



The Randic Index in Neutrosophic Soft Graphs and its Applications

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Abstract. The paper presents and studies the Randic Index for neutrosophic graphs and uncertain graphs. It provides an effective model for uncertainty and imprecision in edge and vertex relations. This study analyzes the Randic Index's properties for neutrosophic soft graphs and the impact of isomorphism between graphs. Additionally, the idea is expanded to directed neutrosophic soft graphs, providing a new method for analyzing directed structures in the presence of uncertainty. A real-world example shows how the Randic Index can be used to help choose the ideal location for a waste management plant. This technique underlines the Randic Index's practical usefulness in urban planning and infrastructure development, illustrating its greater influence across various disciplines.

Keywords: Randic Index; Neutrosophic soft graph; Randic Index in Neutrosophic soft graph.

1. Introduction

Numerous practical problems in computer applications, systems analysis, computer networks, transportation, operations research, and economics may be resolved with the aid of graph theory. In essence, a graph is a relational model used to explain problems in the actual world

that call for interactions between items. The items and their relationships are represented by the vertices and edges of the graph, accordingly. For a variety of causes, such as information loss, a lack of supporting evidence, faulty statistical data, and insufficient information, the information supplied in many optimization scenarios is erroneous or imprecise. Generally speaking, information concerning a real-life situation can be unclear.

Cantor proposed the fundamental concept of classical graph theory. Each vertex or edge in a classical graph can either belong to the graph or not. As a result, uncertain optimization scenarios cannot be represented by ordinary graphs. Since real-world issues are usually unforeseen, it is challenging to replicate them with traditional graphs. Fuzzy set theory was proposed by Zadeh [1]. Neutrosophic sets are a helpful mathematical tool for handling ambiguous, inconsistent, and incomplete data in the real world, according to Smarandache's theory. A truth-membership function (t), an indeterminacy-membership function (i), and a falsity-membership function (f) create neutrosophic sets independently in the real standard or nonstandard unit interval $[0, 1]$. As extensions of the Neutrosophic Set, concepts such as the Double-valued Neutrosophic Set [24] and the SuperHyperNeutrosophic Set [25] are also known. To facilitate the use of NS in real-world applications, Wang developed the notion of single-valued neutrosophic sets, a subclass of neutrosophic sets. The earlier study was motivated by the introduction of the notions of neutrosophic vague graphs and strong neutrosophic vague graphs by Satham Hussain [2]. Neutrosophic Graphs and their extended concepts, such as Neutrosophic HyperGraphs [18], Neutrosophic SuperHyperGraphs [19], Quadripartitioned neutrosophic graphs [22,23], and Plithogenic Graphs [20,21], have been studied in recent years.

In addition to discussing the characteristics of neighborly regular and irregular fuzzy graphs, Nagoor Gani [3, 4] defines regular and irregular fuzzy graphs. Then, Liangsong Huang [5] introduced a study of regular and irregular neutrosophic soft graphs, the two types of degree, d_m -regular, td_m -regular and m -highly irregular neutrosophic soft graphs, and some properties are shown.

The concept of complex neutrosophic soft graphs (CNSGs), which provide a sophisticated framework for simulating indeterminacy, ambiguity, and uncertainty in complex systems, is introduced and developed by Suriya [6–9]. High degrees of neutrosophic uncertainty and complex-valued memberships for both edges and vertices under soft set parameters define a strong complex neutrosophic soft graph (SCNSG). We present a novel measure that considers the cumulative strength and uncertainty of connections within graph characteristics, as well as the complements of a complex neutrosophic soft graph and a strong complex neutrosophic

soft graph.

Yahya [10] introduces the features of ambiguous graphs and subgraphs as well as their Randic Index. This study uses isomorphic properties to investigate the upper and lower boundaries of the Randic Index of vague graphs. We introduce Randic indices for directed ambiguous graphs. Finally, an application of the Randic Index in construction was presented.

We introduce the neutrosophic graph features and the Randic Index. Using a variety of isomorphic features, this paper investigates a truth-membership function, indeterminacy-membership function, and falsity-membership function of the Randic Index in neutrosophic soft graphs. After introducing Randic indices for neutrosophic soft graphs and neutrosophic digraphs, theorems are explored through examples. The Randic Index is used in construction.

2. Preliminaries

This section provides the basic definitions and an example needed to support the main results. We begin with the definition of a fuzzy soft graph.

Definition 2.1. ([11]) *A fuzzy soft graph $G = (G^*, M, N, R)$ is an ordered quadruple that meets the following conditions:*

- $G^* = (V, E)$ is a simple graph,
- R is a non-empty set of parameters,
- (M, R) is a fuzzy set defined over the vertex set V ,
- (N, R) is a fuzzy set defined over the edge set E ,
- For each $a \in R$, the pair $(M(a), N(a))$ forms a fuzzy graph associated with G^* , such that:

$$N(a)(xy) \leq \min\{M(a)(x), M(a)(y)\}$$

holds for all $x, y \in V$.

$\mathfrak{H}(a)$ represents the fuzzy graph $(M(a), N(a))$. A fuzzy soft graph can be thought of as a family of fuzzy graphs parameterized by R . $M(G^*)$ denotes the class of all fuzzy soft graphs over G^* .

Definition 2.2. [12] *An intuitionistic fuzzy soft graph $G = (G^*, F, K, Q)$ is an ordered four tuple if it satisfies the following conditions:*

- $G^* = (V, E)$ is a simple graph,
- Q is a non-empty set of parameters,
- (F, Q) is an intuitionistic fuzzy set over V ,
- (K, Q) is an intuitionistic fuzzy set over E ,
- $(F(a), K(a))$ is an intuitionistic fuzzy graph of G^* for all $a \in Q$. That is

$$\mathfrak{A}_{K(a)}(ab) \leq \min\{\mathfrak{A}_{F(a)}(a), \mathfrak{A}_{F(a)}(b)\}$$

$$\mathfrak{C}_{K(a)}(ab) \geq \max\{\mathfrak{C}_{F(a)}(a), \mathfrak{C}_{F(a)}(b)\}$$

for all $a \in Q$ and $x, y \in V$. The intuitionistic fuzzy graph $(F(a), K(a))$ is denoted by $H(a)$ for convenience. An intuitionistic fuzzy soft graph is a parameterized family of intuitionistic fuzzy graphs.

Definition 2.3. [13] A neutrosophic soft graph $G = (G^*, M, N, R)$ is defined as a quadruple satisfying the following conditions:

- $G^* = (V, E)$ represents a simple (i.e., undirected and without loops or multiple edges) graph,
- R is a non-empty collection of parameters,
- (M, R) denotes a neutrosophic set defined over the vertex set V ,
- (N, R) denotes a neutrosophic set defined over the edge set E ,
- For every $a \in R$, the pair $(M(a), N(a))$ forms a neutrosophic subgraph of G^* . Specifically, the following conditions hold:

$$\mathfrak{A}_{N(a)}(xy) \leq \min\{\mathfrak{A}_{M(a)}(x), \mathfrak{A}_{M(a)}(y)\},$$

$$\mathfrak{B}_{N(a)}(xy) \leq \min\{\mathfrak{B}_{M(a)}(x), \mathfrak{B}_{M(a)}(y)\},$$

$$\mathfrak{C}_{N(a)}(xy) \geq \max\{\mathfrak{C}_{M(a)}(x), \mathfrak{C}_{M(a)}(y)\},$$

For ease of use, $\mathfrak{H}(a)$ is used to represent the neutrosophic fuzzy graph $(M(a), N(a))$. A neutrosophic fuzzy soft graph may therefore be thought of as a family of neutrosophic fuzzy graphs that are parameterized by R . $M(G^*)$ represents the class of all neutrosophic fuzzy soft graphs over G^* , for all $a \in R$ and $x, y \in V$.

Definition 2.4. [10] The Randic Index of a vague graph $G = (S, K)$ is shown by $\mathfrak{RI}(G)$ and explained as follows:

$$\mathfrak{RI}(G) = (\mathfrak{RI}_{\mathfrak{F}}(G), \mathfrak{RI}_{\Omega}(G), \mathfrak{RI}_{\mathfrak{R}}(G))$$

where,

$$\mathfrak{RI}_{\mathfrak{F}}(G) = \sum_{i \neq j, c_i c_j \in E} (\mathfrak{F}_S(c_i) \mathfrak{F}_S(c_j) \mathbb{D}_{\mathfrak{F}}(c_i) \mathbb{D}_{\mathfrak{F}}(c_j))^{-\frac{1}{2}}$$

$$\mathfrak{RI}_{\mathfrak{R}}(G) = \sum_{i \neq j, c_i c_j \in E} (\mathfrak{R}_S(c_i) \mathfrak{R}_S(c_j) \mathbb{D}_{\mathfrak{R}}(c_i) \mathbb{D}_{\mathfrak{R}}(c_j))^{-\frac{1}{2}}$$

where true and false parts of the node c_i degrees are $\mathbb{D}_{\mathfrak{F}}$ and $\mathbb{D}_{\mathfrak{R}}$, respectively.

Definition 2.5. [8] The Randic Index of a neutrosophic graph $G = (S, K)$ is shown by $\mathfrak{R}(G)$ and explained as follows:

$$\mathfrak{R}(G) = (\mathfrak{RI}_{\mathfrak{F}}(G), \mathfrak{RI}_{\Omega}(G), \mathfrak{RI}_{\mathfrak{R}}(G))$$

where,

$$\begin{aligned} \mathfrak{R}_{\mathfrak{A}}(G) &= \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{A}_S(c_i) \mathfrak{P}_S(c_j) \mathbb{D}_{\mathfrak{P}}(c_i) \mathbb{D}_{\mathfrak{P}}(c_j))}} \\ \mathfrak{R}_{\mathfrak{B}}(G) &= \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{B}_S(c_i) \mathfrak{Q}_S(c_j) \mathbb{D}_{\mathfrak{Q}}(c_i) \mathbb{D}_{\mathfrak{Q}}(c_j))}} \\ \mathfrak{R}_{\mathfrak{C}}(G) &= \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{C}_S(c_i) \mathfrak{R}_S(c_j) \mathbb{D}_{\mathfrak{R}}(c_i) \mathbb{D}_{\mathfrak{R}}(c_j))}} \end{aligned}$$

that the true, indeterminacy and false parts of the node c_i degrees are $\mathbb{D}_{\mathfrak{A}}$, $\mathbb{D}_{\mathfrak{B}}$ and $\mathbb{C}_{\mathfrak{R}}$, respectively.

3. The Randic Index in Neutrosophic Soft Graphs

In this section, we investigate the Randic index in neutrosophic soft graphs. The relevant definitions and related materials are presented below.

Definition 3.1. *The Randic Index of a neutrosophic soft graph $G = (S, K)$ is shown by $\mathfrak{R}(G)$ and explained as follows:*

$$\mathfrak{R}(G) = (\mathfrak{R}_{\mathfrak{A}(n)}(G), \mathfrak{R}_{\mathfrak{B}(n)}(G), \mathfrak{R}_{\mathfrak{C}(n)}(G))$$

where,

$$\begin{aligned} \mathfrak{R}_{\mathfrak{A}(n)}(G) &= \sum_{n \in N} \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{A}_{S(n)}(c_i) \mathfrak{A}_{S(n)}(c_j) \mathfrak{D}_{\mathfrak{A}(n)}(c_i) \mathfrak{D}_{\mathfrak{A}(n)}(c_j))}} \\ \mathfrak{R}_{\mathfrak{B}(n)}(G) &= \sum_{n \in N} \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{B}_{S(n)}(c_i) \mathfrak{B}_{S(n)}(c_j) \mathfrak{D}_{\mathfrak{B}(n)}(c_i) \mathfrak{D}_{\mathfrak{B}(n)}(c_j))}} \\ \mathfrak{R}_{\mathfrak{C}(n)}(G) &= \sum_{n \in N} \sum_{i \neq j, c_i c_j \in E} \frac{1}{\sqrt{(\mathfrak{C}_{S(n)}(c_i) \mathfrak{C}_{S(n)}(c_j) \mathfrak{D}_{\mathfrak{C}(n)}(c_i) \mathfrak{D}_{\mathfrak{C}(n)}(c_j))}} \end{aligned}$$

that the true, indeterminacy and false parts of the node c_i degrees are $\mathfrak{D}_{\mathfrak{A}(n)}$, $\mathfrak{D}_{\mathfrak{B}(n)}$ and $\mathfrak{D}_{\mathfrak{C}(n)}$, respectively. For each parameter $n \in N$, the pair $(S(n), K(n))$ forms a neutrosophic soft graph (NSG) on G^* .

Example 3.2. *Let G be the neutrosophic soft graph shown in Figure 1. Then*

$$\begin{aligned} \mathfrak{D}_{n_1}(c_1) &= (0.6, 1.3, 2.0), & \mathfrak{D}_{n_1}(c_2) &= (0.5, 0.9, 1.4), \\ \mathfrak{D}_{n_1}(c_3) &= (0.4, 1.7, 2.2), & \mathfrak{D}_{n_1}(c_4) &= (0.3, 1.1, 1.4), \end{aligned}$$

and

$$\begin{aligned} \mathfrak{D}_{n_2}(c_1) &= (0.4, 1.5, 2.4), & \mathfrak{D}_{n_2}(c_2) &= (0.4, 1.0, 1.6), \\ \mathfrak{D}_{n_2}(c_3) &= (0.5, 1.3, 2.5), & \mathfrak{D}_{n_2}(c_4) &= (0.3, 1.0, 1.5). \end{aligned}$$

Furthermore,

$$\sum_{n \in N} \sum_{\substack{i \neq j \\ c_i, c_j \in E}} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(c_i)\mathfrak{A}_{S(n)}(c_j)\mathfrak{D}_{\mathfrak{A}(n)}(c_i)\mathfrak{D}_{\mathfrak{A}(n)}(c_j)}} = (5.27 + 7.91 + 11.79 + 7.86 + 8.33) \\ + (10.21 + 9.13 + 10.21 + 11.18) \\ = 81.89,$$

$$\sum_{n \in N} \sum_{\substack{i \neq j \\ c_i, c_j \in E}} \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(c_i)\mathfrak{B}_{S(n)}(c_j)\mathfrak{D}_{\mathfrak{B}(n)}(c_i)\mathfrak{D}_{\mathfrak{B}(n)}(c_j)}} = (1.69 + 1.17 + 0.91 + 1.32 + 1.06) \\ + (1.09 + 1.48 + 1.60 + 1.18 + 1.13) \\ = 12.63,$$

and

$$\sum_{n \in N} \sum_{\substack{i \neq j \\ c_i, c_j \in E}} \frac{1}{\sqrt{\mathfrak{C}_{S(n)}(c_i)\mathfrak{C}_{S(n)}(c_j)\mathfrak{D}_{\mathfrak{C}(n)}(c_i)\mathfrak{D}_{\mathfrak{C}(n)}(c_j)}} = (1.09 + 0.96 + 0.96 + 1.09 + 0.74) \\ + (0.79 + 0.72 + 0.82 + 0.89 + 0.55) \\ = 8.61.$$

Hence,

$$\mathfrak{R}(G) = (\mathfrak{R}_{\mathfrak{A}(n)}(G), \mathfrak{R}_{\mathfrak{B}(n)}(G), \mathfrak{R}_{\mathfrak{C}(n)}(G)) = (81.89, 12.63, 8.61).$$

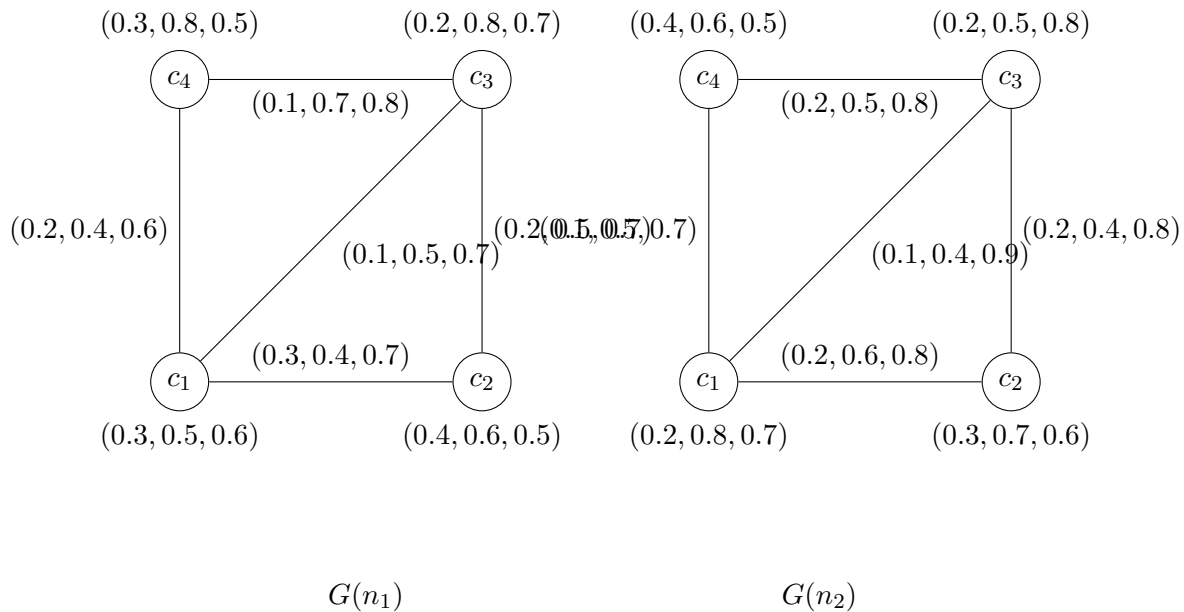


Figure 1. Neutrosophic soft graph G.

Theorem 3.3. Let $G = (S, K)$ be a neutrosophic soft graph on the underlying graph

$$G^* = (V, E),$$

and fix $n \in N$. Let

$$a := |V| \quad \text{and} \quad b := |E|.$$

Assume that all terms appearing in $\mathfrak{A}_{\mathfrak{A}(n)}(G)$, $\mathfrak{A}_{\mathfrak{B}(n)}(G)$, and $\mathfrak{A}_{\mathfrak{C}(n)}(G)$ are well defined. Then

$$\mathfrak{A}_{\mathfrak{A}(n)}(G) \geq \frac{b}{a-1}, \quad \mathfrak{A}_{\mathfrak{B}(n)}(G) \geq \frac{b}{a-1}, \quad \mathfrak{A}_{\mathfrak{C}(n)}(G) \geq \frac{b}{a-1}.$$

Proof. Since G is a neutrosophic soft graph, for every $b_i, b_j \in V$ we have

$$0 \leq \mathfrak{A}_{S(n)}(b_i) \leq 1, \quad 0 \leq \mathfrak{B}_{S(n)}(b_i) \leq 1, \quad 0 \leq \mathfrak{C}_{S(n)}(b_i) \leq 1.$$

Hence,

$$\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j) \leq 1, \quad \mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j) \leq 1, \quad \mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j) \leq 1. \quad (1)$$

Since $|V| = a$, each vertex is adjacent to at most $a - 1$ vertices. Therefore,

$$\begin{aligned} \mathfrak{D}_{\mathfrak{A}(n)}(b_i) &= \sum_{b_i b_j \in E} \mathfrak{A}_{K(n)}(b_i b_j) \leq a - 1, \\ \mathfrak{D}_{\mathfrak{B}(n)}(b_i) &= \sum_{b_i b_j \in E} \mathfrak{B}_{K(n)}(b_i b_j) \leq a - 1, \\ \mathfrak{D}_{\mathfrak{C}(n)}(b_i) &= \sum_{b_i b_j \in E} \mathfrak{C}_{K(n)}(b_i b_j) \leq a - 1 \end{aligned}$$

for all $b_i \in V$. Consequently,

$$\begin{aligned} \mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j) &\leq (a - 1)^2, \\ \mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j) &\leq (a - 1)^2, \\ \mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j) &\leq (a - 1)^2 \end{aligned}$$

for all $b_i, b_j \in V$. Thus,

$$\begin{aligned} \mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j) &\leq (a - 1)^2, \\ \mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j) &\leq (a - 1)^2, \\ \mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j) &\leq (a - 1)^2. \end{aligned} \quad (2)$$

By taking reciprocals and square roots, from (2) we obtain

$$\begin{aligned} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j)}} &\geq \frac{1}{a - 1}, \\ \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j)}} &\geq \frac{1}{a - 1}, \end{aligned}$$

$$\frac{1}{\sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j)}} \geq \frac{1}{a-1}$$

for every edge $b_i b_j \in E$. Summing these inequalities over all b edges yields

$$\mathfrak{R}_{\mathfrak{A}(n)}(G) \geq \frac{b}{a-1}, \quad \mathfrak{R}_{\mathfrak{B}(n)}(G) \geq \frac{b}{a-1}, \quad \mathfrak{R}_{\mathfrak{C}(n)}(G) \geq \frac{b}{a-1}.$$

This completes the proof. \square

Example 3.4. Let G be the neutrosophic soft graph given in Example 3.2. Then

$$a = |V| = 4, \quad b = |E| = 5,$$

and

$$\mathfrak{R}_{\mathfrak{A}(n)}(G) = 81.89, \quad \mathfrak{R}_{\mathfrak{B}(n)}(G) = 12.63, \quad \mathfrak{R}_{\mathfrak{C}(n)}(G) = 8.61.$$

Moreover,

$$\frac{b}{a-1} = \frac{5}{3} \approx 1.67.$$

Therefore,

$$\mathfrak{R}_{\mathfrak{A}(n)}(G) > \frac{b}{a-1}, \quad \mathfrak{R}_{\mathfrak{B}(n)}(G) > \frac{b}{a-1}, \quad \mathfrak{R}_{\mathfrak{C}(n)}(G) > \frac{b}{a-1}.$$

Theorem 3.5. Let $G = (S, K)$ be a strong neutrosophic soft graph so that S is a constant function and $|V| = a$. Then, $\mathfrak{R}_{\mathfrak{A}(n)}(G) \leq \frac{a}{2v_{1(n)}^2}$, $\mathfrak{R}_{\mathfrak{B}(n)}(G) \leq \frac{a}{2v_{2(n)}^2}$ and $\mathfrak{R}_{\mathfrak{C}(n)}(G) \leq \frac{a}{2v_{3(n)}^2}$, where, $(v_{1(n)}, v_{2(n)}, v_{3(n)}) = (\mathfrak{A}_{S(n)}(b_i), \mathfrak{B}_{S(n)}(b_i), \mathfrak{C}_{S(n)}(b_i))$, $b_i \in V$.

Proof: Since a strong neutrosophic soft graph is generally a neutrosophic subgraph of the corresponding complete neutrosophic soft graph, it is simple to prove that $\mathfrak{R}_{\mathfrak{A}(n)}(G) \leq \frac{a}{2v_{1(n)}^2}$, $\mathfrak{R}_{\mathfrak{B}(n)}(G) \leq \frac{a}{2v_{2(n)}^2}$ and $\mathfrak{R}_{\mathfrak{C}(n)}(G) \leq \frac{a}{2v_{3(n)}^2}$.

Theorem 3.6. Let G be a complete neutrosophic soft graph so that S is a constant function. Then $\mathfrak{R}(G) = (\frac{a}{2v_{1(n)}^2}, \frac{a}{2v_{2(n)}^2}, \frac{a}{2v_{3(n)}^2})$, where $a = |V|$ and $(v_{1(n)}, v_{2(n)}, v_{3(n)}) = (\mathfrak{A}_{S(n)}(b_i), \mathfrak{B}_{S(n)}(b_i), \mathfrak{C}_{S(n)}(b_i))$, $b_i \in V$.

Proof: Since S is a constant and $\mathfrak{A}_{S(n)}(b_i) = v_{1(n)}$, $\mathfrak{B}_{S(n)}(b_i) = v_{2(n)}$, $\mathfrak{C}_{S(n)}(b_i) = v_{3(n)}$, $b_i \in V$. Since G is complete neutrosophic soft graph, $\mathfrak{A}_{K(n)}(b_i b_j) = \{\mathfrak{A}_{S(n)}(b_i) \wedge \mathfrak{A}_{S(n)}(b_j)\}$, $\mathfrak{B}_{K(n)}(b_i b_j) = \{\mathfrak{B}_{S(n)}(b_i) \wedge \mathfrak{B}_{S(n)}(b_j)\}$ and $\mathfrak{C}_{K(n)}(b_i b_j) = \{\mathfrak{C}_{S(n)}(b_i) \wedge \mathfrak{C}_{S(n)}(b_j)\}$, $\forall b_i b_j \in E$.

Since G is complete and $|V| = a$, there are $\frac{a(a-1)}{2}$ pairs of vertices and $\frac{a(a-1)}{2}$ edges. Additionally, each vertices in G is adjacent to $(a-1)$ vertices. Then,

$$\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = \sum_{n \in N} \sum_{b_i b_j \in E} \mathfrak{A}_{K(n)}(b_i b_j) = v_{1(n)} \cdot v_{1(n)} \cdots v_{1(n)}(a-1 \text{ times}) = (a-1)v_{1(n)}$$

$$\mathfrak{D}_{\mathfrak{B}(n)}(b_i) = \sum_{n \in N} \sum_{b_i b_j \in E} \mathfrak{B}_{K(n)}(b_i b_j) = v_{2(n)} \cdot v_{2(n)} \cdots v_{2(n)}(a-1 \text{ times}) = (a-1)v_{2(n)}$$

$$\mathfrak{D}_{\mathfrak{C}(n)}(b_i) = \sum_{n \in N} \sum_{b_i, b_j \in E} \mathfrak{C}_{K(n)}(b_i b_j) = v_{3(n)} \cdot v_{3(n)} \cdots v_{3(n)} (a - 1 \text{ times}) = (a - 1)v_{3(n)}$$

Therefore,

$$\begin{aligned} \mathfrak{R}_{\mathfrak{A}(n)}(G) &= \sum_{n \in N} \sum_{i \neq j, b_i, b_j \in E} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j)}} \\ &= \frac{a(a - 1)}{2} \cdot \frac{1}{\sqrt{(v_{1(n)}v_{1(n)}(a - 1)v_{1(n)}(a - 1)v_{1(n)})}} \\ &= \frac{a}{2v_{1(n)}^2} \end{aligned}$$

$$\begin{aligned} \mathfrak{R}_{\mathfrak{B}(n)}(G) &= \sum_{n \in N} \sum_{i \neq j, b_i, b_j \in E} \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j)}} \\ &= \frac{a(a - 1)}{2} \cdot \frac{1}{\sqrt{(v_{2(n)}v_{2(n)}(a - 1)v_{2(n)}(a - 1)v_{2(n)})}} \\ &= \frac{a}{2v_{2(n)}^2} \end{aligned}$$

$$\begin{aligned} \mathfrak{R}_{\mathfrak{C}(n)}(G) &= \sum_{n \in N} \sum_{i \neq j, b_i, b_j \in E} \frac{1}{\sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j)}} \\ &= \frac{a(a - 1)}{2} \cdot \frac{1}{\sqrt{(v_{3(n)}v_{3(n)}(a - 1)v_{3(n)}(a - 1)v_{3(n)})}} \\ &= \frac{a}{2v_{3(n)}^2} \end{aligned}$$

Hence, $\mathfrak{R}(G) = (\frac{a}{2v_{1(n)}^2}, \frac{a}{2v_{2(n)}^2}, \frac{a}{2v_{3(n)}^2})$.

Proposition 3.7. *If two neutrosophic soft graphs G and G' are isomorphic to each other, then, $\mathfrak{R}(G) = \mathfrak{R}(G')$*

Proof: Let $G = (S, K)$ and $G' = (S', K')$ be two isomorphic neutrosophic soft graphs. Then, there is a bijection $l : G \rightarrow G'$ so that

$$\begin{aligned} \mathfrak{A}_{S(n)}(b_i) &= \mathfrak{A}_{S'(n)}(l(b_i)) = \mathfrak{A}_{S(n)}(b'_i), \\ \mathfrak{B}_{S(n)}(b_i) &= \mathfrak{B}_{S'(n)}(l(b_i)) = \mathfrak{B}_{S(n)}(b'_i), \\ \mathfrak{C}_{S(n)}(b_i) &= \mathfrak{C}_{S'(n)}(l(b_i)) = \mathfrak{C}_{S(n)}(b'_i), \end{aligned}$$

$\forall b_i \in V$ and

$$\begin{aligned} \mathfrak{A}_{K(n)}(b_i b_j) &= \mathfrak{A}_{K'(n)}(l(b_i)l(b_j)) = \mathfrak{A}_{K(n)}(b'_i b'_j), \\ \mathfrak{B}_{K(n)}(b_i b_j) &= \mathfrak{B}_{K'(n)}(l(b_i)l(b_j)) = \mathfrak{B}_{K(n)}(b'_i b'_j), \end{aligned}$$

$$\mathfrak{C}_{K(n)}(b_i b_j) = \mathfrak{C}_{K'(n)}(l(b_i)l(b_j)) = \mathfrak{C}_{K(n)}(b'_i b'_j),$$

$\forall b_i b_j \in E$.

So, for each $b_i \in V$ there exists a vertex $b'_i \in V'$ so that $\mathfrak{D}(b_i) = \mathfrak{D}(b'_i)$.

Thus, we have the proof.

Example 3.8. Let G and G' be the neutrosophic soft graphs shown in Figure 2. Assume that there exists an isomorphism

$$\ell : V(G) \rightarrow V(G')$$

such that

$$\mathfrak{A}_{S'(n)}(\ell(b_i)) = \mathfrak{A}_{S(n)}(a_i), \quad \mathfrak{B}_{S'(n)}(\ell(b_i)) = \mathfrak{B}_{S(n)}(a_i), \quad \mathfrak{C}_{S'(n)}(\ell(b_i)) = \mathfrak{C}_{S(n)}(a_i),$$

and

$$\begin{aligned} \mathfrak{A}_{K'(n)}(\ell(b_i)\ell(b_j)) &= \mathfrak{A}_{K(n)}(a_i a_j), \\ \mathfrak{B}_{K'(n)}(\ell(b_i)\ell(b_j)) &= \mathfrak{B}_{K(n)}(a_i a_j), \\ \mathfrak{C}_{K'(n)}(\ell(b_i)\ell(b_j)) &= \mathfrak{C}_{K(n)}(a_i a_j), \end{aligned}$$

for all $1 \leq i, j \leq 3$.

Then

$$\mathfrak{R}_{\mathfrak{A}(n)}(G) = 25.000 + 35.355 + 35.355 = 95.710 = \mathfrak{R}_{\mathfrak{A}(n)}(G'),$$

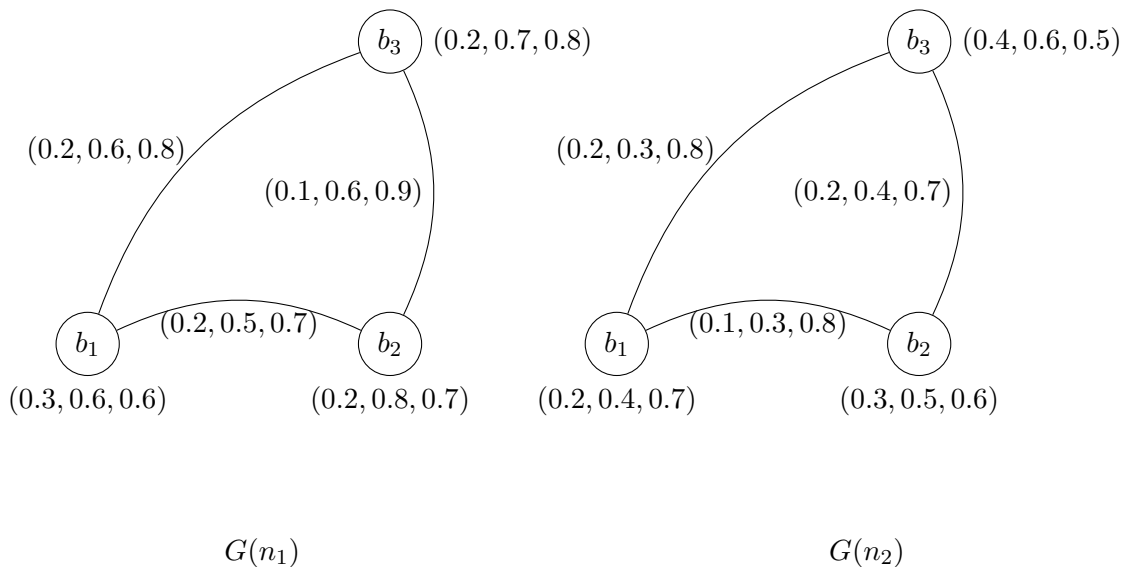
$$\mathfrak{R}_{\mathfrak{B}(n)}(G) = 1.207 + 1.304 + 1.102 = 3.613 = \mathfrak{R}_{\mathfrak{B}(n)}(G'),$$

and

$$\mathfrak{R}_{\mathfrak{C}(n)}(G) = 1.181 + 1.056 + 1.398 = 3.635 = \mathfrak{R}_{\mathfrak{C}(n)}(G').$$

Hence,

$$\mathfrak{R}(G) = \mathfrak{R}(G').$$



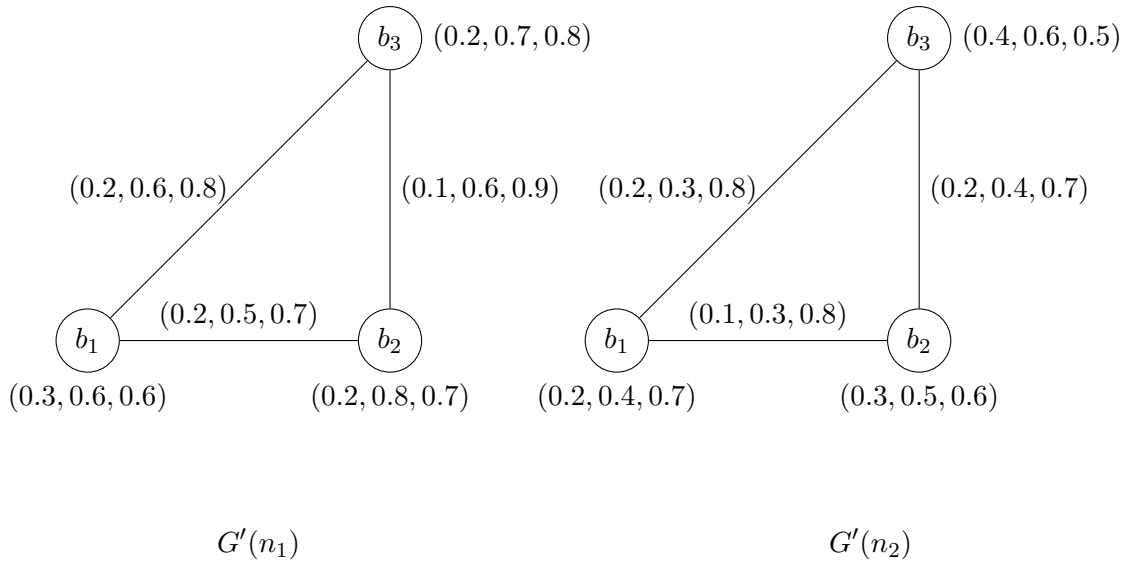


Figure 2. Two isomorphic neutrosophic soft graphs G and G' .

Theorem 3.9. Let $G = (S, K)$ be a neutrosophic soft graph, and fix $n \in N$. Assume that

$$\mathfrak{A}_{S(n)}(b_i) > 0, \quad \mathfrak{B}_{S(n)}(b_i) > 0, \quad \mathfrak{C}_{S(n)}(b_i) > 0 \quad (\forall b_i \in V),$$

and define

$$\begin{aligned} \mu_1 &:= \min_{b_i \in V} \mathfrak{D}_{\mathfrak{A}(n)}(b_i), & \mu_2 &:= \min_{b_i \in V} \mathfrak{D}_{\mathfrak{B}(n)}(b_i), & \mu_3 &:= \min_{b_i \in V} \mathfrak{D}_{\mathfrak{C}(n)}(b_i), \\ \eta_1 &:= \max_{b_i \in V} \mathfrak{D}_{\mathfrak{A}(n)}(b_i), & \eta_2 &:= \max_{b_i \in V} \mathfrak{D}_{\mathfrak{B}(n)}(b_i), & \eta_3 &:= \max_{b_i \in V} \mathfrak{D}_{\mathfrak{C}(n)}(b_i). \end{aligned}$$

Suppose moreover that $\mu_1, \mu_2, \mu_3 > 0$. Define

$$\begin{aligned} f_1 &:= \sum_{b_i, b_j \in E} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)}}, \\ f_2 &:= \sum_{b_i, b_j \in E} \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)}}, \\ f_3 &:= \sum_{b_i, b_j \in E} \frac{1}{\sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)}}. \end{aligned}$$

Then

$$\frac{f_1}{\eta_1} \leq \mathfrak{R}_{\mathfrak{A}(n)}(G) \leq \frac{f_1}{\mu_1}, \quad \frac{f_2}{\eta_2} \leq \mathfrak{R}_{\mathfrak{B}(n)}(G) \leq \frac{f_2}{\mu_2}, \quad \frac{f_3}{\eta_3} \leq \mathfrak{R}_{\mathfrak{C}(n)}(G) \leq \frac{f_3}{\mu_3}.$$

Proof. For each $b_i \in V$, the \mathfrak{A} -, \mathfrak{B} -, and \mathfrak{C} -degrees are

$$\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = \sum_{b_j \in E} \mathfrak{A}_{K(n)}(b_i b_j), \quad \mathfrak{D}_{\mathfrak{B}(n)}(b_i) = \sum_{b_j \in E} \mathfrak{B}_{K(n)}(b_i b_j), \quad \mathfrak{D}_{\mathfrak{C}(n)}(b_i) = \sum_{b_j \in E} \mathfrak{C}_{K(n)}(b_i b_j).$$

By the definitions of μ_r and η_r , we have

$$\mu_1 \leq \mathfrak{D}_{\mathfrak{A}(n)}(b_i) \leq \eta_1, \quad \mu_2 \leq \mathfrak{D}_{\mathfrak{B}(n)}(b_i) \leq \eta_2, \quad \mu_3 \leq \mathfrak{D}_{\mathfrak{C}(n)}(b_i) \leq \eta_3$$

for all $b_i \in V$. Hence, for every edge $b_i b_j \in E$,

$$\mu_1^2 \leq \mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j) \leq \eta_1^2,$$

$$\mu_2^2 \leq \mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j) \leq \eta_2^2,$$

$$\mu_3^2 \leq \mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j) \leq \eta_3^2.$$

Multiplying by the positive quantities $\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)$, $\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)$, and $\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)$, respectively, yields

$$\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mu_1^2 \leq \mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j) \leq \mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\eta_1^2,$$

$$\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mu_2^2 \leq \mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j) \leq \mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\eta_2^2,$$

$$\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mu_3^2 \leq \mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j) \leq \mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\eta_3^2.$$

Taking reciprocals and square roots reverses the inequalities, so for every edge $b_i b_j \in E$,

$$\frac{1}{\eta_1 \sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)}} \leq \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j)}} \leq \frac{1}{\mu_1 \sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)}},$$

$$\frac{1}{\eta_2 \sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)}} \leq \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j)}} \leq \frac{1}{\mu_2 \sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)}},$$

$$\frac{1}{\eta_3 \sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)}} \leq \frac{1}{\sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j)}} \leq \frac{1}{\mu_3 \sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)}}.$$

Summing over all edges gives the desired bounds. \square

Theorem 3.10. *Let $G = (S, K)$ be a neutrosophic soft graph with $|V| = a$, and fix $n \in N$.*

Assume that

$$\mathfrak{A}_{S(n)}(b_i) > 0, \quad \mathfrak{B}_{S(n)}(b_i) > 0, \quad \mathfrak{C}_{S(n)}(b_i) > 0 \quad (\forall b_i \in V).$$

Then

$$\mathfrak{R}_{\mathfrak{A}(n)}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)},$$

$$\mathfrak{R}_{\mathfrak{B}(n)}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)},$$

$$\mathfrak{R}_{\mathfrak{C}(n)}(G) \geq \frac{1}{a-1} \sum_{b_i b_j \in E} \frac{1}{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)}.$$

Proof. Since $|V| = a$, each vertex is incident with at most $a - 1$ edges. Because G is a neutrosophic soft graph, for every edge $b_i b_j \in E$ we have

$$\begin{aligned} \mathfrak{A}_{K(n)}(b_i b_j) &\leq \min\{\mathfrak{A}_{S(n)}(b_i), \mathfrak{A}_{S(n)}(b_j)\}, \\ \mathfrak{B}_{K(n)}(b_i b_j) &\leq \min\{\mathfrak{B}_{S(n)}(b_i), \mathfrak{B}_{S(n)}(b_j)\}, \\ \mathfrak{C}_{K(n)}(b_i b_j) &\leq \min\{\mathfrak{C}_{S(n)}(b_i), \mathfrak{C}_{S(n)}(b_j)\}. \end{aligned}$$

Hence, for each $b_i \in V$,

$$\begin{aligned} \mathfrak{D}_{\mathfrak{A}(n)}(b_i) &= \sum_{b_i b_j \in E} \mathfrak{A}_{K(n)}(b_i b_j) \leq (a - 1)\mathfrak{A}_{S(n)}(b_i), \\ \mathfrak{D}_{\mathfrak{B}(n)}(b_i) &= \sum_{b_i b_j \in E} \mathfrak{B}_{K(n)}(b_i b_j) \leq (a - 1)\mathfrak{B}_{S(n)}(b_i), \\ \mathfrak{D}_{\mathfrak{C}(n)}(b_i) &= \sum_{b_i b_j \in E} \mathfrak{C}_{K(n)}(b_i b_j) \leq (a - 1)\mathfrak{C}_{S(n)}(b_i). \end{aligned}$$

Therefore, for every edge $b_i b_j \in E$,

$$\begin{aligned} \mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j) &\leq (a - 1)^2\mathfrak{A}_{S(n)}(b_i)^2\mathfrak{A}_{S(n)}(b_j)^2, \\ \mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j) &\leq (a - 1)^2\mathfrak{B}_{S(n)}(b_i)^2\mathfrak{B}_{S(n)}(b_j)^2, \\ \mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j) &\leq (a - 1)^2\mathfrak{C}_{S(n)}(b_i)^2\mathfrak{C}_{S(n)}(b_j)^2. \end{aligned}$$

Taking reciprocals and square roots, we obtain

$$\begin{aligned} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j)}} &\geq \frac{1}{(a - 1)\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)}, \\ \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{B}(n)}(b_i)\mathfrak{D}_{\mathfrak{B}(n)}(b_j)}} &\geq \frac{1}{(a - 1)\mathfrak{B}_{S(n)}(b_i)\mathfrak{B}_{S(n)}(b_j)}, \\ \frac{1}{\sqrt{\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{C}(n)}(b_i)\mathfrak{D}_{\mathfrak{C}(n)}(b_j)}} &\geq \frac{1}{(a - 1)\mathfrak{C}_{S(n)}(b_i)\mathfrak{C}_{S(n)}(b_j)}. \end{aligned}$$

Summing over all edges completes the proof. \square

Theorem 3.11. *Let $G = (S, K)$ be a complete, regular, and totally regular neutrosophic soft graph on $|V| = a$ vertices, and fix $n \in N$. Assume that there exist positive constants*

$$x_{1(n)}, x_{2(n)}, x_{3(n)}, h_{1(n)}, h_{2(n)}, h_{3(n)}$$

such that, for every $b_i \in V$,

$$\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = x_{1(n)}, \quad \mathfrak{D}_{\mathfrak{B}(n)}(b_i) = x_{2(n)}, \quad \mathfrak{D}_{\mathfrak{C}(n)}(b_i) = x_{3(n)},$$

and

$$t\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = h_{1(n)}, \quad t\mathfrak{D}_{\mathfrak{B}(n)}(b_i) = h_{2(n)}, \quad t\mathfrak{D}_{\mathfrak{C}(n)}(b_i) = h_{3(n)}.$$

Set

$$v_{1(n)} := h_{1(n)} - x_{1(n)}, \quad v_{2(n)} := h_{2(n)} - x_{2(n)}, \quad v_{3(n)} := h_{3(n)} - x_{3(n)}.$$

Then

$$\mathfrak{R}(G) = (\mathfrak{R}_{\mathfrak{A}(n)}(G), \mathfrak{R}_{\mathfrak{B}(n)}(G), \mathfrak{R}_{\mathfrak{C}(n)}(G)) = \binom{a}{2} \left(\frac{1}{v_{1(n)}x_{1(n)}}, \frac{1}{v_{2(n)}x_{2(n)}}, \frac{1}{v_{3(n)}x_{3(n)}} \right).$$

Proof. Since G is regular, the three degree functions are constant on V , namely,

$$\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = x_{1(n)}, \quad \mathfrak{D}_{\mathfrak{B}(n)}(b_i) = x_{2(n)}, \quad \mathfrak{D}_{\mathfrak{C}(n)}(b_i) = x_{3(n)} \quad (\forall b_i \in V).$$

Since G is totally regular, the total degrees are also constant:

$$t\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = h_{1(n)}, \quad t\mathfrak{D}_{\mathfrak{B}(n)}(b_i) = h_{2(n)}, \quad t\mathfrak{D}_{\mathfrak{C}(n)}(b_i) = h_{3(n)} \quad (\forall b_i \in V).$$

By definition of total degree,

$$t\mathfrak{D}_{\mathfrak{A}(n)}(b_i) = \mathfrak{D}_{\mathfrak{A}(n)}(b_i) + \mathfrak{A}_{S(n)}(b_i),$$

$$t\mathfrak{D}_{\mathfrak{B}(n)}(b_i) = \mathfrak{D}_{\mathfrak{B}(n)}(b_i) + \mathfrak{B}_{S(n)}(b_i),$$

$$t\mathfrak{D}_{\mathfrak{C}(n)}(b_i) = \mathfrak{D}_{\mathfrak{C}(n)}(b_i) + \mathfrak{C}_{S(n)}(b_i).$$

Therefore,

$$\mathfrak{A}_{S(n)}(b_i) = h_{1(n)} - x_{1(n)} = v_{1(n)},$$

$$\mathfrak{B}_{S(n)}(b_i) = h_{2(n)} - x_{2(n)} = v_{2(n)},$$

$$\mathfrak{C}_{S(n)}(b_i) = h_{3(n)} - x_{3(n)} = v_{3(n)}$$

for all $b_i \in V$. Thus the vertex-membership functions are constant.

Because G is complete on a vertices, the number of edges is

$$|E| = \binom{a}{2} = \frac{a(a-1)}{2}.$$

Moreover, every edge $b_i b_j \in E$ contributes the same quantity to each component of the Randic index. Hence

$$\begin{aligned} \mathfrak{R}_{\mathfrak{A}(n)}(G) &= \sum_{b_i b_j \in E} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(b_i)\mathfrak{A}_{S(n)}(b_j)\mathfrak{D}_{\mathfrak{A}(n)}(b_i)\mathfrak{D}_{\mathfrak{A}(n)}(b_j)}} \\ &= \binom{a}{2} \frac{1}{\sqrt{v_{1(n)}v_{1(n)}x_{1(n)}x_{1(n)}}} = \binom{a}{2} \frac{1}{v_{1(n)}x_{1(n)}}, \end{aligned}$$

and similarly,

$$\mathfrak{R}_{\mathfrak{B}(n)}(G) = \binom{a}{2} \frac{1}{v_{2(n)}x_{2(n)}}, \quad \mathfrak{R}_{\mathfrak{C}(n)}(G) = \binom{a}{2} \frac{1}{v_{3(n)}x_{3(n)}}.$$

Combining these equalities gives the result. \square

4. Application of the Randic Index in Construction

A government organization plans to build a waste processing facility in a region that contains a number of medium-sized cities. The ideal site should minimize logistical and environmental issues, have existing infrastructure (partial facilities, land, staff), be easily accessible to nearby cities for effective waste transportation, and have administrative and public support.

The example shows how to use the Randic Index and a neutrosophic soft graph to choose the ideal city for a waste management plant. The selection process takes into account a variety of parameters, including public support, infrastructure availability, road connection and environmental concerns, which are represented as nodes (cities) and edges (roads) with neutrosophic weights signifying a truth, indeterminacy and falsity-membership function. Based on their population statistics, cities like C_1 , C_2 , C_3 , and C_4 are evaluated using imprecise truth, indeterminacy, and falsity-membership ratings. The Randic Index, a graph-theoretic metric, is used to determine node centrality and significance.

Table 1: Weight of vertices in G

G	City 1	City 2	City 3	City 4
$(\mathfrak{A}_{S(1)}, \mathfrak{B}_{S(1)}, \mathfrak{C}_{S(1)})$ Road connection	(0.2,0.6,0.7)	(0.4,0.8,0.5)	(0.3,0.5,0.6)	(0.3,0.7,0.6)
$(\mathfrak{A}_{S(2)}, \mathfrak{B}_{S(2)}, \mathfrak{C}_{S(2)})$ Environment	(0.3,0.4,0.6)	(0.3,0.5,0.7)	(0.4,0.4,0.6)	(0.2,0.3,0.8)

Table 2: Weight of edges in G

G	$C_1 - C_2$	$C_2 - C_3$	$C_3 - C_4$	$C_1 - C_4$	$C_1 - C_3$	$C_2 - C_4$
$(\mathfrak{A}_{K(1)}, \mathfrak{B}_{K(1)}, \mathfrak{C}_{K(1)})$	(0.2,0.6,0.8)	(0.3,0.5,0.6)	(0.2,0.4,0.7)	(0.1,0.5,0.9)	(0.1,0.4,0.8)	(0.2,0.6,0.7)
$(\mathfrak{A}_{K(2)}, \mathfrak{B}_{K(2)}, \mathfrak{C}_{K(2)})$	(0.2,0.3,0.8)	(0.3,0.4,0.7)	(0.2,0.3,0.8)	(0.1,0.2,0.8)	(0.3,0.4,0.7)	(0.2,0.3,0.9)

Table 3: The population of cities

G	City 1	City 2	City 3	City 4
Population	172567	92566	132890	159463

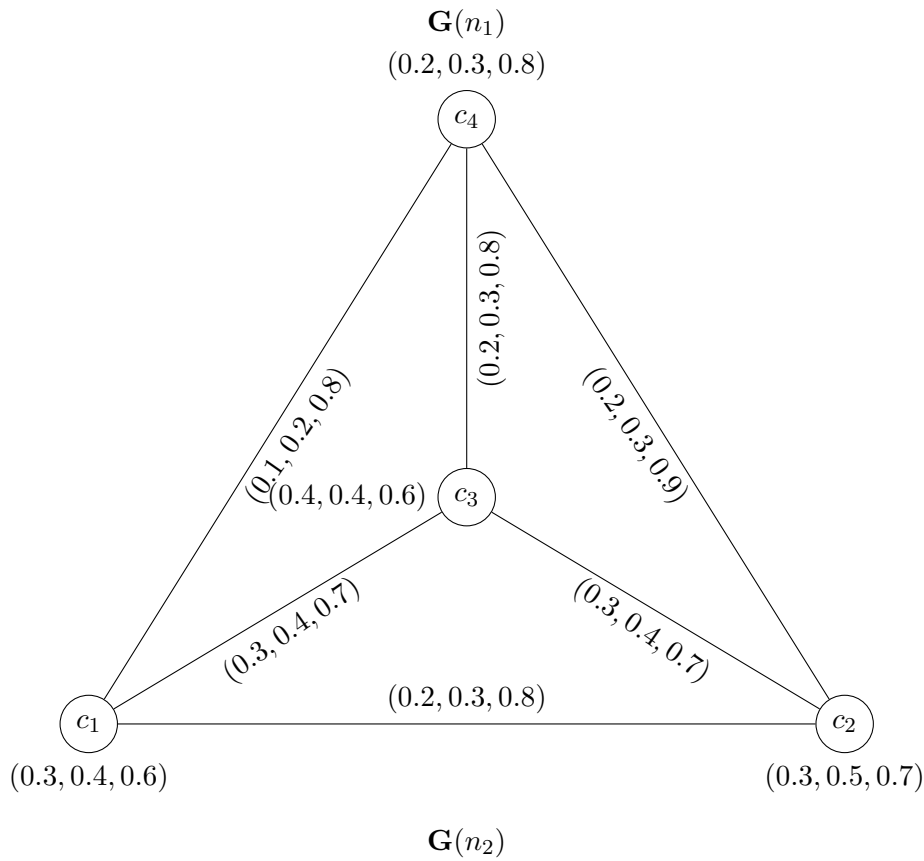
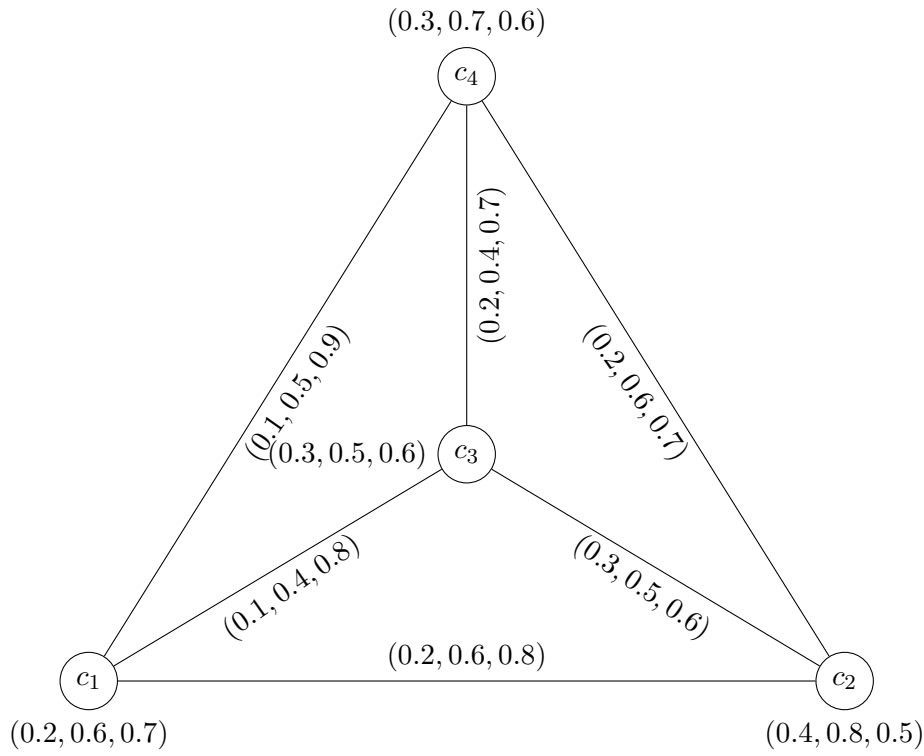


Figure 3: Neutrosophic soft graph G

Let

$$\mathfrak{D}_{n_1}(c_1) = (0.4, 1.5, 2.5), \quad \mathfrak{D}_{n_1}(c_2) = (0.7, 1.7, 2.1),$$

$$\mathfrak{D}_{n_1}(c_3) = (0.6, 1.3, 2.1), \quad \mathfrak{D}_{n_1}(c_4) = (0.5, 1.8, 2.2),$$

and

$$\mathfrak{D}_{n_2}(c_1) = (0.6, 0.9, 2.3), \quad \mathfrak{D}_{n_2}(c_2) = (0.7, 1.0, 2.4),$$

$$\mathfrak{D}_{n_2}(c_3) = (0.8, 1.1, 2.2), \quad \mathfrak{D}_{n_2}(c_4) = (0.5, 0.8, 2.5).$$

Then

$$\begin{aligned} \sum_{n \in N} \sum_{\substack{i \neq j \\ c_i c_j \in E}} \frac{1}{\sqrt{\mathfrak{A}_{S(n)}(c_i) \mathfrak{A}_{S(n)}(c_j) \mathfrak{D}_{\mathfrak{A}(n)}(c_i) \mathfrak{D}_{\mathfrak{A}(n)}(c_j)}} &= (6.68 + 6.30 + 6.09 + 9.13 + 8.33 + 4.88) \\ &+ (5.14 + 3.86 + 5.59 + 7.45 + 4.17 + 6.90) \\ &= 74.52, \end{aligned}$$

$$\begin{aligned} \sum_{n \in N} \sum_{\substack{i \neq j \\ c_i c_j \in E}} \frac{1}{\sqrt{\mathfrak{B}_{S(n)}(c_i) \mathfrak{B}_{S(n)}(c_j) \mathfrak{D}_{\mathfrak{B}(n)}(c_i) \mathfrak{D}_{\mathfrak{B}(n)}(c_j)}} &= (0.90 + 1.06 + 1.11 + 0.94 + 1.31 + 0.76) \\ &+ (2.36 + 2.13 + 3.08 + 3.40 + 2.51 + 2.89) \\ &= 22.45, \end{aligned}$$

and

$$\begin{aligned} \sum_{n \in N} \sum_{\substack{i \neq j \\ c_i c_j \in E}} \frac{1}{\sqrt{\mathfrak{C}_{S(n)}(c_i) \mathfrak{C}_{S(n)}(c_j) \mathfrak{D}_{\mathfrak{C}(n)}(c_i) \mathfrak{D}_{\mathfrak{C}(n)}(c_j)}} &= (0.74 + 0.87 + 0.78 + 0.66 + 0.73 + 0.85) \\ &+ (0.66 + 0.67 + 0.62 + 0.60 + 0.74 + 0.55) \\ &= 8.47. \end{aligned}$$

Hence,

$$\mathfrak{R}(G) = (\mathfrak{R}_{\mathfrak{A}(n)}(G), \mathfrak{R}_{\mathfrak{B}(n)}(G), \mathfrak{R}_{\mathfrak{C}(n)}(G)) = (74.52, 22.45, 8.47).$$

Further,

$$\mathfrak{R}_{\mathfrak{A}(n)}(G - c_1) = 74.52 - 5.14 - 7.45 - 4.17 - 6.68 - 9.13 - 8.33 = 33.62,$$

$$\mathfrak{R}_{\mathfrak{A}(n)}(G - c_2) = 74.52 - 6.68 - 6.30 - 4.88 - 5.14 - 3.86 - 6.90 = 40.76,$$

$$\mathfrak{R}_{\mathfrak{A}(n)}(G - c_3) = 74.52 - 6.30 - 6.09 - 8.33 - 3.86 - 5.59 - 4.17 = 40.18,$$

$$\mathfrak{R}_{\mathfrak{A}(n)}(G - c_4) = 74.52 - 6.09 - 9.13 - 4.88 - 5.59 - 7.45 - 6.90 = 34.48.$$

$$\mathfrak{R}_{\mathfrak{B}(n)}(G - c_1) = 22.45 - 0.90 - 0.94 - 1.31 - 2.36 - 3.40 - 2.51 = 11.03,$$

$$\mathfrak{R}_{\mathfrak{B}(n)}(G - c_2) = 22.45 - 0.90 - 1.06 - 0.76 - 2.36 - 2.13 - 2.89 = 12.35,$$

$$\mathfrak{R}_{\mathfrak{B}(n)}(G - c_3) = 22.45 - 1.06 - 1.11 - 1.31 - 2.13 - 3.08 - 2.51 = 11.25,$$

$$\mathfrak{R}_{\mathfrak{B}(n)}(G - c_4) = 22.45 - 1.11 - 0.94 - 0.76 - 3.08 - 3.40 - 2.89 = 10.27.$$

$$\mathfrak{R}_{\mathfrak{C}(n)}(G - c_1) = 8.47 - 0.74 - 0.66 - 0.73 - 0.66 - 0.60 - 0.74 = 4.34,$$

$$\mathfrak{R}_{\mathfrak{C}(n)}(G - c_2) = 8.47 - 0.74 - 0.87 - 0.85 - 0.66 - 0.67 - 0.55 = 4.13,$$

$$\mathfrak{R}_{\mathfrak{C}(n)}(G - c_3) = 8.47 - 0.87 - 0.78 - 0.73 - 0.67 - 0.62 - 0.74 = 4.06,$$

$$\mathfrak{R}_{\mathfrak{C}(n)}(G - c_4) = 8.47 - 0.78 - 0.66 - 0.85 - 0.62 - 0.60 - 0.55 = 4.41.$$

The calculation above indicates that avoiding nodes C_1 , C_2 , C_3 , and C_4 is inappropriate for building a waste processing facility. City C_1 has the largest population and the greatest number of transportation routes, making it the most crucial and vital factor in maintaining a high Randic Index. City C_1 is the ideal location for the waste management facility.

5. Conclusion

Fuzzy graphs are less accurate, less adaptable, and less compatible than neutrosophic soft graphs. In social networks, neutrosophic soft graphs are frequently used to find the most productive members of a team or organization. The Connectivity Index has numerous uses in psychology, medical science, social groupings, and computer networks. This work presents Randic indices for neutrosophic soft graphs and subgraphs, along with their properties. Using certain isomorphic properties, a truth, indeterminacy, and falsity-membership function of the Randic Index of neutrosophic soft graphs is examined. Eccentricity index and Wiener index for neutrosophic incidence graphs are two things we intend to investigate further. It is also hoped that further research will advance extensions based on HyperGraphs [14, 15] and SuperHyperGraphs [16, 17, 26, 27].

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Use of Generative AI and AI-Assisted Tools

Generative AI and AI-assisted tools were used for tasks such as English grammar checking, and they were not used in any way that violates ethical standards.

Supplementary Information

No supplementary materials accompany this paper.

Disclaimer

The ideas presented here are theoretical and have not yet been validated through empirical testing. While we have strived for accuracy and proper citation, inadvertent errors may remain. Readers should verify any referenced material independently. The opinions expressed are those of the authors and do not necessarily reflect the views of their institutions.

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