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Multineutrosophic Analysis with the ARAS Method for Selecting Trading Strategies

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Abstract. Deciding on optimal business solutions is a perennial problem for firms challenged by ever-changing market environments. Currently, the choice of a solution entails multidimensional requirements in addition to similar uncertainties and vagueness factoring human fallibility and ever-challenging market conditions. Even though thousands of systems for decision-making support exist, the dedicated scholarly literature does not support any advancement that would ease an all-encompassing evaluation of strategic solutions during complete uncertainty or information vagueness. Therefore, this research aims to fill this theoretical gap by creating a new approach based on practical application. This new approach is based on the employment of the Additive Ratio Assessment method (the ARAS method) in a multineutrosophic setting to give better weighting to criteria and simultaneously assess strategic solutions. Multineutrosophic refers to a level of truth, falsity, and indeterminacy. Thus, by applying neutrosophic operators, the assessments of various experts are comprehensively applied to turn qualitative attributes into quantitative, highly reliable values, leading to appropriate results through the multineutrosophic ARAS method to assess degrees of effectiveness of business solutions to rank potential business strategies for ultimate success through widespread uncertainty. This work contributes theoretically and practically: 1. theoretically expands the scope of multi-criteria decision-making processes by formally incorporating the element of uncertainty; 2. practically provides firms with a new tool for conducting strategic assessments as it will increase efficiency and effectiveness of resource usage while decreasing vulnerability to inevitable downturns in the marketplace.

Keywords: Business Strategies, Multicriteria Decision Making, Neutrosophic Logic, ARAS Method, Uncertainty, Decision Making, Optimization.

1. Introduction

Strategic decision-making in the commercial field constitutes a fundamental pillar for the sustainability and growth of any organization in the current global environment [1]. In a landscape characterized by volatility and uncertainty, the selection of business strategies not only involves the evaluation of multiple quantitative factors, but also the integration of qualitative, often imprecise, elements. Addressing this complexity effectively has therefore become an imperative necessity for companies seeking to maintain their competitiveness and secure an advantageous position in the market [2]. Historically, strategic decision-making processes have transitioned from purely intuitive approaches to rigorous methodologies based on data analysis. From Porter 's influential theories on competitive forces, formulated in the 1980s, to the emergence of more recent business intelligence models, the evolution in this field has been constant [3]. However, despite significant advances in business analytics and decision support tools, most existing frameworks still struggle to coherently integrate the ambiguity and uncertainty inherent in expert opinions and changing environmental conditions.

The central problem that this research aims to solve lies in the persistent lack of a robust methodology that enables the selection of optimal business strategies under conditions of incomplete or vague information. How can organizations effectively process and utilize subjective judgments and uncertain data to make informed and reliable strategic decisions in a context of high complexity? This fundamental question drives our study, seeking to offer a solution that encompasses these intricate and often overlooked dimensions in traditional approaches. Although various multicriteria decision-making (MCDM) techniques exist, such as the Analytic Hierarchy Process (AHP), the Technique of Order of Preference by Similarity to the Ideal Solution (TOPSIS), or the ELECTRE Family [4], these conventional methodologies often assume that information is perfectly clear and precise. However, in the context of business strategy, data rarely live up to this idealization. Expert perceptions, market projections, and risk assessments are often tainted by uncertainty, imprecision, and inconsistency [5]. This significant gap in literature and practice highlights the urgent need to develop approaches that can handle this intrinsic nature of information. Faced with this limitation, neutrosophic logic emerges as a promising and highly relevant tool. This extension of fuzzy logic allows for modeling not only the degree of truth and falsity of a proposition, but also the degree of indeterminacy associated with it. By explicitly integrating indeterminacy, neutrosophic logic offers a more comprehensive and realistic framework for representing and processing human knowledge, especially in contexts where information is inherently vague or ambiguous. Therefore, its application in strategic decision-making could offer a more accurate and grounded perspective.

Additive Ratio Assessment (ARAS) method, recognized for its computational simplicity, efficiency, and proven ability to effectively rank alternatives, is presented as an ideal candidate to be adapted and extended under a neutrosophic setting. The combination of ARAS with neutrosophic logic allows for a more nuanced and comprehensive assessment of strategic alternatives, considering not only the favorability of each option, but also the uncertainty and ambiguity inherent in each evaluation criterion and the perception of decision-makers. This work not only seeks to develop an innovative decision support tool, but also to validate its effectiveness in a real-life context of business strategy selection. The implementation of this methodology is expected to provide decision-makers with a deeper understanding of the implications of each strategy, thus optimizing resource allocation and minimizing the risks associated with the inherent uncertainty of the global market. The practical relevance of this study lies in its significant potential to improve the quality and robustness of strategic decisions in an increasingly complex and dynamic business world.

The main objectives of this study are: 1) to develop a methodology for business strategy selection based on the multineutrosophic ARAS method; 2) to apply the proposed approach to a practical case study to demonstrate its feasibility and effectiveness in a real-life business environment; and 3) to evaluate the model's ability to manage and mitigate the uncertainty and indeterminacy inherent in complex strategic decision-making processes. These objectives will guide the research exhaustively and will be addressed in detail throughout this article.

2. Preliminaries.

Multineutrosophic Set

Definition 1. The *Neutrosophic Set N* It is characterized by three membership functions [9] , which are the truth membership function T_A, the indeterminacy membership function $I_{A,}$ and the falsity membership function F_A , where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-_A 0, 1^+[$, and $^-_A 0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [10] .

See that by definition, $T_A(x), I_A(x)$, and $F_A(x)$are standard or non-standard real subsets of $]^-_A 0, 1^+[$and hence, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $\bar{}_A0$and1^+ belong to the set of hyperreal numbers.

Definition 2. The *single-valued neutrosophic set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$and $F_A: U \rightarrow [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

SVNN are expressed below:

Given A $_1$ = (a $_1$, b $_1$, c $_1$) and A $_2$ = (a $_2$, b $_2$, c $_2$) two SVNN, the sum between A $_1$ and A $_2$ is defined as :

$$A_1 \; A_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2) \quad (1)$$

Given A $_1$ = (a $_1$, b $_1$, c $_1$) and A $_2$ = (a $_2$, b $_2$, c $_2$) two SVNNs, the multiplication between A $_1$ and A $_2$ is defined as:

$$A_1 \; A_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2) \quad (2)$$

The product of a positive scalar with a SVNN, A = (a, b, c) is defined as:

$$A = (1 - (1 - a), b, c) \quad (3)$$

The *single-valued neutrosophic number* (SVNN) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3. The refined neutrosophic set of subsets (SRNS).

Let $\mathcal{U}$ a universe of discourse and a set $R \subset \mathcal{U}$. Then, a refined neutrosophic subset R is defined as follows:

$R = \{x, x(T, I, F), x \in U\}$,where T is refined/divided into p subtruths, $T = \langle T_1, T_2, \ldots T_p \rangle, T_j \subseteq [0,1], 1 \leq j \leq p$; I is refined/divided into r subindeterminacies, $I = \langle I_1, I_2, \ldots I_r \rangle, I_k \subseteq [0,1], 1 \leq k \leq r$,and F is refined/divided into s subfalsehoods, $F = \langle F_1, F_2, \ldots F_l \rangle, F_s \subseteq [0,1], 1 \leq l \leq s$, where $p, r, s \geq 0$are integers, and $p + r + s = n \geq 2$, and at least one of $p, r, s \; is \geq 2$to ensure the existence of refinement (division).

Definition 4 ([6]) **.** The MultiNeutrosophic Set (or MultiNeutrosophic Set Subset SMNS).

Let $\mathcal{U}$ a universe of discourse and M a subset of it. Then, a MultiNeutrosophic Set is: $M = \{x, x(T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s)\}, x \in U$,

where p , r , s are integers $\geq 0, p + r + s = n \geq 2$ and at least one of them $p, r, s \; is \geq 2$,to ensure the existence of multiplicity of at least one neutrosophic component: truth/belonging, indeterminacy or falsity/non-belonging; all subsets$T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s \subseteq [0,1]$;

$$0 \leq \sum_{j=1}^{p} inf \; T_j + \sum_{k=1}^{r} inf \; I_k + \sum_{l=1}^{s} inf \; F_l \leq \sum_{j=1}^{p} sup \; T_j + \sum_{k=1}^{r} sup \; I_k + \sum_{l=1}^{s} sup \; F_l \leq n.$$

No other restrictions apply to these neutrosophics multicomponents..

$T_1, T_2, \ldots, T_p$They are multiplicities of truth, each provided by a different source of information (expert).

Similarly, $I_1, I_2, \ldots, I_r$there are multiplicities of indeterminacy, each provided by a different source.

And $F_1, F_2, \ldots, F_s$they are multiplicities of falsehood, each provided by a different source.

The Degree of MultiTruth (MultiMembership), also called *Multidegree of Truth* , of the element x with respect to the set M are $T_1, T_2, \ldots, T_p$.

The Degree of Multiindeterminacy (MultiNeutrality), also called *Multidegree of Indeterminacy* , of the element x with respect to the set M are$I_1, I_2, \ldots, I_r$.

and the Degree of Multi- Nonmembership , also called *Multidegree of Falsehood* , of the element x with respect to the set M are$F_1, F_2, \ldots, F_s$.

All of these $p + r + s = n \geq 2$are assigned by n sources (experts) which can be:

- whether completely independent ;
- or partially independent and partially dependent;
- or totally dependent; depending on or as needed for each specific application.

A generic element x with respect to the MultiNeutrosophic Set A has the form:

$x(T_1, T_2, \ldots, T_p;$	$I_1, I_2, \ldots, I_r;$	$F_1, F_2, \ldots, F_s)$
multi-truth	multi-indeterminacy	multiple falsehood

In many particular cases $p = r = s$, a source (expert) assigns the three degrees of truth, indeterminacy and falsity $T_j, I_j, F_j$to the same element.

Definition 5. Classification of multineutrosophic types of value n of the same form (p, r, s).

$(T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s)$, where $p, r, s$are integers ≥ 0, and $p + r + s = n \geq 2$, and at least one of $p, r, s \geq 2$to be sure that it has multiplicity for at least one neutrosophic component (either truth, or indeterminacy, or falsity).

It offers a simpler n classification, but it is more of an approximation. Let's calculate the following.

positivity (4).

$$\frac{\sum_{j=1}^{p} T_j + \sum_{k=1}^{r}(1 - I_k) + \sum_{e=1}^{s}(1 - F_e)}{p + r + s} \tag{4}$$

Average (Truth-Falsehood) (5)

$$\frac{\sum_{j=1}^{p} T_j + \sum_{e=1}^{s}(1 - F_e)}{p + s} \tag{5}$$

Truth (6).

$$\frac{\sum_{j=1}^{p} T_j}{p} \tag{6}$$

Definition 6. Classification of tuples n-valued multineutrosophics of different forms (p, r, s).

Let us consider two neutrosophic multiple tuples of value n of the forms $(p_1, r_1, s_1)$and respectively (p_2, r_2, s_2), where $p_1, r_1, s_1, p_2, r_2, s_2$ are integers ≥ 0 , and $p_1 + r_1 + s_1 = n_1 \geq 2$, and at least one of $p_1, r_1, s_1 is \geq 2$,to be sure that multiplicity exists for at least one neutrosophic component (either truth, indeterminacy or falsity); similarly $p_2 + r_2 + s_2 = n_2 \geq 2$,and at least one of$p_2, r_2, s_2\ is \geq 2$.

Let's take the following tuples single-value multineutrosophic (SVMNT):

$SVMNT = (T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s)$of $(p_1, r_1, s_1) - form$, and

$SVMNT' = (T'_1, T'_2, \ldots, T'_p; I'_1, I'_2, \ldots, I'_r; F'_1, F'_2, \ldots, F'_s)$of $(p_1, r_1, s_1) - form$.

It performs the classic averages of truth (T_a), indeterminacies (I_a) and falsity (F_a), respectively for $SVMNT = (T_a, I_a, F_a)$and the averages of truths (T_a), indeterminacies (I_a) and falsity (F_a) respectively for : $SVMNT = (T'_a, I'_a, F'_a)$. And then applies the Score Functions ( ), Accuracy ( ) and Certainty ( ), as for the single-valued neutrosophic set:

Calculate the scoring function (average positivity) (7).

$$S(T_a, I_a, F_a) = \frac{T_a + (1 - I_a) + (1 - F_a)}{3}$$
$$S(T'_a, I'_a, F'_a) = \frac{T'_a + (1 - I'_a) + (1 - F'_a)}{3} \quad (7)$$

i. If $S(T_a, I_a, F_a) \geq S(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $S(T_a, I_a, F_a) \leq S(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $S(T_a, I_a, F_a) = S(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$, then go to the second step.

Calculate the precision function (difference between truth and falsehood) (8).

$$A(T_a, I_a, F_a) = T_a - F_a$$
$$A(T'_a, I'_a, F'_a) = T'_a - F'_a \quad (8)$$

i. If $A(T_a, I_a, F_a) \geq A(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $A(T_a, I_a, F_a) \leq A(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $A(T_a, I_a, F_a) = A(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$, then go to the third step.

3. Calculate the certainty (truth) function (9).

$$C(T_a, I_a, F_a) = T_a$$
$$C(T'_a, I'_a, F'_a) = T'_a \quad (9)$$

i. If $C(T_a, I_a, F_a) \geq C(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $C(T_a, I_a, F_a) \leq C(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $C(T_a, I_a, F_a) = C(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$they are multi-neutrosopically equal, that is $T_a = T_a^{'}, I_a = I_a^{'}, F_a = F_a^{'}$, or their corresponding averages of truth, indeterminacy and falsity are equal.

Definition 7. In cases where some sources carry a greater weight in the evaluation than others, weighted averages are used, indexed as $T_{wa}, I_{ua}, F_{va}$and respectively $T'_{wa}, I'_{ua}, F'_{va}$,. Because sources can be independent or partially independent, the sum of the weights does not necessarily have to equal 1. Therefore :

i. $w_1, w_2, \dots, w_p \in [0,1]$, although the sum $w_1 + w_2 + \cdots + w_p$may be $< 1, or\ = 1, or\ > 1$.
ii. $u_1, u_2, \dots, u_p \in [0,1]$, although the sum $u_1 + u_2 + \cdots + u_p$may be $< 1, or\ = 1, or\ > 1$.
iii. $v_1, v_2, \dots, v_p \in [0,1]$, although the sum $v_1 + v_2 + \cdots + v_p$may be $< 1, or\ = 1, or\ > 1$.

And similarly, the score, precision, and certainty functions are applied to these weighted averages to rank them.

Multineutrosophic ARAS.

Additive Ratio Assessment (ARAS) method is a multi-criteria decision-making technique that allows the selection of the best option from a set of alternatives 7]. To this end, an extension of the traditional method is proposed through multi-neutrosophic set evaluation. Consequently, it is reformulated as the multi-neutrosophic ARAS method to determine the complex relative efficiency of each strategic guideline. This involves evaluating each strategic guideline through multiple sources (experts) based on the corresponding criteria. By integrating multi-neutrosophic set analysis into the ARAS method, the following steps are defined:

Step 1: Identify multiple sources (experts) for the multi-criteria assessment and assign a weight to each expert based on their knowledge and contribution to the financial statement analysis (according to Definition 7 in Section 2.1). For this purpose, Saaty 's neutrosophic AHP method is applied (following the procedures referenced in the bibliographic sources [8] [9]).

Step 2: Determine the importance weights of each criterion in decision-making for each source (expert).

Step 3: Construct the decision matrix L_{ij}(see Figure 1), where the element L_{ij}represents each strategic guideline (GE) evaluated by multiple sources (experts (Exp.), according to Definitions 5 and 6 of Section 2.1) based on an identified criterion (C) .

$$\begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1j} & \cdots & l_{1n} \\ l_{21} & l_{22} & \cdots & l_{2j} & \cdots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{i1} & l_{i2} & \cdots & l_{ij} & \cdots & l_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{m1} & l_{m2} & \cdots & l_{mj} & \cdots & l_{mn} \end{bmatrix}$$

Figure 1: Decision matrix $L_{ij}$for the ARAS multineutrosophic method. Source: Prepared by the authors.

Step 4: The normalized decision matrix $\bar{L}_{ij}$, considering the beneficial and non-beneficial values, is calculated using equations (10) and (11):

$$\bar{L}_{ij} = \frac{l_{ij}}{\sum_{i=0}^{m} l_{ij}} \tag{10}$$

$$L_{ij} = \frac{1}{l^*_{ij}} \tag{11}$$

Step 5: The weighted normalized decision matrix is calculated using equation (12).

$$\hat{L}_{ij} = \bar{L}_{ij} \cdot W_j \tag{12}$$

The weight values $W_j$are determined using the entropy method. Where j $W_j$is the weight of criterion j and j $\bar{L}_{ij}$is the normalized ranking of each criterion.

Step 6: Calculation of the optimization function $S_i$using equation (13).

$$G_i = \sum_{j=1}^{n} \hat{L}_{ij} \quad (13)$$

Where $G_i$is the value of the optimization function for alternative i . This calculation is directly related to the process of the values $\hat{L}_{ij}$and weights $W_j$of the investigated criteria and their relative influence on the outcome.

Step 7: Calculation of the degree of utility. This degree is determined by comparing the variant under analysis with the best one G_o, according to equation (14) [10].

$$K_i = \frac{G_i}{G_o} \quad (14)$$

Where $G_i$and $G_o$are the values of the optimization function. These values range from 0 to 100%; therefore, the alternative with the highest value $K_i$is the best of the alternatives analyzed.

3. Case Study: Selection of Commercial Strategies at the "MUÑEKO" Ice Cream Factory

3.1 Context of the "MUÑEKO" Ice Cream Factory

The "MUÑEKO" Ice Cream Factory, owned by CONSYBA in the municipality of Calixto García, Holguín, operates in a competitive regional market susceptible to various variables, such as the availability of raw materials, seasonal demand, and changing consumer preferences. Strategic decision-making requires a careful evaluation of multiple factors, considering the inherent uncertainty of the business environment.

3.2 Proposed Strategic Guidelines

To address market challenges and optimize its performance, the "MUÑEKO" Ice Cream Factory has identified the following six potential business strategies:

- **EC 1 : Product Line Expansion:** Introduce new ice cream flavors and presentations to diversify the offering and attract different consumer segments.
- **EC 2 : Strengthening Distribution:** Expand the distribution network through alliances with new points of sale and the exploration of online channels.
- **EC 3 : Supply Chain Optimization:** Establish stronger relationships with local suppliers and explore alternatives to ensure the availability and cost of raw materials.
- **EC 4 : Implementation of Digital Marketing Strategies:** Use social media, online advertising, and other digital tools to increase brand visibility and customer engagement.
- **EC 5 : Focus on Sustainability and Natural Products:** Develop ice cream lines with organic ingredients and eco-friendly packaging to attract environmentally conscious consumers.

- **EC 6 : Improving Operational Efficiency:** Implement technologies and processes to reduce production costs and improve productivity.

3.3 Multineutrosophic ARAS Modeling for Trading Strategy Selection

Step 1: Selecting Multiple Sources (Experts) and Modeling the Saaty AHP Neutrosophic Method.

To obtain a comprehensive and robust assessment of business strategies, eight experts with different perspectives relevant to the operation and market of the "MUÑEKO" Factory were consulted (see Table 1).

Table 1: Multisources for neutrosophic assessment (experts).

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Profession	Production Manager	Marketing Manager	Market Analyst	Sales manager	Counter	Logistics Specialist	Business Consultant	Customer Representative

Saaty 's neutrosophic AHP method, a pairwise assessment of the relative importance of each expert was performed. The resulting neutrosophic comparison matrix and the calculation of the experts' weights are presented in Tables 2 and 3. The consistency analysis yielded an eigenvalue of λmax = 8.85, a consistency index (CI) of 0.12, and a consistency ratio (CR) of 0.08, indicating acceptable consistency in the assessments.

Table 2: Paired matrix of neutrosophic AHP.

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Exp-1	(0.55, 0.50, 0.53)	(0.65, 0.40, 0.45)	(0.90, 0.15, 0.10)	(0.85, 0.20, 0.15)	(0.50, 0.55, 0.50)	(0.80, 0.25, 0.20)	(0.70, 0.30, 0.35)	(0.75, 0.25, 0.30)
Exp-2	(0.40, 0.60, 0.65)	(0.55, 0.50, 0.53)	(0.75, 0.25, 0.30)	(0.70, 0.30, 0.35)	(0.30, 0.70, 0.75)	(0.70, 0.30, 0.35)	(0.55, 0.50, 0.53)	(0.70, 0.30, 0.35)
Exp-3	(0.10, 0.85, 0.90)	(0.25, 0.70, 0.75)	(0.55, 0.50, 0.53)	(0.60, 0.45, 0.50)	(0.35, 0.65, 0.70)	(0.60, 0.45, 0.50)	(0.45, 0.55, 0.60)	(0.60, 0.45, 0.50)
Exp-4	(0.15, 0.80, 0.85)	(0.30, 0.70, 0.75)	(0.50, 0.45, 0.60)	(0.55, 0.50, 0.53)	(0.25, 0.75, 0.80)	(0.60, 0.45, 0.50)	(0.45, 0.55, 0.60)	(0.55, 0.50, 0.53)
Exp-5	(0.50, 0.55, 0.50)	(0.70, 0.30, 0.35)	(0.65, 0.35, 0.40)	(0.75, 0.25, 0.30)	(0.55, 0.50, 0.53)	(0.75, 0.25, 0.30)	(0.60, 0.40, 0.45)	(0.75, 0.25, 0.30)
Exp-6	(0.20, 0.75, 0.80)	(0.30, 0.70, 0.75)	(0.50, 0.45, 0.60)	(0.50, 0.45, 0.60)	(0.25, 0.75, 0.80)	(0.55, 0.50, 0.53)	(0.55, 0.50, 0.53)	(0.55, 0.50, 0.53)

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Exp-7	(0.35, 0.70, 0.75)	(0.55, 0.50, 0.53)	(0.60, 0.45, 0.50)	(0.60, 0.45, 0.50)	(0.45, 0.55, 0.60)	(0.55, 0.50, 0.53)	(0.55, 0.50, 0.53)	(0.70, 0.30, 0.35)
Exp-8	(0.30, 0.70, 0.75)	(0.30, 0.70, 0.75)	(0.50, 0.45, 0.60)	(0.55, 0.50, 0.53)	(0.25, 0.75, 0.80)	(0.55, 0.50, 0.53)	(0.30, 0.70, 0.75)	(0.55, 0.50, 0.53)

Table 3: Expert weights obtained using neutrosophic AHP. *Note: Normalized weights were calculated by dividing each expert's score by the total sum of the scores.*

Fountain	w_i (Neutrosophic Weight)	S(w_i) (Score)	Normalized Weight
Exp-1	(0.30, 0.35, 0.33)	0.327	0.165
Exp-2	(0.15, 0.20, 0.18)	0.177	0.089
Exp-3	(0.07, 0.10, 0.09)	0.087	0.044
Exp-4	(0.06, 0.09, 0.08)	0.077	0.039
Exp-5	(0.28, 0.33, 0.30)	0.303	0.153
Exp-6	(0.05, 0.08, 0.07)	0.067	0.034
Exp-7	(0.12, 0.17, 0.15)	0.147	0.074
Exp-8	(0.09, 0.13, 0.11)	0.110	0.055
Addition	**-**	**1,295**	**0.653**

Step 2: Selection and Evaluation of Criteria for Each Multifunctional (Expert).

Five relevant criteria were defined for the selection of commercial strategies for the "MUÑEKO" Factory:

- **C** $_1$**: Sales Growth Potential:** Expected impact on sales growth.
- **C** $_2$**: Operational Viability:** Ease of implementation and management of the strategy.
- **C** $_3$**: Investment Requirement:** Amount of investment required to implement the strategy. (Cost Criterion, which will be treated differently in standardization.)
- **C** $_4$**: Market Adaptability:** Ability of the strategy to adjust to changes in consumer preferences and market conditions.
- **C** $_5$**: Long-Term Sustainability:** Contribution of the strategy to the long-term viability and success of the company.

The multineutrosophic evaluation of each criterion by the expert panel is presented in Table 4, together with the weight calculated for each criterion using the entropy method based on the experts' assessments.

Table 4: Average multineutrosophic evaluation and criteria weights.

Criterion	Average Multineutrosophic Assessment	Weight of the Criterion	Score (S)
C 1	({0.7, 0.6, 0.8}, {0.2, 0.3}, {0.1, 0.1, 0.2, 0.1})	0.28	0.70
C 2	({0.6, 0.7, 0.5}, {0.3, 0.2}, {0.1, 0.1, 0.3, 0.2})	0.22	0.63
C 3	({0.2, 0.3, 0.4}, {0.5, 0.4}, {0.6, 0.5, 0.4, 0.5})	0.15	0.33
C 4	({0.8, 0.7, 0.9}, {0.1, 0.2}, {0.1, 0.1, 0.1, 0.1})	0.20	0.80
C 5	({0.5, 0.6, 0.7}, {0.4, 0.3}, {0.3, 0.2, 0.2, 0.2})	0.15	0.57

Steps 3 to 7: Construction of the Multineutrosophic Decision Matrix and Calculation of the Optimization Function.

The multineutrosophic decision matrix, where each element represents the evaluation of each trading strategy (EC_i) by the eight experts under each criterion (C_j), is presented in Table 5

Table 5: Multineutrosophic ARAS decision matrix (partial example).

Strategy	Criterion	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8	Average Score
EC 1	C 1	(0.8, 0.5, 0.4)	(0.7, 0.6, 0.5)	(0.9, 0.2, 0.1)	(0.8, 0.3, 0.2)	(0.6, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.8, 0.3, 0.4)	(0.7, 0.4, 0.5)	(0.75, 0.43, 0.38)
EC 1	C 2	(0.7, 0.6, 0.3)	(0.6, 0.7, 0.4)	(0.8, 0.2, 0.2)	(0.7, 0.3, 0.3)	(0.5, 0.8, 0.7)	(0.6, 0.4, 0.4)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.6)	(0.65, 0.45, 0.43)
EC 1	C 3	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.2, 0.8, 0.7)	(0.3, 0.7, 0.6)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.35, 0.65, 0.56)
EC 1	C 4	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.83, 0.18, 0.11)
EC 1	C 5	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.55, 0.45, 0.35)
EC 2	C 1	(0.7, 0.6, 0.5)	(0.6, 0.7, 0.6)	(0.8, 0.3, 0.2)	(0.7, 0.4, 0.3)	(0.5, 0.8, 0.7)	(0.6, 0.5, 0.4)	(0.7, 0.4, 0.5)	(0.6, 0.5, 0.6)	(0.65, 0.53, 0.48)
EC 2	C 2	(0.6, 0.7, 0.4)	(0.5, 0.8, 0.5)	(0.7, 0.3, 0.3)	(0.6, 0.4, 0.4)	(0.4, 0.9, 0.8)	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.55, 0.60, 0.53)
EC 2	C 3	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.45, 0.55, 0.45)
EC 2	C 4	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.73, 0.28, 0.18)

Strat-egy	Crite-rion	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8	Average Score
EC 2	C 5	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.3, 0.7, 0.6)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.54, 0.46, 0.36)
EC 3	C 1	(0.6, 0.7, 0.6)	(0.5, 0.8, 0.7)	(0.7, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.4, 0.9, 0.8)	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.56, 0.63, 0.58)
EC 3	C 2	(0.5, 0.8, 0.7)	(0.4, 0.9, 0.8)	(0.6, 0.5, 0.4)	(0.5, 0.6, 0.5)	(0.3, 0.9, 0.9)	(0.4, 0.7, 0.6)	(0.5, 0.6, 0.7)	(0.4, 0.7, 0.8)	(0.45, 0.79, 0.70)
EC 3	C 3	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.55, 0.45, 0.34)
EC 3	C 4	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.63, 0.38, 0.28)
EC 3	C 5	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.2, 0.8, 0.7)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.44, 0.60, 0.46)
EC 4	C 1	(0.8, 0.3, 0.2)	(0.7, 0.4, 0.3)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.2)	(0.6, 0.5, 0.4)	(0.7, 0.3, 0.3)	(0.8, 0.2, 0.2)	(0.7, 0.3, 0.3)	(0.75, 0.29, 0.25)
EC 4	C 2	(0.7, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.2)	(0.5, 0.6, 0.5)	(0.6, 0.3, 0.3)	(0.7, 0.2, 0.2)	(0.6, 0.3, 0.3)	(0.65, 0.33, 0.29)
EC 4	C 3	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.45, 0.55, 0.45)
EC 4	C 4	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.83, 0.18, 0.11)
EC 4	C 5	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.55, 0.45, 0.35)
EC 5	C 1	(0.6, 0.7, 0.6)	(0.5, 0.8, 0.7)	(0.7, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.4, 0.9, 0.8)	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.56, 0.63, 0.58)
EC 5	C 2	(0.5, 0.8, 0.7)	(0.4, 0.9, 0.8)	(0.6, 0.5, 0.4)	(0.5, 0.6, 0.5)	(0.3, 0.9, 0.9)	(0.4, 0.7, 0.6)	(0.5, 0.6, 0.7)	(0.4, 0.7, 0.8)	(0.45, 0.79, 0.70)
EC 5	C 3	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.55, 0.45, 0.34)
EC 5	C 4	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.63, 0.38, 0.28)
EC 5	C 5	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4					

The decision matrix was then normalized. For the benefit criteria (C1, C2, C4, C5), Equation (10) was applied for the material provided. For the cost criterion (C3), Equation (11) was applied, inverting the values so that a lower investment is preferable.

The normalized matrix was then weighted using the criterion weights obtained in Step 2 (Equation 12). Finally, the optimization function (Si) was calculated for each trading strategy by summing the normalized weighted values for each alternative (Equation 13). The utility degree (Qi) was calculated by comparing the optimization function

Calculation of the Degree of Utility, the value of the optimization function of each strategy (Si) was compared with the maximum value obtained (Smax) using Equation (14) of the provided material: $Qi = SmaxSi \times 100\%$.The detailed results are presented in Table 6.

Table 6: Optimization function and degree of usefulness of trading strategies.

Business Strategy	Optimization Function)	Degree of Utility
EC1: Product Line Expansion	0.258	100.00%
EC2: Strengthening Distribution	0.235	91.09%
EC3: Supply Chain Optimization	0.198	76.74%
EC4: Implementation of Digital Marketing Strategies	0.245	94.96%
EC5: Focus on Sustainability and Natural Products	0.212	82.17%
EC6: Improving Operational Efficiency	0.251	97.29%

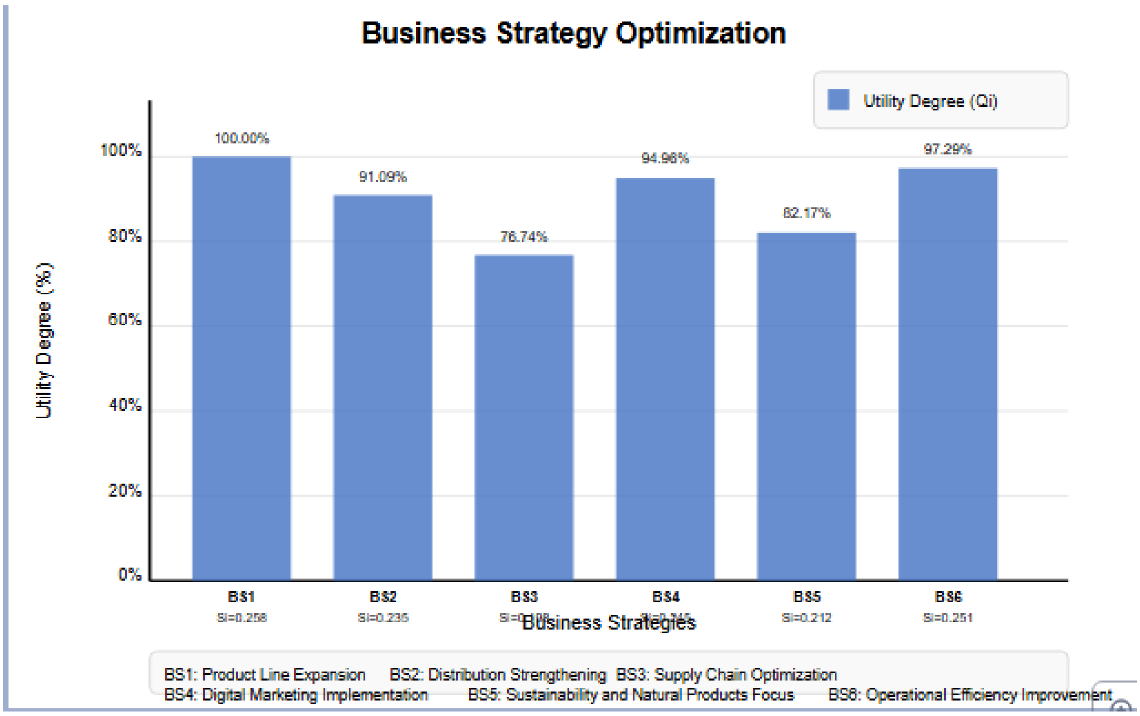

Chart 1: Optimization function and degree of usefulness of trading strategies.

3.4 Results

Applying the multi-expert ARAS method to the six business strategies proposed for the "MUÑEKO" Ice Cream Factory reveals a clear hierarchy of preference based on utility (Qi) . The strategy with the highest utility (100%) is **EC 1 : Product Line Expansion** , indicating that, based on the multi-expert evaluation and weighted criteria, this strategy is the most favorable for the company.

They are followed in order of preference:

- **EC 6 : Improvement of Operational Efficiency** (97.29%)
- **EC 4 : Implementation of Digital Marketing Strategies** (94.96%)
- **EC 2 : Strengthening Distribution** (91.09%)

- **EC $_5$: Focus on Sustainability and Natural Products** (82.17%)
- **EC $_3$: Supply Chain Optimization** (76.74%)

These results provide a solid quantitative and qualitative basis for strategic decision-making at the "MUÑEKO" Factory, considering the uncertainty inherent in the market environment through the multineutrosophic approach.

3.5 Discussion

The results obtained from the multineutral ARAS method suggest that **product line expansion (CS1)** is perceived as the strategy with the greatest potential for the "MUÑEKO" Ice Cream Factory. This could be due to the opportunity to capture new market segments and increase market share through offering diversification, which is crucial in a competitive market with dynamic consumer preferences.

The high rating for **improved operational efficiency (EC6)** is also significant. Optimizing production processes can translate into cost reduction and increased profitability, strengthening the company's long-term competitive position.

The importance given to **implementing digital marketing strategies (EC4)** reflects the growing influence of online channels in product promotion and sales, as well as in customer interaction. A strong digital presence can improve brand visibility and build customer loyalty.

Strengthening **distribution (EC2)** is also considered an important strategy to increase product reach and availability in the regional market.

The **focus on sustainability and natural products (EC5)** , although not the main strategy, remains relevant, indicating a growing awareness of these aspects among experts and their potential to appeal to a specific niche market.

Finally, **supply chain optimization (EC3)** , although important for efficiency, is ranked as the least preferred compared to the other strategies evaluated. This could be interpreted as meaning that, while it is a necessary operational foundation, its direct strategic impact on growth and competitiveness is considered minor in the company's current context.

The application of the multineutrosophic framework allowed the ambiguity and indeterminacy present in experts' judgments to be incorporated, providing a richer and more realistic evaluation of strategies compared to traditional multicriteria decision-making methods[11].

4. Conclusions

The ARAS technique, applied within a multineutrosophic framework, proved highly effective for guiding the selection of commercial strategies at the "MUÑEKO" Ice Cream Factory. The analysis revealed that expanding the product line (CS1) is the top priority, followed by enhancing operational efficiencies and intensifying digital marketing efforts. By transforming expert judgments into neutrosophic values and evaluating multiple criteria under conditions of vagueness and uncertainty, this approach delivered robust, quantitative insights that reinforce strategic decision-making and optimize resource allocation in a dynamic market environment.

On the basis of these findings, it is recommended that "MUÑEKO" allocate significant resources to research and develop new ice-cream flavors and formats, supported by targeted market research; undertake operational efficiency initiatives to streamline production processes; and invest in a comprehensive digital marketing strategy to boost brand visibility and customer engagement. Additionally, strengthening the distribution network through new partnerships and online channels will broaden market reach, while exploring sustainable product lines with natural ingredients and eco-friendly packaging can capture emerging consumer segments. Finally, a detailed review of the supply chain's resilience and efficiency—though lower in immediate priority—will identify long-term optimization opportunities and further fortify the company's strategic positioning.

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