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Article

The Neutral Set as a Re-Notation of the Single-Valued Neutrosophic Set

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Abstract: This short note examines the mathematical relationship between the recently introduced Neutral Set and the established Single-Valued Neutrosophic Set (SVNS). By means of a formal comparison of definitions, notation, and admissibility conditions, we show that the Neutral Set is mathematically equivalent to the SVNS, differing only in symbolic representation. Specifically, the membership functions denoted by truth, indeterminacy, and falsity in the Neutral Set correspond directly to the truth-membership, indeterminacy-membership, and falsity-membership functions of the SVNS under a simple notational transformation. Furthermore, the informational regimes identified for the Neutral Set—sub-ideal (incomplete information), intuitionistic (complete information), and paraconsistent (contradictory information)—coincide with the interpretative framework already established in neutrosophic set theory. We conclude that the Neutral Set should be regarded as a reformulation or particular representation of the Single-Valued Neutrosophic Set rather than a novel mathematical structure.

Keywords: Single-Valued Neutrosophic Set, Neutral Set, neutrosophic logic, neutrosophic set theory, mathematical equivalence, indeterminacy, truth membership, falsity membership, paraconsistent information, uncertainty modeling.

1. Introduction

The introduction of new mathematical structures in uncertainty modeling, fuzzy systems, and generalized set theories requires careful examination in order to distinguish genuinely novel concepts from reformulations of already established frameworks. Such distinctions are particularly important for preserving conceptual clarity, terminological consistency, and the cumulative development of scientific knowledge. In many cases, newly proposed structures may differ only in notation or terminology while remaining mathematically equivalent to previously defined models.

Recently, C. Ardil [1] introduced the concept of a *Neutral Set* as a framework intended to represent truth, indeterminacy, and falsity through three independent mappings taking values in the interval ([0,1]). The proposed structure identifies distinct informational regimes depending on the sum of these components, namely incomplete, intuitionistic, and paraconsistent information.

At the same time, the *Single-Valued Neutrosophic Set* (SVNS), originally developed within neutrosophic set theory [2], is defined through truth-membership, indeterminacy-membership, and falsity-membership functions satisfying the same admissibility condition. The SVNS framework already encompasses incomplete, complete, and contradictory information according to the total sum of its three components.

The objective of this short note is to examine the formal relationship between the Neutral Set and the Single-Valued Neutrosophic Set. By comparing their mathematical definitions and interpretative mechanisms, we demonstrate that the Neutral Set is mathematically equivalent to the Single-Valued Neutrosophic Set, differing primarily in symbolic notation. Furthermore, we show that the assumption of independent components in the Neutral Set corresponds to a particular case within the broader neutrosophic framework. The analysis presented here contributes to maintaining mathematical coherence in the taxonomy of uncertainty models and clarifies the position of the Neutral Set within neutrosophic set theory.

2. Definitions and Formal Comparison

In this section, we present the definitions of the *Neutral Set* and the *Single-Valued Neutrosophic Set* in order to establish a formal comparison between the two mathematical structures.

2.1. Neutral Set

According to Ardil [1], a *Neutral Set* (A) defined on a universe of discourse (X) is represented as:

$$A = \{\langle x, \mu(x), \eta(x), \nu(x)\rangle \mid x \in X\}, \text{ where}$$

$$\mu, \eta, \nu : X \to [0,1]$$

are independent mappings satisfying the condition

$$0 \le \mu(x) + \eta(x) + \nu(x) \le 3$$

The quantities $\mu(x)$, $\eta(x)$, and $\nu(x)$ denote the degrees of truth, indeterminacy, and falsity associated with the element x, respectively. Furthermore, Ardil defines an informational density function:

$$S(x) = \mu(x) + \eta(x) + \nu(x)$$

Based on the value of $S(x)$, three informational regimes are distinguished:

- **Incomplete information**, when $S(x) < 1$
- **Complete (intuitionistic) information**, when $S(x) = 1$
- **Paraconsistent information**, when $S(x) > 1$

2.2. Single-Valued Neutrosophic Set

The *Single-Valued Neutrosophic Set* (SVNS), introduced in neutrosophic set theory [2], is defined on a universe of discourse (U) as:

$$A = \{\langle x, T(x), I(x), F(x)\rangle \mid x \in U\}$$

where

$$T, I, F : U \to [0,1]$$

represent the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively, satisfying the condition

$$0 \le T(x) + I(x) + F(x) \le 3$$

Within the neutrosophic framework, the functions $T(x)$, $I(x)$, and $F(x)$ may be independent, partially independent, partially dependent, or dependent.

Similarly, the informational interpretation depends on the sum of the three components:

- **Incomplete information**, when $T(x) + I(x) + F(x) < 1$
- **Complete information**, when $T(x) + I(x) + F(x) = 1$
- **Contradictory (paraconsistent) information**, when $T(x) + I(x) + F(x) > 1$

2.3. Preliminary Comparison

A direct comparison between the two definitions reveals a one-to-one correspondence between their components:

$$T(x) = \mu(x)$$

$$I(x) = \eta(x)$$

$$F(x) = \nu(x)$$

Moreover, both structures satisfy the same admissibility condition:

$$0 \le T(x) + I(x) + F(x) \le 3$$

which is equivalent to

$$0 \le \mu(x) + \eta(x) + \nu(x) \le 3$$

Likewise, both frameworks adopt the same interpretation of informational regimes, namely incomplete, complete, and contradictory (paraconsistent) information according to the total sum of their respective components.

The only apparent distinction concerns the explicit requirement of independence among the mappings in the Neutral Set definition, whereas the Single-Valued Neutrosophic Set framework permits independent, partially dependent, or dependent components. Therefore, the Neutral Set may be viewed as a particular case within the broader neutrosophic framework.

The following section formally establishes the mathematical equivalence between the Neutral Set and the Single-Valued Neutrosophic Set.

3. Mathematical Equivalence Between the Neutral Set and the Single-Valued Neutrosophic Set

In this section, we formally establish the mathematical relationship between the *Neutral Set* and the *Single-Valued Neutrosophic Set* (SVNS).

Proposition 1

Let

$$A_N = \{\langle x, \mu(x), \eta(x), \nu(x)\rangle \mid x \in X\}$$

be a *Neutral Set* defined on a universe of discourse **X**, and let

$$A_{SVNS} = \{\langle x, T(x), I(x), F(x)\rangle \mid x \in U\}$$

be a *Single-Valued Neutrosophic Set* defined on a universe of discourse **U**.

If the following component-wise correspondence is established:

$$T(x) = \mu(x)$$
$$I(x) = \eta(x)$$
$$F(x) = \nu(x)$$

then the *Neutral Set* is mathematically equivalent to the *Single-Valued Neutrosophic Set*.

Proof

The definition of a Neutral Set requires that the mappings

$$\mu, \eta, \nu : X \to [0,1]$$

satisfy the admissibility condition

$$0 \le \mu(x) + \eta(x) + \nu(x) \le 3$$

Similarly, the definition of a Single-Valued Neutrosophic Set requires that

$$T, I, F : U \to [0,1]$$

satisfy

$$0 \le T(x) + I(x) + F(x) \le 3$$

Substituting the correspondence

$$T(x) = \mu(x)$$
$$I(x) = \eta(x)$$
$$F(x) = \nu(x)$$

into the SVNS condition yields:

$$0 \le \mu(x) + \eta(x) + \nu(x) \le 3$$

which is precisely the admissibility condition of the Neutral Set.

Moreover, the semantic interpretation of the three components coincides in both frameworks:

- **$\mu(x)$** corresponds to **$T(x)$** (truth degree),
- **$\eta(x)$** corresponds to **$I(x)$** (indeterminacy degree),
- **$\nu(x)$** corresponds to **$F(x)$** (falsity degree).

Likewise, the informational regimes are identical in both structures:

- **Incomplete information**, when $\mu(x) + \eta(x) + \nu(x) < 1$
  or equivalently $T(x) + I(x) + F(x) < 1$

- **Complete information**, when $\mu(x) + \eta(x) + \nu(x) = 1$
  or equivalently $T(x) + I(x) + F(x) = 1$
- **Contradictory (paraconsistent) information**, when $\mu(x) + \eta(x) + \nu(x) > 1$
  or equivalently $T(x) + I(x) + F(x) > 1$

Hence, the Neutral Set and the Single-Valued Neutrosophic Set possess identical mathematical structures, identical admissibility constraints, and identical interpretative mechanisms.

The only distinction lies in notation and in the explicit assumption that the three mappings are independent in the Neutral Set framework. Since the Single-Valued Neutrosophic Set already permits independent components as a valid case, the Neutral Set constitutes a particular instance of the more general neutrosophic framework.

Therefore, the so-called *Neutral Set* is mathematically equivalent to the *Single-Valued Neutrosophic Set*. □

4. Discussion

The result established in Proposition 1 demonstrates that the *Neutral Set* does not introduce a new mathematical structure distinct from the *Single-Valued Neutrosophic Set* (SVNS). Rather, the comparison reveals a direct correspondence between the components of the two frameworks, namely:

$$T(x) = \mu(x),\ I(x) = \eta(x),\ F(x) = \nu(x)$$

together with the same admissibility condition:

$$0 \leq T(x) + I(x) + F(x) \leq 3$$

and identical interpretations of informational regimes.

From a mathematical perspective, two set-theoretical models that possess the same functional structure, the same constraints, and the same semantic interpretation should be regarded as equivalent representations of the same concept rather than distinct theories. The introduction of alternative notation, by itself, does not constitute mathematical novelty unless accompanied by new operators, new algebraic properties, generalized domains, or genuinely different semantics.

A possible distinction may be identified in the explicit requirement of independence among the mappings $\mu(x)$, $\eta(x)$, and $\nu(x)$ in the Neutral Set framework. However, this distinction does not invalidate the equivalence established above. In fact, the Single-Valued Neutrosophic Set already includes independent components among its admissible configurations, while also allowing partial dependence or full dependence. Consequently, the Neutral Set can be interpreted as a restricted or particular case of the broader neutrosophic framework rather than an independent mathematical construction.

The present analysis also highlights a broader methodological issue in uncertainty modeling and generalized set theories: the proliferation of terminologies for mathematically equivalent structures may create conceptual fragmentation and unnecessary ambiguity in the literature. For scientific coherence, newly introduced frameworks should be carefully examined in relation to pre-existing theories in order to determine whether they represent genuine mathematical innovations or alternative reformulations of known concepts.

Therefore, based on the formal correspondence established in this paper, the Neutral Set should be understood as a notational reformulation of the Single-Valued Neutrosophic Set and, more specifically, as a particular case included within neutrosophic set theory.

5. Conclusion

This short note examined the mathematical relationship between the recently introduced *Neutral Set* and the established *Single-Valued Neutrosophic Set* (SVNS). Through a formal comparison of definitions, admissibility conditions, and semantic interpretations, we demonstrated that the two frameworks are mathematically equivalent under a direct component-wise correspondence between their respective functions of truth, indeterminacy, and falsity.

The analysis showed that both structures employ identical mappings into the interval **[0,1]**, satisfy the same condition

$$0 \leq T(x) + I(x) + F(x) \leq 3$$

and admit the same informational regimes associated with incomplete, complete, and contradictory (paraconsistent) information. The only distinction identified concerns the explicit assumption of independence among the mappings in the Neutral Set framework, which constitutes a particular case already included within the broader Single-Valued Neutrosophic Set formalism.

Consequently, the Neutral Set should be regarded not as a new mathematical structure, but as a reformulation of the Single-Valued Neutrosophic Set under alternative notation. More generally, this study emphasizes the importance of evaluating newly proposed uncertainty models against established frameworks in order to preserve conceptual consistency and avoid unnecessary terminological proliferation in the scientific literature.

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