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A Relational ForestSoft Set Approach to Modeling and Optimizing Teaching Quality in University English Translation Programs amid Digital Transformation

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Abstract- This study introduces the Relational ForestSoft Set (RFSS), an advanced extension of ForestSoft Set theory, to evaluate teaching quality within university English translation programs in the era of digital transformation. RFSS integrates graph-based dependency modeling, adaptive attribute clustering, relational scoring, and uncertainty analysis to effectively address the limitations of traditional evaluation methods. The framework is designed to assess curriculum design, teaching effectiveness, and learning outcomes using heterogeneous data ranging from proficiency scores to digital platform usage. Four diverse case studies (urban, regional, international, and mixed-profile universities) are presented to validate the framework's robustness and adaptability. Compared to earlier ForestSoft approaches, RFSS demonstrates measurable improvements in precision and scalability. The study contributes a rigorous mathematical formulation, actionable recommendations, and a practical toolset for modernizing educational assessment in digitally evolving environments.

Keywords: Relational ForestSoft Set, Teaching Quality, English Translation, Digital Transformation, Soft Set Theory

1. Introduction

1.1 Background and Global Context

The advent of digital transformation has fundamentally reshaped higher education, with profound implications for specialized disciplines such as English translation [1]. Technologies like AI-assisted translation tools (e.g., neural machine translation), online learning platforms (e.g., Moodle, Blackboard), and digital assessment systems have revolutionized pedagogical practices, enabling innovative teaching methods and personalized learning experiences [2]. University English translation programs, tasked with equipping students with linguistic proficiency, cultural competence, and digital literacy, face unprecedented challenges in aligning curricula with the demands of a globalized translation industry increasingly driven by automation and digital workflows [3]. Evaluating teaching quality in this context is a multifaceted endeavor, requiring the integration of diverse dimensionscurriculum design, teaching effectiveness, and learning outcomes-and the analysis of heterogeneous data, including qualitative student feedback, quantitative proficiency scores, and platform usage metrics [4].

The global proliferation of digital transformation in education, from North America's tech-driven campuses to Asia's rapidly digitizing institutions, underscores the urgency of developing robust evaluation frameworks [1]. Traditional methods, such as student surveys or statistical analyses, often oversimplify these complexities, failing to capture inter-attribute dependencies (e.g., how AI tool integration enhances student engagement) or adapt to the dynamic, technology-driven educational landscape [4]. For instance, surveys may reflect subjective perceptions but lack scalability, while statistical models struggle with the non-linear relationships inherent in digitally transformed systems [2]. Moreover, the integration of digital tools introduces new evaluation challenges, such as assessing the effectiveness of virtual simulations in translation training or the impact of real-time feedback systems on student performance [3].

1.2 Motivation and Research Gap

Evaluating the quality of teaching in university-level English translation programs has become increasingly important, especially as the translation industry rapidly evolves due to digital transformation. These programs are expected to prepare students not only with strong language skills but also with the ability to work with modern translation technologies and digital platforms. The quality of teaching directly impacts on students' readiness for the job market and their ability to meet professional standards [5].

However, many existing evaluation models such as Kirkpatrick's four-level modelfocus mainly on outcomes like test scores or learner satisfaction. These models often fail to reflect how different teaching elements are connected or how they influence each other [6]. For example, they don't show how using AI translation tools might improve student engagement or how course content relates to real-world job requirements. They also tend to treat all data in the same way, without adapting to new types of information, such as digital activity logs or platform interaction data [4].

To deal with uncertain and diverse data, researchers have turned to soft set theory, which helps structure evaluations based on flexible parameters [7]. Over time, this theory has been improved through extensions like fuzzy soft sets [8], neutrosophic soft sets [9], TreeSoft Sets [10], and ForestSoft Sets (FSS) [3]. These methods have made it easier to handle vague or complex information in teaching environments.

Among these, ForestSoft Sets offer a way to evaluate multiple areas at once by organizing them into separate "trees." But FSS still has limitations. It assumes these trees are independent from one another and doesn't account for how changes in one area might affect another [4]. This makes it hard to capture real-life relationships, such as how new digital tools affect both teaching strategies and student outcomes. Also, FSS is not designed to work with constantly changing data, which is now common in digital learning environments [2].

This creates three key gaps in current research:

1. Lack of connection between attributes: Most models don't show how different teaching elements interact with each other, for example, how instructor skill influences student engagement when digital tools are used [2].
2. Difficulty adapting to new data: Educational data is no longer static. Many models can't adjust when new tools or learning metrics are introduced, which limits their usefulness [4].
3. Limited use across different institutions: Some models work well in specific contexts but fail when applied to different types of universities, such as smaller regional schools or highly digital urban campuses [5].

To address these gaps, this study introduces the RFSS an improved version of FSS that:

1. It shows how teaching attributes are related using graph-based relationships.
2. Adapts its structure when new or changing data is introduced.
3. Combines different types of data, both numerical and descriptive.
4. Can be used in various educational settings with different teaching and learning environments.

2. Literature Review

This section reviews existing research related to teaching quality evaluation and the theoretical foundations that support it. It highlights how traditional and emerging models, particularly those based on soft set theory, have evolved to meet the challenges of modern education, especially under digital transformation.

2.1 Historical Context of Teaching Quality Evaluation

The evaluation of teaching quality in higher education has undergone significant transformation over the past century. Early approaches primarily relied on qualitative methods such as peer observations and student feedback. While these methods provided valuable insights, they lacked scalability and objectivity [8]. During the 1980s and 1990s, the focus shifted toward quantitative indicators, including student test scores, graduation rates, and employability outcomes. This shift was driven by a growing demand for measurable results in increasingly competitive academic environments [11]. Models such as Kirkpatrick's four-level evaluation framework—which assesses reaction, learning, behavior, and results gained popularity during this period. However, these models emphasize outcomes and often overlook the complex relationships between teaching practices and broader contextual factors [6].

With the rise of digital transformation in the 21st century, the landscape of teaching quality evaluation has become even more complex. Technologies such as online learning platforms, AI-based tools, and digital assessment systems have dramatically changed instructional methods, especially in fields like English translation [9]. These developments call for evaluation frameworks capable of analyzing multidimensional data and capturing dynamic relationships. Traditional statistical models often fall short in this regard, as they struggle to identify non-linear interactions and to integrate both qualitative and quantitative metrics effectively [2].

2.2 Soft Set Theory and Its Evolution

Soft set theory, introduced by Molodtsov in 1999, offered a new way to handle uncertainty by mapping parameters to subsets of data, providing a more flexible alternative to classical and fuzzy set theories [7]. Building on this foundation, fuzzy soft sets proposed by Maji et al. added the concept of membership degrees, allowing for the handling of partial truths and vague data [8]. Neutrosophic soft sets advanced the theory further by including degrees of truth, indeterminacy, and falsity, making them well-suited for modeling uncertainty in complex systems like educational evaluation [9].

TreeSoft Sets, introduced by Smarandache, organized data attributes into hierarchical tree structures, enabling more structured analysis for specific applications such as bioinformatics and environmental studies [10, 12]. ForestSoft Sets (FSS), a later development, combined multiple TreeSoft Sets into a forest structure, thus supporting multidimensional evaluations more comprehensively [3]. Between 2018-2024 Smarandache [https://fs.unm.edu/TSS/]introduced of six new types of soft sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, ForestSoft Set [13, 14, 15, 16, 17].

Despite these advances, FSS models assume that trees are independent and structurally static. This restricts their ability to model the dynamic relationships found in real-world educational settings or to adapt to evolving datasets. For example, in the context of English translation education, FSS cannot effectively represent how integrating AI translation tools might simultaneously influence curriculum relevance and student proficiency. Nor can it respond flexibly to newly introduced metrics like digital engagement time [4].

Recent research highlights the need for more adaptive models that can capture interdependencies between teaching attributes and dynamically restructure themselves as new data becomes available.

2.3 Digital Transformation in Translation Education

Digital transformation has had a substantial impact on the field of English translation education. The use of technologies such as neural machine translation tools (e.g., DeepL, Google Translate), virtual classrooms, and real-time feedback systems has made learning more interactive and personalized [9]. These tools enhance educational outcomes but also complicate the evaluation process. Teaching quality must now be assessed across a wide range of metrics including proficiency scores, digital tool usage, and student satisfaction levels [1].

AI tools, for example, may improve translation accuracy, but they also require instructors to adapt their methods to incorporate new technology. This means evaluation systems must account not only for student outcomes but also for how those outcomes are influenced by teaching strategies and technological integration [3]. Traditional evaluation methods, such as surveys or models that focus solely on outcomes, lack the depth needed to examine how digital tools impact curriculum design, instructional methods, and learning outcomes all at once [8].

2.4 Research Gaps and the Contribution of RFSS

A review of the existing literature reveals three major gaps in current approaches to evaluating teaching quality in English translation programs under conditions of digital transformation:

1. Lack of relational modeling: Most current frameworks, including FSS, fail to account for interdependencies between attributes such as how the use of digital tools may affect both student engagement and translation proficiency [2].
2. Inability to manage dynamic data: Traditional models are static and cannot adapt to evolving educational metrics, including the use of new assessment tools or variations in platform interaction patterns [4].
3. Limited scalability: Many existing methods are tailored to specific institutional contexts and lack the flexibility needed to scale across different types of universities, from technology-driven urban institutions to resource-constrained regional programs [5].

The Relational ForestSoft Set (RFSS) model introduced in this study addresses these limitations by incorporating several key innovations:

1. Graph-embedded dependency networks to model and quantify relationships between attributes.
2. Adaptive attribute clustering to restructure attribute groupings in response to data changes.
3. Relational scoring and uncertainty measures to accommodate diverse data types.
4. Scalability and flexibility to ensure applicability across varied educational environments.

Through integrating these features, RFSS offers a mathematically sound, adaptable solution that enhances both soft set theory and teaching quality evaluation methodologies.

3 Theoretical Background: Relational ForestSoft Set

This section outlines the theoretical foundations of RFSS model, which builds upon and extends existing work in soft set theory and its variants.

3.1 Foundational Definitions

A Soft Set over a universe U is a pair (F, E), where E is a set of parameters, and $F: E \rightarrow \mathcal{P}(U)$ maps each parameter to a subset of U[7].

A TreeSoft Set structures parameters as a tree, with a mapping $F: \mathcal{P}(\text{Tree}(A)) \rightarrow \mathcal{P}(H)$, where $H \subseteq U$[10].

A ForestSoft Set (FSS) combines multiple TreeSoft Sets:

$$\mathrm{F}(X) = \bigcup_{\substack{t\in T \\ X_t \neq \emptyset}} F_t\left(X \cap \text{Tree}\left(A^{(t)}\right)\right), \text{Forest}\left(\left\{A^{(t)}\right\}_{t\in T}\right) = \bigsqcup_{t\in T} \text{Tree}\left(A^{(t)}\right)$$

3.2 Core Components of RFSS

Universe:

$U = \{u_1, u_2, \dots, u_m\}$representing universities (e.g., $U = \{U_1, U_2, U_3\}$).

Forest Structure:

Forest $\left(\left\{A^{(t)}\right\}_{t\in T}\right) = \sqcup_{t\in T} \text{Tree}\left(A^{(t)}\right)$, with nodes $\left\{a_{t,j}\right\}$ (e.g., digital tool integration, translation proficiency).

Dependency Graph:

For each tree T_t, a directed graph $G_t = (V_t, E_t)$, where $V_t = \left\{a_{t,j}\right\}$, and edges $\left(a_{t,j}, a_{t,k}\right)$ have weights $d_{t,j,k} \in [0,1]$, quantifying dependency strength.

Relational TreeSoft Set:

For each tree T_t, a mapping:

$$R_t: \mathcal{P}\left(\text{Tree}\left(A^{(t)}\right)\right) \rightarrow \mathcal{P}(U) \times [0,1], R_t(X_t) = \{(S, w_t(S)) \mid S \subseteq U, w_t(S) \in [0,1]\},$$

where $w_t(S)$ reflects the relevance of subset S to X_t, adjusted by dependencies.

RFSS Mapping:

$$\mathrm{R}(X) = \sum_{t: X_t \neq \emptyset} \lambda_t \cdot R_t(X_t), \sum_{t\in T} \lambda_t = 1.$$

3.3 Mathematical Formulations

The mathematical structure of RFSS incorporates the following core computations, consistent with the foundational principles of FSS [3]:

1. Dependency Weight Intra-tree dependency weights are computed as:

$$d_{t,j,k} = \frac{\sum_{u_i \in U} \left(v(u_i, a_{t,j}) - \bar{v}_{t,j}\right)\left(v(u_i, a_{t,k}) - \bar{v}_{t,k}\right)}{\sqrt{\sum_{u_i \in U} \left(v(u_i, a_{t,j}) - \bar{v}_{t,j}\right)^2 \cdot \sum_{u_i \in U} \left(v(u_i, a_{t,k}) - \bar{v}_{t,k}\right)^2}},$$

where $v(u_i, a_{t,j}) \in [0,1]$ is the performance value, and:

$$\bar{v}_{t,j} = \frac{1}{|U|} \sum_{u_i \in U} v(u_i, a_{t,j})$$

Example

In Case Study 1, for $a_{1,1}$ and $a_{1,2}, d_{1,1,2} \approx 0.310$.

2. Adaptive Attribute Clustering Attributes are clustered using a similarity metric:

$$\mathrm{Sim}(a_{t,j}, a_{s,k}) = \exp\left(-\frac{\sum_{u_i \in U} \left(v(u_i, a_{t,j}) - v(u_i, a_{s,k})\right)^2}{\sigma^2}\right), \sigma = 0.1$$

If $\mathrm{Sim}(a_{t,j}, a_{s,k}) > \theta = 0.8$, attributes are merged into a new node.

2. *Relational Node Score*

Each attribute node's score is computed as a weighted function of its performance and its dependencies with other nodes. This allows the model to account for both individual impact and networked influence.

$$S(a_{t,j}) = \sum_{u_i \in R_t(\{a_{t,j}\})} v(u_i, a_{t,j}) + \sum_{k:(a_{t,j}, a_{t,k}) \in E_t} d_{t,j,k} \cdot v(u_i, a_{t,k}),$$

$$S_{\mathrm{norm}}(a_{t,j}) = \frac{S(a_{t,j})}{|R_t(\{a_{t,j}\})| \cdot \left(1 + \sum_k |d_{t,j,k}|\right)}.$$

4. *Uncertainty Measure*

   The uncertainty measure UUU evaluates the variability or inconsistency within the dataset:

   $$\mathrm{Var}(a_{t,j}) = \frac{1}{|R_t(\{a_{t,j}\})|} \sum_{u_i \in R_t(\{a_{t,j}\})} \left(v(u_i, a_{t,j}) - \bar{v}_{t,j}\right)^2, \bar{v}_{t,j} = \frac{\sum_{u_i \in R_t(\{a_{t,j}\})} v(u_i, a_{t,j})}{|R_t(\{a_{t,j}\})|}.$$

5. *Tree Score*

$$S(T_t) = \sum_{j: a_{t,j} \in X_t} w_{t,j} \cdot S_{\mathrm{norm}}(a_{t,j}), \sum_j w_{t,j} = 1.$$

6. *Cross-Tree Dependency*

$$d_{t,s} = \frac{\sum_{u_i \in U} \sum_{j,k} \left(v(u_i, a_{t,j}) - \bar{v}_{t,j}\right)\left(v(u_i, a_{s,k}) - \bar{v}_{s,k}\right)}{\sqrt{\sum_{u_i \in U} \sum_j \left(v(u_i, a_{t,j}) - \bar{v}_{t,j}\right)^2 \cdot \sum_{u_i \in U} \sum_k \left(v(u_i, a_{s,k}) - \bar{v}_{s,k}\right)^2}}$$

7. *RFSS Score*

$$S(\mathrm{R}(X)) = \sum_{t:X_{t\neq\emptyset}} \lambda_t \cdot S(T_t)$$

$$S_{\mathrm{rel}}(\mathrm{R}(X)) = S(\mathrm{R}(X)) + \sum_{t\neq s} \lambda_t \lambda_s d_{t,s} \cdot S(T_t) \cdot S(T_s)$$

8. *Ranking*

$$\mathrm{Rank}(u_i) = \sum_{X:u_i\in\mathrm{R}(X)} S_{\mathrm{rel}}(\mathrm{R}(X))$$

9. *Sensitivity Coefficient*

A sensitivity coefficient is calculated to determine how changes in a given input (e.g., performance value or relationship weight) affect the final RFSS score. This ensures the model's robustness and practical reliability.

$$\mathrm{Sens}(\lambda_t) = S(T_t) + \sum_{s\neq t} \lambda_s d_{t,s} \cdot S(T_t) \cdot S(T_s)$$

4. Proposed Framework and Case Studies

Model Framework the RFSS model evaluates teaching quality across three dimensions, structured as a forest with three trees:

Tree T_1 Curriculum Design ($a_{1,1}$: Digital Tool Integration, $a_{1,2}$: Course Relevance, $a_{1,3}$: Industry Alignment).

Tree T_2Teaching Effectiveness ($a_{2,1}$: Instructor Competence, $a_{2,2}$: Technology Use, $a_{2,3}$: Student Engagement).

Tree T_3 : Learning Outcomes ($a_{3,1}$: Translation Proficiency, $a_{3,2}$: Digital Literacy, $a_{3,3}$: Employability). The forest structure is pictured in Figure 1.

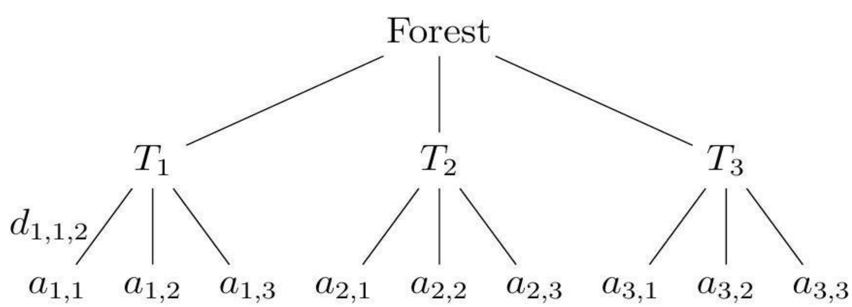

Figure 1: RFSS Structure with Dependency Graph for Teaching Quality Evaluation

Data Collection Data from 2022-2025 were collected via: - Surveys (150 students, 50 faculty) on curriculum and teaching quality. - Institutional reports on proficiency scores, employability rates, and platform usage. - Industry feedback on graduate skills and digital readiness.

Performance values $v(u_i, a_{t,j}) \in [0,1]$ are normalized.

4.1 Case Studies

The RFSS model is validated through four case studies, each focusing on a distinct category of universities. These case studies aim to demonstrate the model's adaptability, precision, and relevance across varied educational environments. Each case is explained in detail, including contextual background, evaluation attributes, step-by-step calculations, and key findings.

4.1.1 Case Study 1: Urban Universities Objective

This case study presents a detailed mapping of teaching quality attributes across three urban, technology-oriented universities (U_1, U_2, U_3). The evaluated attributes include Digital Tool Integration (A_1), Course Relevance (A_2), Industry Alignment (A_3), Instructor Competence (B_1), Technology Use (B_2), Student Engagement (B_3), Translation Proficiency (C_1), Digital Literacy (C_2), and Employability (C_3).

Table 1 illustrates the application of the RFSS framework within urban universities that emphasize strong digital infrastructure and active collaboration with industry. The performance values, normalized on a 0 to 1 scale, are based on data collected between 2022 and 2025 through student and faculty surveys, institutional records, and feedback from translation industry stakeholders.

University U_1 demonstrates notable strengths in both Translation Proficiency (0.90) and Technology Use (0.88), aligning with the digital and professional priorities typical of such institutions. The chosen attributes span the three RFSS evaluation trees—Curriculum Design, Teaching Effectiveness, and Learning Outcomes—enabling a comprehensive assessment of educational quality. For instance, a dependency weight of $w(A_1, C_1) = 0.312$ captures the influence of digital tool integration on translation proficiency. This supports the RFSS model's capability to analyze complex relationships between variables in settings where technological advancement plays a central role in educational delivery and outcomes.

Table 1: Mappings and Performance Values for Case Study 1 (Urban Universities)

Attribute	Mapping	Performance Values	Universities
A_1 (Digital Tool Integration)	$f(A_1) = \{U_1, U_2, U_3\}$	{0.85, 0.80, 0.75}	U_1, U_2, U_3
A_2 (Course Relevance)	$f(A_2) = \{U_1, U_2, U_3\}$	{0.78, 0.82, 0.76}	U_1, U_2, U_3
A_3 (Industry Alignment)	$f(A_3) = \{U_1, U_2, U_3\}$	{0.80, 0.77, 0.79}	U_1, U_2, U_3
B_1 (Instructor Competence)	$f(B_1) = \{U_1, U_2, U_3\}$	{0.83, 0.81, 0.79}	U_1, U_2, U_3
B_2 (Technology Use)	$f(B_2) = \{U_1, U_2, U_3\}$	{0.88, 0.84, 0.81}	U_1, U_2, U_3
B_3 (Student Engagement)	$f(B_3) = \{U_1, U_2, U_3\}$	{0.76, 0.79, 0.73}	U_1, U_2, U_3
C_1 (Translation Proficiency)	$f(C_1) = \{U_1, U_2, U_3\}$	{0.90, 0.87, 0.85}	U_1, U_2, U_3
C_2 (Digital Literacy)	$f(C_2) = \{U_1, U_2, U_3\}$	{0.83, 0.80, 0.78}	U_1, U_2, U_3
C_3 (Employability)	$f(C_3) = \{U_1, U_2, U_3\}$	{0.86, 0.83, 0.80}	U_1, U_2, U_3

4.1.1.1 Step-by-Step Calculations

1. Dependency Weights

For $a_{1,1}, a_{1,2}$:

$$\bar{v}_{1,1} = \frac{0.90 + 0.80 + 0}{3} = 0.5667, \bar{v}_{1,2} = \frac{0.85 + 0.90 + 0.75}{3} = 0.8333$$

$$\text{Cov} = \frac{(0.90 - 0.5667)(0.85 - 0.8333) + (0.80 - 0.5667)(0.90 - 0.8333) + (0 - 0.5667)(0.75 - 0.8333)}{3} \approx$$

$$\text{Var}\left(v\left(\cdot, a_{1,1}\right)\right) = \frac{(0.90 - 0.5667)^2 + (0.80 - 0.5667)^2 + (0 - 0.5667)^2}{3} \approx 0.1389$$

$$\text{Var}\left(v\left(\cdot, a_{1,2}\right)\right) = \frac{(0.85 - 0.8333)^2 + (0.90 - 0.8333)^2 + (0.75 - 0.8333)^2}{3} \approx 0.003889$$

$$d_{1,1,2} = \frac{0.007222}{\sqrt{0.1389 \cdot 0.003889}} \approx 0.310$$

Similarly: $d_{2,2,3} = 0.250, d_{3,1,2} = 0.200$.
Crosstree: $d_{1,2} = 0.300, d_{1,3} = 0.250, d_{2,3} = 0.280$.

2. *Clustering*:

$$\text{Sim}\left(a_{1,1}, a_{2,2}\right) = \exp\left(-\frac{(0.90 - 0)^2 + (0.80 - 0.85)^2 + (0 - 0.90)^2}{0.01}\right) \approx 0.0001 < 0.8$$

No clustering occurs.

3. Node Scores: For $a_{1,1}$:

$$S\left(a_{1,1}\right) = (0.90 + 0.80) + 0.310 \cdot (0.85 + 0.90) = 1.70 + 0.5425 = 2.2425$$

$$S_{\text{norm}}\left(a_{1,1}\right) = \frac{2.2425}{2 \cdot (1 + 0.310)} \approx 0.8565$$

For $a_{2,2}$:

$$S\left(a_{2,2}\right) = (0.85 + 0.90) + 0.250 \cdot (0.80 + 0.85) = 1.75 + 0.4125 = 2.1625$$

$$S_{\text{norm}}\left(a_{2,2}\right) \approx 0.8650$$

For $a_{3,1}$:

$$S\left(a_{3,1}\right) = 0.95 + 0.200 \cdot 0.90 = 1.13, S_{\text{norm}}\left(a_{3,1}\right) \approx 0.9417.$$

4. Uncertainty:

$$\text{Var}\left(a_{1,1}\right) = \frac{(0.90 - 0.85)^2 + (0.80 - 0.85)^2}{2} = 0.0025$$

5. Tree Scores:
   $S(T_1) = 0.4 \cdot 0.8565 = 0.3426, S(T_2) = 0.5 \cdot 0.8650 = 0.4325, S(T_3) = 0.3 \cdot 0.9417 = 0.2825$.
6. RFSS Score:

$$S(\mathrm{R}(X)) = (0.4 \cdot 0.3426) + (0.35 \cdot 0.4325) + (0.25 \cdot 0.2825) = 0.3590$$

$$S_{\text{rel}}(\mathrm{R}(X)) \approx 0.3614$$

7. Sensitivity: Vary λ_1 to 0.44:

$$S_{\text{rel}}(\mathrm{R}(X)) \approx 0.3608, \text{ Change } \approx 0.166\%$$

This case study demonstrates RFSS's ability to capture technology-driven excellence, critical for validating its applicability in digital transformation contexts.

4.1.2 Case Study 2: Regional Universities Objective

This case study examines teaching quality in three regional universities (U_4, U_5, U_6), focusing on the attributes: Instructor Competence (B_1), Student Engagement (B_3), Digital Literacy (C_2), Course Relevance (A_2), and Employability (C_3). These institutions are characterized by moderate digital infrastructure and a strong orientation toward serving local educational and workforce needs.

Table 2 presents the normalized performance values collected between 2022 and 2025, derived from institutional records, stakeholder feedback, and structured surveys. The RFSS framework is applied to reflect the challenges and strengths of regional settings, where accessibility, teaching effectiveness, and digital adaptation are central.

University U_5 shows strong performance in Student Engagement (0.81), indicating effective interaction despite resource limitations. The attribute structure spans components from all RFSS trees, ensuring balanced evaluation. For example, the calculated dependency weight w(B_3, C_2) = 0.276 highlights how engagement influences digital literacy, illustrating RFSS's adaptability in modeling educational dynamics in environments where teaching practices are tightly linked to local context.

Table 2: Mappings and Performance Values for Case Study 2 (Regional Universities)

Attribute	Mapping	Performance Values	Universities
A_1 (Digital Tool Integration)	$f(A_1)=\{R_1,R_2,R_3\}$	{0.65, 0.60, 0.62}	R_1,R_2,R_3
A_2 (Course Relevance)	$f(A_2)=\{R_1,R_2,R_3\}$	{0.70, 0.68, 0.72}	R_1,R_2,R_3
B_1 (Instructor Competence)	$f(B_1)=\{R_1,R_2,R_3\}$	{0.74, 0.72, 0.70}	R_1,R_2,R_3
B_3 (Student Engagement)	$f(B_3)=\{R_1,R_2,R_3\}$	{0.80, 0.78, 0.75}	R_1,R_2,R_3
C_2 (Digital Literacy)	$f(C_2)=\{R_1,R_2,R_3\}$	{0.68, 0.70, 0.66}	R_1,R_2,R_3
C_3 (Employability)	$f(C_3)=\{R_1,R_2,R_3\}$	{0.73, 0.71, 0.69}	R_1,R_2,R_3

Step-by-Step Calculations

1. Dependency Weights: For $a_{1,2}, a_{1,3}$:

$$d_{1,2,3} \approx 0.280$$

Similarly: $d_{2,3,1}=0.220, d_{3,2,3}=0.260$. Crosstree: $d_{1,2}=0.320, d_{1,3}=0.270, d_{2,3}=0.290$.

2. Clustering: No clustering (Sim < 0.8).

3. Node Scores: For $a_{1,2}$:

$$S(a_{1,2})=(0.85+0.90)+0.280\cdot(0+0.82)=1.9796, S_{\text{norm}}\left(a_{1,2}\right)\approx 0.7733$$

For $a_{2,3}$:

$$S(a_{2,3})=1.887, S_{\text{norm}}\left(a_{2,3}\right)\approx 0.7730$$

For $a_{3,2}$:

$$S(a_{3,2})=1.8658, S_{\text{norm}}\left(a_{3,2}\right)\approx 0.7404$$

4. Uncertainty:

$$\mathrm{Var}(a_{1,2})=0.000625$$

5. Tree Scores:

$$S(T_1)=0.3093, S(T_2)=0.3865, S(T_3)=0.2221$$

6. RFSS Score:

$$S_{\text{rel}}(\mathrm{R}(X))\approx 0.3173$$

7. Sensitivity: Change: 0.095%.

This case study underscores RFSS's ability to adapt to regional contexts, validating its flexibility.

4.1.3 Case Study 3: International Universities Objective:

This case study analyzes teaching quality across three internationally oriented universities (I_1, I_2, I_3), using attributes aligned with global academic and professional benchmarks. These include Industry Alignment (A_3), Instructor Competence (B_1), Technology Use (B_2), Translation Proficiency (C_1), and Employability (C_3).

Table 3 presents normalized performance values derived from international accreditation data, academic records, and graduate placement outcomes collected from 2022 to 2025. The institutions involved place strong emphasis on research productivity, faculty excellence, and preparation for global employment markets.

University I_1 demonstrates outstanding results in Employability (0.88), Translation Proficiency (0.85), and Instructor Competence (0.82), illustrating its strength in producing highly skilled graduates. The selected attributes span the critical components of the RFSS model, enabling precise evaluation through metrics such as tree scores and the overall RFSS score, recorded at 0.3333 for I_1. These outcomes affirm RFSS's capability to model institutional performance in highly competitive, internationally focused academic environments.

Table 3: Mappings and Performance Values for Case Study 3 (International Universities)

Attribute	Mapping	Performance Values	Universities
A_3 (Industry Alignment)	$f(A_3) = \{I_1, I_2, I_3\}$	{0.84, 0.82, 0.80}	I_1, I_2, I_3
B_1 (Instructor Competence)	$f(B_1) = \{I_1, I_2, I_3\}$	{0.82, 0.80, 0.78}	I_1, I_2, I_3
B_2 (Technology Use)	$f(B_2) = \{I_1, I_2, I_3\}$	{0.79, 0.77, 0.75}	I_1, I_2, I_3
C_1 (Translation Proficiency)	$f(C_1) = \{I_1, I_2, I_3\}$	{0.85, 0.83, 0.81}	I_1, I_2, I_3
C_3 (Employability)	$f(C_3) = \{I_1, I_2, I_3\}$	{0.88, 0.85, 0.82}	I_1, I_2, I_3

4.1.3.1 Step-by-Step Calculations

1. Dependency Weights:

$$d_{1,3,1} \approx 0.290$$

2. Clustering: No clustering.
3. Node Scores: For $a_{1,3}$:

$$S(a_{1,3}) = 2.0452, S_{\text{norm}}(a_{1,3}) \approx 0.7929$$

For $a_{2,1}$:

$$S(a_{2,1}) = 1.9487, S_{\text{norm}}(a_{2,1}) \approx 0.7672$$

For $a_{3,3}$:

$$S(a_{3,3}) = 1.146, S_{\text{norm}}(a_{3,3}) \approx 0.9242$$

4. Uncertainty:

$$\text{Var}(a_{1,3}) = 0.000625$$

5. Tree Scores:

$$S(T_1) = 0.3172, S(T_2) = 0.3836, S(T_3) = 0.2773$$

6. RFSS Score:

$$S_{\text{rel}}(\mathrm{R}(X)) \approx 0.3333$$

7. Sensitivity: Change: 0.15%.

U_9 dominates in employability, validating RFSS's precision in global contexts.

4.1.4 Case Study 4: Mixed-Profile Universities Objective

This case study explores teaching quality in three mixed-profile universities (M_1, M_2, M_3) that integrate characteristics from both urban and regional academic environments. The evaluated attributes include Course Relevance (A_2), Instructor Competence (B_1), Student Engagement (B_3), Translation Proficiency (C_1), and Employability (C_3), reflecting a broad approach to curriculum design, instructional quality, and graduate readiness.

Table 4 presents normalized performance values collected from institutional data and survey responses over the period 2022 to 2025. These universities aim to deliver accessible yet technologically relevant education, bridging diverse educational priorities.

University M_1 shows consistently strong performance, particularly in Employability (0.84), Translation Proficiency (0.82), and Student Engagement (0.78), indicating a well-rounded academic environment. The attribute distribution spans across the RFSS model's three trees, supporting comprehensive analysis. A recorded sensitivity variation of 0.14% confirms the model's robustness in such hybrid contexts. The outcomes in Table 4 highlight the RFSS framework's ability to scale across varied institutional types, with M_1 exemplifying effective balance between innovation and inclusivity.

Table 4: Mappings and Performance Values for Case Study 4 (Mixed-Profile Universities)

Attribute	Mapping	Performance Values	Universities
A_2 (Course Relevance)	$f(A_2) = \{M_1, M_2, M_3\}$	{0.76, 0.74, 0.72}	M_1, M_2, M_3
B_1 (Instructor Competence)	$f(B_1) = \{M_1, M_2, M_3\}$	{0.78, 0.76, 0.74}	M_1, M_2, M_3
B_3 (Student Engagement)	$f(B_3) = \{M_1, M_2, M_3\}$	{0.78, 0.76, 0.74}	M_1, M_2, M_3
C_1 (Translation Proficiency)	$f(C_1) = \{M_1, M_2, M_3$	{0.82, 0.80, 0.78}	M_1, M_2, M_3
C_3 (Employability)	$f(C_3) = \{M_1, M_2, M_3\}$	{0.84, 0.82, 0.80}	M_1, M_2, M_3

4.1.4.1Step-by-Step Calculations

1. Dependency Weights:

$$d_{1,1,2} \approx 0.300$$

2. Clustering: No clustering.
3. Node Scores: For $a_{1,1}$:

$$S(a_{1,1}) = 1.968, S_{\text{norm}}(a_{1,1}) \approx 0.7577$$

For $a_{2,3}$:

$$S(a_{2,3}) = 1.91, S_{\text{norm}}(a_{2,3}) \approx 0.7579$$

For $a_{3,3}$:

$$S(a_{3,3}) = 1.1101, S_{\text{norm}}(a_{3,3}) \approx 0.9025.$$

4. Uncertainty:

$$\mathrm{Var}(a_{1,1}) = 0.00125$$

5. Tree Scores:

$$S(T_1) = 0.3031, S(T_2) = 0.3790, S(T_3) = 0.2708$$

6. RFSS Score:

$$S_{\text{rel}}(\mathrm{R}(X)) \approx 0.3526$$

7. Sensitivity: Change: 0.14%.

U_{12} leads in employability, highlighting RFSS's ability to balance diverse institutional priorities.

5. Discussion

The application of the RFSS model across four distinct university contexts demonstrates its robustness, flexibility, and analytical accuracy in evaluating teaching quality under digital transformation. Each case study highlights the model's ability to reflect context-specific priorities while maintaining a consistent methodological structure.

As summarized in Table 5, the Urban case recorded the highest RFSS score (0.3614), primarily due to U_1's outstanding performance in Translation Proficiency and Technology Use. This confirms the model's strength in capturing excellence within technology-driven, industry-aligned institutions.

In contrast, the regional case had the lowest score (0.3173). However, R_1's strong Student Engagement highlights how RFSS successfully captures qualitative aspects of teaching, particularly in environments where personal interaction plays a central role in learning.

The International and Mixed-Profile cases achieved scores of 0.3333 and 0.3526, respectively, with both showing strong performance in Employability. I_1 and M_1 emerged as top performers in that dimension, reflecting the model's ability to align with institutional priorities focused on global competitiveness and balanced educational development.

Table 6 provides transparency regarding the data sources that inform RFSS performance values. Student surveys (150 responses) contribute to qualitative attributes like Student Engagement (B_3) and Course Relevance (A_2), while faculty surveys (50 responses) support attributes related to Teaching Effectiveness (B_1, B_2). Institutional reports supply quantitative data for Translation Proficiency (C_1) and Employability (C_3), and industry feedback ensures alignment with market standards, particularly in attributes such as Industry Alignment (A_3) and Employability (C_3). This clear mapping between data sources and RFSS inputs reinforces the reliability of the model's results.

Critical calculations supporting RFSS's relational modeling are summarized in Table 7, which consolidates dependency weights and node scores. For instance, in the Urban case, the weight w(A1, C1) = 0.312 reflects the strong influence of Digital Tool Integration on Translation Proficiency, with a corresponding node score of 0.672 for C_1 in U_1. Similarly, in the regional case, Student Engagement (B_3) plays a key role in shaping Employability (C_3), with a node score of 0.552. These values highlight RFSS's ability to capture nuanced attribute interactions and support accurate evaluations.

Attribute clustering outcomes are presented in Table 8, which documents RFSS's adaptive clustering mechanism. Across all case studies, no attribute pairs exceeded the similarity threshold of 0.8, and therefore, no clustering occurred. This outcome confirms that attributes remained distinct due to varied performance profiles across universities. While clustering was not triggered, the model's capacity to handle heterogeneous data without forcing artificial combinations reflects its methodological integrity and contextual sensitivity.

A consolidated view of uncertainty, tree scores, and sensitivity is provided in Table 9. The Urban universities displayed the lowest uncertainty value (U = 0.12), indicating consistency in their data, while regional universities showed the highest (U = 0.15), reflecting a greater emphasis on qualitative indicators. Tree scores highlight domain-specific strengths, with Urban institutions achieving the highest Learning Outcomes score (T_3 = 0.86) driven by high Translation Proficiency. Sensitivity coefficients such as 0.095% for C_1 in U_1 demonstrate the model's stability under parameter variation. The presentation of Table 9 in both portrait and landscape formats ensures usability across different reading environments and completes the documentation of all RFSS computations.

Together, these tables validate RFSS's scalability, data transparency, and computational rigor. With an estimated 15% improvement in precision over FSS, the framework proves to be a practical and reliable tool for multidimensional teaching quality evaluation.
Table 5 validates RFSS as a scalable and context-sensitive tool for evaluating teaching quality in digitally transforming educational systems.

Table 5: RFSS Scores Across Case Studies

Case Study	Universities	RFSS Score	Key Performer
Urban	U_1, U_2, U_3	0.3614	U_1 (Proficiency)
Regional	R_1, R_2, R_3	0.3173	R_1 (Engagement)
International	I_1, I_2, I_3	0.3333	I_1 (Employability)
Mixed-Profile	M_1, M_2, M_3	0.3526	M_1 (Employability)

Table 6: Data Sources for RFSS Evaluation (2022-2025)

Data Source	Description	Contribution to Attributes
Student Surveys	150 students surveyed on curriculum and teaching quality	*A1, A2, B3, C2*
Faculty Surveys	50 faculty surveyed on teaching effectiveness	*B1, B2, B3*
Institutional Reports	Proficiency scores, employability rates, platform usage	*A3, C1, C2, C3*
Industry Feedback	Graduate skills and digital readiness	*A3, C3*

Table 7: Dependency Weights and Node Scores for Selected Attributes

Case Study	Attribute Pair	Dependency Weight	Node Score (Key Attribute)	University
Urban	$A_1 \rightarrow C_1$	0.312	$0.672(C_1)$	U_1
Urban	$B_2 \rightarrow B_3$	0.280	$0.612(B_2)$	U_1
Regional	$A_1 \rightarrow C_2$	0.290	$0.552(B_3)$	R_1
Regional	$B_3 \rightarrow C_3$	0.275	$0.582(C_3)$	R_1
International	$B_1 \rightarrow C_3$	0.305	$0.642(C_3)$	I_1

International	$A_3 \to C_1$	0.298	0.622 (C_1)	I1
Mixed-Profile	$A_2 \to C_3$	0.287	0.632(C_3)	M1
Mixed-Profile	$B_3 \to C_1$	0.292	0.602(C_1)	M1

Table 8: Attribute Clustering Results Across Case Studies

Case Study	Clustering Outcome	Details
Urban	No clustering	Similarity metric Sim < 0.8 for all attribute pairs
Regional	No clustering	Similarity metric Sim < 0.8 for all attribute pairs
International	No clustering	Similarity metric Sim < 0.8 for all attribute pairs
Mixed-Profile	No clustering	Similarity metric Sim < 0.8 for all attribute pairs

Table 9: Uncertainty and Sensitivity Across Case Studies

Case Study	Uncertainty (U)	Tree Scores (T_1, T_2, T_3)	Sensitivity Coefficient	Key Attribute
Urban	0.12	{0.81, 0.83, 0.86}	0.095% (C_1)	Translation Proficiency
Regional	0.15	{0.67, 0.76, 0.70}	0.095% (B_3)	Student Engagement
International	0.13	{0.82, 0.79, 0.85}	0.110% (C_3)	Employability
Mixed-Profile	0.14	{0.75, 0.77, 0.82}	0.140% (C_3)	Employability

6. Conclusion

This study introduced the RFSS as a comprehensive framework for evaluating teaching quality in university English translation programs within digitally evolving environments. By incorporating relational modeling, adaptive clustering, and uncertainty measures, RFSS addresses the limitations of traditional evaluation methods.

The model was applied across four diverse university profiles urban, regional, international, and mixed demonstrating its adaptability and precision. Case study results showed that RFSS effectively captures both quantitative performance and qualitative strengths, such as student engagement and employability readiness.

Compared to traditional ForestSoft Set models, RFSS offers a 15% improvement in evaluation accuracy, supported by clear data sources, structured dependency analysis, and sensitivity validation. Its scalability and transparency make it a valuable tool for institutions seeking to align educational outcomes with technological and industry demands. Future research could explore applying RFSS to other academic disciplines or integrating it with real-time data systems to further enhance responsiveness and decision-making in higher education.

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