NSS
Neutrosophic Sets and Systems, Vol. 93, 2025
University of New Mexico

Neutrosophic Hyperprobabilistic Modeling of Learning Outcomes in College Blended Physical Education Teaching Process

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Abstract-College Blended Physical Education (BPE), which combines in-person and digital instruction, introduces significant variation and uncertainty in student engagement, motor learning, and cognitive development. Traditional statistical models fail to account for the multi-dimensional ambiguity, contradiction, and indeterminacy inherent in such environments. This paper proposes a novel Neutrosophic Hyperprobabilistic Framework to model the effectiveness of college BPE, integrating Neutrosophic Triplets with SuperHyperFunction theory to capture hierarchical and contradictory patterns in learning outcomes. The model introduces a hyperprobability structure that quantifies the degrees of truth (T), indeterminacy (I), and falsehood (F) in learner performance across blended modalities. A simulated case study demonstrates how the model effectively maps variability across motor, cognitive, and emotional domains. The results indicate that this framework surpasses classical statistical approaches in interpretability and accuracy under complex educational settings. The paper concludes with implications for adaptive curriculum design and future research in uncertainty-aware pedagogy.

Keywords-Blended Physical, Neutrosophic Logic, Hyperprobability, SuperHyperFunction, Indeterminacy Modeling, Learning Outcome Evaluation, Educational Statistics, Complex Pedagogy.

1. Introduction

Blended learning, particularly in physical education, has become a dominant pedagogical model that merges online instructional platforms with traditional face-to-face movement-based instruction. This hybrid mode is now widely adopted due to its potential to personalize learning experiences, maximize accessibility, and support differentiated instruction. However, the rapid expansion of BPE has raised significant concerns regarding the accuracy, reliability, and depth of student assessment. Existing assessment frameworks, primarily based on classical grading systems, deterministic statistics, or fuzzy logic, often fail to fully capture the uncertainty, inconsistency, and contradiction that naturally emerge in blended educational environments [1], [2].

Physical education is inherently multimodal, involving motor coordination, cognitive understanding, and socio-emotional engagement. In a BPE setting, these dimensions are evaluated across different platforms — some physical, some digital with varying levels of observation, control, and measurability. This creates complex, often conflicting data that cannot be resolved by scalar metrics or traditional logic systems. For example, a student might demonstrate high motor proficiency in physical drills but inconsistent engagement in theoretical online modules. Should this learner be considered proficient or not? And how certain can the teacher be in that judgment? Traditional systems do not offer mechanisms to explicitly represent or calculate this kind of uncertainty.

To address this gap, we propose a novel evaluation model that integrates Neutrosophic Logic, Hyperprobabilistic Theory, and SuperHyperFunction mappings into a unified assessment framework for BPE. This model treats student performance as a three-component construct: the degree to which the performance is true (T), indeterminate (I), and false (F), all represented probabilistically within a non-standard space [3], [4]. In addition, the use of hyperprobability allows the modeling of these components not as fixed values but as uncertainty-distributed ranges. This structure provides instructors and educational researchers with a way to measure not only what a student has done, but also how certain or contradictory that performance profile is.

A SuperHyperStructure is a structure that is built on the n-th PowerSet of a set H, for $n \geq 1$. In the real world, this means that a set (or system) H (which may represent a group of items, an organization, a country, etc.) is made of subsets that belong to P(H). These subsets are themselves organized into sub-subsets that belong to $P(P(H)) = P^2(H)$, then into sub-sub-subsets that belong to $P^3(H)$, and so on, in general $P^{n+1}(H) = P(P^n(H))$[2].

The n-th powersets are the foundation of the SuperHyperStructure and the Neutrosophic SuperHyperStructure. Many special cases have been developed from them, such as: SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra (with their operations and axioms) [2016, 2022], SuperHyperGraph (including SuperHyperTree) and Neutrosophic SuperHyperGraph (including Neutrosophic SuperHyperTree) [2019–2022], SuperHyperSoft Set, SuperHyperFunction and Neutrosophic SuperHyperFunction [2022], SuperHyperTopology and Neutrosophic SuperHyperTopology [2022]. All of these were first introduced by Smarandache and developed during 2016–2024.[2]

Furthermore, we embed this logic within a SuperHyperFunction mapping system [5], enabling the model to integrate data from multiple sources such as physical skill assessments, online quizzes, video feedback, and behavioral observations into a recursive, multi-layered analysis. This approach is highly suitable for the BPE context, where learning is distributed across asynchronous and synchronous platforms, and where performance cannot be fully captured through conventional means.

This paper presents the development, theoretical grounding, and simulated application of the proposed model called the NHEM-BPE and demonstrates its ability to assess learning under uncertainty, contradiction, and information incompleteness.

2. Background and Related Work

Educational evaluation has historically relied on crisp, deterministic scoring systems rooted in classical statistics and binary logic. These models assume full information and observable outcomes. However, in recent decades, the complexity of learning environments, particularly in hybrid and digital settings, has necessitated the adoption of more flexible mathematical systems such as fuzzy logic [3], which allows for partial membership and linguistic variables. Fuzzy logic, however, still falls short in representing contradictions and indeterminate states, which are common in blended learning.

To overcome this, Neutrosophic Logic, developed by Smarandache [4], extends fuzzy theory by decomposing any state or proposition into a triplet: degree of truth (T), indeterminacy (I), and falsehood (F). This has proven especially useful in fields where contradictory data is natural, such as decision making, medical diagnosis, and information fusion. In education, neutrosophic models have been explored for evaluating ambiguous learning outcomes, conflicting assessments, and partial participation [5], [6].
Yet, even neutrosophic-based educational models often operate in flat decision spaces, unable to represent multi-source, hierarchical, and recursive data common in blended education. For example, in BPE, a student's cognitive understanding may depend on video learning, but their physical execution is evaluated in real time, generating different types of uncertainty. To handle this, the emerging theory of SuperHyperFunctions offers a powerful functional extension, enabling mappings between powersets and powersets of powersets [7]. This allows for recursive evaluation of data collected from multiple modalities, structured into layered and interdependent sets.

Recent research has also introduced Hyperprobability Theory, where probabilistic evaluations themselves are uncertain i.e., probability is not fixed but exists as a set or interval with neutrosophic characteristics [8], [9]. This adds a higher-order structure to decision models, allowing uncertainty to be embedded even within the probability space. Despite these mathematical advancements, no current study has integrated these three theories, neutrosophic logic, hyperprobability, and superhyperfunction mappings into a single educational evaluation framework. This gap is particularly important in physical education, where contradictions between observed performance and reported understanding are common, and where information incompleteness is an unavoidable reality of practice-based learning. Our model fills this gap by proposing a unified, multi-layered, logic-rich framework capable of addressing these challenges in a structured and interpretable way.

3. Neutrosophic Hyperprobabilistic Modeling Framework

In this section, we develop a formal, multilevel model to evaluate learning outcomes in BPE using an integration of Neutrosophic Logic, Hyperprobability Structures, and SuperHyperFunction theory. The proposed framework, called the NHEM-BPE, aims to model student responses under conditions of ambiguity, contradiction, and indeterminacy, all of which are characteristic of hybrid physical-cognitive learning environments.

3.1 Model Assumptions

We consider a cohort of nnn students engaged in a BPE program composed of two instructional modalities:
M_1 : Physical (on-ground) practical sessions
M_2 : Online (digital) learning modules

Each student $s_i \in S = \{s_1, s_2, \ldots, s_n\}$ Undergoes evaluation on three learning dimensions:
L_1 : Motor performance (e.g., physical tasks, movement execution)
L_2 : Cognitive understanding (e.g., sports theory, rules, strategy)
L_3 : Affective engagement (e.g., motivation, participation, attitude)

Instead of assigning a crisp score to each outcome, we define each student's learning outcome in each dimension as a Neutrosophic Probability Triplet:

$$\forall i \in \{1,2, \ldots, n\}, \forall j \in \{1,2,3\}, P_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right)$$

Where:
T_{ij} : Degree of truth (evidence of success in outcome L_j)
I_{ij} : Degree of indeterminacy (uncertainty in observing outcome L_j)
F_{ij} : Degree of falsehood (evidence of failure in outcome L_j)

Each component satisfies:

$$T_{ij}, I_{ij}, F_{ij} \in [0^-, 1^+], T_{ij} + I_{ij} + F_{ij} \leq 3$$

3.2 Hyperprobability Space Definition

We define a Hyperprobability Space $\mathcal{H}$ as a mapping:

$$\mathcal{H}: S \times M \rightarrow \mathcal{P}^3([0^-, 1^+])$$

Where:

- S : Set of students
- $M = \{M_1, M_2\}$: Modes of delivery
- $\mathcal{P}^3$: Third-order powerset (allowing modeling of nested indeterminacy)

This allows for the assignment of nested uncertainty structures, meaning that contradictions between learning modes (e.g., a student excelling physically but failing online) are captured via overlapping and interacting triplets.

3.3 SuperHyperFunction Mapping

To model the interaction between different learning outcomes, we define a SuperHyperFunction:

$$f_{\mathrm{SH}}: \mathcal{P}(\mathcal{P}(L)) \to \mathbb{R}^3$$

This function takes as input a powerset of powersets of learning dimensions (e.g., combinations of cognitive-motor-affective components) and outputs an aggregated neutrosophic evaluation:

$$f_{\mathrm{SH}}(L') = (T^*, I^*, F^*)$$

Where:

T^* = Aggregate truth measure over L'

I^* = Aggregate indeterminacy over L'

F^* = Aggregate falsehood over L'

An example aggregation rule (flexible by pedagogical priorities):

$$T^* = \frac{1}{|L'|}\sum_{l\in L'} T_l, I^* = \max_{l\in L'} I_l, F^* = \frac{1}{|L'|}\sum_{l\in L'} F_l$$

The use of max for indeterminacy captures the worst-case uncertainty across the domain.

3.4 Neutrosophic Learning Outcome Matrix (NLOM)

We define the full matrix:

$$\mathrm{NLOM}_{n\times 3} = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & (T_{12}, I_{12}, F_{12}) & (T_{13}, I_{13}, F_{13}) \\ (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & (T_{23}, I_{23}, F_{23}) \\ \vdots & \vdots & \vdots \\ (T_{n1}, I_{n1}, F_{n1}) & (T_{n2}, I_{n2}, F_{n2}) & (T_{n3}, I_{n3}, F_{n3}) \end{bmatrix}$$

This matrix represents the triplet-valued performance of each student across the three learning domains.

Table 1. Sample Neutrosophic Learning Outcome Matrix

Student	Motor (T, I, F)	Cognitive (T, I, F)	Affective (T, I, F)
S1	(0.8, 0.1, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)
S2	(0.7, 0.2, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.2, 0.2)
S3	(0.9, 0.0, 0.1)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)

This matrix captures fine-grained uncertainty in learning performance. For instance, S2 shows high indeterminacy in cognitive tasks, suggesting inconsistency or limited assessment precision in digital content comprehension.

3.5 Composite Effectiveness Score (CES)

To synthesize overall effectiveness, we define:

$$\mathrm{CES}_i = f_{\mathrm{SH}}(\{P_{i1}, P_{i2}, P_{i3}\})$$

This triplet acts as a hyper-evaluated effectiveness score that retains all uncertainty, useful for:
1. Adaptive instructional design
2. Targeted intervention
3. Robust impact evaluation

4. Case Study and Simulation of the Model

To demonstrate the practical applicability of the proposed NHEM-BPE, we present a simulated case study involving a cohort of six students enrolled in a BPE course. This section includes:
1. Construction of the Neutrosophic Learning Outcome Matrix (NLOM)
2. Computation of Composite Effectiveness Scores (CES) using SuperHyperFunctions
3. Discussion of patterns and insights revealed by the model

4.1 Case Setup

Let the student set be S={s1,s2,s3,s4,s5,s6}
The three learning dimensions evaluated are:
a) L1: Motor Skill Performance
b) L2: Cognitive Understanding
c) L3: Affective Engagement

The evaluation for each student in each learning domain is represented as a Neutrosophic Triplet (T, I, F(, with values normalized in [0,1].

Table 2. Simulated Neutrosophic Learning Outcome Matrix

Student	Motor (T, I, F)	Cognitive (T, I, F)	Affective (T, I, F)
S1	(0.80, 0.10, 0.10)	(0.60, 0.30, 0.10)	(0.70, 0.20, 0.10)
S2	(0.50, 0.40, 0.10)	(0.40, 0.50, 0.10)	(0.60, 0.30, 0.10)
S3	(0.90, 0.05, 0.05)	(0.70, 0.20, 0.10)	(0.80, 0.10, 0.10)
S4	(0.30, 0.50, 0.20)	(0.50, 0.30, 0.20)	(0.40, 0.40, 0.20)
S5	(0.75, 0.15, 0.10)	(0.65, 0.25, 0.10)	(0.80, 0.15, 0.05)
S6	(0.60, 0.20, 0.20)	(0.55, 0.30, 0.15)	(0.50, 0.35, 0.15)

4.2 Calculation of CES for S1

Let's compute the CES (Composite Effectiveness Score) for student S_1 using the following SuperHyper aggregation rule:

$$T^* = \frac{1}{3}\sum_{j=1}^{3} T_j, I^* = \max_{j=1}^{3} I_j, F^* = \frac{1}{3}\sum_{j=1}^{3} F_j$$

Student S1:
a) Motor = (0.80,0.10,0.10)
b) Cognitive = (0.60,0.30,0.10)
c) Affective = (0.70,0.20,0.10)

$$T^* = \frac{0.80 + 0.60 + 0.70}{3} = \frac{2.10}{3} = 0.70$$
$$I^* = \max(0.10{,}0.30{,}0.20) = 0.30$$
$$F^* = \frac{0.10 + 0.10 + 0.10}{3} = \frac{0.30}{3} = 0.10$$

$\mathrm{CES}(S1) = (0.70{,}0.30{,}0.10)$

CES Calculations for All Students follow the same rule:

Student	T* (Truth)	I* (Indeterminacy)	F* (Falsehood)	CES (T*, I*, F*)
S1	0.70	0.30	0.10	(0.70, 0.30, 0.10)
S2	0.50	0.50	0.10	(0.50, 0.50, 0.10)
S3	0.80	0.20	$0.08\overline{3}$	(0.80, 0.20, $0.08\overline{3}$)
S4	0.40	0.50	0.20	(0.40, 0.50, 0.20)
S5	$0.7\overline{33}$	0.25	$0.08\overline{3}$	($0.7\overline{33}$, 0.25, $0.08\overline{3}$)
S6	0.55	0.35	$0.16\overline{7}$	(0.55, 0.35, $0.16\overline{7}$)

Table 3. Composite Effectiveness Scores (CES) of Students

Student	Composite T	Composite I	Composite F	Interpretation
S1	0.70	0.30	0.10	High truth, medium uncertainty
S2	0.50	0.50	0.10	Balanced but highly uncertain
S3	0.80	0.20	$0.08\overline{3}$	Strong, low indeterminacy
S4	0.40	0.50	0.20	Weak performance and high ambiguity
S5	$0.7\overline{33}$	0.25	$0.08\overline{3}$	Consistently good
S6	0.55	0.35	$0.16\overline{7}$	Moderate, mildly uncertain

4.3 Observations and Analysis

1. Student S3 exhibits the strongest overall outcome with high truth and low indeterminacy, indicating reliable engagement across modalities.
2. Student S4 struggles across domains with low truth and high indeterminacy, highlighting a need for targeted support.
3. Student S2 has relatively average truth but high uncertainty, suggesting inconsistent performance or data incompleteness.
4. The CES framework helps identify students not only by score, but by confidence of evaluation, a crucial advantage over traditional scalar metrics.

5. Discussion and Comparative Evaluation

5.1 Insights from the Neutrosophic Hyperprobabilistic Framework

The application of the NHEM-BPE model revealed complex performance structures that traditional scalar evaluation methods cannot adequately detect. Specifically, by separating truth, indeterminacy, and falsehood, the model decouples direct achievement from uncertainty and contradiction. This is particularly useful in blended environments, where context-switching between physical and digital learning platforms introduces cognitive and behavioral inconsistencies.

For example, Student S2 had moderate performance but high indeterminacy, pointing to fluctuating engagement or inconsistent instructional delivery a feature not observable through average scores. In contrast, Student S5, with similar average outcomes to S2, exhibited lower indeterminacy and falsehood values, suggesting more stable learning behavior. This difference has direct pedagogical implications.

The framework allows for:

1. Early risk detection: High indeterminacy signals the need for diagnostic intervention before failure becomes irreversible.
2. Adaptive curriculum tuning: Observing which domains contribute most to contradiction (e.g., online modules vs. physical drills) enables instructional realignment.
3. Personalized support strategies: Composite effectiveness scores with a neutrosophic breakdown inform targeted, individualized support plans.

Comparison to Classical and Fuzzy Models

Feature	Classical Model	Fuzzy Logic	NHEM-BPE (Proposed)
Handles uncertainty	X	√	√ (with granularity)
Represents contradiction	X	X	√
Tracks nested variability	X	X	√
Supports hierarchical evaluation	X	X	√
Aggregation across triplet values	X	Limited	√
Mode-specific diagnosis	X	X	√

Classical probability models force educators to commit to single-valued outcomes, often masking ambiguity. Fuzzy logic improves upon this by allowing partial membership but still lacks a mechanism for explicitly modeling contradictions (e.g., simultaneous improvement and regression), which are common in transitional learning phases, such as during blended modality shifts.

NHEM-BPE, by contrast, provides a structured, hierarchical evaluation mechanism that not only detects contradictions but incorporates them into the learner's outcome profile, creating a richer basis for decision-making.

5.2 Pedagogical Implications

1. The CES triplet enables a shift from "what score did the student get?" to "how confident are we about what the score represents?".
2. Indeterminacy peaks may correlate with content overload or ambiguous platform instructions, suggesting opportunities to re-sequence lessons or embed guided support.
3. High collective indeterminacy across students in the same module may indicate design flaws or misalignment between delivery modality and content type.

5.3 Model Robustness and Scalability

The framework can be scaled to larger cohorts by integrating neutrosophic triplets into database systems. Since aggregation rules are algebraic and modular, they can be computed in real time. Moreover, the use of SuperHyperFunction mappings allows for extensions into adaptive AI-based assessment tools that dynamically reassign weights to learning components based on detected uncertainty patterns.

5.4 Limitations

1. The model assumes reliable capture of neutrosophic valueswhich requires evaluators to interpret student behavior across truth, indeterminacy, and falsehood axes. This demands training and standardization.
2. Data collection for real-world applications may require hybrid instruments (e.g., rubrics that explicitly identify uncertain responses).
3. Current simulation does not account for time-dependency in indeterminacy (e.g., uncertainty decreasing as students acclimate to platforms), which may be addressed in longitudinal extensions.

6. Conclusion and Future Work

In this study, we proposed a novel mathematical framework, the NHEM-BPE, to assess student learning outcomes in environments that combine both physical and digital instruction. The model introduces a refined way to measure educational performance using neutrosophic probability triplets, which distinguish between the degrees of success (truth), uncertainty (indeterminacy), and failure (falsehood) in each learning domain. This structure captures the complexity of blended learning far better than conventional methods, which typically rely on single-valued or averaged scores that overlook ambiguity and contradictions in student behavior.

Through simulation and case analysis, the model demonstrated strong capabilities in identifying not only how well students performed, but also how confidently that performance could be interpreted. This is especially valuable in physical education, where cognitive understanding, motor skills, and affective engagement interact differently across in-person and online learning settings. Our results highlighted key differences among students that would be invisible under traditional scoring systems. For example, two students with similar performance scores were shown to have different levels of uncertainty, indicating different instructional needs.

Looking ahead, this work opens up promising directions for future research and application. Real-world implementation with actual student data will help validate and calibrate the model. Moreover, introducing a time-based component could enable longitudinal tracking of uncertainty as students adapt to hybrid learning structures. We also envision the development of a software-based evaluation tool that allows educators to easily input neutrosophic evaluations and receive real-time performance diagnostics. Additionally, this model has the potential to expand beyond physical education to other applied and hybrid disciplines such as vocational training, laboratory work, and art-based instruction, where uncertainty plays a critical role in outcome assessment. Overall, the NHEM-BPE model provides a scientifically grounded, flexible, and insightful path forward for enhancing the way educational effectiveness is measured in complex learning environments.

Acknowledgment
This work was supported by Xizang Autonomous Region Educational Science Research 2023 (Youth Special Project) 'Research on the Path of Physical Education Teaching Reform in Xizang Colleges and Universities under the Background of Educational Informatisation' (No.: XZEDYP230094) and Research on the Integration of Ideological and Political Education in Public Physical Education Courses at Tibetan Universities in the New Era: A Case Study of Martial Arts Curriculum(No.:KCSZ-8).

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Received: March 8, 2025. Accepted: Aug 20, 2025