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Type-1 Neutrosophic-Topology Based on Type-1 Neutrosophic Sets

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Abstract: This paper develops a topological framework based on type-1 neutrosophic sets, extending the study of uncertainty beyond classical and intuitionistic fuzzy structures. We investigate fundamental operators — interior, closure, and exterior — on type-1 neutrosophic sets X[I], and establish separation axioms for type-1 topological spaces Ti[I], i = 0, 1, …, 4. Furthermore, we analyze continuous and contra type-1 neutrosophic functions, exploring their structural properties within this framework. By isolating type-1 neutrosophic sets and formalizing their topological behaviour, our results provide new insights into neutrosophic topology and lay the groundwork for broader applications in mathematical analysis, algebra, and related fields.

Keywords: Type-1 Neutrosophic Sets; Type-1 Neutrosophic Topology; Type-1 Neutrosophic Interior Sets; Type-1 Neutrosophic Closure Sets; Type-1 Neutrosophic Exterior Sets; Type-1 Neutrosophic Boundary Sets; Separation Axioms for Ti[I], i = 0, 1, …, 4; Continuity on Type-1 Neutrosophic Topological Spaces

1. Introduction

Neutrosophy, introduced by Smarandache in the 1980s [38], is a branch of philosophy that investigates the origin and characteristics of neutrality in nature. The mathematical study of uncertainty has evolved through several foundational milestones. In 1965, Zadeh proposed fuzzy sets as an extension of classical crisp sets, establishing the framework for fuzzy mathematics [42]. This was followed in 1968 by Chang, who extended fuzzy concepts to topological spaces, exploring notions such as open and closed sets, neighborhoods, interiors, continuity, and compactness [17]. Lowen further advanced this structure in 1976 by studying fuzzy compactness [31]. In 1986, Atanassov enriched fuzzy theory with intuitionistic fuzzy sets, introducing degrees of membership and non-membership [12,13], and in 1997, Coker extended these ideas to intuitionistic topological spaces [18]. Building on these developments, Smarandache introduced neutrosophic sets [39] as a generalization of intuitionistic fuzzy sets [12,13], incorporating three components for each element in X: the degree of membership, the degree of indeterminacy, and the degree of non-membership. Subsequent research investigated neutrosophic set operations and their topological properties. Lupianez [29,30] and Salama [35,36] defined various topologies on neutrosophic sets, though some did not satisfy De Morgan's laws. Also, neutrosophic closed sets and neutrosophic continuous functions were investigated in [37]. Karataş and Kuru [28] redefined neutrosophic operations and introduced neutrosophic topology, studying properties such as closure, interior, exterior, boundary, and subspace. In algebraic structures, groups play a central role [21,23], and neutrosophic sets have been applied in diverse areas including analysis [15,16], neutrosophic crisp topology [11], algebra [9,10,19], uncertainty graph theory [24,25,41], and practical domains such as sentiment analysis and medical diagnostics [1]. Smarandache and Kandasamy [26] extended neutrosophic theory to algebraic structures by introducing an indeterminate element I. The concept of neutrosophic groups was introduced in [33], while others explored algebraic operations from different viewpoints [27]. Cetkin and Aygun [19] proposed an approach to neutrosophic subgroups, and Abdalla and Saleem [22] studied neutrosophic group structures further. Recently, Al-Odhari [2–9] developed neutrosophic sets generated by classical set theory, identifying three types. In a revised work, he isolated type-1 neutrosophic sets and investigated their properties to construct new mathematical structures [2]. In this article, we build a topological framework based on type-1 neutrosophic sets and examine properties such as interior, closure, exterior, and boundary operators on type-1 neutrosophic sets X[I]. Furthermore, we study separation axioms for type-1 topological spaces (Ti[I], i = 0, 1, …, 4), and analyse continuous and contra type-1 neutrosophic functions along with their properties.

2. Type-1 Neutrosophic Topological Spaces

In this section, we introduce the concept of type-1 topological spaces, which is based on type-1 neutrosophic sets in [2–9]. A neutrosophic set of type-1 is induced from a neutrosophic element a + bI, where I is called the neutrosophic element with property $I^2 = I$ and $0I = 0$, as appeared in the work of Kandasamy and Smarandache on neutrosophic algebraic structures in [26,27]. An attempt to present an intuitive mathematical framework that includes type-1 neutrosophic set theory was presented in previous works and is still under development and mathematical construction.

Definition 2.1. Let X[I] be a type-1 neutrosophic set and τ[I] be a collection of neutrosophic subsets of X[I]. Then τ[I] is a type-1 neutrosophic topology on X[I] (in short T1NTS), if and only if:

(1) ∅[I] and X[I] ∈ τ[I].

(2) If A1[I] and A2[I] ∈ τ[I], then A1[I] ∩ A2[I] ∈ τ[I].

(3) Every neutrosophic union of neutrosophic members of τ[I] is also a neutrosophic member of τ[I].

The couple (X[I], τ[I]) is called a type-1 topological space on X[I]. Moreover, the members of τ[I] are said to be type-1 neutrosophic open sets in τ[I]. A subset A[I] ⊆ X[I] is called a type-1 neutrosophic closed set in (T1NTS) (X[I], τ[I]) if X[I] − A[I] ∈ τ[I].

Example 2.2. Let X[I] = {1 + I, 1 + 2I, 2 + I, 2 + 2I} be a type-1 neutrosophic set and τ[I] = {∅[I], X[I], {1+I, 1+2I}, {1+I, 2+I}, {1+I}, {1+I, 1+2I, 2+I}} be a collection of neutrosophic subsets of X[I]. Then (X[I], τ[I]) is a type-1 neutrosophic topological space.

Example 2.3. Let τ[I] = {∅[I], P(X[I])} and τ[I] = {∅[I], X[I]}. These are called, respectively, a type-1 neutrosophic discrete topological space and a type-1 neutrosophic indiscrete topological space over X[I].

Theorem 2.4. Let (X[I], τ[I]) be a type-1 neutrosophic topological space. Then: (1) ∅[I] and X[I] are type-1 neutrosophic open (or closed) sets over X[I]. (2) The intersection of any finite number of type-1 neutrosophic open (or closed) sets is a type-1 neutrosophic open (or closed) set over X[I]. (3) The union of any two type-1 neutrosophic open (or closed) sets is a type-1 neutrosophic open (or closed) set over X[I].

Proof. Straightforward. □

Theorem 2.5. Let (X[I], τ1[I]) and (X[I], τ2[I]) be two type-1 neutrosophic topological spaces over X[I]. Then (X[I], τ1[I] ∩ τ2[I]) is a type-1 neutrosophic topological space over X[I].

Proof. (1) Since ∅[I] and X[I] ∈ τ1[I] and τ2[I], then ∅[I] and X[I] ∈ τ1[I] ∩ τ2[I]. (2) Suppose A1[I], A2[I] ∈ τ1[I] ∩ τ2[I]. Then A1[I], A2[I] ∈ τ1[I] and A1[I], A2[I] ∈ τ2[I], which implies A1[I] ∩ A2[I] ∈ τ1[I] and A1[I] ∩ A2[I] ∈ τ2[I]. Therefore A1[I] ∩ A2[I] ∈ τ1[I] ∩ τ2[I]. (3) Assume {Ai[I] : i ∈ J} ⊆ τ1[I] ∩ τ2[I]. Then Ai[I] ∈ τ1[I] and Ai[I] ∈ τ2[I] for all i ∈ J. Therefore ∪i∈J Ai[I] ∈ τ1[I] and ∪i∈J Ai[I] ∈ τ2[I], so ∪i∈J Ai[I] ∈ τ1[I] ∩ τ2[I]. Consequently, (X[I], τ1[I] ∩ τ2[I]) is a type-1 neutrosophic topological space. □

Corollary 2.6. Let {(X[I], τi[I]) : i ∈ J} be a family of type-1 neutrosophic topological spaces over X[I]. Then (X[I], ∩i∈J τi[I]) is a type-1 neutrosophic topological space over X[I].

Proof. By the method similar to Theorem 2.5. □

Remark 2.1. The union of type-1 neutrosophic topological spaces may not be a type-1 neutrosophic topological space.

Example 2.7. Let X[I] = {a+aI, a+bI, b+aI, b+bI} be a type-1 neutrosophic set, and let τ1[I] = {∅[I], X[I], {a+aI}}, τ2[I] = {∅[I], X[I], {b+aI}} be two type-1 neutrosophic topologies on X[I]. Then τ1[I] ∪ τ2[I] is not a type-1 neutrosophic topological space.

Definition 2.8. Let (X[I], τ[I]) be a type-1 neutrosophic topological space, N[I] ⊆ X[I], and x ∈ X[I]. N[I] is said to be a neighborhood (in short nbd) of a neutrosophic point x ∈ X[I] if there exists O[I] ∈ τ[I] such that x ∈ O[I] ⊆ N[I], denoted Nnbd[I](x). The set which contains all such N[I] is called the neighborhood neutrosophic system and denoted by Nnbd[I](x) = {N[I] ⊆ X[I] : ∃ O[I] ∈ τ[I] such that x ∈ O[I] ⊆ N[I]}.

Example 2.9. Let X[I] = {0, 1, 2, I, 2I, 1+I, 1+2I, 2+I, 2+2I} be a type-1 neutrosophic set and consider the type-1 neutrosophic sub-collection τ[I] of X[I] given by τ[I] = {∅[I], X[I], {0}, {0, 1, I, 2I, 1+I, 1+2I}}, where (X[I], τ[I]) is a type-1 neutrosophic topological space. Then X[I] is a neighborhood of the point 0, since 0 ∈ X[I] ⊆ X[I]; {0} is a neighborhood of 0, since 0 ∈ {0} ⊆ {0}; and, more generally, every superset of {0} within X[I], such as {0,1}, {0,2}, {0,I}, {0,2I}, {0,1+I}, {0,1+2I}, {0,2+I}, {0,2+2I}, {0,1,2}, {0,1,2,I}, {0,1,2,I,2I}, {0,1,2,I,2I,1+I}, {0,1,2,I,2I,1+I,1+2I}, and {0,1,2,I,2I,1+I,1+2I,2+I}, is also a neighborhood of the point 0, because in each case 0 ∈ {0} ⊆ the given set.

3. Properties of Type-1 Neutrosophic Subsets on Type-1 Topological Spaces

In this section, we investigate the main properties of the type-1 neutrosophic subspace of type-1 topological spaces. We study type-1 neutrosophic interior, closure, and exterior operators via the X[I] set and their properties.

3.1. Type-1 Neutrosophic Interior Points

This subsection is devoted to studying the interior operator via X[I] sets and their properties.

Definition 3.1. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and A[I] ⊆ X[I]. The interior functor of A[I] is denoted Intn(A[I]) and defined by Intn(A[I]) = ∪{O[I] ⊆ X[I] : O[I] ⊆ A[I] and O[I] ∈ τ[I]}.

Remark 3.1. Intn(A[I]) is the largest type-1 neutrosophic open set contained in A[I], so Intn(A[I]) ⊆ A[I].

Example 3.2. Consider the type-1 neutrosophic topological space in Example 2.2. Let A[I] = {1+I, 1+2I, 2+2I} and B[I] = {1+I, 1+2I, 2+I} be two type-1 neutrosophic subsets of X[I]. Then Intn(A[I]) = {1+I, 1+2I} ∪ {1+I} = {1+I, 1+2I}, and Intn(B[I]) = {1+I,1+2I} ∪ {1+I,2+I} ∪ {1+I} ∪ {1+I,1+2I,2+I} = {1+I, 1+2I, 2+I}.

Theorem 3.3. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and A[I] ⊆ X[I]. Then Intn(A[I]) = A[I] if and only if A[I] is a type-1 neutrosophic open set.

Proof. If A[I] is a type-1 neutrosophic open set over X[I], then A[I] is itself a type-1 neutrosophic open set that contains A[I]; so A[I] is the largest type-1 neutrosophic open set contained in A[I], and Intn(A[I]) = A[I]. Conversely, if Intn(A[I]) = A[I], then Intn(A[I]) = A[I] ∈ τ[I]. □

Theorem 3.4. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and A[I] ⊆ X[I]. Then x ∈ Intn(A[I]) if and only if there is a type-1 neutrosophic open set B[I] such that x ∈ B[I] ⊆ A[I].

Proof. Let x ∈ Intn(A[I]) and take B[I] = Intn(A[I]). Then by the definition of Intn(A[I]), B[I] is a type-1 neutrosophic open set and, by Remark 3.1, x ∈ B[I] ⊆ A[I]. Conversely, let B[I] be a type-1 neutrosophic open set such that x ∈ B[I] ⊆ A[I]. Then by the definition of Intn(A[I]), x ∈ B[I] ⊆ Intn(A[I]). □

Theorem 3.5. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and A[I], B[I] ⊆ X[I]. Then the following axioms hold: (1) Intn(∅[I]) = ∅[I] and Intn(X[I]) = X[I]. (2) Intn(Intn(A[I])) = Intn(A[I]). (3) If A[I] ⊆ B[I] then Intn(A[I]) ⊆ Intn(B[I]). (4) Intn(A[I]) ∪ Intn(B[I]) ⊆ Intn(A[I] ∪ B[I]). (5) Intn(A[I] ∩ B[I]) = Intn(A[I]) ∩ Intn(B[I]).

Proof. (1) is obvious. (2) Let Intn(A[I]) = B[I]. Then Intn(B[I]) = B[I] by Theorem 3.3, and so Intn(Intn(A[I])) = Intn(A[I]). (3) Let x ∈ Intn(A[I]). Then by Theorem 3.4, there is a type-1 neutrosophic open set O[I] such that x ∈ O[I] ⊆ A[I]. Since A[I] ⊆ B[I], x ∈ O[I] ⊆ B[I], hence x ∈ Intn(B[I]); that is, Intn(A[I]) ⊆ Intn(B[I]). (4) Since A[I] ⊆ A[I] ∪ B[I] and B[I] ⊆ A[I] ∪ B[I], by part (3), Intn(A[I]) ⊆ Intn(A[I] ∪ B[I]) and Intn(B[I]) ⊆ Intn(A[I] ∪ B[I]); hence Intn(A[I]) ∪ Intn(B[I]) ⊆ Intn(A[I] ∪ B[I]). (5) Since A[I] ∩ B[I] ⊆ A[I] and A[I] ∩ B[I] ⊆ B[I], by part (3), Intn(A[I] ∩ B[I]) ⊆ Intn(A[I]) ∩ Intn(B[I]) … (i). Conversely, let x ∈ Intn(A[I]) ∩ Intn(B[I]); then there are two type-1 neutrosophic open sets M[I] and N[I] such that x ∈ M[I] ⊆ A[I] and x ∈ N[I] ⊆ B[I], so x ∈ (M[I] ∩ N[I]) ⊆ (A[I] ∩ B[I]), which implies x ∈ Intn(A[I] ∩ B[I]) by Theorem 3.4, giving Intn(A[I]) ∩ Intn(B[I]) ⊆ Intn(A[I] ∩ B[I]) … (ii). From (i) and (ii), we deduce that Intn(A[I] ∩ B[I]) = Intn(A[I]) ∩ Intn(B[I]). □

Example 3.6. Consider the type-1 neutrosophic topological space in Example 2.2. Let A[I] = {1+I,1+2I,2+2I} and B[I] = {1+I,1+2I,2+I} be two type-1 neutrosophic subsets of X[I]. Then Intn(A[I]) = {1+I,1+2I} and Intn(B[I]) = {1+I,1+2I,2+I}. Also, (A[I]∪B[I]) = X[I], while Intn(A[I]∪B[I]) = X[I]; therefore Intn(A[I]) ∪ Intn(B[I]) ⊆ Intn(A[I] ∪ B[I]).

3.2. Type-1 Neutrosophic Cluster Points

This subsection addresses the closure operator via X[I] sets and their properties.

Definition 3.7. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and H[I] ⊆ X[I]. The closure operator of H[I] is denoted Cln(H[I]) and defined by Cln(H[I]) = ∩{B[I] ⊆ X[I] : H[I] ⊆ B[I] and B[I] is a type-1 neutrosophic closed set}.

Example 3.8. Consider the type-1 neutrosophic topological space in Example 2.2 and H[I] = {1+2I, 2+I} ⊆ X[I]. The type-1 neutrosophic closed sets are F = {∅[I], X[I], {2+2I}, {2+I,2+2I}, {1+2I,2+2I}, {1+2I,2+I,2+2I}}. Then Cln(H[I]) = X[I] ∩ {1+2I,2+I,2+2I} = {1+2I, 2+I, 2+2I}.

Remark 3.2. Cln(H[I]) is the smallest type-1 neutrosophic closed set containing H[I], therefore H[I] ⊆ Cln(H[I]). Consequently, for any H[I] ⊆ X[I], Intn(H[I]) ⊆ H[I] ⊆ Cln(H[I]), by Remarks 3.1 and 3.2, respectively.

Theorem 3.9. For a type-1 neutrosophic topological space (X[I], τ[I]) and H[I] ⊆ X[I], Cln(H[I]) = H[I] if and only if H[I] is a type-1 neutrosophic closed set.

Proof. Suppose H[I] is a type-1 neutrosophic closed set. Since H[I] ⊆ H[I], Cln(H[I]) is a type-1 neutrosophic closed set, and H[I] ⊆ Cln(H[I]); therefore Cln(H[I]) = H[I]. Conversely, if Cln(H[I]) = H[I], then Cln(H[I]) is a type-1 neutrosophic closed set, and consequently H[I] is a type-1 neutrosophic closed set. □

Theorem 3.10. For a type-1 neutrosophic topological space (X[I], τ[I]) and H[I] ⊆ X[I], x ∈ Cln(H[I]) if and only if, for every type-1 neutrosophic open set B[I] containing x, B[I] ∩ H[I] ≠ ∅[I].

Proof. Let x ∈ Cln(H[I]) and let B[I] be any type-1 neutrosophic open set containing x. If B[I] ∩ H[I] = ∅, then H[I] ⊆ X[I] − B[I]. Since X[I] − B[I] is a type-1 neutrosophic closed set containing H[I], we get Cln(H[I]) ⊆ X[I] − B[I], and so x ∈ X[I] − B[I], contradicting x ∈ B[I]. Therefore B[I] ∩ H[I] ≠ ∅[I]. Conversely, let x ∉ Cln(H[I]). Then X[I] − Cln(H[I]) is a type-1 neutrosophic open set containing x. By hypothesis, [X[I] − Cln(H[I])] ∩ H[I] ≠ ∅[I], contradicting [X[I] − Cln(H[I])] ⊆ [X[I] − H[I]]. □

Theorem 3.11. For a type-1 neutrosophic topological space (X[I], τ[I]) and H[I], B[I] ⊆ X[I], the following axioms hold: (1) Cln(∅[I]) = ∅[I] and Cln(X[I]) = X[I]. (2) Cln(Cln(H[I])) = Cln(H[I]). (3) If H[I] ⊆ B[I] then Cln(H[I]) ⊆ Cln(B[I]). (4) Cln(H[I]) ∪ Cln(B[I]) = Cln(H[I] ∪ B[I]). (5) Cln(H[I] ∩ B[I]) ⊆ Cln(H[I]) ∩ Cln(B[I]).

Proof. (1) is obvious. (2) Follows directly from Theorem 3.9. (3) Let x ∈ Cln(H[I]). Then by Theorem 3.10, for every type-1 neutrosophic open set A[I] containing x, A[I] ∩ H[I] ≠ ∅[I]. Since H[I] ⊆ B[I], A[I] ∩ B[I] ≠ ∅[I], hence x ∈ Cln(B[I]); that is, Cln(H[I]) ⊆ Cln(B[I]). (4) Since H[I] ⊆ H[I] ∪ B[I] and B[I] ⊆ H[I] ∪ B[I], by part (3), Cln(H[I]) ∪ Cln(B[I]) ⊆ Cln(H[I] ∪ B[I]). Since H[I] ⊆ Cln(H[I]) and B[I] ⊆ Cln(B[I]), H[I] ∪ B[I] ⊆ Cln(H[I]) ∪ Cln(B[I]), and since Cln(H[I]) ∪ Cln(B[I]) is the smallest type-1 neutrosophic closed set containing H[I] ∪ B[I], Cln(H[I] ∪ B[I]) ⊆ Cln(H[I]) ∪ Cln(B[I]); combining both inclusions gives Cln(H[I]) ∪ Cln(B[I]) = Cln(H[I] ∪ B[I]). (5) Since H[I] ∩ B[I] ⊆ H[I] and H[I] ∩ B[I] ⊆ B[I], by part (3), Cln(H[I] ∩ B[I]) ⊆ Cln(H[I]) ∩ Cln(B[I]). □

Example 3.12. Consider the type-1 neutrosophic topological space in Example 2.2. Let H[I] = {1+I} and B[I] = {2+I} be two type-1 neutrosophic subsets of X[I]. Then Cln(H[I]) = X[I] and Cln(B[I]) = {2+I, 2+2I}. Therefore (H[I] ∩ B[I]) = ∅ (since H[I] and B[I] are disjoint singletons), Cln(H[I] ∩ B[I]) = ∅[I], and Cln(H[I]) ∩ Cln(B[I]) = {2+I, 2+2I}; we deduce that Cln(H[I] ∩ B[I]) ⊆ Cln(H[I]) ∩ Cln(B[I]).

Theorem 3.13. For a type-1 neutrosophic topological space (X[I], τ[I]) and H[I] ⊆ X[I], the following hold: (1) Intn(X[I] − H[I]) = X[I] − Cln(H[I]). (2) Cln(X[I] − H[I]) = X[I] − Intn(H[I]).

Proof. (1) Since H[I] ⊆ Cln(H[I]), X[I] − Cln(H[I]) ⊆ X[I] − H[I]. Since Cln(H[I]) is a type-1 neutrosophic closed set, X[I] − Cln(H[I]) is a type-1 neutrosophic open set, so X[I] − Cln(H[I]) = Intn[X[I] − Cln(H[I])] ⊆ Intn(X[I] − H[I]). Conversely, let x ∈ Intn(X[I] − H[I]); then there is a type-1 neutrosophic open set A[I] such that x ∈ A[I] ⊆ X[I] − H[I]. Then X[I] − A[I] is a type-1 neutrosophic closed set containing H[I], and x ∉ X[I] − A[I]; hence x ∉ Cln(H[I]), that is, x ∈ X[I] − Cln(H[I]). (2) Since Intn(H[I]) ⊆ H[I], X[I] − H[I] ⊆ X[I] − Intn(H[I]). Since Intn(H[I]) is a type-1 neutrosophic open set, X[I] − Intn(H[I]) is a type-1 neutrosophic closed set, so Cln(X[I] − H[I]) ⊆ Cln[X[I] − Intn(H[I])] = X[I] − Intn(H[I]). On the other side, let x ∉ Cln(X[I]−H[I]); then by Theorem 3.10, there is a type-1 neutrosophic open set A[I] containing x such that A[I] ∩ (X[I]−H[I]) = ∅[I], so x ∈ A[I] ⊆ H[I], that is, x ∈ Intn(H[I]); hence x ∉ X[I] − Intn(H[I]). Therefore X[I] − Intn(H[I]) ⊆ Cln(X[I] − H[I]). □

Theorem 3.14. For a type-1 neutrosophic subset H[I] ⊆ X[I] of a type-1 neutrosophic topological space (X[I], τ[I]), the following holds: (1) If B[I] is a type-1 neutrosophic open set in X[I], then Cln(H[I]) ∩ B[I] ⊆ Cln(H[I] ∩ B[I]). (2) If B[I] is a type-1 neutrosophic closed set in X[I], then Intn(H[I] ∪ B[I]) ⊆ Intn(H[I]) ∪ B[I].

Proof. (1) Let x ∈ Cln(H[I]) ∩ B[I], so x ∈ Cln(H[I]) and x ∈ B[I]. Let C[I] be any type-1 neutrosophic open set containing x. Then C[I] ∩ B[I] is a type-1 neutrosophic open set containing x. Since x ∈ Cln(H[I]), (C[I] ∩ B[I]) ∩ H[I] ≠ ∅, so C[I] ∩ (B[I] ∩ H[I]) ≠ ∅[I], which implies x ∈ Cln(H[I] ∩ B[I]). That is, Cln(H[I]) ∩ B[I] ⊆ Cln(H[I] ∩ B[I]). (2) Since B[I] is a type-1 neutrosophic closed set in X[I], by part (1) and Theorem 3.13, X[I] − [Intn(H[I]) ∪ B[I]] = [X[I]−Intn(H[I])] ∩ [X[I]−B[I]] = [Cln(X[I]−H[I])] ∩ [X[I]−B[I]] ⊆ Cln[(X[I]−H[I]) ∩ (X[I]−B[I])] = Cln(X[I]−(H[I]∪B[I])) = X[I] − (Intn(H[I]∪B[I])). Hence Intn(H[I] ∪ B[I]) ⊆ Intn(H[I]) ∪ B[I]. □

Theorem 3.15. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and H[I], B[I] ⊆ X[I]. Then: (1) $Intn(H[I]^c) = (Cln(H[I]))^c$. (2) $Cln(H[I]^c) = (Intn(H[I]))^c$.

Proof. (1) Cln(H[I]) = ∩{B[I] ⊆ X[I] : H[I] ⊆ B[I] and B[I] is a type-1 neutrosophic closed set}. Therefore $(Cln(H[I]))^c = \cup\{B[I]^c \subseteq X[I] : B[I]^c \subseteq H[I]^c$ and B[I] is a type-1 neutrosophic closed set}. The right-hand side of this equality is $Intn(H[I]^c)$, thus $Intn(H[I]^c) = (Cln(H[I]))^c$. (2) Substituting $H[I]^c$ for H[I] in part (1) gives $Cln(H[I]^c)^c = (Intn(H[I]^c))^c = Intn(H[I])$, so $Cln(H[I]^c) = (Intn(H[I]))^c$. □

3.3. Type-1 Neutrosophic Exterior and Boundary Points

This subsection studies the exterior and boundary operators via X[I] sets and their properties.

Definition 3.16. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and H[I] ⊆ X[I]. The exterior of H[I] over X[I] is denoted Extn(H[I]) and defined as $Extn(H[I]) = Intn(H[I]^c)$.

Example 3.17. Consider the type-1 neutrosophic topological space in Example 2.2. Let H[I] = {1+I, 1+2I, 2+2I}, so $H[I]^c$ = {2+I}, and $Intn(H[I]^c)$ = ∅[I] (since {2+I} ∉ τ[I] and contains no nonempty open subset). Then $Extn(H[I]) = Intn(H[I]^c)$ = ∅[I].

Theorem 3.18. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and A[I], B[I] ⊆ X[I]. Then: (1) Extn(A[I] ∪ B[I]) = Extn(A[I]) ∩ Extn(B[I]). (2) Extn(A[I]) ∪ Extn(B[I]) ⊆ Extn(A[I] ∩ B[I]).

Proof. By Definition 3.16 and Theorem 3.5, part (2): (1) $Extn(A[I]\cup B[I]) = Intn((A[I]\cup B[I])^c) = Intn(A[I]^c \cap B[I]^c) = Intn(A[I]^c) \cap Intn(B[I]^c)$ = Extn(A[I]) ∩ Extn(B[I]). (2) The proof is similar to (1). □

Definition 3.19. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and B[I] ⊆ X[I]. The type-1 neutrosophic boundary of B[I] over X[I] is denoted Γn(B[I]) and defined by $Γn(B[I]) = Cln(B[I]) \cap Cln(B[I]^c)$. It is noted that $Γn(B[I]) = Γn(B[I]^c)$.

Example 3.20. Consider the type-1 neutrosophic topological space in Example 2.2. If H[I] = {1+2I, 2+I}, then $H[I]^c$ = {1+I, 2+2I}, Cln(H[I]) = {1+2I, 2+I, 2+2I}, and $Cln(H[I]^c)$ = X[I]. Therefore Γn(H[I]) = $Cln(H[I]) \cap Cln(H[I]^c)$ = {1+2I, 2+I, 2+2I}.

Theorem 3.21. Let (X[I], τ[I]) be a type-1 neutrosophic topological space and A[I] ⊆ X[I]. Then $(Γn(A[I]))^c$ = Extn(A[I]) ∪ Intn(A[I]).

Proof. Since $Intn(A[I]^c) = (Cln(A[I]))^c$ by Theorem 3.15, $(\Gamma n(A[I]))^c = (Cln(A[I]) \cap Cln(A[I]^c))^c = (Cln(A[I]))^c \cup (Cln(A[I]^c))^c = (Cln(A[I]))^c \cup Intn(A[I]) = Extn(A[I]) \cup Intn(A[I])$. □

Theorem 3.22. For a type-1 neutrosophic topological space (X[I], τ[I]) and $H[I] \subseteq X[I]$, $\Gamma n(Intn(H[I])) \subseteq \Gamma n(H[I])$.

Proof. Since $Intn(H[I]) \subseteq H[I]$, we have $\Gamma n(Intn(H[I])) = Cln(Intn(H[I])) \cap Cln(Intn(H[I])^c) \subseteq Cln(H[I]) \cap Cln(H[I]^c) = \Gamma n(H[I])$. □

Theorem 3.23. For a subset $H[I] \subseteq X[I]$ of a type-1 neutrosophic topological space (X[I], τ[I]), the following holds: (1) $Cln(H[I]) = \Gamma n(H[I]) \cup Intn(H[I])$. (2) $\Gamma n(H[I]) \cap Intn(H[I]) = \emptyset[I]$. (3) $\Gamma n(H[I]) = Cln(H[I]) \cap Cln(X[I] - H[I])$.

Proof. (1) $\Gamma n(H[I]) \cup Intn(H[I]) = (Cln(H[I]) - Intn(H[I])) \cup Intn(H[I]) = [Cln(H[I]) \cap (X[I]-Intn(H[I]))] \cup Intn(H[I]) = [Cln(H[I]) \cup Intn(H[I])] \cap [(X[I]-Intn(H[I])) \cup Intn(H[I])] = Cln(H[I]) \cap X[I] = Cln(H[I])$. (2) This is clear from the definition of $\Gamma n(H[I])$. (3) By the definition, $\Gamma(H[I]) = Cln(H[I]) - Intn(H[I]) = Cln(H[I]) \cap (X[I]-Intn(H[I])) = Cln(H[I]) \cap Cln(X[I]-H[I])$, as required. □

Corollary 3.24. For a type-1 neutrosophic subset $H[I] \subseteq X[I]$ of a type-1 neutrosophic topological space (X[I], τ[I]), $\Gamma n(H[I])$ is a type-1 neutrosophic closed set in (X[I], τ[I]).

Proof. By Theorems 3.11 and 3.23. □

Theorem 3.25. For a subset $H[I] \subseteq X[I]$ of a type-1 neutrosophic topological space (X[I], τ[I]), the following holds: (1) H[I] is a type-1 neutrosophic open set if and only if $\Gamma n(H[I]) \cap H[I] = \emptyset[I]$. (2) H[I] is a type-1 neutrosophic closed set if and only if $\Gamma n(H[I]) \subseteq H[I]$. (3) H[I] is both a type-1 neutrosophic open set and closed set if and only if $\Gamma n(H[I]) = \emptyset[I]$.

Proof. (1) Let H[I] be a type-1 neutrosophic open set. Then Intn(H[I]) = H[I], and by Theorem 3.23, $\Gamma n(H[I]) \cap H[I] = \Gamma n(H[I]) \cap Intn(H[I]) = \emptyset[I]$. Conversely, suppose $\Gamma n(H[I]) \cap H[I] = \emptyset[I]$. Then $H[I] - Intn(H[I]) = [H[I] \cap Cln(H[I])] - [H[I] \cap Intn(H[I])] = H[I] \cap (Cln(H[I]) - Intn(H[I])) = H[I] \cap \Gamma n(H[I]) = \emptyset[I]$, so Intn(H[I]) = H[I]; hence H[I] is a type-1 neutrosophic open set. (2) Let H[I] be a type-1 neutrosophic closed set. Then Cln(H[I]) = H[I], so $\Gamma n(H[I]) = Cln(H[I]) - Intn(H[I]) = H[I] - Intn(H[I]) \subseteq H[I]$. Conversely, if $\Gamma n(H[I]) \subseteq H[I]$, then by Theorem 3.23, $Cln(H[I]) = Intn(H[I]) \cup \Gamma n(H[I]) \subseteq Intn(H[I]) \cup H[I] \subseteq H[I]$, so Cln(H[I]) = H[I]; hence H[I] is a type-1 neutrosophic closed set. (3) Let H[I] be both closed and open. Then Cln(H[I]) = H[I] = Intn(H[I]), so $\Gamma n(H[I]) = Cln(H[I]) - Intn(H[I]) = H[I] - H[I] = \emptyset[I]$. Conversely, if $\Gamma n(H[I]) = \emptyset[I]$, then $Cln(H[I]) - Intn(H[I]) = \emptyset[I]$. Since $Intn(H[I]) \subseteq Cln(H[I])$, we have Cln(H[I]) = Intn(H[I]); since $Intn(H[I]) \subseteq H[I] \subseteq Cln(H[I])$, we have Cln(H[I]) = H[I] = Intn(H[I]). Hence H[I] is both a type-1 neutrosophic closed set and a type-1 neutrosophic open set. □

Definition 3.26. For a type-1 neutrosophic topological space (X[I], τ[I]) and $Y[I] \subseteq X[I]$, the relativized type-1 neutrosophic topology of τ[I] on Y[I] is denoted τY[I] and defined by τY[I] = {H[I] ∩ Y[I] : H[I] is a type-1 neutrosophic open set in X[I]}. The pair (Y[I], τY[I]) is called a type-1 neutrosophic topological subspace of (X[I], τ[I]).

Example 3.27. Let X = {1, 2, 3}, Y = {1, 2} ⊂ X, X[I] = {1+1I, 1+2I, 1+3I, 2+1I, 2+2I, 2+3I, 3+1I, 3+2I, 3+3I}, and Y[I] = {1+1I, 1+2I, 2+1I, 2+2I}. Let τ[I] = {∅[I], X[I], {1+1I}, {1+3I}, {1+1I,1+3I}} be a type-1 neutrosophic topology on X[I]. Then τY[I] = {∅[I], Y[I], {1+1I}∩Y[I], {1+3I}∩Y[I], {1+1I,1+3I}∩Y[I]}, so τY[I] = {∅[I], Y[I], {1+1I}} is a type-1 neutrosophic relative topology on Y[I].

4. Separation Axioms

Definition 4.1. A type-1 neutrosophic topological space is called:

(1) T0[I] type-1 neutrosophic space if, for two points $x \neq y \in X[I]$, there is a type-1 neutrosophic open set G[I] in X[I] such that $x \in G[I]$ and $y \notin G[I]$.

(2) T1[I] type-1 neutrosophic space if, for two points $x \neq y \in X[I]$, there are two type-1 neutrosophic open sets G[I] and U[I] in X[I] such that $x \in G[I]$, $y \notin G[I]$, $y \in U[I]$, and $x \notin U[I]$.

(3) T2[I] type-1 neutrosophic space, or type-1 neutrosophic Hausdorff space, if, for two points $x \neq y \in X[I]$, there are two type-1 neutrosophic open sets G[I] and U[I] in X[I] such that $x \in G[I]$, $y \in U[I]$, and $U[I] \cap G[I] = \emptyset[I]$.

(4) Regular type-1 neutrosophic space if, for each type-1 neutrosophic closed set F[I] in (X[I], τ[I]) and each x ∉ F[I], there are two type-1 neutrosophic open sets G[I] and U[I] in (X[I], τ[I]) such that F[I] ⊆ G[I], x ∈ U[I], and U[I] ∩ G[I] = ∅[I].

(5) A type-1 neutrosophic topological space (X, τ[I]) is called type-1 neutrosophic T3[I] space if it is a regular type-1 neutrosophic space and a T1[I] type-1 neutrosophic space.

(6) Normal type-1 neutrosophic space if, for each two disjoint type-1 neutrosophic closed sets F[I] and M[I] in (X, τ[I]), there are two type-1 neutrosophic open sets G[I] and U[I] in (X[I], τ[I]) such that F[I] ⊆ G[I], M[I] ⊆ U[I], and U[I] ∩ G[I] = ∅[I]. A type-1 neutrosophic topological space (X[I], τ[I]) is called T4[I] type-1 neutrosophic space if it is a normal type-1 neutrosophic space and a T1[I] type-1 neutrosophic space.

Theorem 4.2. Let (X[I], τ[I]) be a T1[I] type-1 neutrosophic space and x a type-1 neutrosophic element. Then {x} is a type-1 neutrosophic closed set.

Proof. We show that $\{x\}^c$ is a type-1 neutrosophic open set. Let y be a type-1 neutrosophic element with y ∈ (X[I] − x), so x ≠ y. By the definition of T1[I], for the two distinct elements x and y, there exists a type-1 neutrosophic open set U[I]y such that y ∈ U[I]y and x ∉ U[I]y. Since x ∉ U[I]y, U[I]y lies entirely within the complement of x, so U[I]y ⊆ (X[I]−x). Hence (X[I]−x) = ∪{U[I]y : x ≠ y}. Since (X[I]−x) is a type-1 neutrosophic open set, {x} is a type-1 neutrosophic closed set. □

Theorem 4.3. Every T3[I] type-1 neutrosophic space is a T2[I] type-1 neutrosophic space.

Proof. Let (X[I], τ[I]) be a T3[I] type-1 neutrosophic space and x ≠ y ∈ X[I] be any points. Since (X[I], τ[I]) is a T1[I] type-1 neutrosophic space, by Theorem 4.2, {x} is a type-1 neutrosophic closed set and y ∉ {x}. Since (X[I], τ[I]) is a regular type-1 neutrosophic space, there are two type-1 neutrosophic open sets G[I] and U[I] in (X[I], τ[I]) such that x ∈ {x} ⊆ G[I], y ∈ U[I], and U[I] ∩ G[I] = ∅[I]. Hence (X[I], τ[I]) is a T2[I] type-1 neutrosophic space. □

Theorem 4.4. Every T4[I] type-1 neutrosophic space is a T3[I] type-1 neutrosophic space.

Proof. Let (X[I], τ[I]) be a T4[I] type-1 neutrosophic space. Let F[I] be any type-1 neutrosophic closed set in (X[I], τ[I]) and x ∉ F[I] be any point in (X[I], τ[I]). Since (X[I], τ[I]) is a T1[I] type-1 neutrosophic space, by Theorem 4.2, {x} is a type-1 neutrosophic closed set in (X[I], τ[I]) and F[I] ∩ {x} = ∅[I]. Since (X[I], τ[I]) is a normal type-1 neutrosophic space, there are two type-1 neutrosophic open sets G[I] and U[I] in (X[I], τ[I]) such that x ∈ {x} ⊆ G[I], F[I] ⊆ U[I], and U[I] ∩ G[I] = ∅[I]. Hence (X[I], τ[I]) is a T3[I] type-1 neutrosophic space. □

Theorem 4.5. A type-1 neutrosophic topological space (X[I], τ[I]) is a T0[I] type-1 neutrosophic space if and only if, for two points x ≠ y ∈ X[I], Cln({x}) ≠ Cln({y}).

Proof. Suppose (X[I], τ[I]) is a T0[I] type-1 neutrosophic space. Then for two points x ≠ y, there is a type-1 neutrosophic open set G[I] such that x ∈ G[I], y ∉ G[I]. Since X[I]−G[I] is a type-1 neutrosophic closed set and y ∈ X[I]−G[I], Cln({y}) ⊆ Cln(X[I]−G[I]) = X[I]−G[I]. Since x ∈ Cln({x}) and x ∉ X[I]−G[I], x ∉ Cln({y}); that is, Cln({x}) ≠ Cln({y}). Conversely, let x ≠ y ∈ X[I]. By hypothesis, Cln({x}) ≠ Cln({y}), so there is some point z ∈ Cln({x}) with z ∉ Cln({y}). If x ∈ Cln({y}), then z ∈ Cln({x}) ⊆ Cln({y}), a contradiction; hence x ∉ Cln({y}). Thus G[I] = X[I]−Cln({y}) is a type-1 neutrosophic open set in (X[I], τ[I]) such that x ∈ G[I] and y ∉ G[I]. Therefore (X[I], τ[I]) is a T0[I] type-1 neutrosophic space. □

Theorem 4.6. A type-1 neutrosophic topological space (X[I], τ[I]) is a T1[I] type-1 neutrosophic space if and only if {x} is a type-1 neutrosophic closed set in (X[I], τ[I]) for all x ∈ X[I].

Proof. Suppose (X[I], τ[I]) is a T1[I] type-1 neutrosophic space. Let x ∈ X[I]; we show X[I]−{x} is a type-1 neutrosophic open set. Let y ∈ X[I]−{x}, so x ≠ y. Since (X[I], τ[I]) is T1[I], there are type-1 neutrosophic open sets G[I] and U[I] such that x ∈ G[I], y ∉ G[I], y ∈ U[I], and x ∉ U[I]. Then y ∈ U[I] ⊆ X[I]−{x}. Hence X[I]−{x} is a type-1 neutrosophic open set, so {x} is a type-1 neutrosophic closed set. Conversely, let x ≠ y ∈ X[I]. By hypothesis, {x} and {y} are type-1 neutrosophic closed sets, so X[I]−{x} and X[I]−{y} are type-1 neutrosophic open sets, with x ∈ X[I]−{y}, y ∉ X[I]−{y}, x ∉ X[I]−{x}, and y ∈ X[I]−{x}. Therefore (X[I], τ[I]) is a T1[I] type-1 neutrosophic space. □

Theorem 4.7. A type-1 neutrosophic topological space (X[I], τ[I]) is a T2[I] type-1 neutrosophic space if and only if, for each x ∈ X[I] and each y ≠ x ∈ X[I], there is a type-1 neutrosophic open set H[I] in (X[I], τ[I]) containing x such that y ∉ Cln(H[I]).

Proof. Suppose (X[I], τ[I]) is T2[I]. Let x ∈ X[I] and y ≠ x. There are two type-1 neutrosophic open sets G[I] and U[I] such that x ∈ G[I], y ∈ U[I], and U[I] ∩ G[I] = ∅[I]. Take H[I] = G[I]; then H[I] is a type-1 neutrosophic open set containing x, and y ∉ H[I] ⊆ Cln(H[I]) (since U[I] is disjoint from H[I] and, being a neighborhood of y, excludes y from Cln(H[I])). Conversely, let x ≠ y ∈ X[I]. By hypothesis, there is a type-1 neutrosophic open set H[I] containing x such that y ∉ Cln(H[I]). Then X[I]−Cln(H[I]) is a type-1 neutrosophic open set containing y, with x ∈ H[I], y ∈ X[I]−Cln(H[I]), and H[I] ∩ (X[I]−Cln(H[I])) = ∅[I]. Hence (X[I], τ[I]) is T2[I]. □

Theorem 4.8. A type-1 neutrosophic topological space (X[I], τ[I]) is a regular type-1 neutrosophic space if and only if, for each x ∈ X[I] and each type-1 neutrosophic open set M[I] in (X[I], τ[I]) containing x, there is a type-1 neutrosophic open set H[I] in (X[I], τ[I]) containing x such that Cln(H[I]) ⊆ M[I].

Proof. Suppose (X[I], τ[I]) is regular. Let x ∈ X[I] and M[I] be a type-1 neutrosophic open set containing x. Then X[I]−M[I] is a type-1 neutrosophic closed set with x ∉ X[I]−M[I]. By regularity, there are type-1 neutrosophic open sets G[I] and U[I] such that (X[I]−M[I]) ⊆ G[I], x ∈ U[I], and U[I] ∩ G[I] = ∅[I]. Take H[I] = U[I]; then H[I] ⊆ (X[I]−G[I]), so Cln(H[I]) ⊆ Cln(X[I]−G[I]) ⊆ (X[I]−G[I]) ⊆ M[I]. Conversely, let F[I] be a type-1 neutrosophic closed set and x ∉ F[I]. Then x ∈ X[I]−F[I], a type-1 neutrosophic open set containing x. By hypothesis, there is a type-1 neutrosophic open set H[I] containing x with Cln(H[I]) ⊆ (X[I]−F[I]). Then F[I] ⊆ [X[I]−Cln(H[I])], and X[I]−Cln(H[I]) is a type-1 neutrosophic open set with H[I] ∩ [X[I]−Cln(H[I])] = ∅[I]; hence (X[I], τ[I]) is regular. □

Theorem 4.9. A type-1 neutrosophic topological space (X[I], τ[I]) is a normal type-1 neutrosophic space if and only if, for each type-1 neutrosophic closed set F[I] in (X[I], τ[I]) and each type-1 neutrosophic open set G[I] containing F[I], there is a type-1 neutrosophic open set V[I] in (X[I], τ[I]) containing F[I] such that Cln(V[I]) ⊆ G[I].

Proof. Suppose (X[I], τ[I]) is normal. Let F[I] be a type-1 neutrosophic closed set and G[I] a type-1 neutrosophic open set containing F[I]. Then X[I]−G[I] is a type-1 neutrosophic closed set with F[I] ∩ (X[I]−G[I]) = ∅[I]. By normality, there are type-1 neutrosophic open sets H[I] and U[I] such that (X[I]−G[I]) ⊆ U[I], F[I] ⊆ H[I], and U[I] ∩ H[I] = ∅[I]. Take V[I] = H[I]; then V[I] ⊆ (X[I]−U[I]), so Cln(V[I]) ⊆ Cln(X[I]−U[I]) ⊆ (X[I]−U[I]) ⊆ G[I]. Conversely, let F[I] and H[I] be disjoint type-1 neutrosophic closed sets. Then H[I] ⊆ (X[I]−F[I]), a type-1 neutrosophic open set containing H[I]. By hypothesis, there is a type-1 neutrosophic open set V[I] containing H[I] such that Cln(V[I]) ⊆ (X[I]−F[I]). Then F[I] ⊆ X[I]−Cln(V[I]), and X[I]−Cln(V[I]) is type-1 neutrosophic open, with V[I] ∩ [X[I]−Cln(V[I])] = ∅[I]; hence (X[I], τ[I]) is normal. □

5. Continuous Type-1 Neutrosophic Functions

Definition 5.1. A type-1 neutrosophic function fn : (X[I], τ[I]) → (Y[I], τ1[I]) between two type-1 neutrosophic topological spaces is called continuous if fn^{-1}(U[I]) is a type-1 neutrosophic open set in (X[I], τ[I]) for every type-1 neutrosophic open set U[I] in Y[I].

Example 5.2. Let X = {a, b} and Y = {1, 2, 3} be two classical sets with a classical function fc : X → Y such that fc(a) = 1 and fc(b) = 2. The type-1 neutrosophic sets generated by X and Y are X[I] = {a+aI, a+bI, b+aI, b+bI} and Y[I] = {1+1I,1+2I,1+3I,2+1I,2+2I,2+3I,3+1I,3+2I,3+3I}. The type-1 neutrosophic function fn is given by fn(a+aI) = fc(a)+fc(a)fn(I) = 1+1I, fn(a+bI) = fc(a)+fc(b)fn(I) = 1+2I, fn(b+aI) = fc(b)+fc(a)fn(I) = 2+1I, and fn(b+bI) = fc(b)+fc(b)fn(I) = 2+2I. Let fn : (X[I],τ[I]) → (Y[I],τ1[I]) with τ[I] = {∅[I],X[I],{a+aI},{a+bI},{a+bI,a+aI}} and τ1[I] = {∅[I],Y[I],{1+1I},{1+2I},{1+1I,1+2I}}. This type-1 neutrosophic function is continuous, since for every type-1 neutrosophic open set in (Y[I],τ1[I]) we have fn^{-1}({1+1I}) = {a+aI}, fn^{-1}({1+2I}) = {a+bI}, fn^{-1}({1+1I,1+2I}) = {a+bI,a+aI}, fn^{-1}(Y[I]) = X[I], and fn^{-1}(∅[I]) = ∅[I].

Theorem 5.3. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) between two type-1 neutrosophic topological spaces is continuous if and only if fn^{-1}(F[I]) is a type-1 neutrosophic closed set in (X[I], τ[I]) for every type-1 neutrosophic closed set F[I] in Y[I].

Proof. Let fn be continuous and F[I] be a type-1 neutrosophic closed set in Y[I]. Then fn^{-1}(Y[I]−F[I]) = X[I]−fn^{-1}(F[I]) is a type-1 neutrosophic open set in (X[I],τ[I]), so fn^{-1}(F[I]) is a type-1 neutrosophic closed set. Conversely, suppose $fn^{-1}(F[I])$ is a type-1 neutrosophic closed set for every type-1 neutrosophic closed set F[I] in Y[I]. Let U[I] be any type-1 neutrosophic open set in Y[I]. Then $fn^{-1}(Y[I]-U[I]) = X[I]-fn^{-1}(U[I])$ is a type-1 neutrosophic closed set in (X[I],τ[I]), that is, $fn^{-1}(U[I])$ is a type-1 neutrosophic open set. Hence fn is continuous. □

Theorem 5.4. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is continuous if and only if, for each x ∈ X[I] and each type-1 neutrosophic open set U[I] in Y[I] with fn(x) ∈ U[I], there exists a type-1 neutrosophic open set V[I] in (X[I],τ[I]) such that x ∈ V[I] and fn(V[I]) ⊆ U[I].

Proof. Suppose fn is continuous. Let x ∈ X[I] and U[I] be a type-1 neutrosophic open set containing fn(x). Put $V[I] = fn^{-1}(U[I])$; since fn is continuous, V[I] is type-1 neutrosophic open, x ∈ V[I], and fn(V[I]) ⊆ U[I]. Conversely, let U[I] be a type-1 neutrosophic open set in Y[I] and $x \in fn^{-1}(U[I])$. Then fn(x) ∈ U[I], and by hypothesis there is a type-1 neutrosophic open set V[I] with x ∈ V[I] and fn(V[I]) ⊆ U[I]. Hence $x \in V[I] \subseteq fn^{-1}(U[I])$, so $fn^{-1}(U[I])$ is type-1 neutrosophic open, and fn is continuous. □

Theorem 5.5. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is continuous if and only if fn[Cln(A[I])] ⊆ Cln[fn(A[I])] for all A[I] ⊆ X[I].

Proof. Let fn be continuous and A[I] ⊆ X[I]. Then Cln[fn(A[I])] is type-1 neutrosophic closed in Y[I], so by Theorem 5.3, $fn^{-1}[Cln(fn(A[I]))]$ is type-1 neutrosophic closed in (X[I],τ[I]), that is, $Cln[fn^{-1}[Cln[fn(A[I])]]] = fn^{-1}[Cln[fn(A[I])]]$. Since fn(A[I]) ⊆ Cln[fn(A[I])], $A[I] \subseteq fn^{-1}[Cln[fn(A[I])]]$, so $Cln(A[I]) \subseteq Cln[fn^{-1}[Cln[fn(A[I])]]] = fn^{-1}[Cln[fn(A[I])]]$. Hence fn[Cln(A[I])] ⊆ Cln[fn(A[I])]. Conversely, let H[I] be a type-1 neutrosophic closed set in Y[I], i.e., Cln(H[I]) = H[I]. Since $fn^{-1}(H[I]) \subseteq X[I]$, by hypothesis $fn[Cln[fn^{-1}(H[I])]] \subseteq Cln[fn(fn^{-1}(H[I]))] \subseteq Cln(H[I]) = H[I]$, so $Cln[fn^{-1}(H[I])] \subseteq fn^{-1}(H[I])$; hence $Cln[fn^{-1}(H[I])] = fn^{-1}(H[I])$, that is, $fn^{-1}(H[I])$ is type-1 neutrosophic closed, and fn is continuous. □

Theorem 5.6. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is continuous if and only if $Cln[fn^{-1}(B[I])] \subseteq fn^{-1}[Cln(B[I])]$ for all B[I] ⊆ Y[I].

Proof. Let fn be continuous and B[I] ⊆ Y[I]. Then Cln(B[I]) is type-1 neutrosophic closed in Y[I], so by Theorem 5.3, $fn^{-1}[Cln(B[I])]$ is type-1 neutrosophic closed, i.e., $Cln[fn^{-1}[Cln(B[I])]] = fn^{-1}[Cln(B[I])]$. Since B[I] ⊆ Cln(B[I]), $fn^{-1}(B[I]) \subseteq fn^{-1}[Cln(B[I])]$, so $Cln[fn^{-1}(B[I])] \subseteq Cln[fn^{-1}[Cln(B[I])]] = fn^{-1}[Cln(B[I])]$. Conversely, let H[I] be type-1 neutrosophic closed in Y[I], i.e., Cln(H[I]) = H[I]. Since H[I] ⊆ Y[I], by hypothesis $Cln[fn^{-1}(H[I])] \subseteq fn^{-1}[Cln(H[I])] = fn^{-1}(H[I])$; hence $Cln[fn^{-1}(H[I])] = fn^{-1}(H[I])$, that is, $fn^{-1}(H[I])$ is type-1 neutrosophic closed, and fn is continuous. □

Theorem 5.7. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is continuous if and only if $fn^{-1}(Intn(B[I])) \subseteq Intn[fn^{-1}(B[I])]$ for all B[I] ⊆ Y[I].

Proof. Let fn be continuous and B[I] ⊆ Y[I]. Then Intn(B[I]) is type-1 neutrosophic open in Y[I], so $fn^{-1}(Intn(B[I]))$ is type-1 neutrosophic open in (X[I],τ[I]), i.e., $Intn[fn^{-1}[Intn(B[I])]] = fn^{-1}[Intn(B[I])]$. Since Intn(B[I]) ⊆ B[I], $fn^{-1}[Intn(B[I])] \subseteq fn^{-1}(B[I])$, so $fn^{-1}[Intn(B[I])] \subseteq Intn[fn^{-1}(B[I])]$. Conversely, let U[I] be type-1 neutrosophic open in Y[I], i.e., Intn(U[I]) = U[I]. Since U[I] ⊆ Y[I], by hypothesis $fn^{-1}(U[I]) = fn^{-1}[Intn(U[I])] \subseteq Intn[fn^{-1}(U[I])]$, so $fn^{-1}(U[I]) = Intn[fn^{-1}(U[I])]$, that is, $fn^{-1}(U[I])$ is type-1 neutrosophic open, and fn is continuous. □

Definition 5.8. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is called a type-1 neutrosophic closed function if fn(G[I]) is a type-1 neutrosophic closed set in (Y[I],τ1[I]) for every type-1 neutrosophic closed set G[I] in (X[I],τ[I]).

Theorem 5.9. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is a type-1 neutrosophic closed function if and only if Cln[fn(A[I])] ⊆ fn[Cln(A[I])] for all A[I] ⊆ X[I].

Proof. Suppose fn is a type-1 neutrosophic closed function and A[I] ⊆ X[I]. Since Cln(A[I]) is type-1 neutrosophic closed in (X[I],τ[I]) and fn is closed, fn[Cln(A[I])] is type-1 neutrosophic closed in Y[I], i.e., Cln[fn[Cln(A[I])]] = fn[Cln(A[I])]. Since A[I] ⊆ Cln(A[I]), fn(A[I]) ⊆ fn[Cln(A[I])], so Cln[fn(A[I])] ⊆ Cln[fn[Cln(A[I])]] = fn[Cln(A[I])]. Conversely, let F[I] be type-1 neutrosophic closed in (X[I],τ[I]), i.e., Cln(F[I]) = F[I]. By hypothesis Cln[fn(F[I])] ⊆ fn[Cln(F[I])] = fn(F[I]), so Cln[fn(F[I])] = fn(F[I]), that is, fn(F[I]) is type-1 neutrosophic closed in Y[I]. Hence fn is a type-1 neutrosophic closed function. □

Definition 5.10. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is called a type-1 neutrosophic open function if fn(G[I]) is a type-1 neutrosophic open set in (Y[I],τ1[I]) for every type-1 neutrosophic open set G[I] in (X[I],τ[I]).

Theorem 5.11. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is a type-1 neutrosophic open function if and only if fn[Intn(A[I])] ⊆ Intn[fn(A[I])] for all A[I] ⊆ X[I].

Proof. Suppose fn is a type-1 neutrosophic open function and A[I] ⊆ X[I]. Since Intn(A[I]) is type-1 neutrosophic open in (X[I],τ[I]) and fn is open, fn[Intn(A[I])] is type-1 neutrosophic open in Y[I], i.e., Intn[fn[Intn(A[I])]] = fn[Intn(A[I])]. Since Intn(A[I]) ⊆ A[I], fn[Intn(A[I])] ⊆ fn(A[I]), so Intn[fn[Intn(A[I])]] ⊆ Intn[fn(A[I])] = fn[Intn(A[I])]. Hence fn[Intn(A[I])] ⊆ Intn[fn(A[I])]. Conversely, let F[I] be type-1 neutrosophic open in (X[I],τ[I]), i.e., Intn(F[I]) = F[I]. By hypothesis fn[Intn(F[I])] ⊆ Intn[fn(F[I])] = fn(F[I]), so fn(F[I]) ⊆ Intn[fn(F[I])], hence Intn[fn(F[I])] = fn(F[I]), that is, fn(F[I]) is type-1 neutrosophic open in Y[I]. Hence fn is a type-1 neutrosophic open function. □

6. Contra Type-1 Neutrosophic Functions

Definition 6.1. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is called a contra continuous type-1 neutrosophic function if fn^{-1}(V[I]) is a type-1 neutrosophic closed set in (X[I],τ[I]) for every type-1 neutrosophic open set V[I] in Y[I].

Theorem 6.2. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is contra continuous if and only if fn^{-1}(F[I]) is a type-1 neutrosophic open set in (X[I],τ[I]) for every type-1 neutrosophic closed set F[I] in Y[I].

Proof. Suppose fn is contra continuous. Let G[I] be a type-1 neutrosophic closed set in Y[I]. Then Y[I]−G[I] is type-1 neutrosophic open, so X[I]−fn^{-1}(G[I]) = fn^{-1}(Y[I]−G[I]) is type-1 neutrosophic closed in (X[I],τ[I]), that is, fn^{-1}(G[I]) is type-1 neutrosophic open. Conversely, let G[I] be type-1 neutrosophic open in Y[I]. Then Y[I]−G[I] is type-1 neutrosophic closed, and by hypothesis, fn^{-1}(Y[I]−G[I]) = X[I]−fn^{-1}(G[I]) is type-1 neutrosophic open, that is, fn^{-1}(G[I]) is type-1 neutrosophic closed. Hence fn is contra continuous. □

Theorem 6.3. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is contra continuous if and only if, for each x ∈ X[I] and each type-1 neutrosophic closed set G[I] in Y[I] containing fn(x), there is a type-1 neutrosophic open set U[I] in (X[I],τ[I]) containing x such that fn(U[I]) ⊆ G[I].

Proof. Suppose fn is contra continuous. Let x ∈ X[I] and G[I] be a type-1 neutrosophic closed set in Y[I] containing fn(x). By Theorem 6.2, U[I] = fn^{-1}(G[I]) is a type-1 neutrosophic open set in (X[I],τ[I]). Since fn(x) ∈ G[I], x ∈ fn^{-1}(G[I]) = U[I], and fn(U[I]) = fn(fn^{-1}(G[I])) ⊆ G[I]. Conversely, let G[I] be a type-1 neutrosophic closed set in Y[I]. For each x ∈ fn^{-1}(G[I]), fn(x) ∈ G[I], so by hypothesis there is a type-1 neutrosophic open set U[I]x containing x with fn(U[I]x) ⊆ G[I]. Therefore fn^{-1}(G[I]) = ∪{U[I]x : x ∈ fn^{-1}(G[I])}, which is type-1 neutrosophic open. Hence, by Theorem 6.2, fn is contra continuous. □

Theorem 6.4. The set of all points x ∈ X[I] at which fn : (X[I],τ[I]) → (Y[I],τ1[I]) is not contra continuous is identical with the union of the boundaries of the inverse images of type-1 neutrosophic closed sets of Y[I] containing fn(x).

Proof. Suppose fn is not contra continuous at x ∈ X[I]. By Theorem 6.3, there is a type-1 neutrosophic closed set G[I] in Y[I] containing fn(x) such that fn(U[I]) ⊈ G[I] for every type-1 neutrosophic open set U[I] containing x. That is, for every such U[I], fn(U[I]) ∩ (Y[I]−G[I]) ≠ ∅[I], which implies U[I] ∩ fn^{-1}(Y[I]−G[I]) ≠ ∅[I]. Therefore x ∈ Cln[fn^{-1}(Y[I]−G[I])] = Cln[X[I]−fn^{-1}(G[I])]. However, since fn(x) ∈ G[I], x ∈ fn^{-1}(G[I]) ⊆ Cln[fn^{-1}(G[I])]. Thus x ∈ Cln[X[I]−fn^{-1}(G[I])] ∩ Cln[fn^{-1}(G[I])] = Γn[fn^{-1}(G[I])]. Conversely, suppose x ∈ X[I] and x ∈ Γn[fn^{-1}(G[I])] for some type-1 neutrosophic closed set G[I] in Y[I] containing fn(x). If fn were contra continuous at x, there would be a type-1 neutrosophic open set U[I] containing x such that fn(U[I]) ⊆ G[I], giving x ∈ U[I] ⊆ fn^{-1}(G[I]), that is, x ∈ Intn[fn^{-1}(G[I])] ⊆ Γn[fn^{-1}(G[I])]^c, contradicting x ∈ Γn[fn^{-1}(G[I])]. Hence, by Theorem 6.3, fn is not contra continuous at x. □

Definition 6.5. The kernel of a subset H[I] of a type-1 neutrosophic topological space (X[I],τ[I]) is denoted Kern(H[I]) and defined by Kern(H[I]) = ∩{U[I] : H[I] ⊆ U[I] and U[I] is a type-1 neutrosophic open set in X[I]}.

Lemma 6.6. Let H[I] and G[I] be subsets of a type-1 neutrosophic topological space (X[I],τ[I]). The following holds: (1) x ∈ Kern(H[I]) if and only if H[I] ∩ U[I] ≠ ∅[I] for any type-1 neutrosophic closed set U[I] containing x. (2) H[I] ⊆ Kern(H[I]), and H[I] = Kern(H[I]) if H[I] is a type-1 neutrosophic open set in X[I]. (3) If H[I] ⊆ G[I] then Kern(H[I]) ⊆ Kern(G[I]).

Proof. (1) Let V[I] be any type-1 neutrosophic closed subset of X[I] containing x. Suppose H[I] ∩ V[I] = ∅[I]. Then H[I] ⊆ X[I]−V[I], and since X[I]−V[I] is type-1 neutrosophic open, Kern(H[I]) ⊆ X[I]−V[I]; hence x ∈ X[I]−V[I], contradicting x ∈ V[I]. Therefore H[I] ∩ V[I] ≠ ∅[I]. Conversely, suppose x ∉ Kern(H[I]). Then there is a type-1 neutrosophic open set V[I] containing H[I] with x ∉ V[I]. Then X[I]−V[I] is a type-1 neutrosophic closed set containing x with H[I] ∩ (X[I]−V[I]) = ∅[I], contradicting the hypothesis. Hence x ∈ Kern(H[I]). (2) By the definition of Kern(H[I]), H[I] ⊆ Kern(H[I]). If H[I] is type-1 neutrosophic open, H[I] is itself the smallest type-1 neutrosophic open set containing H[I], so Kern(H[I]) = H[I]. (3) Let x ∈ Kern(H[I]). By part (1), H[I] ∩ V[I] ≠ ∅[I] for any type-1 neutrosophic closed set V[I] containing x. Since H[I] ⊆ G[I], G[I] ∩ V[I] ≠ ∅[I], so x ∈ Kern(G[I]); that is, Kern(H[I]) ⊆ Kern(G[I]). □

Theorem 6.7. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is contra continuous if and only if fn[Cln(H[I])] ⊆ Kern[fn(H[I])] for every subset H[I] of X[I].

Proof. Suppose fn is contra continuous. Let H[I] be any subset of X[I], and let y ∉ Kern[fn(H[I])]. By Lemma 6.6, there is a type-1 neutrosophic closed set F[I] in Y[I] containing y such that fn(H[I]) ∩ F[I] = ∅[I], so H[I] ∩ fn^{-1}(F[I]) = ∅[I]. Since fn is contra continuous, by Theorem 6.2, fn^{-1}(F[I]) is a type-1 neutrosophic open set in (X[I],τ[I]), so X[I]−fn^{-1}(F[I]) is type-1 neutrosophic closed, i.e., Cln[X[I]−fn^{-1}(F[I])] = X[I]−fn^{-1}(F[I]). Since H[I] ∩ fn^{-1}(F[I]) = ∅[I], H[I] ⊆ X[I]−fn^{-1}(F[I]), so Cln(H[I]) ⊆ Cln[X[I]−fn^{-1}(F[I])] = X[I]−fn^{-1}(F[I]); hence Cln(H[I]) ∩ fn^{-1}(F[I]) = ∅[I], so fn[Cln(H[I])] ∩ F[I] = ∅[I], giving y ∉ fn[Cln(H[I])]. Therefore fn[Cln(H[I])] ⊆ Kern[fn(H[I])]. Conversely, suppose fn[Cln(H[I])] ⊆ Kern[fn(H[I])] for every subset H[I] of X[I]. Let V[I] be any type-1 neutrosophic open subset of Y[I]. Then fn^{-1}(V[I]) ⊆ X[I], and by hypothesis fn[Cln(fn^{-1}(V[I]))] ⊆ Kern[fn(fn^{-1}(V[I]))] ⊆ Kern(V[I]) = V[I] (the last equality by Lemma 6.6, since V[I] is type-1 neutrosophic open). This implies Cln(fn^{-1}(V[I])) ⊆ fn^{-1}(V[I]), that is, fn^{-1}(V[I]) is type-1 neutrosophic closed in (X[I],τ[I]). Hence fn is contra continuous. □

Theorem 6.8. A type-1 neutrosophic function fn : (X[I],τ[I]) → (Y[I],τ1[I]) is contra continuous if and only if Cln[fn^{-1}(G[I])] ⊆ fn^{-1}[Kern(G[I])] for every subset G[I] of Y[I].

Proof. Suppose fn is contra continuous. Let G[I] be any type-1 neutrosophic subset of Y[I]. Since fn^{-1}(G[I]) ⊆ X[I] and fn is contra continuous, by Theorem 6.7, fn[Cln[fn^{-1}(G[I])]] ⊆ Kern[fn(fn^{-1}(G[I]))] ⊆ Kern(G[I]); hence Cln[fn^{-1}(G[I])] ⊆ fn^{-1}[Kern(G[I])]. Conversely, suppose Cln[fn^{-1}(G[I])] ⊆ fn^{-1}[Kern(G[I])] for every subset G[I] of Y[I]. Let V[I] be any type-1 neutrosophic open subset of Y[I]. By hypothesis and Lemma 6.6, Cln[fn^{-1}(V[I])] ⊆ fn^{-1}[Kern(V[I])] ⊆ fn^{-1}(V[I]), so Cln[fn^{-1}(V[I])] = fn^{-1}(V[I]), that is, fn^{-1}(V[I]) is type-1 neutrosophic closed in (X[I],τ[I]). Therefore fn is contra continuous. □

7. Conclusion

This paper has developed a topological framework based on type-1 neutrosophic sets, formalizing the interior, closure, exterior, and boundary operators on type-1 neutrosophic sets X[I] and establishing separation axioms for type-1 topological spaces Ti[I], i = 0, 1, …, 4. Continuous and contra continuous type-1 neutrosophic functions were introduced and characterized through several equivalent formulations involving closure, interior, and kernel operators. These results isolate type-1 neutrosophic sets as a distinct and tractable framework and lay the groundwork for broader applications in mathematical analysis, algebra, and related fields.

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