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Design of the Bartlett and Hartley tests for homogeneity of variances under indeterminacy environment

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ABSTRACT
The existing Bartlett's test and Hartley's test under classical statistics can be applied only when all observations in the sample are determined, precise and determinate. In some complex situations, it may not possible to measure the exact observations. In this case, the neutrosophic statistics is applied for the decision. In this paper, we present Bartlett's test and Hartley's test under the neutrosophic statistics. We present the designing for the proposed tests under neutrosophic statistical interval method. We present an example and compare the proposed neutrosophic Bartlett's test and Hartley's test over the existing tests under classical statistics. From the comparative study, we conclude that the proposed tests are quite effective, informative and flexible to be applied under the indeterminate environment.

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1. Introduction

The homogeneity of variances is required for the testing the performance in a verity of fields such as educational methods, manufacturing processes, textile industry, agricultural production system and ecology [1]. In classical statistics, analysis of variance (ANONA) and regression analysis is done on the assumption that the population is normal and samples from various methods have equal variance. Bartlett's test and Hartley's test under classical statistics have been widely for the testing of the equality of variances in a variety of fields, see for example [1–7]. More application of statistical tests can be seen in [8–10].

The existing Bartlett's test and Hartley's test cannot be applied for the testing of the homogeneity of the population variances when some observations are fuzzy and unclear. In this case, the testing is done using Bartlett's test and Hartley's test under the fuzzy logic. Wu [11] discussed the analysis of variance for fuzzy data. Kruse et al. [12] provided testing of variances procedure under the fuzzy logic. Ramos-Guajardo et al. [13] provided the testing procedure of equal variances for fuzzy random data. More details can be seen in [14].

Smarandache [15] mentioned that neutrosophic logic which considers the measure of indeterminacy is the generalization of traditional fuzzy logic. More details on the neutrosophic can be seen in [16–21]. Based on the neutrosophic logic, Smarandache [22] gave the idea of neutrosophic statistics. The neutrosophic statistics, which is, analysed the neutrosophic numbers is the generalization of classical statistics, which deals with the determined numbers [22,23]. The neutrosophic statistics is an alternative to classical statistics is applied in the presence of Neutrosophy in the sample. The statistics based on neutrosophic numbers is more effective and adequate than classical statistics under uncertainty, see for example [24,25]. The neutrosophic statistics perform better than classical statistics in control charts and inspection schemes, see for example [26,27]. Recently, Aslam [28] developed the neutrosophic analysis of variance (NANOVA) for testing of several population means. More information on neutrosophic logic can be seen in [29–31].

Neutrosophic numbers are recorded when the sample is selected from the population having uncertain observations or parameters. The existing Bartlett's test and Hartley's test under classical statistics and fuzzy logic cannot be applied for testing the homogeneity of variances of the data when observations are neutrosophic numbers, in interval or uncertain. According to the best of our knowledge, there is no work on Bartlett's test and Hartley's test under neutrosophic statistics. In this paper, we will develop Bartlett's test and Hartley's test under neutrosophic statistics for testing the homogeneity for variances. We will present the designing and decision criteria of these tests in the presence of Neutrosophy. We will present an example with the expectation that proposed tests are more effective, adequate, and flexible and information under uncertainty environment.

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2. Preliminaries

Let X be a random variable having determined values under classical statistics which is drawn from the normal distribution with mean μ and variance σ^2. Let uI; $I\epsilon[inf, sup]$ be an indeterminate part. Suppose that $X_N = X + uI; X_N\epsilon[inf, sup]$ be neutrosophic random variable having neutrosophic numbers. Note here that $uI;I\epsilon$ is an indeterminate part and X is a determinate part of $X_N\epsilon[X_L, X_U]$, where X_L and X_U denote the lower and upper values of indeterminacy interval. We assumed that $X_N\epsilon[X_L, X_U]$ is drawn from the neutrosophic normal distribution with a neutrosophic population mean $\mu_N\epsilon[\mu_L, \mu_U]$ and neutrosophic population variance $\sigma_N^2\epsilon[\sigma_L^2, \sigma_U^2]$, see [22,23]. In practice, $\mu_N\epsilon[\mu_L, \mu_U]$ and $\sigma_N^2\epsilon[\sigma_L^2, \sigma_U^2]$ are unknown and estimated based on sample size $n_N\epsilon[n_L, n_U]$ using the neutrosophic sample mean $\bar{X}_N\epsilon\left[\bar{X}_L, \bar{X}_U\right]$ and neutrosophic sample variance $s_N^2\epsilon[s_L^2, s_U^2]$, where $\bar{X}_N = \sum_{i=1}^{n_N} X_N/n_N$ and $s_N^2 = \sum_{i=1}^{n_N} (X - \bar{X}_N)^2/(n_N - 1)$.

3. The proposed neutrosophic Bartlett's test

Suppose there are k normally distributed populations and it is required to test either the variances are k populations are homogenous or not. The Bartlett's test is one of the popular tests in classical statistics for the testing of equality of variances. This test cannot be applied when the populations have neutrosophic numbers. In this section, we extend Bartlett's test under classical statistics using the neutrosophic statistics. Like the existing test, it is assumed that $X_N\epsilon[X_L, X_U]$ follows the neutrosophic normal distribution. Let $s_{jN}^2\epsilon[s_{jL}^2, s_{jU}^2]$ presents neutrosophic sample variance $n_{jN}\epsilon[n_{jL}, n_{jU}]$ items from jth neutrosophic population ($j = 1, 2, 3, \ldots, k_N$). The neutrosophic pooled sample variance for k_N populations is defined as

$$s_{pN}^2 = \frac{\sum_{j=1}^{k_N}(n_{jN} - 1).s_{jN}^2}{\sum_{j=1}^{k_N}(n_{jN} - 1)}; s_{jN}^2\epsilon[s_{jL}^2, s_{jU}^2], \times n_{jN}\epsilon[n_{jL}, n_{jU}],\ s_{pN}^2\epsilon[s_{pL}^2, s_{pU}^2]. \quad (1)$$

The test statistic for testing the null hypothesis $H_{0N}: \sigma_{1N}^2 = \sigma_{2N}^2 = \ldots = \sigma_{kN}^2$ versus the alternative hypothesis that at least one population variance is different. The statistics under neutrosophic statistical interval method (NSIM) is defined by

$$B_N = \frac{[2.30259, 2.30259]}{C_N}\left[\left(\sum_{j=1}^{k_N}(n_{jN} - 1).\log s_{pN}^2\right) - \sum_{j=1}^{k_N}(n_{jN} - 1)\log s_{jN}^2\right]; B_N\epsilon[B_L, B_U], \quad (2)$$

where

$$C_N = 1 + \frac{1}{3(k_N)}\left[\sum_{j=1}^{k_N}\frac{1}{(n_{jN} - 1)} - \frac{1}{\sum_{j=1}^{k_N}(n_{jN} - 1)}\right]; \times n_{jN}\epsilon[n_{jL}, n_{jU}];\ C_N\epsilon[C_L, C_U]. \quad (3)$$

By following [32], when $n_{jN} > [6, 6]$, the statistic $B_N\epsilon[B_L, B_U]$ follows the neutrosophic Chi-square distribution with $k_N - 1$ neutrosophic degree of freedom. The $H_{0N}: \sigma_{1N}^2 = \sigma_{2N}^2 = \ldots = \sigma_{kN}^2$ is rejected if $B_N\epsilon[B_L, B_U] > c$, where c is tabulated value. On the other hand, when $n_{jN} \leq [6, 6]$, the neutrosophic statistic $B_N C_N = M_N; B_N\epsilon[B_L, B_U], C_N\epsilon[C_L, C_U], M_N\epsilon[M_L, M_U]$ exceed the table.

4. The proposed neutrosophic Hartley's test

In classical statistics, the purpose of Hartley's test is to test the hypothesis of equal variances when the sample is drawn from the normal distribution. In this section, we generalize the existing Hartley's test under neutrosophic statistics under the assumption that $X_N\epsilon[X_L, X_U]$ is drawn from a neutrosophic normal population having equal $n_{jN}\epsilon[n_{jL}, n_{jU}]$. Let $s_{\max N}^2; s_{\max N}^2\epsilon[s_{\max L}^2, s_{\max U}^2]$ and $s_{\min N}^2; s_{\min N}^2\epsilon[s_{\min L}^2, s_{\min U}^2]$ be the largest and smaller sample variances of the k_N population. The proposed Neutrosophic Hartley's test is defined by

$$F_{\max N} = \frac{s_{\max N}^2}{s_{\min N}^2}; s_{\max N}^2\epsilon[s_{\max L}^2, s_{\max U}^2]; \times s_{\min N}^2\epsilon[s_{\min L}^2, s_{\min U}^2];\ F_{\max N}\epsilon[F_{\max L}, F_{\max U}]. \quad (4)$$

The $H_{0N}: \sigma_{1N}^2 = \sigma_{2N}^2 = \ldots = \sigma_{kN}^2$ is will rejected in favour of an alternative hypothesis if $F_{\max N}\epsilon[F_{\max L}, F_{\max U}]$ exceeds the critical value selected from the table [32].

5. Applications of the proposed tests

A reputed textile mill is located in Faisalabad; Pakistan is weaving the cloth using the looms. Due to some external factors such as the electrical or gas load shedding, the mill is concern about the strength of the cloth that is producing. The quality control engineers want to test either there is a significant variation in cloth strength from various looms. For this testing, they selected five looms at random and measured the strength of cloths produced by them in a unit of measure momme. During the strength measurement, the quality control engineers found that some observations are not determined and they have to choose the approximate value or note them in an interval. The data is shown in Table 1. The data in Table 1 reduces to data given in (www.math.montana.edu > jobo) if no uncertainty is recorded. From Table 1, we note that several observations have Neutrosophy and cannot be analysed using

Table 1. The neutrosophic strength data.

Looms				
1	2	3	4	5
[14.0,14.1]	[13.9,13.9]	[14.1,13.7]	[13.6,13.8]	[13.8,13.9]
[14.1,14.3]	[13.8,13.9]	[14.2,13.8]	[13.8,13.8]	[13.6,13.8]
[14.2,14.2]	[13.9,13.9]	[14.1,14.2]	[14.0,14.0]	[13.9,14.0]
[14.0,14.1]	[14.0,14.2]	[14.0,13.9]	[13.9,14.0]	[13.8,13.9]
[14.1,14.3]	[14.0,14.1]	[13.9,13.9]	[13.7,13.7]	[14.0,14.2]

Table 2. Neutrosophic descriptive statistics.

$\bar{X}_N$	[14.08,14.20]	[13.92,14.0]	[14.06,13.90]	[13.80,13.86]	[13.82,13.96]
s_{jN}^2	[0.007,0.01]	[0.007,0.02]	[0.013,0.035]	[0.025,0.018]	[0.022,0.023]

existing Bartlett's test and Hartley's test under classical statistics. To test the variations in cloth strength, we will apply the proposed neutrosophic Bartlett's test and neutrosophic Hartley's test.

5.1. Application of proposed neutrosophic Bartlett's test

The testing procedure of the proposed test is given as

Step 1: We state $H_{0N}:\sigma_{1N}^2=\sigma_{2N}^2=\ldots=\sigma_{kN}^2$ Vs the alternative hypothesis that at least one neutrosophic population variance is significant.

Step 2: Let the level of significance for the test is 5%.

Step 3: the necessary neutrosophic descriptive statistics are given in Table 2

The values of $s_{pN}^2\epsilon[s_{pL}^2,s_{pU}^2]$, $B_N\epsilon[B_L,B_U]$ and $C_N\epsilon[C_L,C_U]$ are given by

$$s_{pN}^2=\frac{\sum_{j=1}^{[5,5]}(n_{jN}-1).s_{jN}^2}{\sum_{j=1}^{[5,5]}(n_{jN}-1)};s_{pN}^2\epsilon[0.296,0.424],$$

$$B_N=\frac{[2.30259,2.30259]}{C_N}\left[\left(\sum_{j=1}^{[5,5]}(n_{jN}-1).\log s_{pN}^2\right)-\sum_{j=1}^{k_N}(n_{jN}-1)\log s_{jN}^2\right];B_N\epsilon[6.8375,5.6519]$$

and

$$C_N=1+\frac{1}{3([5,5])}\left[\sum_{j=1}^{[5,5]}\frac{1}{(n_{jN}-1)}-\frac{1}{\sum_{j=1}^{[5,5]}(n_{jN}-1)}\right];\times C_N\epsilon[6.8375,5.65].$$

Step 4: As $n_{jN}\le[6,6]$, the statistics test is $M_N\epsilon[7.28,6.02]$.

Step 5: The table value from [32] is 2. We will reject $H_{0N}:\sigma_{1N}^2=\sigma_{2N}^2=\ldots=\sigma_{kN}^2$ and conclude that at least one population variance is significantly different.

5.2. Application of the proposed neutrosophic Hartley's test

The testing procedure for this test is given as

Step 1: We state $H_{0N}:\sigma_{1N}^2=\sigma_{2N}^2=\ldots=\sigma_{kN}^2$ Vs the alternative hypothesis that at least one neutrosophic population variance is significant.

Step 2: Let the level of significance for the test is 5%.

Step 3: The proposed statistic is computed as $F_{\max N}=\frac{s_{\max N}^2}{s_{\min N}^2}=\frac{[0.007,0.01]}{[0.025,0.018]};F_{\max N}\epsilon[3.57,1.8]$.

Step 4: The table value from [32] is 2.61. By [22], as critical value is between 1.8 and 3.57. Therefore, there is indeterminacy about the rejection of $H_{0N}:\sigma_{1N}^2=\sigma_{2N}^2=\ldots=\sigma_{kN}^2$.

6. Discussion and comparison

As mentioned earlier, the proposed tests are the generalization of the existing Bartlett's test and Hartley's test. As mentioned by [24,25], a method having the data analysis values in an indeterminacy interval under uncertainty environment than the determined values under classical statistics is said be more effective and adequate to be used in uncertainty. From the analysis given in the last section, we can see that the proposed tests provide the values in indeterminacy intervals. For example, the values of $M_N\epsilon[7.28,6.02]$ and $F_{\max N}\epsilon[3.57,1.8]$ are in the indeterminacy interval. The statistics are reduced to existing tests under classical statistics when all the observations are determined. When we compare the results under the neutrosophic statistics with existing statistics, we found that values of M_N can be from 6.02 to 7.28. The existing test statistic under classical statistics provides only the determined value which is 7.28. On the other hand, the values of $F_{\max N}$ will be from 1.8 to 3.57 while it is 3.57 for the existing test under classical statistics. From this comparison, it is concluded that the theory of the proposed tests coincides with the theory given by [24,25].

7. Concluding remarks

In a complex system, it may not possible that the measured sample has determined and precise observations. To deal with the indeterminacy, the neutrosophic statistics is the best alternative. In this paper, we presented Bartlett's test and Hartley's test under the neutrosophic statistics. We presented the necessary calculations of both tests for under neutrosophic statistical interval method. By comparing the proposed tests with the test under classical statistics, it is found that the proposed tests provide the results in indeterminacy intervals which are required for analysing the data is obtained from the complex systems. We conclude that the proposed tests are more adequate, effective and informative to be applied under an uncertainty environment. We recommend the use of the proposed tests in biostatistics, medical sciences, educational methods, manufacturing processes, textile industry, agricultural production system, and ecology. The other homogeneity tests of variances can be considered future research. The efficiency of the proposed tests using simulation can be a good area for future research.

NSIM: neutrosophic statistical interval method
ANONA: analysis of variance
$n_N \epsilon [n_L, n_U]$: Neutrosophic sample size
μ: mean under classical statistics
σ^2: Variance under classical statistic
X_N: Neutrosophic random variable
X_L: Lower value of indeterminacy interval
X_U: Upper value of indeterminacy interval
$\mu_N \epsilon [\mu_L, \mu_U]$: Neutrosophic population mean
$\sigma_N^2 \epsilon [\sigma_L^2, \sigma_U^2]$: Neutrosophic population variance
$\overline{X}_N \epsilon [\overline{X}_L, \overline{X}_U]$: Neutrosophic sample mean
$s_N^2 \epsilon [s_L^2, s_U^2]$: Neutrosophic sample variance
$s_{pN}^2 \epsilon [s_{pL}^2, s_{pU}^2]$: Neutrosophic pooled sample variance
$B_N \epsilon [B_L, B_U]$: Neutrosophic Bartlett's statistic
$F_{\max N} \epsilon [F_{\max L}, F_{\max U}]$: Neutrosophic Hartley's statistic

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References

[1] Jayalath KP, Ng HKT, Manage AB, et al. Improved tests for homogeneity of variances. Commun Stat-Simul Comput. 2017;46:7423–7446.
[2] Conover WJ, Johnson ME, Johnson MM. A comparative study of tests for homogeneity of variances, with applications to the outer continental shelf bidding data. Technometrics. 1981;23:351–361.
[3] Madsen J, Tabanera R. Bartlett's test applied to variance component models. Scand J Stat. 2003;30:609–616.
[4] Gastwirth JL, Gel YR, Miao W. The impact of Levene's test of equality of variances on statistical theory and practice. Stat Sci. 2009;24:343–360.
[5] Ma X-B, Lin F-C, Zhao Y. An adjustment to the Bartlett's test for small sample size. Commun Stat-Simul Comput. 2015;44:257–269.
[6] El-Shanshoury H, El-Shanshoury G, Abaza A. Evaluation of low dose ionizing radiation effect on some blood components in animal model. J Radiat Res Appl Sci. 2016;9:282–293.
[7] Wang Y, Rodríguez de Gil P, Chen Y-H, et al. Comparing the performance of approaches for testing the homogeneity of variance assumption in one-factor ANOVA models. Educ Psychol Meas. 2017;77:305–329.
[8] Daramola M, Mtshali K, Senokoane L, et al. Influence of operating variables on the transesterification of waste cooking oil to biodiesel over sodium silicate catalyst: A statistical approach. J Taibah Univ Sci. 2016;10: 675–684.
[9] Belhassan A, Chtita S, Lakhlifi T, et al. QSPR study of the retention/release property of odorant molecules in pectin gels using statistical methods. J Taibah Univ Sci. 2017;11:1030–1046.
[10] Oyeyiola AO, Akinyemi MI, Chiedu IE, et al. Statistical analyses and risk assessment of potentially toxic metals (PTMS) in children's toys. J Taibah Univ Sci. 2017;11:842–849.
[11] Wu H-C. Analysis of variance for fuzzy data. Int J Syst Sci. 2007;38:235–246.
[12] Kruse R, Berthold MR, Moewes C, et al. Synergies of soft computing and statistics for intelligent data analysis. Verlag Berlin Heidelberg: Springer; 2013.
[13] Ramos-Guajardo AB, Lubiano MA, González-Rodríguez G. Bootstrap comparison of statistics for testing the homoscedasticity of random fuzzy sets*Synergies of soft computing and statistics for intelligent data analysis*. Berlin, Heidelberg: Springer; 2013. p. 125–133.
[14] Ramos-Guajardo AB, Lubiano MA. K-sample tests for equality of variances of random fuzzy sets. Comput Stat Data Anal. 2012;56:956–966.
[15] Smarandache F. Neutrosophy. neutrosophic probability, set, and logic, ProQuest information & learning. Ann Arbor, Michigan, USA. 1998;105:118–123.
[16] Guo Y, Şengür A, Tian J-W. A novel breast ultrasound image segmentation algorithm based on neutrosophic similarity score and level set. Comput Methods Programs Biomed. 2016;123:43–53.
[17] Nabeeh NA, Abdel-Basset M, El-Ghareeb HA, et al. Neutrosophic multi-criteria decision making approach for iot-based enterprises. IEEE Access. 2019;7:59559–59574.
[18] Nabeeh NA, Smarandache F, Abdel-Basset M, et al. An integrated neutrosophic-topsis approach and its application to personnel selection: A new trend in brain processing and analysis. IEEE Access. 2019;7:29734–29744.
[19] Abdel-Basset M, Mohamed M, Elhoseny M, et al. Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artif Intell Med. 2019;101:101735.
[20] Abdel-Basset M, El-hoseny M, Gamal A, et al. A novel model for evaluation Hospital medical care systems based on plithogenic sets. Artif Intell Med. 2019;100:101710.
[21] Abdel-Baset M, Chang V, Gamal A. Evaluation of the green supply chain management practices: A novel neutrosophic approach. Comput Ind. 2019;108: 210–220.
[22] Smarandache F. Introduction to neutrosophic statisticsInfinite Study; 2014 https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf.

[23] Aslam M. A new sampling plan using neutrosophic process loss consideration. Symmetry. 2018;10:132.
[24] Chen J, Ye J, Du S. Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. Symmetry. 2017;9:208.
[25] Chen J, Ye J, Du S, et al. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. Symmetry. 2017;9:123.
[26] Aslam M. Design of sampling plan for exponential distribution under neutrosophic statistical interval method. IEEE Access. 2018;6:64153–64158.
[27] Aslam M. Attribute control chart using the repetitive sampling under neutrosophic system. IEEE Access. 2019;7:15367–15374.
[28] Aslam M. Neutrosophic analysis of variance: application to university students. Complex Intell Syst. 2019;5(4):403–407.
[29] Abdel-Basset M, Atef A, Smarandache F. A hybrid neutrosophic multiple criteria group decision making approach for project selection. Cogn Syst Res. 2018;57:216–227.
[30] Abdel-Basset M, Gunasekaran M, Mohamed M, et al. A novel method for solving the fully neutrosophic linear programming problems. Neural Comput Appl. 2018;31(5):1595–1605.
[31] Abdel-Basset M, Manogaran G, Gamal A, et al. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. Des Autom Embedded Syst. 2018;22(3):257–278.
[32] Kanji GK. 100 statistical tests. London: Sage; 2006.