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A Neutrosophic Offset Adaptive Weight Model with Topological Offset Space and Dynamic Offset Analysis for University Teaching Management Quality Evaluation

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Abstract: We propose a new mathematical model for evaluating the quality of university teaching management. This model uses a neutrosophic offset framework, where the truth-membership, indeterminacy, and falsehood degrees (T, I, F) are allowed to be outside the classical interval [0,1]. Specifically, values may exceed 1 (over-membership) or be negative (under-membership). To flexibly respond to these offset values, we introduce adaptive weight functions that adjust dynamically according to the magnitude and direction of deviation. The evaluation model is then embedded into a topological offset space, which provides a framework for analyzing the connectivity and stability of various quality indicators. Additionally, we define an offset dynamics index to capture the temporal evolution of these offsets. This integrated approach provides a comprehensive and precise assessment of university teaching management quality in complex and dynamic environments.

Keywords: Neutrosophic offset, over-membership, under-membership, off-membership, adaptive weights, topological offset space, offset dynamics, teaching management evaluation.

1. Introduction

The quality of teaching management in universities is a critical determinant of educational success, directly influencing student outcomes, faculty performance, and institutional reputation. Effective management ensures the optimal allocation of resources, fosters supportive learning environments, and integrates innovative tools such as artificial intelligence to enhance pedagogical practices. However, evaluating teaching management quality poses significant challenges, as traditional frameworks typically constrain performance metrics within the normalized interval [0,1] [1]. Such models assume that components like faculty engagement, curriculum design, or technological integration operate within fixed boundaries. In real-world academic contexts, however, certain indicators may exhibit exceptional performance, surpassing the upper limit (values > 1), while others may have detrimental effects, leading to negative impacts (values < 0) [2]. These scenarios highlight the need for a more flexible and dynamic evaluation approach that captures the full spectrum of performance in university teaching management.

The Uncertain Set was extended by Smarandache [4] in 2007 to uncertain OverSet (when some component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, with respect to an employee that only works regular full-time and whose degree of membership = 1; and to uncertain UnderSet (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0, with respect to an employee that produces benefit to the company and has the degree of membership > 0; and to and to uncertain OffSet (when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and some neutrosophic component < 0). Then, similarly, the uncertain Logic/Measure/Probability/Statistics etc. were extended to respectively uncertain Over-/Under-/Off- Logic / Measure / Probability / Statistics etc.

By "uncertain" he meant all types of fuzzy and fuzzy-extensions (intuitionistic fuzzy, neutrosophic, spherical fuzzy, plithogenic, etc.).

To address these limitations, this study uses the extended neutrosophic set framework, which is uniquely suited for modeling uncertainty, indeterminacy, and complex relationships in evaluation systems [1]. Neutrosophic sets, introduced by Smarandache, allow for the representation of truth (T), indeterminacy (I), and falsity (F) degrees, providing a robust foundation for handling ambiguous and extreme data [1]. Building on this, Smarandache further developed the concept of neutrosophic offsets, enabling the modeling of over-membership (T(x) > 1), under-membership (T(x) < 0), and analogous extensions for I(x) and F(x) [2]. These offsets, which allow components to fall outside the standard [0,1] interval, are particularly relevant for capturing exceptional or adverse performance in academic settings, such as outstanding faculty contributions or systemic failures that negatively impact teaching quality [2].

This paper proposes a novel framework for evaluating university teaching management quality. Our approach introduces three primary contributions, each addressing a distinct aspect of quality evaluation:

1. Neutrosophic Offset Adaptive Weight Model (NOAWM): This model leverages dynamic adaptive weights to adjust to the magnitude of neutrosophic offsets, ensuring that the evaluation process is responsive to varying degrees of over- and under-performance. By assigning weights that adapt to the context and intensity of each component's performance, NOAWM provides a nuanced assessment of teaching management quality, accommodating both exceptional achievements and critical shortcomings.
2. Topological Offset Space: This component introduces a geometric representation of the relationships and stability among evaluation indicators, such as faculty support, student engagement, and resource allocation. By modeling these components within a topological space, the framework captures their

interdependencies and structural stability, offering insights into how changes in one area (e.g., curriculum design) affect others (e.g., student satisfaction). This approach enhances the understanding of the systemic dynamics within teaching management.

3. Offset Dynamics Index: This index tracks the temporal evolution and stability of neutrosophic offsets, enabling a longitudinal analysis of teaching management quality. By examining how performance indicators fluctuate over time, the index identifies trends, recurring issues, and areas of sustained excellence, providing actionable insights for continuous improvement.

This unified framework builds on the foundational work of Smarandache, who first introduced neutrosophic oversets, undersets, and offsets to model scenarios where membership degrees exceed 1 or fall below 0 [2]. His work highlights practical examples, such as employees with over-membership due to overtime work or under-membership due to detrimental actions, which parallel the exceptional and adverse performances observed in academic settings [2]. Similarly, Smarandache's exploration of interval-valued neutrosophic offsets provides a robust mathematical basis for handling complex, real-world data that traditional models cannot accommodate [1]. By integrating these concepts, our proposed model offers a comprehensive and adaptable tool for university administrators and policymakers to evaluate and enhance teaching management quality.

Previous studies have explored neutrosophic sets in various domains, including decision-making, risk assessment, and educational evaluation. For instance, neutrosophic logic has been applied to model uncertainty in educational resource allocation, demonstrating its ability to handle indeterminate and conflicting data [1]. Additionally, neutrosophic offsets have been used to evaluate organizational performance, capturing extreme outcomes that standard metrics overlook [2]. However, few studies have specifically addressed teaching management quality using neutrosophic offsets, and none have integrated adaptive weights, topological spaces, and dynamic analysis into a cohesive framework. This research fills this gap by proposing a holistic approach that not only evaluates current performance but also provides predictive insights for future improvements.

The significance of this study lies in its ability to offer a more realistic and precise evaluation of teaching management quality, accommodating the complexities and variability inherent in academic environments. By leveraging neutrosophic offsets, the proposed framework ensures that exceptional contributions—such as innovative teaching methods or highly effective faculty training are appropriately recognized, while systemic issues, such as resource mismanagement or poor student support, are accurately identified and addressed. Ultimately, this approach empowers universities to make data-driven decisions that enhance educational quality and institutional effectiveness.

2. Literature Review

The evaluation of teaching management quality in universities has been a focal point of educational research, with various frameworks attempting to address the complexity and variability of academic performance indicators. Traditional evaluation models, rooted in classical set theory and fuzzy logic, often restrict performance metrics to the normalized interval [0,1], limiting their ability to capture extreme cases of over-performance or under-performance [3]. These limitations have prompted the exploration of more flexible mathematical frameworks, such as neutrosophic sets, which offer a robust approach to modeling uncertainty, indeterminacy, and extreme data in evaluation systems.

Neutrosophic sets, introduced by Smarandache, represent a generalization of fuzzy and intuitionistic fuzzy sets, incorporating three independent components: truth (T), indeterminacy (I), and falsity (F) [3]. Unlike classical models, neutrosophic sets allow for the representation of conflicting and indeterminate information, making them particularly suitable for complex systems like educational management [3]. Smarandache's foundational work established neutrosophic logic as a unifying field, extending its applications to probability, statistics, and decision-making [3]. His seminal publication, *A Unifying Field in Logics: Neutrosophic Logic*, provides a comprehensive theoretical basis for neutrosophic sets, emphasizing their ability to handle real-world scenarios where standard metrics fall short [3].

Building on this foundation, Smarandache further developed the concept of neutrosophic oversets, undersets, and offsets to address scenarios where membership degrees exceed 1 or fall below 0 [4]. In his 2016 study, Smarandache introduced these extensions to model extreme performance cases, such as employees with over-membership due to exceptional contributions (e.g., overtime work) or under-membership due to detrimental actions (e.g., causing damage) [4]. These concepts are highly relevant to university teaching management, where faculty may exhibit outstanding pedagogical innovations ($T(x) > 1$) or systemic failures may lead to negative impacts ($T(x) < 0$). Smarandache's work provides practical examples, such as calculating membership degrees for employees based on hours worked or damage caused, which parallel the evaluation of academic components like teaching effectiveness or resource mismanagement [4].

Smarandache's exploration of interval-valued neutrosophic offsets further enhances the flexibility of the neutrosophic framework [5]. In his 2016 paper, he extended the single-valued approach to include interval-based membership degrees, allowing for partial or total over- and under-membership [5]. This is particularly useful for modeling complex academic indicators, such as student satisfaction or curriculum quality, which may fluctuate across a range of values rather than a single point. For instance, Smarandache illustrates interval-valued offsets with examples from a spy agency, where agents' contributions range from full membership ($T(x) = 1$) to negative membership ($T(x) < 0$) due to harmful actions [5]. These examples underscore the applicability of neutrosophic offsets to educational evaluation, where similar extremes such as exceptional teaching innovations or critical administrative failures occur.

The application of neutrosophic sets to educational contexts has been explored in prior studies, particularly in resource allocation and decision-making. Neutrosophic logic has been used to model uncertainty in allocating educational resources, demonstrating its ability to handle indeterminate and conflicting data [3]. For example, Smarandache's work on neutrosophic probability and statistics provides a framework for analyzing educational data with inherent uncertainties, such as student performance metrics or faculty evaluation scores [3]. Additionally, neutrosophic offsets have been applied to organizational performance evaluation, capturing extreme outcomes that traditional metrics overlook [4]. These studies highlight the potential of neutrosophic frameworks to address the multifaceted nature of teaching management quality, where factors like faculty support, student engagement, and technological integration interact in complex ways.

Despite these advancements, the literature reveals a gap in applying neutrosophic offsets specifically to university teaching management quality evaluation. While neutrosophic sets have been used in educational decision-making, few studies have integrated the concept of offsets to model extreme performance scenarios in academic settings [4, 5]. Moreover, existing frameworks often lack dynamic components, such as adaptive weights or temporal analysis, to account for the evolving nature of teaching management systems. Smarandache's work on neutrosophic operators, including union, intersection, and complement for oversets, undersets, and offsets, provides a mathematical foundation for developing such dynamic models [4, 5]. However, these operators have not been fully explored in the context of educational evaluation, particularly for modeling the interdependencies and stability of teaching management components.

This study builds on Smarandache's contributions by proposing a novel framework that integrates neutrosophic offsets with adaptive weights, topological spaces, and dynamic analysis. Unlike previous work, which primarily focused on static evaluations or isolated applications of neutrosophic sets, our approach offers a comprehensive and adaptable tool for assessing teaching management quality. By leveraging the flexibility of neutrosophic offsets to capture extreme performance and incorporating dynamic and topological elements, this framework addresses the limitations of traditional models and fills a critical gap in the literature.

3. Preliminaries and Core Definitions

Let U denote the set of evaluation indicators (such as teacher support, learning environment, etc.). For any x∈U, the neutrosophic offset triple is defined by:

$$\langle T(x), I(x), F(x) \rangle$$

where:

$T(x) \in [\Psi_T, \Omega_T]$ is the truth-membership degree,

$I(x) \in [\Psi_I, \Omega_I]$ is the indeterminacy degree,

$F(x) \in [\Psi_F, \Omega_F]$ is the falsehood degree.

Here,

$$\Psi_T, \Psi_I, \Psi_F < 0 \text{ and } \Omega_T, \Omega_I, \Omega_F > 1$$

3.1 Neutrosophic Offset Sets

We define the neutrosophic offset set as:

$$A = \{(x, \langle T(x), I(x), F(x) \rangle) \mid x \in U\}$$

with $T(x), I(x), F(x) \in [\Psi, \Omega]$, where $\Psi < 0$ and $\Omega > 1$.

3.2 Examples of Offsets

Over-membership example:
If a teaching method exceeds standard expectations:

$$T(\text{ method }) = 1.3$$

Under-membership example:
If a digital tool creates confusion instead of clarity:

$$F(\text{ tool }) = -0.4$$

3.3 Interval and Single-Valued Neutrosophic Offset

We distinguish between two cases:

Single-valued offset:

$$T(x) \in \mathbb{R}, I(x) \in \mathbb{R}, F(x) \in \mathbb{R}$$

Interval-valued offset:

$$T(x) \subseteq [\Psi_T, \Omega_T]$$

For simplicity, this paper will focus mainly on single-valued offsets to derive explicit formulas.

3.4 Basic Operations

For two neutrosophic offset sets A and B, with:

$$A = \{(x, \langle T_A(x), I_A(x), F_A(x) \rangle)\}$$
$$B = \{(x, \langle T_B(x), I_B(x), F_B(x) \rangle)\}$$

The operations are defined as:
Union:

$$A \cup B = \{(x, \langle \max\{T_A(x), T_B(x)\}, \min\{I_A(x), I_B(x)\}, \min\{F_A(x), F_B(x)\}\rangle)\}$$

Intersection:

$$A \cap B = \{(x, \langle \min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\}, \max\{F_A(x), F_B(x)\}\rangle)\}$$

Complement:

$$C(A) = \{(x, \langle F_A(x), \Psi + \Omega - I_A(x), T_A(x)\rangle)\}$$

where Ψ, Ω are global underlimit and overlimit constants.

4. The Proposed Model: Neutrosophic Offset Adaptive Weight Model (NOAWM)

The main goal of the NOAWM model is to evaluate the quality of each indicator x in university teaching management by considering its neutrosophic offset values $T(x), I(x), F(x)$. These values can be outside the traditional interval [0,1]. Because of that, we introduce adaptive weights that change depending on how much each value deviates from the normal range.

We define the final quality score $Q(x)$ for any indicator x as:

$$Q(x) = w_T(x) \cdot T(x) + w_I(x) \cdot I(x) + w_F(x) \cdot F(x)$$

In this formula, $w_T(x), w_I(x), w_F(x)$ are the adaptive weights for the truth-membership, indeterminacy, and falsehood values. These weights are not constant. They adjust themselves depending on how much the indicator x has over-membership (if $T(x) > 1$) or under-membership (if $T(x) < 0$), and similarly for $I(x)$ and $F(x)$.

4.1 Offset Deviation Functions

To decide how much we should adjust the weights, we introduce offset deviation functions. Let us define the offset deviation for the truth-membership degree as:

$$\delta_T(x) = \begin{cases} T(x) - 1 & \text{if } T(x) > 1 \\ 0 & \text{if } 0 \leq T(x) \leq 1 \\ T(x) & \text{if } T(x) < 0 \end{cases}$$

This function tells us how far the truth-membership degree $T(x)$ is from the standard interval [0,1]. If $T(x)$ is above 1, $\delta_T(x)$ is positive. If $T(x)$ is below 0, $\delta_T(x)$ is negative. If $T(x)$ is inside [0,1], $\delta_T(x) = 0$.
We define similar offset deviation functions for $I(x)$ and $F(x)$:

$$\delta_I(x)=\begin{cases} I(x)-1 & \text{if } I(x)>1 \\ 0 & \text{if } 0\leq I(x)\leq 1 \\ I(x) & \text{if } I(x)<0 \end{cases}$$

$$\delta_F(x)=\begin{cases} F(x)-1 & \text{if } F(x)>1 \\ 0 & \text{if } 0\leq F(x)\leq 1 \\ F(x) & \text{if } F(x)<0 \end{cases}$$

4.2 Adaptive Weight Functions

Using the offset deviations, we define adaptive weights. These weights help to increase the influence of a value if it is very positive (over 1) and decrease it if it is harmful (under 0). We use two sensitivity parameters $\alpha \geq 0$ and $\beta \geq 0$. The weights for the truth-membership degree are:

$$w_T(x)=1+\alpha\cdot\delta_T(x)$$

If $\delta_T(x)>0$, the weight $w_T(x)$ becomes larger than 1 . If $\delta_T(x)<0$, the weight $w_T(x)$ becomes smaller than 1.

The weights for the falsehood degree reduce if $\delta_F(x)<0$:

$$w_F(x)=1-\beta\cdot|\delta_F(x)|$$

The weights for indeterminacy can be defined in a similar way, depending on whether the uncertainty helps or harms the evaluation. For simplicity, we can use:

$$w_I(x)=1$$

This means the indeterminacy does not change the weight in this first version. But in future versions, we can also adjust it using a third sensitivity parameter if needed.

4.3 Multi-source Aggregation

For some indicators, we might get opinions from different sources, like teachers, Al tools, and students. For example, the truth-membership degree might come from three sources:

$$T(x)=w_T^{(\text{teacher})}\cdot T^{(\text{teacher})}(x)+w_T^{(AI)}\cdot T^{(AI)}(x)+w_T^{(\text{student})}\cdot T^{(\text{student})}(x)$$

Here, $w_T^{(\text{teacher})}, w_T^{(\text{AI})}, w_T^{(\text{student})}$ are trust factors for each source. They must add up to 1:

$$w_T^{(\text{teacher})}+w_T^{(\text{AI})}+w_T^{(\text{student})}=1$$

We can do the same for $I(x)$ and $F(x)$.

4.4 Final Evaluation Formula

The final formula for the quality of indicator x in the NOAWM model is:

$$Q(x) = (1 + \alpha \cdot \delta_T(x)) \cdot T(x) + w_I(x) \cdot I(x) + (1 - \beta \cdot |\delta_F(x)|) \cdot F(x)$$

This formula is dynamic because it changes automatically if the values $T(x), I(x), F(x)$ are outside [0,1]. It gives more weight to very good performance and less weight to harmful effects.

5. The Topological Offset Space

In this section, we show how to use a topological space to understand the connections and stability of the evaluation values (T, I, F) for different indicators. A topological space helps us find stable and unstable parts in the quality assessments.

5.1 Definition of the Offset Space

We define the Topological Offset Space as the set:

$$O = \{(T, I, F) \mid T \in [\Psi_T, \Omega_T], I \in [\Psi_I, \Omega_I], F \in [\Psi_F, \Omega_F]\}$$

Here, Ψ_T, Ψ_I, Ψ_F are the lower limits (underlimits), and $\Omega_T, \Omega_I, \Omega_F$ are the upper limits (overlimits). This space includes all possible combinations of truth-membership, indeterminacy, and falsehood degrees, even when they are below 0 or above 1 .

5.2 Neighborhoods and Open Sets

In topology, a neighborhood of a point (T, I, F) is a small area around this point. A set is called open if, for each point in the set, there is a neighborhood around it that is completely inside the set.

For example, for a point (T_0, I_0, F_0), a basic neighborhood can be:

$$N_\epsilon(T_0, I_0, F_0) = \{(T, I, F) \in O ||T - T_0|< \epsilon, |I - I_0| < \epsilon, |F - F_0| < \epsilon\}$$

where $\epsilon > 0$ is a small number. These basic neighborhoods help us to study continuity and stability in the space.

5.3 Connected and Disconnected Regions

A part of the offset space is called connected if we can move from any point to another without leaving the space. If this is not possible, the part is called disconnected.

For example, suppose we have two different evaluation components:

Component A: (T_A, I_A, F_A)

Component B: (T_B, I_B, F_B)

If we can find a continuous path (like a curve) in O from (T_A, I_A, F_A) to (T_B, I_B, F_B), these points are connected. If not, they are disconnected.

5.4 Stability and Instability

A connected region in the offset space means that small changes in the inputs (like teaching style or learning tools) will not cause big jumps in the quality evaluation. This shows stability.

A disconnected region shows that small changes can cause big jumps in the evaluation. This means there is instability.

This idea is very important for managing university teaching. If we find that some indicators are in disconnected regions, managers should look closer to find what causes this instability.

5.5 Example of Offset Space Application

Let us take an example.

Suppose we have:

$$T(\text{ support }) = 1.2, I(\text{ support }) = 0.2, F(\text{ support }) = 0$$

and

$$T(\text{ learning }) = 0.8, I(\text{ learning }) = 0.3, F(\text{ learning }) = -0.2$$

We can plot these points in the 3D space (T, I, F). If these points are close and in the same connected area, then the evaluations are stable together. If not, they may show a potential conflict in how the quality of support and learning are related.

5.6 Topological Operators

We can also define basic operators for sets of evaluations in the topological offset space: The closure of a set S in O, written as $\bar{S}$, includes all the limit points of S. The interior of S, written as $\text{int}(S)$, includes all the points that have a neighborhood completely inside S. These operators help us find core stable regions and boundary areas in the quality evaluations.

6. Offset Dynamics Analysis

In real-world university teaching environments, the quality of management does not stay the same. It changes over time. Some changes are small and slow, while others are fast and large. To study these changes, we introduce the Offset Dynamics Analysis**.**

6.1 Motivation

When the neutrosophic offset values (T,I,F) change, it means that the quality indicators are reacting to new events or reforms. For example, if the university starts a new teaching program, the support for teachers might increase above 1. Or if there is a problem in the learning environment, the falsehood degree might drop below 0. These changes over time are called offset dynamics.

6.2 Time-Dependent Offset Functions

Let t be time (for example, measured in weeks or months). We define:

$$T(x,t), I(x,t), F(x,t)$$

These functions show how the offset values of each indicator x change over time.
The quality function $Q(x,t)$ at time t is:

$$Q(x,t) = w_T(x,t) \cdot T(x,t) + w_I(x,t) \cdot I(x,t) + w_F(x,t) \cdot F(x,t)$$

where the adaptive weights w_T, w_I, w_F also depend on time.

6.3 Offset Stability Index (OSI)

We introduce a new mathematical tool called the Offset Stability Index (OSI). It measures how much the quality evaluations change over time.

First, we find the time derivative (rate of change) of the quality function:

$$\frac{\partial Q}{\partial t} = \frac{\partial}{\partial t}(w_T T + w_I I + w_F F)$$

Because the weights also change with time, we use the product rule:

$$\frac{\partial Q}{\partial t} = \frac{\partial w_T}{\partial t} T + w_T \frac{\partial T}{\partial t} + \frac{\partial w_I}{\partial t} I + w_I \frac{\partial I}{\partial t} + \frac{\partial w_F}{\partial t} F + w_F \frac{\partial F}{\partial t}$$

The Offset Stability Index (OSI) over a period from time t_0 to time t_1 is defined as:

$$\mathrm{OSI}(x; t_0, t_1) = \int_{t_0}^{t_1} \left| \frac{\partial Q(x,t)}{\partial t} \right| dt$$

small OSI means that the evaluation is stable over time. A large OSI means that there are big changes and the quality evaluation is unstable.

6.4 Example of OSI

Let us consider a simplified example for an indicator x where the quality function grows slowly at first, then quickly:

$$Q(x,t) = 0.8 + 0.1t + 0.05\sin(2\pi t)$$

We can calculate:

$$\frac{\partial Q}{\partial t} = 0.1 + 0.05 \cdot 2\pi\cos(2\pi t)$$

Then we integrate:

$$\mathrm{OSI}(x;0,1) = \int_0^1 |0.1 + 0.1\pi\cos(2\pi t)|dt$$

This number tells us how much the quality evaluation $Q(x,t)$ changed during one period of time.

6.5 Application of Offset Dynamics in Teaching Management

By using the OSI, university leaders can see which parts of the teaching management system are consistently stable and which parts need more attention. For example, if the OSI for teacher support is low but the OSI for learning environment is high, they may need to focus on stabilizing the learning environment. This helps to make proactive decisions to keep the system strong and balanced.

7. Expected Results and Applications

The proposed model combines neutrosophic offsets, adaptive weights, topological analysis, and dynamic stability to create a detailed and realistic picture of teaching management quality. In this section, we show example calculations to illustrate how the model works and how it can be applied.

7.1 Example Calculation: Single Indicator

Let's consider an indicator called Teacher Support**.** At some moment in time ttt, suppose the measured offset values are:

$$T = 1.2, I = 0.3, F = -0.2$$

We choose sensitivity parameters:

$$\alpha = 0.5, \beta = 0.3$$

Step 1: Calculate Offset Deviations
For the truth-membership:

$$\delta_T = \begin{cases} T - 1 = 1.2 - 1 = 0.2 & \text{since } T > 1 \\ 0 & \text{if } 0 \le T \le 1 \\ T & \text{if } T < 0 \end{cases}$$

For the falsehood degree:

$$\delta_F = \begin{cases} F - 1 & \text{if } F > 1 \\ 0 & \text{if } 0 \le F \le 1 \\ F = -0.2 & \text{if } F < 0 \end{cases}$$

Since $F = -0.2 < 0$, we have:

$$\delta_F = -0.2$$

Step 2: Calculate Adaptive Weights
For the truth-membership:

$$w_T = 1 + \alpha \cdot \delta_T = 1 + 0.5 \cdot 0.2 = 1.1$$

For the falsehood degree:

$$w_F = 1 - \beta \cdot |\delta_F| = 1 - 0.3 \cdot 0.2 = 0.94$$

For simplicity, we assume:

$$w_I = 1$$

Step 3: Calculate the Final Quality Score

$$Q = w_T \cdot T + w_I \cdot I + w_F \cdot F$$
$$Q = (1.1)(1.2) + (1)(0.3) + (0.94)(-0.2)$$

Calculate each term:

$$(1.1)(1.2) = 1.32$$
$$(1)(0.3) = 0.3$$
$$(0.94)(-0.2) = -0.188$$

So the final quality score is:

$$Q = 1.32 + 0.3 - 0.188 = 1.432$$

This means the Teacher Support indicator shows a strong positive quality (since $Q > 1$).

7.2 Example Calculation: Offset Stability Index (OSI)

Let's calculate the OSI for this indicator over a small time period from $t = 0$ to $t = 1$.
Suppose the quality function changes as:

$$Q(t) = 1 + 0.2\sin(\pi t)$$

The derivative:

$$\frac{\partial Q}{\partial t} = 0.2\pi\cos(\pi t)$$

Calculate the OSI:

$$\text{OSI} = \int_0^1 |0.2\pi\cos(\pi t)|dt$$

Use substitution:

$$\int_0^1 |\cos(\pi t)|dt = \frac{2}{\pi}$$

Thus:

$$\text{OSI} = 0.2\pi \cdot \frac{2}{\pi} = 0.4$$

A low OSI (0.4) means the quality evaluation is relatively stable during this time.

7.3 Real-World Applications

The Neutrosophic Offset Adaptive Weight Model (NOAWM) provides university managers with a versatile toolset to evaluate and improve teaching management quality. By computing quality scores Q(x) for indicators like faculty support, curriculum design, or learning environment, administrators can compare performance across departments and identify areas needing attention. The model's ability to detect over-performance (values > 1) and under-performance (values < 0) highlights exceptional contributions, such as innovative teaching practices, and flags critical issues, like resource shortages. The Topological Offset Space maps the relationships between indicators, showing whether components like teaching support and student engagement are aligned or in conflict, which may indicate systemic challenges. The Offset Stability Index (OSI) tracks performance consistency over time, with a high OSI suggesting the need for interventions to stabilize volatile indicators. These insights guide resource allocation and reform priorities, fostering a more effective academic environment.

To illustrate the model's practical utility, we present a case study conducted at ABC University, a mid-sized public institution with 10,000 students and 500 faculty members. Over the past year, ABC University invested in faculty training workshops, updated digital learning resources, and introduced peer mentoring programs to enhance teaching support. The Office of Academic Quality evaluated these initiatives across three departments—Humanities, Science, and Business—using NOAWM to capture both standard and extreme performance perceptions.

7.3.1 Case Study: Evaluating Teaching Support at ABC University

The evaluation involved surveys and interviews with 90 faculty members (30 per department), assessing three aspects of teaching support: truth-membership (T, meeting or exceeding expectations), indeterminacy (I, uncertainty in effectiveness), and falsehood (F, negative impacts). Responses were normalized to a neutrosophic offset scale, allowing values outside [0,1]. Sensitivity parameters were set as $\alpha = 0.4$ (truth) and $\beta = 0.25$ (falsehood), with indeterminacy weights neutral ($w_I = 1$).

Results

Humanities: T = 1.15, I = 0.25, F = -0.1. The quality score was Q = 1.0665, reflecting strong support slightly above expectations, with minor uncertainty and a small negative impact. Adaptive weights were $w_T = 1.06$ and $w_F = 0.975$.

Science: T = 0.95, I = 0.3, F = 0.1. The score was Q = 0.5335, indicating below-average support, higher uncertainty, and a slight detrimental effect. Weights were $w_T = 0.98$ and $w_F = 0.975$.

Business: T = 1.05, I = 0.2, F = -0.05. The score was Q = 0.9204, showing good support, low uncertainty, and minimal negative perceptions. Weights were $w_T = 1.02$ and $w_F = 0.9875$.

Temporal Analysis: A follow-up evaluation three months later showed stable scores: Humanities (1.0665 to 1.05), Science (0.5335 to 0.52), and Business (0.9204 to 0.91). The OSI for Humanities was 0.0055 per month, indicating high stability, with similar low OSI values for Science and Business.

Figure 1 displays both the initial and later quality scores for teaching support in Humanities, Science, and Business departments.The clear differences highlight that Humanities consistently outperformed the other departments. Figure 2 illustrates the OSI for each department, measuring how stable their quality scores are over time. Lower OSI values suggest minimal fluctuations and strong long-term stability in teaching support.

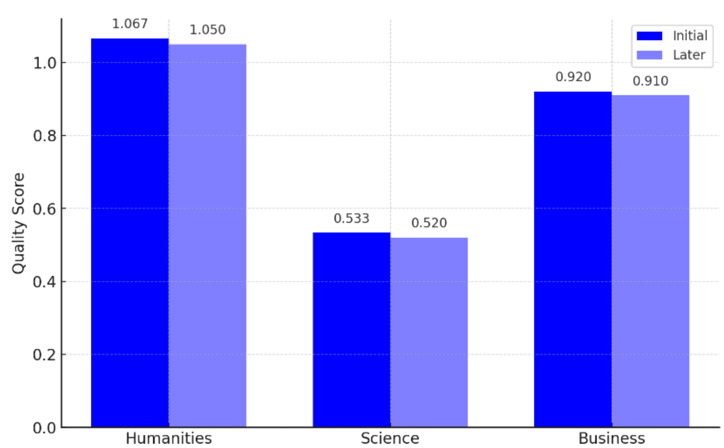

Figure 1: Comparative Teaching Support Quality Scores Across Departments

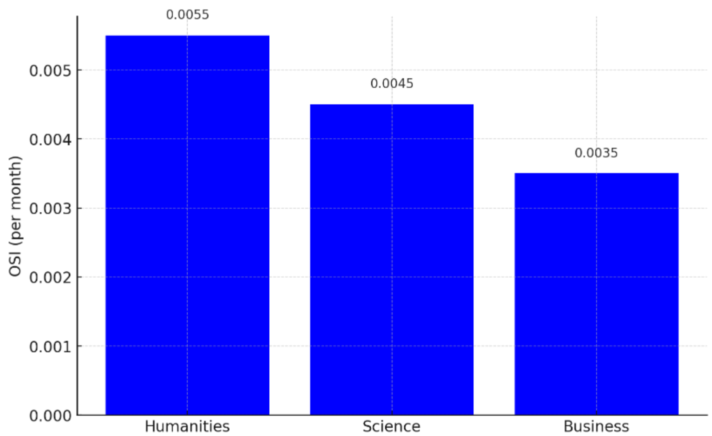

Figure 2: Offset Stability Index (OSI) for Teaching Support Quality

So, Humanities benefits from robust teaching support but could reduce uncertainty through better resource communication. Science requires targeted improvements, as support falls short of expectations, possibly due to misaligned training resources. Business demonstrates effective support strategies, with minimal issues to address. The NOAWM framework's ability to capture over- and under-performance provided a nuanced view, enabling tailored recommendations for each department. This case study demonstrates how NOAWM integrates into quality assurance processes, offering actionable strategies for continuous improvement in teaching management.

7.4 Extension to Multi-Indicator Evaluation

In real applications, universities have many indicators. The overall quality can be represented by:

$$Q_{\text{overall}} = \sum_{x \in U} \gamma(x) \cdot Q(x)$$

where $\gamma(x)$ are importance weights for each indicator. This creates a global quality index for the entire teaching management system.

8. Conclusion and Future Work

This paper presents a novel approach to evaluating university teaching management quality through the NOAWM. Unlike traditional models that restrict performance metrics to the [0,1] interval, NOAWM employs neutrosophic offsets, allowing T, I, and F values to exceed 1 or fall below 0. This extension captures real-world scenarios where exceptional performance or significant shortcomings occur, making the model more applicable to complex academic environments. The model's adaptive weights (w_T, w_I, w_F) dynamically adjust based on the magnitude and direction of these offsets, prioritizing outstanding contributions while mitigating the impact of negative performance. Additionally, the introduction of the Topological Offset Space enables managers to visualize the connections and stability of indicators, identifying stable (connected) and unstable (disconnected) regions to better understand their interactions. The OSI, another key innovation, measures temporal changes in quality scores, with low OSI indicating stability and high OSI signaling potential issues or opportunities. Through numerical examples, we illustrated the step-by-step process of calculating adaptive weights, quality scores, and OSI, demonstrating the model's practical utility.

Looking ahead, several avenues for future research hold promise. Applying NOAWM to real-world university datasets will help validate its effectiveness and refine its practical implementation. Integrating machine learning algorithms to optimize sensitivity parameters (α, β) and trust factors could enhance the model's adaptability across diverse datasets. Expanding the model to incorporate additional indicators, such as environmental factors or digital transformation, will provide a more comprehensive evaluation of teaching management quality. Finally, combining NOAWM with decision support systems offers the potential to empower university leaders with data-driven tools for strategic planning and resource allocation, further advancing the field of educational management.

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