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Time-Truncated Group Plan under a Weibull Distribution based on Neutrosophic Statistics

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Abstract: The aim of reducing the inspection cost and time using acceptance sampling can be achieved by utilizing the features of allocating more than one sample item to a single tester. Therefore, group acceptance sampling plans are occupying an important place in the literature because they have the above-mentioned facility. In this paper, the designing of a group acceptance sampling plan is considered to provide assurance on the product's mean life. We design the proposed plan based on neutrosophic statistics under the assumption that the product's lifetime follows a Weibull distribution. We determine the optimal parameters using two specified points on the operating characteristic curve. The discussion on how to implement the proposed plan is provided by an illustrative example.

Keywords: time-truncated test; Weibull distribution; risk; uncertainty; neutrosophic

1. Introduction

The ambition of each producer is to globalize their business by means of marketing the products. However, few producers reach this goal since they only make sincere efforts in improving and controlling the product's quality to accomplish this target. The producer who enhances the product's quality need not concern its globalization because the continuous improvement in quality helps to increase the positive opinion of the products and to fulfill the consumer's expectations. Hence, the involvement of the producers with great efforts supports to attain the desired result and to achieve the ambition. For quality improvement and maintenance purposes, the producer uses certain statistical techniques, namely control charts and acceptance sampling (see Montgomery [1] and Schilling and Neubauer [2]). In spite of the application of control charts in quality maintenance via monitoring the manufacturing process, it is not suitable for assuring the quality of the finished products. But there is a necessity to provide quality assurance for the products before they are received by the consumer. Under this situation, the manufacturers may prefer complete inspection. However, complete inspections are not appropriate for all situations because they are costly, require quality inspectors, and are time consuming. Therefore, in most of the cases, manufacturers adopt sampling inspections to provide quality assurance. In sampling inspection, a sample of items is selected randomly from the entire lot for inspection.

Acceptance sampling is also a form of sampling inspection, in which the decision to accept or reject a lot is made based on the results of sample items taken from the concerned lot. Obviously, acceptance sampling overcomes the drawbacks of complete inspections, such as inspection cost and time consumption, since it inspects only a part of the items of the lot for making decisions. Acceptance sampling plans yield the sample size and acceptance criteria associated with the sampling rules to be implemented. For further details on acceptance sampling, one may refer to Dodge [3] and Schilling and

Neubauer [2]. In the literature, several sampling plans are available for lot sentencing with different sampling procedures; however, a single-sampling plan (SSP) is the most basic, as well as the easiest, sampling plan in terms of the implementation process. In SSP, a single sample of size n is taken for lot sentencing, and the acceptance/rejection decision is made immediately by comparing the sample results with acceptance numbers determined from attribute inspections or with acceptance criteria from variables inspections. Many authors have investigated SSPs under various situations (see, for example, Loganathan et al. [4], Liu and Cui [5], Govindaraju [6], and Hu and Gui [7]).

In SSP implementation, a sample of n items is distributed to n testers, and the decision is made after consolidating the information obtained from all the testers. Obviously, it requires much time to make a decision, and the inspection cost is also high. One can overcome these drawbacks by implementing a group acceptance sampling plan (GASP) instead of using SSP. In GASP, a certain number of sample items are allocated to a single tester, and the test is conducted simultaneously on the sample items. Therefore, the testing time and inspection cost are reduced automatically under GASP when compared to SSP. It is to be mentioned that the number of testers involved in the inspection is frequently referred to as the number of groups, and the number of sample items allocated to each group is defined as the group size. For the purposes of making a decision on the lot by utilizing minimum cost and time, GASP has been used for the inspection of different quality characteristics by several authors (see, for example, Aslam and Jun [8]).

When industrial practitioners are uncertain about the parameters, the inspection cannot be done using traditional sampling plans. In this case, the use of fuzzy-based sampling plans is the best alternative to traditional sampling plans. Fuzzy-based sampling plans have been widely used for lot sentencing. Kanagawa and Ohta [9] proposed a single-attribute plan using fuzzy logic. More details on fuzzy sampling plans can be seen in Chakraborty [10], Jamkhaneh and Gildeh [11], Turanoğlu et al. [12], Jamkhaneh and Gildeh [13], Tong and Wang [14], Uma and Ramya [15], Afshari and Gildeh [16], and Khan et al. [17].

The fuzzy approach has been used to compute the degree of truth. Fuzzy logic is a special case of neutrosophic logic. The later approach computes measures of indeterminacy in addition to the first approach (see Smarandache [18]). Abdel-Basset et al. [19] discussed the application of neutrosophic logic in decision making. Abdel-Basset et al. [20] worked on linear programming using the idea of neutrosophic logic. Broumi et al. [21] provided the minimum spanning tree using neutrosophic logic. More details can be seen in [22,23]. Neutrosophic statistics is treated as an extension of classical statistics, in which set values are considered rather than crisp values. Sometimes, the data may be imprecise, incomplete, and unknown, and exact computation is not possible. Under these situations, the neutrosophic statistics concept is used (see Smarandache [24]). Broumi and Smarandache [25] discussed the correlations of sets using neutrosophic logic. More details about the use of neutrosophic logic in sets can be seen in [26–28]. But one can use a set of values (that respectively approximates these crisp numbers) for a single variable using neutrosophic statistics. Chen et al. [29,30] introduced neutrosophic numbers to solve rock engineering problems. Patro and Smarandache [31] and Alhabib et al. [32] discussed some basicsofprobablity distribution under neutrosophic numbers. Nowadays, the neutrosophic statistics concept is used for quality control purposes. When designing the control chart and sampling plans under classical statistics, it is assumed that the value which represents the quality of the product is known. But in neutrosophic statistics, such value is indeterminate or lies between an interval. Some researchers have designed the control chart and acceptance sampling plans under these statistics (see, for example, Aslam et al. [33]). Aslam [34] introduced neutrosophic statistics in the area of acceptance sampling plans. Aslam and Arif [35] proposed a sudden death testing plan under uncertainty.

As mentioned earlier, Aslam and Jun [8] designed GASP to ensure the Weibull-distributed mean life of the products under classical statistics. They determined the optimal parameters for some calculated values of failure probability; however, they did not consider the case where the failure probability is uncertain. Therefore, in this paper, we attempted to design GASP for providing

Weibull-distributed mean life assurance where the values of shape parameters and failure probabilities are uncertain. That is, we considered the design of GASP under neutrosophic statistics, which is the main difference between the proposed work and the work done by Aslam and Jun [8]. We will compare the proposed plan with the existing sampling plan under classical statistics in terms of the sample size required for inspection. We expect that the proposed plan will be quite effective, adequate, and efficient compared to the existing plan in an uncertainty environment.

2. Design of the Proposed Plan using Neutrosophic Statistics

The method to design the proposed GASP for providing quality assurance of the product in terms of mean life is discussed in this section. The ratio between the true mean life and the specified mean life of the product is considered as the quality of the product. A Weibull distribution is considered as an appropriate model to express the lifetime of the product because of its flexible nature. So, we assume that the lifetime of the product $t_N \in \{t_L, t_U\}$ under study follows a neutrosophic Weibull distribution, which has the shape parameter $\delta_N \in \{\delta_L, \delta_U\}$ and scale parameter $\lambda_N \in \{\lambda_L, \lambda_U\}$. Then, the cumulative distribution function (cdf) of the Weibull distribution is obtained as follows.

$$F(t_N; \lambda_N, \delta_N) = 1 - \exp\left(-\left(\frac{t_N}{\lambda_N}\right)^{\delta_N}\right), t_N \geq 0, \lambda_N > 0, \delta_N > 0. \quad (1)$$

In this study, it is assumed that the scale parameter λ_N is unknown and the shape parameter δ_N is known. It can be seen that the cdf depends only on t_N/λ_N since the shape parameter is known. One can estimate the shape parameter from the available history of the production process when it is unknown. The true mean life of the product under the neutrosophic Weibull distribution is calculated by the following equation

$$\mu_N = \left(\frac{\lambda_N}{\delta_N}\right)\Gamma\left(\frac{1}{\delta_N}\right), \quad (2)$$

where Γ(.) represents the complete gamma function. Then, the probability that the product will fail before it reaches the experiment time t_{N0},is denoted by p_N and is given as follows

$$p_N = 1 - \exp\left(-\left(\frac{t_{N0}}{\lambda_N}\right)^{\delta_N}\right). \quad (3)$$

As pointed out by Aslam and Jun [8], we can write t_{N0} as a constant multiple of the specified mean life μ_{N0}, such as $t_{N0} = t_0 = \mathrm{a}\mu_0 a\mu_{N0}$ where 'a' is called the experiment termination ratio. Also, we can express the unknown scale parameter in terms of the true mean life and known shape parameters. After t_{N0}, λ_N value substitution, and possible simplification, one can obtain the probability that the product will fail before attaining the experiment time $t_{N0,}$ using the following equation.

$$p_N = 1 - \exp\left(-a^{\delta_N}\left(\frac{\mu_{N0}}{\mu_N}\right)^{\delta_N}\left(\frac{\Gamma\left(\frac{1}{\delta_N}\right)}{\delta_N}\right)^{\delta_N}\right), \quad (4)$$

With respect to the ratios between the true mean life and the specified mean life, μ_N/μ_{N0}, the acceptable quality level (AQL, i.e., p_{N1}) and limiting quality level (LQL, i.e., p_{N2}) are defined. That is, the failure probabilities obtained when the mean ratio values greater than one are taken as AQL and the same are obtained at a mean ratio equal to one are considered as LQL. The operating procedure of the proposed GASP for a time-truncated life test is described as follows:

Step 1. Take a sample of $n_N \in \{n_L, n_U\}$ items randomly from the submitted lot and distribute $r_N \in \{r_L, r_U\}$ items into $g_N \in \{g_L, g_U\}$ groups. Then, conduct the life test on the sample items for the specified time t_{N0}.

Step 2. Observe the test and count number of sample items failed in each group before reaching experiment time t_{N0} and denote it as $d_N \in \{d_L, d_U\}$.

Step 3. If at most, c_N sample items found to be failed in each of all g_N groups, then accept the lot where $c_N \in \{c_L, c_U\}$. Otherwise, reject the lot.

Two parameters used to characterize the proposed plan are number of groups g_N and the acceptance number c_N. It is to be noted that $r_N \in \{r_L, r_U\}$ denotes the number of items in each group and is called the group size. The operating procedure of the proposed GASP is represented by a flow chart and is shown in Figure 1.

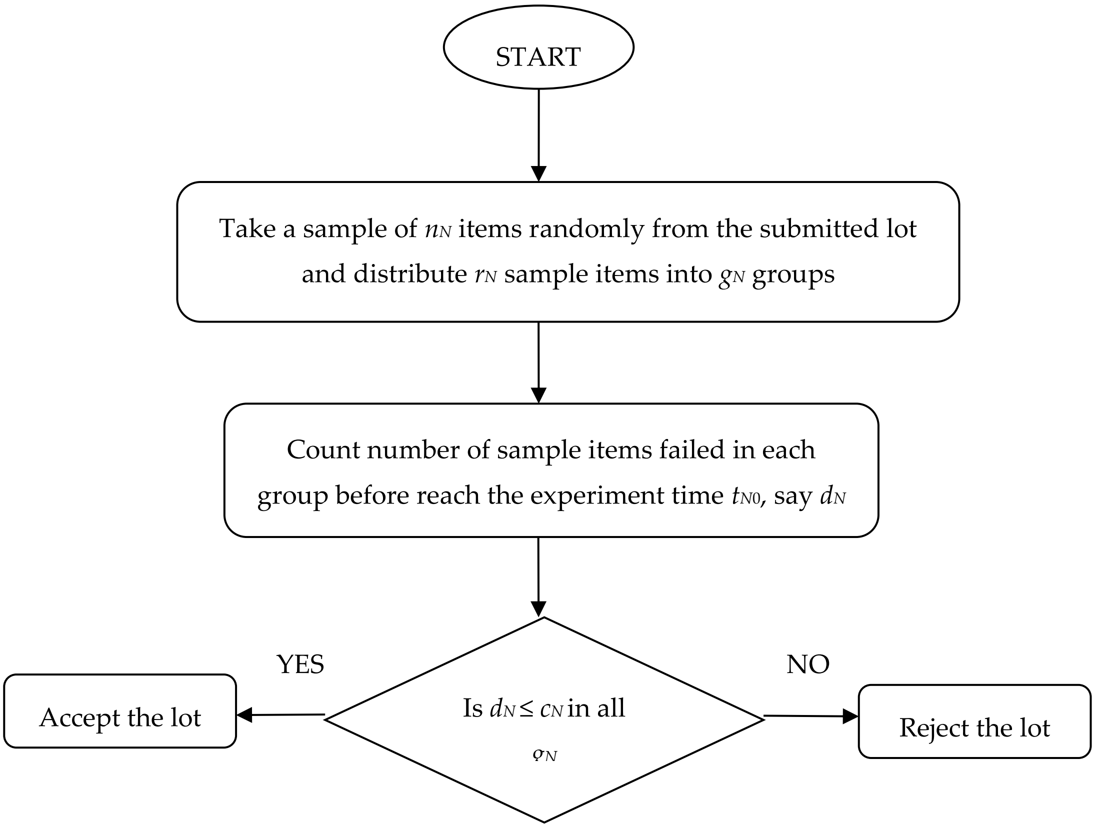

Figure 1. Operating procedure of the proposed group acceptance sampling plan (GASP) under a truncated life test.

In general, an operating characteristic (OC) function helps to investigate the performance of the sampling plan. The OC function of the proposed GASP under a Weibull model based on time-truncated test is given by

$$P_{aN}(p_N) = \left[\sum_{d_N=0}^{c_N} \binom{r_N}{d_N} p_N^{d_N} (1-p_N)^{r_N-d_N} \right]^{g_N}. \tag{5}$$

Generally, each producer wishes that the sampling plan should provide a chance greater than $(1-\alpha)$ to accept the product when the product quality is at AQL, where α is the producer's risk, whereas the consumer wants that the chance to accept the lot to be less than β when quality of the product is at LQL, where β is the consumer's risk. Obviously, the sampling plan that involves the minimum risks to both producer and consumer will be favorable. The design of the sampling plan by considering AQL and LQL, along with producer and consumer risks is known as two points on the OC curve approach and this approach is considered as the most important among others. Similarly, the sampling plan that makes its decision on the submitted lot using minimum sample size or average sample number (ASN) will be attractive. Therefore, in this study, we design GASP with the intention of assuring a Weibull-distributed mean life of the products with minimum sample size and minimum cost using two points on the OC curve approach. It should be mentioned that the ASN of the proposed plan is the product of the number of groups and group size (i.e., $n_N = g_N r_N$). For determining the optimal parameters, we use the following optimization problem.

$$\begin{gathered}\text{Minimize } g_N \\ \text{Subject to } P_a(p_{N1}) \geq 1-\alpha, \\ P_a(p_{N2}) \leq \beta, \\ g_N \geq 1, r_N > 1,\ c_N \geq 0,\end{gathered} \tag{6}$$

where p_{N1} and p_{N2} are the failure probabilities obtained from the following equations

$$p_{N1} = 1-\exp\left(-a^{\delta_{N1}}\left(\frac{\mu_{N0}}{\mu_N}\right)^{\delta_{N1}}\left(\frac{\Gamma\left(\frac{1}{\delta_{N1}}\right)}{\delta_{N1}}\right)^{\delta_{N1}}\right), \delta_{N1} \in \{\delta_{L1}, \delta_{U1}\}, \tag{7}$$

$$p_{N2} = 1-\exp\left(-a^{\delta_{N2}}\left(\frac{\mu_{N0}}{\mu_N}\right)^{\delta_{N2}}\left(\frac{\Gamma\left(\frac{1}{\delta_{N2}}\right)}{\delta_{N2}}\right)^{\delta_{N2}}\right), \delta_{N2} \in \{\delta_{L2}, \delta_{U2}\}, \tag{8}$$

$$P_{aN}(p_{N1}) = \left[\sum_{d_N=0}^{c_N}\binom{r_N}{d_N} p_{N1}^{d_N}\left(1-p_{N1}\right)^{r_N-d_N}\right]^{g_N}, \tag{9}$$

$$P_{aN}(p_{N2}) = \left[\sum_{d_N=0}^{c_N}\binom{r_N}{d_N} p_{N2}^{d_N}\left(1-p_{N2}\right)^{r_N-d_N}\right]^{g_N}. \tag{10}$$

In this designing, we define AQL as the failure probability corresponding to the mean ratios μ_N/μ_{N0} = 2, 4, 6, 8, 10. Similarly, the LQL is defined as the failure probability corresponding to the mean ratio $\mu_N/\mu_{N0} = 1$. The optimal parameters of the proposed GASP are determined for various combinations of group size, shape parameter, and producer's risk. We used the grid search method under neutrosophic statistics to find the optimal values of parameters $[g_L, g_U]$ and $[c_L, c_U]$. We selected those values of parameters from several combinations of parameters that satisfy the given conditions where the range between g_L and g_U is at a minimum. For this determination, we considered two sets of group sizes, such as r_N = {10, 12} and r_N = {4, 6}, and two sets of shape parameters, such as δ_N = {0.9, 1.1} and δ_N = {1.9, 2.1}. Similarly, the producer risks are assumed to be $\alpha = 0.1$ and $\alpha = 0.05$, and four values of the consumer's risk, namely β = 0.25, 0.10, 0.05, 0.01, are used. The experiment termination ratios involved in this determination are $a = 0.5$ and $a = 1$. Then, the optimal parameters are reported in Tables 1–4. We can observe the following trends from tables.

i. In most of the cases, the number of groups required for inspection decreases if the constant 'a' increases from 0.5 to 1.
ii. For fixed values of δ_N, α, β, a, and μ_N/μ_{N0}, the number of groups increases when group size decreases. There is no particular change in the number of groups when the mean ratio increases.

Table 1. Optimal parameters of the proposed GASP under neutrosophic statistics when r_N = [10, 12] and δ_N = [0.9, 1.1].

		a = 0.5			**a = 1.0**		
β	μ_N/μ_{N0}	g_N	c_N	$Pa_N(p_{N1})$	g_N	c_N	$Pa_N(p_{N1})$
0.25	2	[19, 44]	[6, 7]	[0.919, 0.983]	[5, 7]	[7, 8]	[0.904, 0.951]
	4	[2, 4]	[3, 4]	[0.905, 0.987]	[1, 3]	[4, 5]	[0.910, 0.957]
	6	[1, 3]	[2, 4]	[0.919, 0.999]	[1, 3]	[4, 6]	[0.974, 0.999]
	8	[1, 3]	[2, 4]	[0.956, 1.000]	[1, 3]	[4, 7]	[0.990, 1.000]
	10	[1, 3]	[2, 3]	[0.974, 0.998]	[1, 3]	[3, 7]	[0.974, 1.000]
0.1	2	*	*	*	[26, 28]	[8, 9]	[0.924, 0.969]
	4	[2, 4]	[3, 4]	[0.905, 0.987]	[1, 3]	[4, 6]	[0.910, 0.992]
	6	[2, 4]	[3, 4]	[0.969, 0.998]	[1, 3]	[4, 6]	[0.974, 0.999]
	8	[2, 4]	[2, 3]	[0.915, 0.994]	[1, 3]	[3, 5]	[0.951, 0.999]
	10	[2, 4]	[3, 4]	[0.994, 1.000]	[1, 3]	[3, 5]	[0.974, 1.000]
0.05	2	*	*	*	[34, 36]	[8, 9]	[0.902, 0.960]
	4	[7, 11]	[4, 5]	[0.935, 0.996]	[3, 5]	[5, 7]	[0.930, 0.998]
	6	[3, 6]	[3, 4]	[0.954, 0.997]	[2, 4]	[4, 5]	[0.949, 0.993]
	8	[2, 4]	[2, 3]	[0.915, 0.994]	[1, 3]	[3, 4]	[0.951, 0.991]
	10	[2, 4]	[2, 3]	[0.948, 0.998]	[1, 3]	[3, 4]	[0.974, 0.997]
0.01	2	*	*	*	*	*	*
	4	[9, 17]	[4, 5]	[0.917, 0.994]	[4, 6]	[5, 6]	[0.908, 0.984]
	6	[5, 8]	[3, 4]	[0.925, 0.996]	[2, 4]	[4, 5]	[0.949, 0.993]
	8	[5, 8]	[3, 4]	[0.968, 0.999]	[2, 4]	[3, 4]	[0.905, 0.987]
	10	[3, 5]	[2, 3]	[0.923, 0.997]	[2, 4]	[3, 6]	[0.948, 1.000]

*: Plan does not exist.

Table 2. Optimal parameters of the proposed GASP under neutrosophic statistics when r_N = [10, 12] and δ_N = [1.9, 2.1].

		a = 0.5			**a = 1.0**		
β	μ_N/μ_{N0}	g_N	c_N	$Pa_N(p_{N1})$	g_N	c_N	$Pa_N(p_{N1})$
0.25	2	[5, 11]	[2, 3]	[0.926, 0.988]	[2, 4]	[5, 6]	[0.990, 0.995]
	4	[2, 4]	[1, 2]	[0.981, 0.999]	[1, 3]	[3, 4]	[0.998, 1.000]
	6	[2, 4]	[1, 2]	[0.996, 1.000]	[1, 3]	[2, 4]	[0.998, 1.000]
	8	[2, 4]	[1, 2]	[0.999, 1.000]	[1, 3]	[1, 5]	[0.990, 1.000]
	10	[2, 4]	[1, 2]	[0.999, 1.000]	[1, 3]	[2, 3]	[1.000, 1.000]
0.1	2	[39, 65]	[3, 4]	[0.942, 0.995]	[2, 4]	[4, 6]	[0.945, 0.995]
	4	[4, 7]	[1, 2]	[0.962, 0.999]	[1, 3]	[2, 3]	[0.985, 0.997]
	6	[3, 7]	[1, 2]	[0.994, 1.000]	[1, 3]	[2, 4]	[0.998, 1.000]
	8	[5, 7]	[1, 2]	[0.996, 1.000]	[1, 3]	[1, 3]	[0.990, 1.000]
	10	[3, 7]	[1, 2]	[0.999, 1.000]	[1, 3]	[2, 4]	[1.000, 1.000]
0.05	2	[67, 84]	[3, 4]	[0.902, 0.994]	[3, 5]	[4, 6]	[0.918, 0.994]
	4	[4, 8]	[1, 2]	[0.962, 0.998]	[1, 3]	[2, 4]	[0.985, 1.000]
	6	[5, 8]	[1, 2]	[0.989, 1.000]	[1, 3]	[2, 5]	[0.998, 1.000]
	8	[4, 8]	[1, 2]	[0.997, 1.000]	[1, 3]	[2, 5]	[1.000, 1.000]
	10	[5, 8]	[1, 2]	[0.998, 1.000]	[1, 3]	[1, 5]	[0.996, 1.000]
0.01	2	[59, 129]	[3, 4]	[0.914, 0.990]	[7, 9]	[5, 6]	[0.964, 0.989]
	4	[6, 13]	[1, 2]	[0.944, 0.997]	[2, 4]	[2, 5]	[0.970, 1.000]
	6	[9, 13]	[1, 2]	[0.981, 1.000]	[1, 3]	[1, 3]	[0.973, 1.000]
	8	[7, 13]	[1, 2]	[0.995, 1.000]	[1, 3]	[1, 2]	[0.990, 0.999]
	10	[7, 13]	[1, 2]	[0.998, 1.000]	[1, 3]	[1, 2]	[0.996, 1.000]

Table 3. Optimal parameters of the proposed GASP under neutrosophic statistics when r_N = [4, 6] and δ_N = [0.9, 1.1].

		a = 0.5			a = 1.0		
β	μ_N/μ_{N0}	g_N	c_N	$Pa_N(p_{N1})$	g_N	c_N	$Pa_N(p_{N1})$
0.25	2	*	*	*	*	*	*
	4	[6, 10]	[2, 3]	[0.932, 0.990]	[8, 10]	[3, 4]	[0.964, 0.988]
	6	[7, 10]	[2, 3]	[0.970, 0.998]	[2, 4]	[2, 3]	[0.955, 0.988]
	8	[2, 4]	[1, 2]	[0.928, 0.994]	[2, 4]	[2, 3]	[0.977, 0.996]
	10	[2, 4]	[1, 2]	[0.950, 0.997]	[1, 3]	[1, 2]	[0.923, 0.980]
0.1	2	*	*	*	*	*	*
	4	[68, 88]	[3, 4]	[0.967, 0.997]	[12, 14]	[3, 4]	[0.947, 0.983]
	6	[13, 17]	[2, 3]	[0.945, 0.997]	[3, 5]	[2, 3]	[0.933, 0.985]
	8	[13, 17]	[2, 3]	[0.973, 0.999]	[3, 5]	[2, 3]	[0.965, 0.995]
	10	[3, 5]	[1, 2]	[0.926, 0.996]	[3, 5]	[2, 3]	[0.980, 0.998]
0.05	2	*	*	*	*	*	*
	4	[103, 115]	[3, 4]	[0.951, 0.996]	[16, 18]	[3, 4]	[0.930, 0.978]
	6	[19, 22]	[2, 3]	[0.920, 0.996]	[4, 6]	[2, 3]	[0.911, 0.982]
	8	[17, 22]	[2, 3]	[0.965, 0.999]	[4, 6]	[2, 3]	[0.954, 0.994]
	10	[4, 7]	[1, 2]	[0.902, 0.994]	[4, 6]	[2, 3]	[0.973, 0.998]
0.01	2	*	*	*	*	*	*
	4	[170, 176]	[3, 4]	[0.920, 0.993]	*	*	*
	6	[23, 34]	[2, 3]	[0.904, 0.994]	[24, 26]	[3, 4]	[0.970, 0.996]
	8	[32, 34]	[2, 3]	[0.934, 0.998]	[6, 8]	[2, 3]	[0.932, 0.992]
	10	[28, 34]	[2, 3]	[0.967, 0.999]	[6, 8]	[2, 3]	[0.960, 0.997]

*: Plan does not exist.

Table 4. Optimal parameters of the proposed GASP under neutrosophic statistics when r_N = [4, 6] and δ_N = [1.9, 2.1].

		a = 0.5			a = 1.0		
β	μ_N/μ_{N0}	g_N	c_N	$Pa_N(p_{N1})$	g_N	c_N	$Pa_N(p_{N1})$
0.25	2	[156, 164]	[2, 3]	[0.902, 0.993]	[3, 5]	[2, 3]	[0.929, 0.958]
	4	[8, 23]	[1, 2]	[0.989, 1.000]	[1, 3]	[1, 3]	[0.983, 1.000]
	6	[20, 23]	[1, 2]	[0.994, 1.000]	[1, 3]	[1, 3]	[0.996, 1.000]
	8	[8, 23]	[1, 2]	[0.999, 1.000]	[1, 3]	[1, 2]	[0.999, 1.000]
	10	[9, 23]	[1, 2]	[1.000, 1.000]	[1, 3]	[1, 2]	[0.999, 1.000]
0.1	2	*	*	*	[25, 27]	[3, 4]	[0.966, 0.983]
	4	[26, 37]	[1, 2]	[0.965, 0.999]	[2, 4]	[1, 2]	[0.966, 0.995]
	6	[18, 37]	[1, 2]	[0.995, 1.000]	[2, 4]	[1, 2]	[0.992, 1.000]
	8	[18, 37]	[1, 2]	[0.998, 1.000]	[2, 4]	[1, 2]	[0.997, 1.000]
	10	[33, 37]	[1, 2]	[0.999, 1.000]	[2, 4]	[1, 2]	[0.999, 1.000]
0.05	2	*	*	*	[32, 34]	[3, 4]	[0.957, 0.978]
	4	[27, 48]	[1, 2]	[0.964, 0.999]	[3, 5]	[1, 2]	[0.949, 0.994]
	6	[23, 48]	[1, 2]	[0.993, 1.000]	[3, 5]	[1, 2]	[0.988, 0.999]
	8	[45, 48]	[1, 2]	[0.995, 1.000]	[3, 5]	[1, 2]	[0.996, 1.000]
	10	[28, 48]	[1, 2]	[0.999, 1.000]	[3, 5]	[1, 2]	[0.998, 1.000]
0.01	2	*	*	*	[49, 51]	[3, 4]	[0.935, 0.968]
	4	[27, 74]	[1, 2]	[0.964, 0.999]	[4, 6]	[1, 2]	[0.933, 0.992]
	6	[64, 74]	[1, 2]	[0.981, 1.000]	[4, 6]	[1, 2]	[0.984, 0.999]
	8	[71, 74]	[1, 2]	[0.993, 1.000]	[4, 6]	[1, 2]	[0.995, 1.000]
	10	[29, 74]	[1, 2]	[0.999, 1.000]	[4, 6]	[1, 2]	[0.998, 1.000]

*: Plan does not exist.

3. Illustrative Example

Suppose that a manufacturer wants to provide the mean life assurance for his product, and he claims that the true mean life of the product is μ_N = 500 h. The quality inspector decides to check whether the manufacture's claim on the lifetime of the product is true or not and, therefore, specifies the experiment time as t_{N0} = 500 h. Hence, the experiment termination ratio is calculated as a = 1.0. The failure probability corresponding to the mean ratio μ_N/μ_{N0} = 4 is considered as AQL and the same at mean ratio 1 is taken as LQL. The consumer risk is assumed to be β = 0.25. The shape parameter of the Weibull distribution is specified as δ_N = 0.9. Suppose the quality inspector wants to implement the proposed GASP under neutrosophic statistics, and he decides to allocate r_N = 6 items to each tester. Therefore, in order to execute the proposed plan for the above-specified conditions, we obtain the optimal parameters g_N = [8, 10] and c_N = [3, 4] from Table 3. The input values (or specified values) and the optimal values determined for those input values are reported in Table 5 for easy identification.

Table 5. Summary of input values and output parameters.

Input Values							Output Parameters	
μ_N	t_{N0}	a	μ_N/μ_{N0}	β	δ_N	r_N	g_N	c_N
500(h)	500(h)	1.0	4	0.25	0.9	6	[8, 10]	[3, 4]

This shows that the number of groups for the inspection lies between 8 and 10. Suppose the quality inspector chooses eight groups. Then, implementation procedure of the proposed plan is as follows.

A random sample of 48 items is chosen from the submitted lot, and 6 sample items are distributed to 8 groups. The sample items are included in the life test, and the test is conducted up to the specified time 500 h. The submitted lot is accepted if there are, at most, 3 sample items that failed before the time 500 h in each of all 8 groups. Otherwise, the lot is rejected.

4. Comparison

To show the efficiency of the proposed plan in terms of number of groups (sample size) over the existing SSP, we tabulated the optimal parameters determined for some specified values of a, r_N, and δ_N. The minimum number of groups required for inspecting the lot under neutrosophic statistics and classical statistics is shown in Table 6. We note from Table 6 that the proposed sampling plan under neutrosophic statistics has the smaller number of groups compared to the time-truncated plan under classical statistics. For example, when a = 0.5 and μ_N/μ_{N0} = 2, the number of groups in an indeterminate interval under classical statistics is larger than the proposed sampling plan under neutrosophic statistics. The same efficiency of the proposed plan can be observed for all other specified parameters. By comparing both sampling plans, it can be noted that time-truncated group sampling plan under neutrosophic statistics is better than the plan using classical statistics. Hence, the proposed plan is more economical than the existing plan in saving cost, time, and efforts in uncertainty environments.

Table 6. Values of the proposed GASP and single-sampling plan (SSP) under neutrosophic statistics when r_N = [10, 12] and δ_N = [0.9, 1.1].

		a = 0.5		a = 1.0	
β	μ_N/μ_{N0}	GASP	SSP	GASP	SSP
		g_N	n_N	g_N	n_N
0.25	2	[19, 44]	[34, 38]	[5, 7]	[20, 23]
	4	[2, 4]	[11, 14]	[1, 3]	[7, 8]
	6	[1, 3]	[9, 10]	[1, 3]	[5, 6]
	8	[1, 3]	[6, 7]	[1, 3]	[5, 4]
	10	[1, 3]	[6, 7]	[1, 3]	[4, 4]

Table 6. *Cont.*

		$a = 0.5$		$a = 1.0$	
β	μ_N/μ_{N0}	GASP	SSP	GASP	SSP
		g_N	n_N	g_N	n_N
0.1	2	*	[54, 59]	[26, 28]	[36, 35]
	4	[2, 4]	[17, 20]	[1, 3]	[10, 11]
	6	[2, 4]	[14, 13]	[1, 3]	[9, 7]
	8	[2, 4]	[11, 13]	[1, 3]	[7, 7]
	10	[2, 4]	[8, 10]	[1, 3]	[7, 5]
0.05	2	*	[68, 73]	[34, 36]	[48, 42]
	4	[7, 11]	[22, 27]	[3, 5]	[13, 14]
	6	[3, 6]	[16, 19]	[2, 4]	[12, 10]
	8	[2, 4]	[13, 16]	[1, 3]	[10, 8]
	10	[2, 4]	[13, 16]	[1, 3]	[8, 8]
0.01	2	*	[103, 108]	*	[68, 59]
	4	[9, 17]	[36, 36]	[4, 6]	[22, 21]
	6	[5, 8]	[23, 28]	[2, 4]	[16, 15]
	8	[5, 8]	[20, 24]	[2, 4]	[12, 13]
	10	[3, 5]	[20, 20]	[2, 4]	[12, 10]

*: Plan does not exist.

5. Conclusions

In this paper, we have designed a group acceptance sampling plan for cases where the quality of the product is in determinate and vague. Therefore, neutrosophic statistics has been used in this design instead of classical statistics. The optimal parameters determined for different combinations of group sizes and shape parameters have been tabulated. It is concluded from this study that one can use the proposed plan if there is uncertainty in the product's quality. The proposed sampling plan using some other neutrosophic distributions or sampling schemes can be considered in future research. The extension of the proposed plan for big data is also a fruitful area for future research.

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Glossary

$t_N \in \{t_L, t_U\}$	lifetime of the product, where t_L is the lower value of the lifetime and t_U is the upper value of the lifetime
$\delta_N \in \{\delta_L, \delta_U\}$	shape parameter of the Weibull distribution, where δ_L is the lower value of the shape parameter and δ_U is the upper value of the shape parameter
$\lambda_N \in \{\lambda_L, \lambda_U\}$	scale parameter, where λ_L is the lower value of the scale parameter and λ_U is the upper value of the scale parameter
t_{N0}	experiment time
μ_N	true mean life
μ_{N0}	specified mean life
a	experiment termination ratio (i.e., $a = t_{N0}/\mu_{N0}$)
μ_N/μ_{N0}	ratio between the true mean life and the specified mean life

p_{N1}	acceptable quality level (AQL)
p_{N2}	limiting quality level (LQL)
$n_N \in \{n_L, n_U\}$	sample size (i.e., $n_N = g_N r_N$), where n_L is the lower value of the sample size and n_U is the upper value of the sample size
$r_N \in \{r_L, r_U\}$	group size, where r_L is the lower value of the group size and r_U is the upper value of the group size
$g_N \in \{g_L, g_U\}$	number of groups, where g_L is the lower value of the number of groups and g_U is the upper value of the number of groups
$d_N \in \{d_L, d_U\}$	number of failure items in the sample, where d_L is the lower value of the number of failure items and d_U is the upper value of the number of failure items
$c_N \in \{c_L, c_U\}$	acceptance number, where c_L is the lower value of the acceptance number and c_U is the upper value of the acceptance number
α	producer's risk
β	consumer's risk
$P_{aN}(p_N)$	Probability of acceptance at failure probability p_N
$P_{aN}(p_{N1})$	Probability of acceptance at failure probability p_{N1} or at AQL
$P_{aN}(p_{N2})$	Probability of acceptance at failure probability p_{N2} or at LQL

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