Uncertainty: two probabilities for the three states of neutrosophy

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Abstract

Uncertainty is inherent to the real world: everything is only probable, precision like in measurements is finite, noise is everywhere... Also, science is based on a modeling of reality that can only be approximate. Therefore we postulate that uncertainty should be considered in our models, and for making this more easy we propose a simple operational conceptualization of uncertainty. Starting from the simple model of associating a probability p to a statement supposed to be true our proposed modeling bridges the gap towards the most complex representation proposed by neutrosophy as a triplet of probabilities. The neutrosophic representation consists in using a triplet of probabilities (t,i,f) instead of just a single probability. In this triplet, t represents the probability of the statement to be true, and f it's the probability to be false. The specific point of neutrosophy it that the probability i represents the probability of the statement to be uncertain, imprecise, or neutral among other significations according to the application.

Our proposed representation uses only 2 probabilities instead of 3, and it can be easily translated into the neutrosophic representation. By being simpler we renounce to some power of representing the uncertain but we encourage the modeling of uncertainty (instead of ignoring it) by making this simpler. Briefly said, the prepare the path towards using neutrosophy. Our proposed representation of uncertainty consist, for a statement, not only to add its probability to be true p, but also a second probability pp to model the confidence we have in the first probability p. This second parameter pp represents the plausibility of p, therefore the opposite of its uncertainty. This is the confidence given to the value of p, in short pp is the probability of p (hence the name pp), This is simple to understand, and that allows calculations of combined events using classical probability such as based on the concepts of mean and variance. The stringent advantage of our modeling by the couple (p,pp) is that experts can be easily interrogated to provide their expertise by asking them simply the chance they give to an event a occur (this is p) and the confidence they have in that prediction (which is pp). We give also a formula to transform from our model to the neutrosophic representation. Finally, a short discussion on the entropy as a measure of uncertainty is done.

Keywords: Uncertainty. neutrosophy, probability, representation of uncertainty, entropy

1.Introduction

We propose a modeling of uncertainty that is intermediate between a simple probability and one of the most general representation, known under the name of neutrosophy[1]. By uncertainty we mean an event that can occur or not, an event that is not certain. This includes also the uncertainty inherent to any data such as the measure of a signal which is most often imprecise, fuzzy and noisy. This idea of an intermediate approach is based both on the desire to properly represent uncertainty for some kind of approximate calculations in an easy way, but also to facilitate later the transition to the more complete approach of neutrosophy, if this becomes required for a better estimation.

In the real world many applications have to compute from uncertain informations such values affected by the imprecision of the sensors or signals degraded by noise and simply probabilistic situations. The usual true/false logic and arithmetic numbers can only represent crisp values, leaving completely aside the uncertain. Then the statistics have been developed to be still able to make predictions for probable events, like to estimate the chance of rain tomorrow. For most real-world modeling the uncertainty it includes can not be modeled precisely enough just with a single probability for each data. Such highly uncertain data in complex situations should be modeled with more parameters, as proposed by fuzzy logic for example.

Multi-valued representation add the number of variables required for a finer fitting to the reality, similarly as a real number use more bits than an integer to trade in a better approximation. The final user of any application being a human, a representation adapted to humans must be envisaged in the last steps of processing to present the result. Has humans have their cognition taking place on a background of emotions, the concepts that are manipulated are often colored by a sentiment: I like it, I dislike it, or I am indifferent to this. The human by its emotional dimension has a cognition which is for the most often considering 3-state values and not boolean logical values (binary true/false). There is a characteristically human third state, the neutral state, it's not only yes or no but also I don't know, to not repeat the like/dislike/indifferent terms. So there is a 3-state based logic extending the classical binary logic. Those states are associated by the human to linguistic terms, having a semantical value to him. And more than 3 states can be considered when we make categories, but rarely more than 5 to 7, else we mix-up everything. Learning is a case of such a 3 state processing: a new situation will update the knowledge acquired from previous one by changes that are either reinforcement, inhibition or neutral (no change).

Many situations of the real-world are also intrinsically 3-state, or considered as such by the humans: for example the ambient temperature can be warm, cold, or agreeable. You can be positive, negative or indifferent about a proposition. A value can be positive, null or negative, like the one from a sensor. By driving a car you can be accelerating, braking, or just maintain a constant speed (having set on the automatic cruise-control). A regulator will add, subtract or do nothing on the value to be regulated. This extra third state is the most often a neutral state. This was roughly the inspiration that leads to the conception of neutrosophy, the taking into account of the neutral situation.

So therefore neutrosophy use 3 variables to represent these human 3 states, a value can have a truth of t percent as well as simultaneously a falsity of f percent and also a indetermination of i percent, using a triplet (t ,i, f) those values are between 0 and 1. These values are also independent, unrelated, by opposition of the classical probability were the truth is given by t and the falsity by $1-t$. Here f is not independent of t, but dependent as $f=1-t$. Namely for the extreme cases: if it's 100% sure then t = 1 and therefore f = 0 (it's absolutely not false), and oppositely if it's 100% false then f = 1 and therefore t = 0 (it's not true).

Simply said, the representation by a probability uses 1 variable and the neutrosophic representation uses 3 variables, this makes the second more difficult to use, as an human expert has to decide instead for 3 values for the triplet (t,i,f). We believe there are cases or applications were something more complex than just a probability will be beneficial, and also still not requiring the full generality of neutrosophy. By using 2 variables one should be able to model in a finer way than with just one, and hopefully in way more simple than neutrosophy. This will have also the benefit to prepare the path for using neutrosophy by having with a small effort open the path to multi-valued representations.

We will start with a simple hypothesis, that we have to deal with uncertainty because it is inherent to the real-world. Then we will look at how the qualitative human cognition can be feed of information from the quantitative reality. After some considerations about the uncertainty will serve to define our representation using 2 variables, 2 probabilities instead of the usual single one. Then we will make the link between our representation and the basics of the theory of neutrosophy. Our concrete results will be, aside our 2 variables model, some extensions of it that are easy to make. Finally, we will link our representation more closely to the notion of uncertainty by evaluating it from an extension of the concept of entropy, which is closely related to the unexpected content of a signal.

2. Hypothesis

Our starting assumption H1 is that "real world situations involve some uncertainty". More precisely, this means that our measures of the real situation are both incomplete and fraught with uncertainty, in particular imprecision and noise. The understanding of the situation is necessarily incomplete and so is its perception. As a result, the modeling elaborated will be imperfect, as it is only a limited approximation of reality. Thus, what is being calculated involves an element of uncertainty in relation to reality. The objective of science has always been to reduce this gap between model and reality (to make more useful predictions).

However, the hypothesis H1 we made is demonstrated by the consideration of concrete practical cases, a few examples of which are sufficient to convince. To be short, we will retain only one example, and therefore we will continue to consider that it is just a hypothesis for the rest of this text. In the implementation of an expert system, a case of artificial intelligence, a set of calculation rules must be constructed about knowledge obtained from human experts in this field. Sometimes several experts have divergent opinions, or warn against the uncertainty of their statements. With a methodology capable of dealing with uncertainty, however, such information can be used when otherwise it should be rejected, or if nothing else is available, it can still be considered but with inconsiderate risks.

However, it is also possible to choose H1's opposite hypothesis, which claims that ¬H1 "a solution can be calculated by working on data considered accurate, without explicitly involving uncertainty". This is the preferred approach to date, except perhaps in these complex situations where it does not lead to an optimal result, or even very often to a suboptimal result acceptable in precision and certainty.

The H1 hypothesis results in a more complete one, a more precise variant H1', which is that H1' "modeling real-world situations involves some uncertainty". Indeed, the search for a numerical solution is based on a model in addition to measurements.

3. From qualitative to quantitative

Given this situation, we are coming up with the solutions that are available to us. They are of two types that can be considered together:

S1: Handle in a better way the uncertainty.

S2: Reduce uncertainty, for example by adding more sensors.

In some situations S2 is not possible, mainly because it is not feasible for economic reasons, or because to go further than what has already been done and which has proven unsatisfactory. Thus, there is always some uncertainty in the system on which a result is calculated. Then it is desirable to look for a good S1 type solution, therefore a solution able to address uncertainty explicitly.

Often the desired result is a simple "yes or no" type answer, i.e. of a binary type. It therefore seems tempting to consider the whole problem in a binary approach. In terms of human decision-makers, the desired outcome is more of a three-state type. It is a question of discriminating between three cases: for example, I am confident in the project, I do not have a clear opinion, I am rather not confident in this project. Sometimes the decision maker's choice is not only between doing or not doing, but also between doing one thing or doing the opposite, or doing nothing at all. Typically for a stock market investor, he must choose according to his current valuation of a security he owns whether he buys more or sells it or whether he thinks it is better to wait.

The way of thinking of a human expert is mainly qualitative and not quantitative. In general, the results of a calculation are quantitative, and a decision threshold is set to choose whether or not to do so. Binary logic is somehow too square, it cuts too sharply, it's not flexible enough: true or false. If a calculation is performed in binary logic, each elementary operation has this defect. In a three-state logic, certain inputs or intermediate results can be

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neutralized so that in specific circumstances they no longer affect the final result. This is an additional flexibility that is central to neural networks in particular.

In a numerical calculation, each value has an effect, possibly very small, on the final result. This corresponds better to the case of reality where there are many cross-influences. A particular quantitative approach is the probabilistic approach. Each entry or intermediate result is no longer either true or false, but a probability of being true. So we won't say "it will rain tomorrow", but "we have a 65% chance of rain tomorrow", or "it should fall 4 mm of water". For a specialist a numerical value speaks for itself, and for many more people a percentage will be easier to get, and even more will understand a fractional form like "there are about 3 chances out of 5 that it will rain tomorrow".

The case of this example of rainfall forecasting is interesting for two other aspects. The people most interested in this information are farmers who have to decide whether it is necessary to water their crops, or whether what comes from the sky will be enough. So they want to know the probabilities of different rainfall amounts, for the coming days, in order to make the average over the week, for example, or to determine the worst case scenario. For example, the probability of one millimeter of rain, that of 2, that of 4, that of 6 or more and also that there will be no rain. The models used for forecasting often offer this information by principle. This gives the probabilities for a whole series of values (in fact categories centred on these values), such as 0, 1, 2, 4, 6 or more mm of rain. The total of these probabilities should be 100%, for each day. This is a more complete approach, than considering only the maximum and its probability as in our previous approach, such of all situations (global) evaluation corresponds to the new extension of neutrosophy recently proposed by its creator because it is even more general: plithogenesis [2]. Indeed, these forecasting simulations are carried out using multidimensional matrix models running on a large number of vector calculation units (or matrix or tensor depending on the name we prefer to give them) in parallel. In concrete terms, the farmer wishes to reduce the uncertainty about the future that is specific to nature, to control it as good as possible in order to maintain the quantity and quality of his production with a minimum of intervention and costs.

Thus, a quantitative rather than qualitative approach, often probabilistic, is preferable.

4. Consideration of uncertainty

In the case of general public meteorological forecasts, forecasts of up to 7 days have been given since several years. So for the forecast of each coming day it is emphasized that the accuracy or certainty of the forecast decreases as the day is further away. For example, we can say that for the seventh day "the prediction is estimated to be 65% correct". So, for example, that it is estimated that "it will rain next Wednesday 4 mm of water with a probability of 65%". Here it is a question of the degree of certainty of the prediction, therefore of its uncertainty, but in the problems of the real world there are also uncertainties of measurement, modeling, calculation...

So we have a probability of a first probability, here the accuracy of the rain probability, a probability of the second order in a way.

In the case of a human expert, it is useful, and this extra parameter can be estimated also by the expert: he will be asked his prognosis, then how strongly he believes in it, what certainty he attributes to it. To do this, experts can be asked to choose between linguistic values such as "a little, medium, a lot, completely" and then convert this qualitative judgment (his confidence) into a percentage.

In this way, a model incorporating uncertainty is formed, using two variables. Which we call p and pp. The first variable p is the probability of a certain state, for example rain, the second pp is the probability of p, so the certainty that we give to the probability p. Using just two words, it's a probability and its plausibility (or confidence).

These two variables are between 0 and 1 (or in an equivalent way between 0% and 100%). If p = 1 then the expert estimates that the fact will occur. If p = 0 that it will not occur, then this corresponds to the binary logic, true and respectively false. Such a classical binary logic is therefore representable in this finer model. Since not everything is black or white, the intermediate values specify the probability that it is true, this is a very classical probability. Then the expert gives an estimate of his prognosis, the plausability (his confidence). He can consider himself certain (pp = 1) or totally uncertain (pp = 0), and with all the gradations in between.

If $pp = 1$, only the probability p is considered, and the calculation of the usual probabilities will be used. If $pp = 0$, it is not possible to say anything and the p value is not relevant.

Now uncertainty is simply the opposite of certainty pp. If the certainty pp is 0 then the uncertainty i will be $1 - pp$, and in this case will be 1, it will be total uncertainty.

So in summary we propose à simple model to consider uncertainly using only 2 variables, that can be easily and intuitively be used by human experts, considering 2 classical probabilities:

p the probability of the event estimated by the expert

pp the confidence of the expert in the above probabiliy

This second variable pp is also the opposite of the estimated uncertainty I: $pp = 1 - i$

For example, the event can be team A wins over team B in a sport game to come. The expert will express his opinion by saying there is a 75% of chances that team A is the winner, so p(teamA) = 0,75. Then he can add to be more informative that he believes so at a confidence of 9 /10 (the note he attributes to the quality of his prevision). So, here pp(p(teamA)) = 0.9

The refined prevision of the expert is given by the couple (p, pp) = (0.75,0,9). That means is strongly believe that team A has 75% of winning.

Another expert could say that he believe also that team A wins, but with of probability of 80%, but he can add he is not a specialist of team A either he knows very well team B, and that he is prudent about that prediction a little more than mildly. His opinion can be represented by the couple (p, pp) = (0.8, 0.6)

Now the goal will be to aggregate the opinions of all experts in a way taking into account the weight each one attach to his own prediction. This can be done by a simple weighted mean calculation, that is the way the center of gravity is determined.

5. Link with neutrosophy

Neutrosophy, as its name suggests, considers the case of neutrality in addition to the true and false cases, giving it a central importance in modeling. Depending on the applications, different meanings will be chosen for the three states: true, neutral, false, or positive, neutral, negative (as in the previous example of the investor), or true, uncertain, false, when modeling the existing uncertainty in the system.

In addition to considering just three states, it is possible to consider a variant called a simple neutrosophical number (SNn), that is actually a triplet of three probabilities. Such a SNn relates to a statement and is a triplet (t,i,f) where each component t, i and f is a value between 0 and 1, a probability. These probabilities are respectively the degree of belonging to (the state of) truth, the one relating to (the state of) uncertainty and also the one concerning (the state of falsity) of the statement. It should be noted that unlike the representation by a single probability, here the probability of falsity is not necessarily the complement of the probability of truth (it can be specified independently; this gives the expert additional freedom, but also this makes it is more complex to collect his opinion).

For example, true and false binary logic values are represented in an SNn by (1,0,0) and (0,0,1) respectively.

For the information described in our previously presented alternative representation of p and pp, i.e. a probability of validity p (of a statement) and an estimate of its certainty pp, we can imagine a transformation in the neutrosophical representation which is more general. On the basis of the representations of the true and false binary logical states (which can be considered as the extremes of the representation domain) we must choose a correspondence that produces, respectively for true and false, the values according to the table below.

Table 1 Correspondence between the values indicated for the first and second representation.

	p	pp	t	i	f
True	1	1	1	0	0
False	0	1	0	0	1

We can choose as a transformation satisfying these constraints, simply as;

$$t = p$$

$$i = 1 - pp$$

$$f = 1 - p$$

By the way, note that the separate processing of rules for t and f in fuzzy logic [3] with as here $f = 1 - t$ corresponds to the variant called intuitutionistic fuzzy logic [4], and therefore our proposal is an extension of it. What is also interesting, we can define, in fact arbitrarily, a transformation from the neutrosophical representation to our p and pp representation, i.e. the probability and its plausibility (or confidence).

Then, we can choose, although it is not one-to-one (bi-univoc):

$$p = \frac{(t+1-f)}{2}$$

$$pp = 1 - i$$

It should be noted that in general uncertainty, as considered in these transformations and especially in the representation by a probability and its estimate (p,pp), is not a neutral element for the combination operations that are frequently used, such as logical AND and logical OR, but rather a special kind of projection that partially eliminates a dimension. When the estimate pp is 0, the value of the probability p vanishes, it no longer makes sense. The independence of t and f is also destroyed by the conversion of the neutrosophical representation to that of a probability and its plausibility.

Except in this case of degeneration, the proposed representation has all the characteristics of probabilities. Thus it is possible to calculate the logical AND as the joint-probability per a simple arithmetic product. Here, this will be done for p and pp separately.

6. Extension

A slightly extended representation can be considered to deal with uncertainty, if not better, at least a little differently. This idea is used in fuzzy logic and it is taken up in neutrosophy also at the level of the functions of membership. For example, p or t are the value of membership to the veracity of the proposal, the probability that it is true.

Instead of giving an estimate of the probability, it is possible, which ultimately amounts to the same thing, to describe the probability by an interval, for example p is between pmin and pmax. This interval denotes a confidence (an interval of confidence), a confidence in the value of p (the narrower the interval, the higher the confidence), which can be equally expressed by a standard deviation (or variance) Δp. For example p and Δp with Δp = pmax - pmin instead of the representation of the interval type pmin and pmax.

On the assumption, quite often approximately verified in practice, of a Gaussian distribution of the value around the mean p, then we have a "normal" representation that is well treated in statistics, especially for the conjunction of two measurements of the same signal. Then the result of combining the two mean values taking into account their respective certainties (via a representation as standard deviation, as variance) will be the estimate produced by the Kalman filter [5]. This filter calculates an optimal estimate according to the rules for combining the means and standard deviations of two measurements [6]. Since the half standard deviation Δp added to or subtracted from the mean p gives the location of the probability values 0.5 = 50%, a correspondence exists with certainty and therefore uncertainty. That uncertainty is further discussed in the next section.

7. Entropy

A specific addition to this (p, pp) representation has been communicated to us by Vasile Patrascu: to calculate the uncertainty in a more precise way from the pair of probabilities (p,pp). That more precise undestanding of uncertainty is nothing else than the entropy that he has defined in [7] for various multi-valued representations of neutrosophic information. He kindly provided us the formula for our p and pp probabilistic representation. In this article Patrascu proposed a calculation formula for the entropy E (uncertainty) of the neutrosophic information given in triplet (t,i,f), that is:

$$E(t,i,f) = 1 - \frac{|t-f|}{1+|t+f-1|+i} \qquad (1.1)$$

Replacing in it t, i and f by the expressions used before, that were:

$$t = p \qquad (1.2)$$

$$i = 1 - pp \qquad (1.3)$$

$$f = 1 - p \qquad (1.4)$$

we obtain the following formula for calculating the uncertainty (or entropy) E of the pair (p,pp):

$$E(p,pp) = 1 - \frac{|2p-1|}{2-pp} \qquad (1.5)$$

In the particular case of the two boolean logical constants we obtain:

$$E(True) = E(1,1) = 0 \qquad (1.6)$$

$$E(False) = E(0,1) = 0 \qquad (1.7)$$

Further, on the basis of another of his articles [8] Patrascu propose to associate a complete neutrosophic information (t,i,f) to the probability pair (p,pp). He even provided us 2 variants.

- Variant (I)

$$t = \left(\frac{1+pp}{2}\right)p - \frac{1-pp}{4}(1-|2p-1|) \qquad (2.1)$$

$$i = \frac{1-pp}{2}(2-|2p-1|) \qquad (2.2)$$

$$f = \left(\frac{1+pp}{2}\right)(1-p) - \frac{1-pp}{4}(1-|2p-1|) \qquad (2.3)$$

with: $t + i + f = 1$

- Variant (II)

$$t = \frac{p - \frac{1-pp}{2}(1-|2p-1|)}{1+(1-pp)|2p-1|} \qquad (2.4)$$

$$i = \frac{1-pp}{1+(1-pp)|2p-1|} \qquad (2.5)$$

$$f = \frac{1-p-\frac{1-pp}{2}(1-|2p-1|)}{1+(1-pp)|2p-1|} \qquad (2.6)$$

With $t + i + f = 1$

8. Conclusion

An intermediate representation between fuzzy logic in its intuitutionistic variant and neutrosophy can be considered with more generality than the former and less complexity than the latter. It is an extension of the basic probability by considering also its plausibility (or confidence). Thus it may be appropriate in situations where a less detailed modeling of uncertainty than the one of neutrosophy is considered satisfactory. Then the treatments are simpler but, above all, it is easier to collect the opinions of the experts by simply asking them for their forecast and also to estimate he certainty they attach to their prediction. This representation is equivalent to that of an interval (of probabilities), or to that used in probability theory with the mean value and variance. So this representation allows to remain for operations at well-known numerical calculations that do not involve functions with degrading effect on information such as min and max.

Also however, on the basis of this easier questioning of the experts, it will be possible to produce a neutrosophical representation that is more general by the simple transformations proposed, and thus to spare the experts of the burden of a higher complexity of thinking while still exploiting the power of neutrosophy to better treat the uncertainty inherent in real systems

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