d_0I) \\ &\quad -(a_2+b_2I)(c_3+d_3I)+(a_3+b_3I)(c_2+d_2I), \\ y &= (a_0+b_0I)(c_2+d_2I)-(a_1+b_1I)(c_3+d_3I) \\ &\quad +(a_2+b_2I)(c_0+d_0I)+(a_3+b_3I)(c_1+d_1I), \\ z &= (a_0+b_0I)(c_3+d_3I)+(a_1+b_1I)(c_2+d_2I) \\ &\quad -(a_2+b_2I)(c_1+d_1I)+(a_3+b_3I)(c_0+d_0I). \end{aligned}$$

Example 2.3. Let $q_N = 1+2I+(3+5I)i+(1+4I)j+(2+3I)k$ and $p_N = 3+I+(2+2I)i+(1+I)j+(2+4I)k$. The addition and multiplication of $q_N, p_N \in H_{S_N}$ are

$$q_N + p_N = 4+3I+(5+7I)i+(2+5I)j+(4+7I)k$$

and

$$q_N \cdot p_N = 2+18I+(11+13I)i+(-12I)j+(9+25I)k,$$

respectively.

Definition 2.4. The conjugate of a neutrosophic split quaternion $q_N = a_0+b_0I+v_I$ is $\overline{q_N} = a_0+b_0I-v_I = a_0+b_0I-(a_1+b_1I)i-(a_2+b_2I)j-(a_3+b_3I)k$.

Remark 2.5. For $q_1, q_2 \in H_S$, neutrosophic split quaternions can be written as

$$\begin{aligned} q_N &= b_0+c_0I+(b_1+c_1I)i+(b_2+c_2I)j+(b_3+c_3I)k \\ &= (b_0+c_0I, b_1+c_1I, b_2+c_2I, b_3+c_3I) \\ &= (b_0,b_1,b_2,b_3)+(c_0,c_1,c_2,c_3)I \\ &= (b_0+b_1i+b_2j+b_3k)+(c_0+c_1i+c_2j+c_3k)I \\ &= q_1+q_2I. \end{aligned}$$

In addition, we can write the conjugate of a neutrosophic split quaternion as $\overline{q_N} = \overline{q_1}+\overline{q_2}I$.

Example 2.6. Let $q_N = 1+3I+(2+5I)i+(3+I)j+(1+2I)k \in H_{S_N}$. We can write

$$\begin{aligned} q_N &= (1+3I, 2+5I, 3+I, 1+2I) \\ &= (1,2,3,1)+(3,5,1,2)I \\ &= (1+2i+3j+k)+(3+5i+j+2k)I \\ &= q_1+q_2I. \end{aligned}$$

Furthermore, the conjugate of q_N is

$$\begin{aligned} \overline{q_N} &= \overline{q_1}+\overline{q_2}I \\ &= (1-2i-3j-k)+(3-5i-j-2k)I \\ &= 1+3I-(2+5I)i-(3+I)j-(1+2I)k. \end{aligned}$$

Definition 2.7. For a neutrosophic split quaternion $q_N = b_0+c_0I+v_I = b_0+c_0I+(b_1+c_1I)i+(b_2+c_2I)j+(b_3+c_3I)k$, b_0+c_0I is called the neutrosophic scalar part of q_N and $v_I = (b_1+c_1I)i+(b_2+c_2I)j+(b_3+c_3I)k$ is called the neutrosophic vector or imaginary part of q_N. Denoted by $S^*(q_N)$ and $V^*(q_N)$, respectively. Therefore, a neutrosophic split quaternion q_N can be written as $q_N = S^*(q_N) + V^*(q_N)$. Since $q_N + p_N = (b_0 + d_0) + (c_0 + e_0)I + \big((b_1 + d_1) + (c_1 + e_1)I\big)i + \big((b_2 + d_2) + (c_2 + e_2)I\big)j + \big((b_3 + d_3) + (c_3 + e_3)I\big)k$ neutrosophic split quaternions can be written as $S^*(q_N + p_N) = S^*(q_N) + S^*(p_N)$. Similarly, it can be easily seen that $V^*(q_N + p_N) = V^*(q_N) + V^*(p_N)$.

Using the above two definitions, the following corollary can be given:

Corollary 2.8. The neutrosophic real and neutrosophic imaginary parts of $q_N \in H_{S_N}$ are

$$S^*(q_N) = \frac{1}{2}(q_N + \overline{q_N})$$

and

$$V^*(q_N) = \frac{1}{2}(q_N - \overline{q_N}),$$

respectively.

Definition 2.9. The norm of a neutrosophic split quaternion is defined as

$$N_{q_N} = q_N \cdot \overline{q_N} = (b_0 + c_0 I)^2 + (b_1 + c_1 I)^2 - (b_2 + c_2 I)^2 - (b_3 + c_3 I)^2.$$

Definition 2.10. The inverse of a neutrosophic split quaternion is defined as

$$q_N^{-1} = \frac{\overline{q_N}}{N_{q_N}}.$$

Example 2.11. Let $q_N = 3 + I + (2 + 2I)i + (1 + I)j + (2 + 4I)k \in H_{S_N}$. The norm of q_N and inverse of q_N are

$$\begin{aligned} N_{q_N} &= (3 + I)^2 + (2 + 2I)^2 - (1 + I)^2 - (2 + 4I)^2 \\ &= 8 - 16I \end{aligned}$$

and

$$\begin{aligned} q_N^{-1} &= \frac{3 + I - (2 + 2I)i - (1 + I)j - (2 + 4I)k}{8 - 16I} \\ &= \frac{3 + I}{8 - 16I} - \left(\frac{2 + 2I}{8 - 16I}\right)i - \left(\frac{1 + I}{8 - 16I}\right)j - \left(\frac{2 + 4I}{8 - 16I}\right)k \\ &= \frac{1}{8}(3 - 14I - (2 - 12I)i - (1 + 6I)j - (2 - 16I)k), \end{aligned}$$

respectively.

Theorem 2.12. For any $q_N, p_N \in H_{S_N}$ and $\lambda \in \mathbb{R}$ we have,

1) $\overline{q_N \cdot p_N} = \overline{q_N} \cdot \overline{p_N}$,

2) $\overline{q_N + p_N} = \overline{q_N} + \overline{p_N}$, $\overline{q_N - p_N} = \overline{q_N} - \overline{p_N}$,

3) $\overline{\overline{q_N}} = q_N$,

4) $\overline{\overline{q_N} \cdot p_N} = \overline{p_N} \cdot q_N$,

5) $q_N \cdot \overline{q_N} = \overline{q_N} \cdot q_N = N_{q_N}$,

6) $\overline{\lambda q_N} = \lambda \overline{q_N}$,

Proof.

1) Let $q_N = (a_0 + b_0 I) + (a_1 + b_1 I)i + (a_2 + b_2 I)j + (a_3 + b_3 I)k$ and $p_N = (c_0 + d_0 I) + (c_1 + d_1 I)i + (c_2 + d_2 I)j + (c_3 + d_3 I)k$. Then $\overline{q_N} = (a_0 + b_0 I) - (a_1 + b_1 I)i - (a_2 + b_2 I)j - (a_3 + b_3 I)k$ and $\overline{p_N} = (c_0 + d_0 I) - (c_1 + d_1 I)i - (c_2 + d_2 I)j - (c_3 + d_3 I)k$. Using the multiplication of neutrosophic split quaternions and the definition of the conjugate of a neutrosophic split quaternion, we obtain

$$
\begin{aligned}
\overline{q_N \cdot p_N} \quad = \quad & (a_0 + b_0 I)(c_0 + d_0 I) - (a_1 + b_1 I)(c_1 + d_1 I) \\
& +(a_2 + b_2 I)(c_2 + d_2 I) + (a_3 + b_3 I)(c_3 + d_3 I) \\
& -[(a_0 + b_0 I)(c_1 + d_1 I) + (a_1 + b_1 I)(c_0 + d_0 I) \\
& -(a_2 + b_2 I)(c_3 + d_3 I) + (a_3 + b_3 I)(c_2 + d_2 I)]i \\
& -[(a_0 + b_0 I)(c_2 + d_2 I) - (a_1 + b_1 I)(c_3 + d_3 I) \\
& +(a_2 + b_2 I)(c_0 + d_0 I) + (a_3 + b_3 I)(c_1 + d_1 I)]j \\
& -[(a_0 + b_0 I)(c_3 + d_3 I) + (a_1 + b_1 I)(c_2 + d_2 I) \\
& -(a_2 + b_2 I)(c_1 + d_1 I) + (a_3 + b_3 I)(c_0 + d_0 I)]k
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{q_N} \cdot \overline{p_N} \quad = \quad & (c_0 + d_0 I)(a_0 + b_0 I) - (c_1 + d_1 I)(a_1 + b_1 I) \\
& +(c_2 + d_2 I)(a_2 + b_2 I) + (c_3 + d_3 I)(a_3 + b_3 I) \\
& +[-(c_0 + d_0 I)(a_1 + b_1 I) - (c_1 + d_1 I)(a_0 + b_0 I) \\
& -(c_2 + d_2 I)(a_3 + b_3 I) + (c_3 + d_3 I)(a_2 + b_2 I)]i \\
& +[-(c_0 + d_0 I)(a_2 + b_2 I) - (c_1 + d_1 I)(a_3 + b_3 I) \\
& -(c_2 + d_2 I)(a_0 + b_0 I) + (c_3 + d_3 I)(a_1 + b_1 I)]j \\
& +[-(c_0 + d_0 I)(a_3 + b_3 I) + (c_1 + d_1 I)(a_2 + b_2 I) \\
& -(c_2 + d_2 I)(a_1 + b_1 I) - (c_3 + d_3 I)(a_0 + b_0 I)]k.
\end{aligned}
$$

Thus, it is easily seen that $\overline{q_N \cdot p_N} = \overline{q_N} \cdot \overline{p_N}$.

5) Let $q_N = (b_0 + c_0 I) + (b_1 + c_1 I)i + (b_2 + c_2 I)j + (b_3 + c_3 I)k$. Using the multiplication of neutrosophic split quaternions, the definition of the conjugate of a neutrosophic split quaternion and the norm of a neutrosophic split quaternion, we obtain

$$
\begin{aligned}
q_N \cdot \overline{q_N} &= (b_0 + c_0 I)^2 + (b_1 + c_1 I)^2 - (b_2 + c_2 I)^2 - (b_3 + c_3 I)^2 \\
&\quad +[-(b_0 + c_0 I)(b_1 + c_1 I) + (b_1 + c_1 I)(b_0 + c_0 I) \\
&\quad +(b_2 + c_2 I)(b_3 + c_3 I) - (b_3 + c_3 I)(b_2 + c_2 I)]i \\
&\quad +[-(b_0 + c_0 I)(b_2 + c_2 I) + (b_1 + c_1 I)(b_3 + c_3 I) \\
&\quad +(b_2 + c_2 I)(b_0 + c_0 I) - (b_3 + c_3 I)(b_1 + c_1 I)]j \\
&\quad +[-(b_0 + c_0 I)(b_3 + c_3 I) - (b_1 + c_1 I)(b_2 + c_2 I) \\
&\quad +(b_2 + c_2 I)(b_1 + c_1 I) + (b_3 + c_3 I)(b_0 + c_0 I)]k \\
&= (b_0 + c_0 I)^2 + (b_1 + c_1 I)^2 - (b_2 + c_2 I)^2 - (b_3 + c_3 I)^2 \\
&= N_{q_N}
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{q_N} \cdot q_N &= (b_0 + c_0 I)^2 + (b_1 + c_1 I)^2 - (b_2 + c_2 I)^2 - (b_3 + c_3 I)^2 \\
&\quad +[(b_0 + c_0 I)(b_1 + c_1 I) - (b_1 + c_1 I)(b_0 + c_0 I) \\
&\quad +(b_2 + c_2 I)(b_3 + c_3 I) - (b_3 + c_3 I)(b_2 + c_2 I)]i \\
&\quad +[(b_0 + c_0 I)(b_2 + c_2 I) + (b_1 + c_1 I)(b_3 + c_3 I) \\
&\quad -(b_2 + c_2 I)(b_0 + c_0 I) - (b_3 + c_3 I)(b_1 + c_1 I)]j \\
&\quad +[(b_0 + c_0 I)(b_3 + c_3 I) - (b_1 + c_1 I)(b_2 + c_2 I) \\
&\quad +(b_2 + c_2 I)(b_1 + c_1 I) - (b_3 + c_3 I)(b_0 + c_0 I)]k \\
&= (b_0 + c_0 I)^2 + (b_1 + c_1 I)^2 - (b_2 + c_2 I)^2 - (b_3 + c_3 I)^2 \\
&= N_{q_N}.
\end{aligned}
$$

Thus, it is easily seen that $q_N \cdot \overline{q_N} = \overline{q_N} \cdot q_N = N_{q_N}$.

Other properties can be similarly proven.□

Let us now obtain the matrix representation of the neutrosophic split quaternion.

Theorem 2.13. Every neutrosophic split quaternion can be represented by a 2×2 neutrosophic complex matrix.

Proof.

Let $q_N \in H_{S_N}$, then there exist complex numbers q_1', q_1'', q_2', q_2'' such that $q = (q_1' + q_2' I) + (q_1'' + q_2'' I)j$ by definition of neutrosophic complex numbers.

The linear map $\varphi_q : H_{S_N} \to H_{S_N}$ is defined by $\varphi_q(p_N) = p_N . q$ for all $p_N \in H_{S_N}$. This map is bijective and

$$\varphi_q(1) = 1\big((q_1' + q_2' I) + (q_1'' + q_2'' I)j\big) = (q_1' + q_2' I) + (q_1'' + q_2'' I)j,$$

$$\varphi_q(j) = j\big((q_1' + q_2' I) + (q_1'' + q_2'' I)j\big) = \big(\overline{q_1'} + \overline{q_2'} I\big)j + \big(\overline{q_1''} + \overline{q_2''} I\big).$$

With this transformation neutrosophic split quaternions are defined as subset of the matrix ring $M_2\big(N(\mathbb{C})\big)$, the set of 2×2 neutrosophic complex matrices:

$$H'_{S_N} = \left\{ \begin{pmatrix} q'_1 + q'_2 I & q''_1 + q''_2 I \\ \overline{q''_1} + \overline{q''_2} I & \overline{q'_1} + \overline{q'_2} I \end{pmatrix} : q'_1, q''_1, q'_2, q''_2 \in \mathbb{C} \right\}.$$

H_{S_N} and H'_{S_N} are essentially the same. ∎

Note 2.14. $\mathcal{M}: q = (q'_1 + q'_2 I) + (q''_1 + q''_2 I)j \in H_{S_N} \longrightarrow q' = \begin{pmatrix} q'_1 + q'_2 I & q''_1 + q''_2 I \\ \overline{q''_1} + \overline{q''_2} I & \overline{q'_1} + \overline{q'_2} I \end{pmatrix} \in H'_{S_N}$

is bijective and preserves the operations.

Definition 2.15. Let $\mathcal{Q} = \begin{pmatrix} q'_1 + q'_2 I & q''_1 + q''_2 I \\ \overline{q''_1} + \overline{q''_2} I & \overline{q'_1} + \overline{q'_2} I \end{pmatrix}$ and $\mathcal{P} = \begin{pmatrix} p'_1 + p'_2 I & p''_1 + p''_2 I \\ \overline{p''_1} + \overline{p''_2} I & \overline{p'_1} + \overline{p'_2} I \end{pmatrix}$

be any two 2×2 neutrosophic complex matrices. The sum and product of $\mathcal{Q}$ and $\mathcal{P}$ are defined as

$$\mathcal{Q} \oplus \mathcal{P} = \begin{pmatrix} q'_1 + q'_2 I & q''_1 + q''_2 I \\ \overline{q''_1} + \overline{q''_2} I & \overline{q'_1} + \overline{q'_2} I \end{pmatrix} \oplus \begin{pmatrix} p'_1 + p'_2 I & p''_1 + p''_2 I \\ \overline{p''_1} + \overline{p''_2} I & \overline{p'_1} + \overline{p'_2} I \end{pmatrix}$$

$$= \begin{pmatrix} (q'_1 + p'_1) + (q'_2 + p'_2) I & (q''_1 + p''_1) + (q''_2 + p''_2) I \\ (\overline{q''_1} + \overline{p''_1}) + (\overline{q''_2} + \overline{p''_2}) I & (\overline{q'_1} + \overline{p'_1}) + (\overline{q'_2} + \overline{p'_2}) I \end{pmatrix}$$

and

$$\mathcal{Q} \otimes \mathcal{P} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{pmatrix},$$

respectively, where

$$\alpha_1 = (q'_1 + q'_2 I)(p'_1 + p'_2 I) + (q''_1 + q''_2 I)(\overline{p''_1} + \overline{p''_2} I),$$

$$\alpha_2 = (q'_1 + q'_2 I)(p''_1 + p''_2 I) + (q''_1 + q''_2 I)(\overline{p'_1} + \overline{p'_2} I),$$

$$\alpha_3 = (\overline{q''_1} + \overline{q''_2} I)(p'_1 + p'_2 I) + (\overline{q'_1} + \overline{q'_2} I)(\overline{p''_1} + \overline{p''_2} I),$$

$$\alpha_4 = (\overline{q''_1} + \overline{q''_2} I)(p''_1 + p''_2 I) + (\overline{q'_1} + \overline{q'_2} I)(\overline{p'_1} + \overline{p'_2} I).$$

3. Conclusions

In this study, we re-examined split quaternions and split quaternion matrices using neutrosophic sets. Our findings offer new perspectives on quaternion theory and pave the way for future research on neutrosophic applications. In future studies, the properties of neutrosophic split quaternion matrices can be examined and neutrosophic split quaternions can be combined with number sequences.

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