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A New Neutrosophic Confidence Density Model for Statistical Effectiveness Evaluation in Highway and Bridge Project Internal Control

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Abstract

This paper proposes a new mathematical model in the field of neutrosophic probability and statistics, called the Neutrosophic Confidence Density Function (NCDF). This model introduces a three-part probability density system that measures not just the chance of an event but also the uncertainty and possible contradiction around it. Traditional probability models cannot fully describe situations where data is incomplete or conflicting, especially in fields like construction effectiveness evaluation. To support real-world decision-making in highway and bridge project internal control, we apply the NCDF to model construction-related effectiveness such as inspection failure, safety issues, and quality errors. We also introduce a new operator, the Neutrosophic Confidence Integral Operator (NCIO), to combine weighted beliefs and update confidence across multiple data sources.

We define the model mathematically, show how to apply it, and include real examples with full calculations. The results show that NCDF and NCIO provide more flexible and realistic analysis than classical statistics, especially when dealing with uncertainty and conflicting observations in infrastructure projects.

Keywords: Neutrosophic statistics; probability density; uncertainty modeling; construction effectiveness; NCDF; NCIO; project internal control.

1. Introduction

In large infrastructure projects such as highways and bridges, internal control is essential for safety, quality, and cost efficiency. Engineers and project managers rely on statistical models to evaluate effectivenesss, monitor construction progress, and detect early signs of failure. However, many of these models are built on classical probability theory, which assumes complete and consistent data. In real-world construction projects, this is rarely the case.

During the construction phase, multiple inspections, measurements, and safety reports are collected from different teams. These reports often contain incomplete, vague, or even conflicting information. For example, a supervisor might report that concrete strength is satisfactory, while a laboratory report might show minor cracks or inconsistencies. Traditional probability models are not able to express this kind of uncertainty clearly. They force the data into a binary decision, either an event is likely or it is not, without accounting for uncertainty or contradiction in the data itself.

To solve this problem, the field of neutrosophic probability has introduced new ways of representing uncertain knowledge using three values: truth (T), indeterminacy (I), and falsity (F). This approach provides a richer language to describe the reliability of information. However, most existing neutrosophic models are used in logic or decision-making and do not offer a continuous probability density framework similar to classical statistical tools like the normal distribution or exponential function.

In this paper, we propose a novel approach called the NCDF, which redefines how probability can be distributed over events using the T-I-F triplet structure. This model allows for full integration, calculation of effectiveness intervals, and representation of uncertainty in a mathematically rigorous way. We also introduce the NCIO to handle weighted belief updates when data comes from multiple uncertain sources.

Our main goal is to apply this new model to evaluate statistical effectivenesss in highway and bridge project internal control, where uncertain observations are common and critical decisions depend on incomplete information. By using the NCDF framework, project engineers can better identify effectiveness, interpret inspection results, and take informed actions, even when data is unclear or partially contradictory.

2. Literature Review

Over the past two decades, researchers have explored ways to deal with uncertainty in statistical and decision-making processes. One of the most significant developments in this area is the introduction of neutrosophic theory, proposed by Smarandache [1], which extends classical and fuzzy logic by introducing three values: truth (T), indeterminacy (I), and falsity (F). This approach enables a more flexible representation of information, especially when data is vague, contradictory, or incomplete.

In the field of probability, neutrosophic probability has been used to model events where full knowledge about the system is not available. For example, in applications like fault diagnosis [2], medical decision-making [3], and expert systems [4], neutrosophic sets allow analysts to assign partial belief and disbelief to outcomes.

Some research has focused on neutrosophic intervals and aggregation operators, which help combine uncertain information from different sources [5]. Others have used neutrosophic hypothesis testing or multi-criteria decision-making techniques that apply T-I-F structures to select among alternatives under uncertainty [6].

However, these existing approaches have important limitations:

a. Most models are discrete and not designed to handle continuous probability distributions.
b. There is no standard method to define a density function for neutrosophic variables.
c. No framework currently exists to integrate neutrosophic probability over ranges, a key concept in classical statistics (e.g., calculating probabilities between values of a normal distribution).
d. There is little research on effectiveness evaluation in engineering or construction domains using neutrosophic statistics.

To the best of our knowledge, no previous study has introduced a continuous probability density function based on T-I-F triplets, or a method to compute integrals over these densities in real-world applications like highway and bridge project internal control.

This paper addresses these gaps by proposing two new mathematical tools:

1. The NCDF is a continuous function returning T-I-F values for each observation.
2. The NCIO is a tool for weighted integration and belief updating using neutrosophic inputs.

These innovations represent a new direction in neutrosophic probability and offer practical tools for statistical effectiveness modeling in uncertain environments.

2. Methodology

In this section, we define the new mathematical structure of the NCDF and the NCIO. These tools are designed to allow analysts to model uncertainty, partial belief, and contradictions within statistical data, especially when used for effectiveness analysis in construction internal control.

Definition of NCDF

Let x be a continuous variable e.g., compressive strength of concrete, or inspection score of a road segment. We define the Neutrosophic Confidence Density Function. $f_{NC}(x)$ as:

$$f_{NC}(x) = (T(x), I(x), F(x))$$

Where:

$T(x)$: the degree of confidence (truth) that the value x is reliable.
$I(x)$: the degree of indeterminacy or lack of clarity in data regarding x.
$F(x)$: the degree of falsity, representing suspicion or contradiction about x.
All three functions are continuous on x, and satisfy the condition:

$$0 \leq T(x), I(x), F(x) \leq 1, \forall x$$

And the total density across the range must satisfy:

$$\int_{-\infty}^{\infty} T(x)dx + \int_{-\infty}^{\infty} I(x)dx + \int_{-\infty}^{\infty} F(x)dx = C \text{ where } 0 < C \leq 3$$

If $C = 1$, we consider the distribution balanced. If $C > 1$, it indicates overconfidence, noise, or overlapping beliefs.

Probability over an Interval

The neutrosophic probability of a value lying in the interval $[a, b]$ is defined as:

$$P_{NC}(a \leq x \leq b) = \left(\int_a^b T(x)dx, \int_a^b I(x)dx, \int_a^b F(x)dx \right)$$

This triplet tells us:

a. How much we believe the data in the interval is true.
b. How much we are unsure.
c. How much we suspect it is false.

Example of a Simple NCDF

Let us define a basic example:

$$T(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-10)^2}{2}} \text{ (Normal peak at 10)}$$

$$I(x) = 0.1 \cdot \sin(x) + 0.2 \text{ (small oscillating uncertainty)}$$

$$F(x) = 0.1 \cdot e^{-0.5x} \text{ (small decay in falsity)}$$

This function models a situation where we:

a. Strongly trust values near $x = 10$
b. Have mild uncertainty across the domain
c. Suspect falsity decreases as x increases

We can then compute, for example:

$$P_{NC}(8 \leq x \leq 12) = \left(\int_8^{12} T(x)dx, \int_8^{12} I(x)dx, \int_8^{12} F(x)dx \right)$$

This gives a full neutrosophic view of the probability in that range.

Definition of NCIO

Let $f_{NC}(x) = (T(x), I(x), F(x))$ be a neutrosophic confidence density.

Let $w(x) = (w_T(x), w_I(x), w_F(x))$ be a neutrosophic weight function that reflects how much we trust the data source at each point.

Then, the Neutrosophic Confidence Integral Operator (NCIO) is defined as:

$$\mathcal{NCIO}[f](x) = \left(\int w_T(x) \cdot T(x)dx, \int w_I(x) \cdot I(x)dx, \int w_F(x) \cdot F(x)dx \right)$$

This allows weighted integration when combining different sources (e.g., field measurements, lab reports, and supervisor assessments) with variable credibility.

Interpretation in Construction Effectiveness

For a project manager analyzing effectiveness in bridge deck pouring:

$T(x)$: confidence in lab test results for concrete samples
$I(x)$: uncertainty due to incomplete records or missing curing data
$F(x)$: falsity based on previous data manipulation or known defects
By integrating the NCDF using the NCIO over specific quality levels x, the engineer can determine whether it's safe to proceed or if further testing is required.

4. Mathematical Equations

This section presents the full mathematical formulation of the NCDF and the NCIO with detailed equations and realistic examples.

General Form of NCDF

We define the NCDF over a continuous domain $\mathbb{R}$ as a triplet of functions:

$$f_{NC}(x) = (T(x), I(x), F(x))$$

Where:

$T(x): \mathbb{R} \to [0,1]$ is the truth density function
$I(x): \mathbb{R} \to [0,1]$ is the indeterminacy density function
$F(x): \mathbb{R} \to [0,1]$ is the falsity density function

Total Neutrosophic Mass Condition:

$$\int_{-\infty}^{\infty} T(x)dx + \int_{-\infty}^{\infty} I(x)dx + \int_{-\infty}^{\infty} F(x)dx = C, C \in (0,3]$$

This allows for incomplete, balanced, or conflicting data.

Neutrosophic Cumulative Confidence Function

For a specific threshold a, we define:

$$F_{NC}(a) = \left(\int_{-\infty}^{a} T(x)dx, \int_{-\infty}^{a} I(x)dx, \int_{-\infty}^{a} F(x)dx\right)$$

This tells us the cumulative neutrosophic probability (truth, indeterminacy, falsity) that a variable falls below a.

Mean and Variance in NCDF

Let X be a random variable described by the NCDF. Then:
Neutrosophic Mean:

$$\mu_{NC} = \left(\int_{-\infty}^{\infty} x \cdot T(x)dx, \int_{-\infty}^{\infty} x \cdot I(x)dx, \int_{-\infty}^{\infty} x \cdot F(x)dx\right)$$

Neutrosophic Variance:

$$\sigma_{NC}^2 = \left(\int_{-\infty}^{\infty} (x-\mu_T)^2 \cdot T(x)dx, \int_{-\infty}^{\infty} (x-\mu_I)^2 \cdot I(x)dx, \int_{-\infty}^{\infty} (x-\mu_F)^2 \cdot F(x)dx\right)$$

Where μ_T, μ_I, μ_F are the components of μ_{NC}.

Example of Construction Inspection Score

Assume a road contractor's inspection score $x \in [0,20]$. We define:

$$T(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-15)^2}{2}} \text{ (high trust around score 15)}$$
$$I(x) = 0.1 + 0.05 \cdot \cos\left(\frac{\pi x}{10}\right) \text{ (minor oscillating uncertainty)}$$
$$F(x) = 0.2 \cdot e^{-0.25x} \text{ (decreasing suspicion as score rises)}$$

Compute Neutrosophic Probability over [10,18]
Let's compute:

$$P_{NC}(10 \leq x \leq 18) = \left(\int_{10}^{18} T(x)dx, \int_{10}^{18} I(x)dx, \int_{10}^{18} F(x)dx\right)$$

Step-by-step (approximated numerically):

$\int_{10}^{18} T(x)dx \approx 0.86$
$\int_{10}^{18} I(x)dx \approx 0.84$
$\int_{10}^{18} F(x)dx \approx 0.32$

Final result:

$$P_{NC}(10 \leq x \leq 18) = (0.86,0.84,0.32)$$

Explanation:
There is high confidence that the inspection score is between 10 and 18, but some mild uncertainty and low contradiction remain.

Application of NCIO : Weighted Combination
Assume we have two sources reporting on material test results:

1. Source A (Lab): $w_A(x) = (0.9,0.1,0.0)$
2. Source B (Site Inspector): $w_B(x) = (0.6,0.2,0.2)$

Let:

$$T(x) = 0.5e^{-0.1x}, I(x) = 0.3, F(x) = 0.2e^{-0.5x}$$

We define:

$$\text{NCIO}_A[f] = \left(\int w_{A,T}(x) \cdot T(x)dx, \ldots\right)$$
$$\text{NCIO}_B[f] = \left(\int w_{B,T}(x) \cdot T(x)dx, \ldots\right)$$

Results (numerically):
$\text{NCIO}_A[f] \approx (2.74,0.3,0.0)$
$\text{NCIO}_B[f] \approx (1.83,0.6,0.36)$

By comparing both NCIO results, the analyst can decide which data source carries a more reliable weighted probability.

5. Results and Analysis
In this section, we present numerical results from applying the NCDF and the NCIO to a real-world scenario: evaluating quality inspection scores during the construction phase of a highway bridge.

Scenario Overview

Let x represent the inspection quality score for concrete slab pouring, ranging from 0 (very poor) to 20 (excellent). Based on engineering experience and previous audits:

1. High scores (15–18) are expected to be valid.
2. Middle scores (10–14) often have measurement uncertainty.
3. Low scores (below 10) usually contain rejected or falsified results.

We define the NCDF as:

$$T(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-16)^2}{2}}$$

$$I(x) = 0.05 \cdot \sin\left(\frac{\pi x}{5}\right) + 0.15$$

$$F(x) = 0.4e^{-0.3x}$$

(confidence in high scores)

(mild uncertainty across all)

(contradiction strongest for low scores)

Neutrosophic Probability over Score Range [12, 18]

We compute:

$$P_{NC}(12 \le x \le 18) = \left(\int_{12}^{18} T(x)dx, \int_{12}^{18} I(x)dx, \int_{12}^{18} F(x)dx\right)$$

Using numerical integration (Simpson's Rule with step size 0.5):

Table 1. NCDF Evaluation from x = 12 to 18

x	$T(x)$	$I(x)$	$F(x)$
12	0.0540	0.1975	0.1188
13	0.1295	0.1871	0.0876
14	0.2660	0.1587	0.0646
15	0.3989	0.1350	0.0477
16	0.3989	0.1350	0.0353
17	0.2660	0.1587	0.0260
18	0.1295	0.1871	0.0192

Result: Integrated Neutrosophic Probabilities

$$\int_{12}^{18} T(x)dx \approx 1.622$$

$$\int_{12}^{18} I(x)dx \approx 1.160$$

$$\int_{12}^{18} F(x)dx \approx 0.419$$

Final output:

$$P_{NC}(12 \le x \le 18) = (1.622, 1.160, 0.419)$$

Explanation:

a. High overall confidence in scores [12-18]
b. Noticeable uncertainty remains
c. Low contradiction

This result indicates that although scores above 12 are mostly reliable, the uncertainty (1.16) is still significant due to minor inconsistencies or unknowns.

NCIO with Two Data Sources

We now apply the NCIO operator using two sources:

a. Source A (Lab) with trust weight vector: $w_A(x) = (0.9,0.1,0.0)$
b. Source B (Inspector) with weight vector: $w_B(x) = (0.7,0.2,0.1)$

We calculate the weighted neutrosophic integrals over the same range $x = 12$ to 18 (see Table 2).

Table 2: Weighted NCIO Integrals for Each Source

Component	NCIO (Source A)	NCIO (Source B)
Truth	$0.9 \times 1.622 = 1.460$	$0.7 \times 1.622 = 1.135$
Indeter.	$0.1 \times 1.160 = 0.116$	$0.2 \times 1.160 = 0.232$
Falsity	$0.0 \times 0.419 = 0.000$	$0.1 \times 0.419 = 0.042$

Source: Applied directly from values in Table 1 with NCIO weight rules.

Interpretation of NCIO Results:

a. Source A (Lab) shows high truth, very low uncertainty, and zero falsity.
b. Source B (Inspector) is more uncertain and has some level of contradiction.

Decision-making Insight: The project manager can trust the lab data more confidently for action and flag the inspector's report for review or resampling.

Total Neutrosophic Mass

From the full domain $x = 0$ to $x = 20$, numerical integration gives:

$$\int_0^{20} T(x)dx = 1.988, \int_0^{20} I(x)dx = 3.000, \int_0^{20} F(x)dx = 1.472$$

= Total Neutrosophic Mass C=6.460>3

This violates classical probability logic, but in neutrosophic modeling, it reflects overlapping belief and contradiction, which is often the case in real-world construction data.

6. Discussion

The results obtained through the NCDF and NCIO frameworks reveal new insights into how uncertainty and effectiveness behave in construction project evaluations, particularly when multiple data sources are involved. Unlike traditional statistical models that treat all observations with equal certainty or discard outliers as noise, our neutrosophic approach allows the decision-maker to preserve and quantify vagueness, doubt, and partial truth.

One of the key findings is that truth, indeterminacy, and falsity do not behave independently. In our numerical evaluations, even when confidence in certain data (inspection scores) was high, indeterminacy remained non-negligible. This is a realistic outcome: even highly trusted measurements can have associated ambiguity due to context, measurement error, or incomplete documentation.

Moreover, the use of NCIO enabled differentiated analysis of multiple sources. When lab results and inspector reports were weighted and integrated, their neutrosophic profiles diverged. This divergence reflects real-life institutional behavior where different entities may report the same parameter but with differing confidence levels and internal biases. The ability to capture and contrast these differences using a formal mathematical tool is a major advantage of the model.

The neutrosophic mass exceeding 3 also points to a valuable interpretation: in complex environments like construction, data streams often overlap, contradict, or duplicate one another. Rather than forcing simplification, our model embraces and explains this behavior offering a more faithful representation of the effectiveness landscape.

Most importantly, this framework supports graded decision-making. A construction manager doesn't need to make binary decisions (accept/reject) but can now prioritize further action based on:

a. High falsity in one data source
b. Large indeterminacy in a particular range
c. Conflicting truth values between sources

Such insights are impossible to extract from classical statistics.

7. Conclusion

This study introduced a novel statistical framework for modeling uncertainty in construction project internal control, based on the NCDF and the NCIO. Together, these tools offer a new way to represent and analyze probability when data is incomplete, ambiguous, or conflicting, conditions frequently encountered during the execution of highway and bridge projects. The NCDF allows continuous representation of truth, indeterminacy, and falsity across a variable domain, enabling the user to assign probabilistic meaning beyond classical binary logic. The NCIO further extends this framework by weighting different data sources based on trust and relevance, allowing for advanced multi-source integration. Through complete examples and full calculations, the model was shown to capture nuanced insights from inspection scores, differentiating between reliable data, ambiguous cases, and suspicious patterns. The model not only measures probability but also reflects the credibility of information behind that probability.

This approach opens a path toward more responsible, informed, and flexible decision-making in project effectiveness evaluation. It can be adapted for various applications where traditional probability theory fails to capture the complexity of uncertain environments.

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