Applications of Neutrosophic Topology in Gold Rate Analysis in India

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Abstract: Gold plays a significant role in the Indian economy due to its cultural, financial and investment value. However, gold price behaviour is highly uncertain and influenced by multiple indeterminate factors such as market sentiments, macroeconomic variables and regional variations. This study introduces an innovative framework using Neutrosophic Topology to analyse gold rate fluctuations in India. Incorporating the concept of neutrosophic topology provides a more flexible and realistic modelling approach compared to classical and fuzzy systems. This paper develops a neutrosophic topological structure on gold price data and demonstrates its application in analysing uncertainty, continuity, and clustering of price movements across major Indian cities.

Keywords: Neutrosophic Topology; Gold Price; Uncertainty; Neutrosophic Sets

1. Introduction

Gold is widely regarded as a safe haven asset and plays a crucial role in India's financial systems. Its demand is influenced by cultural practices, inflation hedging, and investment diversification. Studies show that gold prices in India are affected by household financial conditions, macroeconomic variables, and global trends. Traditional statistical and econometric models often fail to capture uncertainty and indeterminacy inherent in gold price movements. To overcome these limitations, neutrosophic approaches have been introduced, which extend fuzzy logic by including a third component, indeterminacy. Recent studies on neutrosophic statistical analysis have shown that gold prices vary significantly across cities and time periods, making uncertainty modelling essential.

2. Preliminaries

Definition 2.1 [9]. Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{\langle x, \mu A(x), \sigma A(x), \gamma A(x)\rangle : x \in X\}$, where $\mu A(x)$, $\sigma A(x)$, and $\gamma A(x)$ represent the degree of membership function, the degree of indeterminacy, and the degree of non-membership function, respectively, of each element $x \in X$ to the set A.

Remark 2.2 [9]. A neutrosophic set $A = \{\langle x, \mu A(x), \sigma A(x), \gamma A(x)\rangle : x \in X\}$ can be identified with an ordered triple $A = \{\langle \mu A(x), \sigma A(x), \gamma A(x)\rangle\}$ in $]{-0}, 1^{+}[$ on X.

Remark 2.3 [9]. For the sake of simplicity, we shall use the symbol $A = \{\langle \mu(x), \sigma(x), \gamma(x)\rangle\}$ for the neutrosophic set $A = \{\langle x, \mu A(x), \sigma A(x), \gamma A(x)\rangle : x \in X\}$.

Example 2.4 [9]. Every intuitionistic fuzzy set A on a non-empty set X is obviously a Neutrosophic set having the form $A = \{\langle \mu A(x), 1 - \mu A(x) + \sigma A(x), \gamma A(x)\rangle : x \in X\}$. Since the main purpose is to construct the tools for developing Neutrosophic Set and Neutrosophic topology, the neutrosophic sets 0N and 1N in X must be introduced as follows.

0N may be defined as follows:

(01) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$
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Definition 2.5. A Neutrosophic topology τN on a set X is a family of neutrosophic subsets satisfying the following conditions:

- $0, X \in \tau N$.
- Arbitrary union of neutrosophic open sets is neutrosophic open.
- Finite intersection of neutrosophic open sets is neutrosophic open.

Remark 2.6. This structure allows modelling of uncertain open sets, which is useful in financial data analysis.

Example 2.7 [9]. Any fuzzy topological space $(X, \tau 0)$ in the sense of Chang is obviously an NTS in the form of $\{A : \mu A \in \tau 0\}$, wherever a fuzzy set in X whose membership function is μA is identified with its counterpart. The following is an example of a Neutrosophic topological space.

Example 2.8 [9]. Let $X = \{x\}$ and $A = \{\langle x, 0.5, 0.5, 0.4\rangle : x \in X\}$, $B = \{\langle x, 0.4, 0.6, 0.8\rangle : x \in X\}$, $C = \{\langle x, 0.5, 0.6, 0.4\rangle : x \in X\}$, $D = \{\langle x, 0.4, 0.5, 0.8\rangle : x \in X\}$. Then the family $\tau N = \{0_N, 1_N, A, B, C, D\}$ is called a neutrosophic topological space on X.

Definition 2.9 [9]. The complement of A [C(A)] of an NOS is called a neutrosophic closed set [NCS for short] in X.

Let us define the neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces:

Definition 2.10 [9]. Let $(X, \tau N)$ be an NTS and $A = \{(\mu A, \sigma A, \gamma A)\}$ be a Neutrosophic set in X. Then the neutrosophic closure and neutrosophic interior of A are defined by:

$NCl(A) = \cap\{K : K \text{ is an NCS in X and } A \subseteq K\}$

$NInt(A) = \cup\{G : G \text{ is an NOS in X and } G \subseteq A\}$

It can be shown that NCl(A) is an NCS and NInt(A) is an NOS in X. A is NOS if and only if A = NIntA. A is NCS if and only if A = NClA.

3. Literature Review

Existing research highlights the importance of gold price modelling in India. Econometric studies show long-run relationships between household finance and gold prices. The gold market exhibits volatility, seasonality, and policy-driven fluctuations. Neutrosophic statistical methods have been applied to analyse gold rate variations across cities.

4. Methodology

Gold price data is collected from major cities of India. The data includes daily minimum and maximum prices, monthly averages, and regional variations. Indeterminacy is defined as $I(t) = N_{indet} / N_{total}$, where N_{indet} is the number of conflicting headlines from the World Gold Council (WGC), weighted by importance, such that $I \in [0,1]$ and $0 \le T + F + I \le 3$.

Data Collection

Table 5.1. Neutrosophic data collection – Thoothukudi / Chennai / Delhi

Date	City	22kt Low L	22kt High U	U − L	Headline Total	Indet. Headlines	I = Nindet/Ntotal (GN)
31/08/2026	Thoothukudi	9713	9930	217	10	3	0.3 (9778.1)
31/08/2026	Chennai	9700	9925	225	10	2	0.2 (9745.0)
15/08/2026	Delhi	6016	6032	16	8	3	[0.2,0.4] (interval)

Neutrosophic Representation of Gold Price

Let the gold price be represented as a neutrosophic number given by GN = GL + GU.IN, where GL represents the lower price, GU represents the upper price, and IN represents the indeterminacy interval, IN ∈ [IL, IU].

Construction of Neutrosophic Topology

Let X be the set of gold price observations, and let τN be the collection of neutrosophic open sets that represent the price intervals.

Remark 7.1. Neutrosophic open sets may represent stable price changes, high-volatility zones, and uncertain transition regions.

Definition 7.2. A function $f : X \to Y$ is neutrosophically continuous if the pre-image of every neutrosophic open set is neutrosophically open.

Remark 7.3. The above function is used to model smooth price transitions and sudden fluctuations.

Application of Gold Rate Analysis

(a) Modelling Price Uncertainty:

Neutrosophic topology captures daily fluctuations, regional differences, and market unpredictability. Unlike classical models, it explicitly represents indeterminacy, which is crucial in financial markets.

(b) Clustering of Gold Prices:

- Stable clusters (low volatility): zones with I < 0.30, corresponding to low volatility. Here T ≈ 1, F ≈ 0, the price trend is clearly defined, and classical forecasting holds.
- Boundary regions (high uncertainty): zones with I > 0.40, corresponding to high uncertainty where 0.40 ≤ T, I, F ≤ 0.6. This is maximum indeterminacy, and the recommendation is clearly to hold — neither to sell nor to buy.
- Transition zones (potential price shifts): the neighbourhood of boundary points where 0.30 ≤ I ≤ 0.40 indicates potential price shifts (breakout/breakdown). This serves as an early warning for investors before T switches to F.

(c) Decision-Making Support:

This model helps identify optimal buying periods, detect unstable market phases, and support investment strategies.

Table 8.1. Neutrosophic clustering and decision support

Region	I	Zone	⟨T, I, F⟩	Cluster / Decision
Chennai	0.22	IntN(A)	⟨.78, .22, .20⟩	Stable / Buy
Thoothukudi	0.32	Transition	⟨.60, .32, .35⟩	Potential Shift / Hold

Delhi (6016–6032)	0.35	Neutrosophic Boundary	⟨.55, .35, .40⟩	Boundary / Range

Theorems and Proofs in Neutrosophic Topology for Gold Rate Analysis

Notations 9.1. Let X be the set of gold price observations, GN(x) = ⟨T(x), I(x), F(x)⟩ be a neutrosophic representation of gold price, and τN be the neutrosophic topology on X.

Theorem 9.2 (Neutrosophic Topology from Gold Price Intervals).

Let X be the set of gold prices in India, and define a family τN consisting of all neutrosophic subsets representing price intervals with uncertainty. Then τN forms a neutrosophic topology on X.

Proof. The empty set 0 corresponds to no price observation, and X represents all gold prices; both can be expressed as 0 = ⟨0,1,1⟩, X = ⟨1,0,0⟩. Hence 0, X ∈ τN. Let {Ui} ⊆ τN; each Ui represents a price interval with uncertainty, and their union U = ⋃i Ui still represents a valid uncertain price region, preserving the truth, indeterminacy, and falsity bounds. Hence U ∈ τN. Let U1, U2 ∈ τN; then U1 ∩ U2 corresponds to overlapping price intervals, which is also a neutrosophic open set. Thus all axioms hold, so τN is a neutrosophic topology. □

Theorem 9.3 (Neutrosophic Continuity of the Gold Price Function).

Let f : X → R represent the mapping from time to gold prices. If small changes in input produce bounded neutrosophic variations in output, then f is neutrosophically continuous.

Proof. A function is neutrosophically continuous if for every open set V ⊆ R the pre-image $f^{-1}(V)$ is neutrosophically open. Let V represent a stable gold price interval; then $f^{-1}(V)$ consists of the time points where the price falls within this interval. Since price changes are gradual over short intervals, the pre-image corresponds to the cluster of neighbouring time points with bounded indeterminacy. Thus $f^{-1}(V)$ ∈ τN. Hence f is neutrosophically continuous. □

Theorem 9.4 (Neutrosophic Closure Represents Maximum Price Stability).

The neutrosophic closure of a gold price set A ⊆ V represents the smallest closed set containing all stable and near-stable price observations.

Proof. The closure is defined as ClN(A) = ∩{C / A ⊆ C, where C is a neutrosophic closed set}. In gold price analysis, let A represent the stable prices. Closure adds boundary points, since the intersection of closed sets is closed, so ClN(A) is closed and minimal. Hence the closure captures both exact and approximate stability regions in gold pricing. □

Theorem 9.5 (Neutrosophic Interior and Market Stability).

The neutrosophic interior of a gold price set represents the most stable core region of prices.

Proof. The interior is defined as IntN(A) = ∪{U / U ⊆ A, U is neutrosophically open}. Here U represents the stable price intervals, and their union gives the largest stable region inside A. Thus IntN(A) excludes boundary fluctuations and captures highly reliable price zones. □

Theorem 9.6 (Boundary Region Captures Maximum Uncertainty).

The neutrosophic boundary of a gold price set represents maximum uncertainty in price transitions.

Proof. The boundary is defined as ∂N(A) = ClN(A) − IntN(A). This includes the points that are not fully stable and the points not completely outside the set. Thus these are transition points where prices may rise or fall and indeterminacy is highest. Hence the boundary models volatile gold market behaviour. □

Theorem 9.7 (Compactness and Bounded Gold Price Behaviour).

If the neutrosophic topological space (X, τN) of gold prices is compact, then every sequence of gold prices has a convergent subsequence.

Proof. By definition, a compact space ensures every open cover has a finite subcover. In the gold price context, the open cover is the collection of price intervals covering all data. Thus price fluctuations remain bounded, ensuring the existence of a convergent subsequence (stability over time). Hence gold prices exhibit bounded behaviour under the compactness assumption. □

Theorem 9.8 (Connectedness of the Gold Market).

If X is neutrosophically connected, it cannot be divided into two disjoint stable regions.

Proof. Assume X = U ∪ V, disjoint open sets, implying two separate price regimes. But real market prices are interdependent, and transitions exist. Thus separation is impossible, so X is connected. □

5. Results and Discussion

This neutrosophic topological framework reveals that gold price behaviour is nonlinear and uncertain.

6. Future Scope of This Research

This framework can be integrated with AI and deep learning. It can be applied to stock and cryptocurrency markets, and a comparative study with fuzzy topology can also be implemented.

7. Conclusion

Neutrosophic topology provides a powerful framework to model gold rate behaviour under uncertainty. It helps investors, especially in Tamil Nadu, to decide whether to buy, sell, or hold by quantifying not only bullish and bearish sentiment but also confusion.

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