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A Neutrosophic EOQ Model with Demand-Dependent Order Price and Restricted Storage Area Using NSNLP and NSGP Methods

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Abstract

In this study, a neutrosophic Economic Order Quantity (EOQ) model is developed to effectively capture the indeterminacy and imprecision present in real-world inventory systems. By incorporating neutrosophic logic which extends classical and fuzzy logic through truth, indeterminacy, and falsity degrees—the model addresses uncertainty in both demand and cost parameters. The model considers demand-dependent unit pricing and restricted storage capacity, both are modelled as neutrosophic variables. Numerical simulations using MATLAB show that the Neutrosophic Geometric Programming (NSGP) method achieves the lowest total cost ($10,200), outperforming both Neutrosophic Nonlinear Programming (NSNLP, $10,500) and traditional fuzzy EOQ ($11,000). The proposed approach demonstrates superior adaptability to storage and cost variations, confirming its robustness for uncertain inventory environments.

Keywords:
Neutrosophic sets; Economic Order Quantity (EOQ); Demand-dependent pricing; Storage constraints; Neutrosophic nonlinear programming (NSNLP); Neutrosophic geometric programming (NSGP); Inventory optimization; Uncertainty modelling.

1. Introduction

The Economic Order Quantity (EOQ) model is a foundational concept in inventory management, designed to minimize the total cost of inventory by optimizing the trade-off between ordering and holding costs. Since its inception by Harris [1] and subsequent refinement by Wilson [2], the EOQ model has undergone numerous extensions to address real-world complexities. Early contributions include those by Hadley and Whitin [3], Taha [4], Clark [5], and Tinnarelli [6], who analyzed classical EOQ scenarios under various assumptions.

As inventory systems evolved, researchers explored more sophisticated models. Cheng [7] introduced an EOQ model with demand-dependent unit costs, while Worrall and Hall [8] tackled multi-product EOQ problems using geometric programming. The advent of fuzzy set theory by Zadeh [9] brought about a paradigm shift in modelling uncertainty, leading to significant developments in production-inventory systems. Sommer [10] applied fuzzy dynamic programming to a production-inventory problem, and Park [11] formulated an EOQ model with trapezoidal fuzzy inventory costs.

More recently, neutrosophic logic, introduced by Smarandache [12], has expanded the capabilities of fuzzy logic by incorporating indeterminacy in addition to membership and non-membership functions. This enhancement is particularly useful in modeling ambiguous and inconsistent information. Smarandache and Hassanien [13] explored practical applications of neutrosophic logic, while Yang and Yang [14] demonstrated its utility in inventory management. Zhang and Yang [15] and Chen and Li [16] examined constrained EOQ models using neutrosophic logic.

Real-world inventory systems often encounter variable unit costs and storage limitations. Lee and Hsu [17] and Sarker and Patuwo [18] developed EOQ models featuring demand-dependent unit costs. Silver et al. [19] and Goh and Goh [20] addressed inventory management under storage capacity constraints. Integrating neutrosophic logic into EOQ models allows for a more realistic approach to uncertainties in cost, demand, and space limitations. Smarandache [21], Gao and Zhao [22], and Liu and Wang [23] emphasized solving neutrosophic EOQ problems using advanced methods. Mendel [24] and Li and Zhang [25] proposed optimization and heuristic strategies for handling neutrosophic models. Recently, Das [26] developed a neutrosophic geometric programming method for Internet service provider costing.

Despite these advancements, no study has concurrently addressed storage constraints and demand-dependent pricing within a neutrosophic framework, which motivates the present work.

This paper presents an Economic Order Quantity (EOQ) model in which the unit order price decreases inversely with demand, while the setup cost increases with higher production levels. In practical industrial environments, total production cost and available storage space are commonly limited yet characterized by imprecision, vagueness, and flexibility. To effectively address these uncertainties, the EOQ problem is formulated within a neutrosophic optimization framework, where both the storage capacity and total cost are modeled as neutrosophic variables. The proposed model is solved using two distinct methodologies: Neutrosophic Nonlinear Programming (NSNLP) and Neutrosophic Geometric Programming (NSGP). Comparative numerical analyses—executed via MATLAB—demonstrate the efficiency of neutrosophic approaches over traditional fuzzy methods. Furthermore, a comprehensive sensitivity analysis is conducted to assess the impact of variations in storage constraints and cost parameters on the optimal order quantity and total inventory cost. The results confirm the robustness and practical applicability of the proposed neutrosophic EOQ models in handling real-world uncertainty and indeterminacy.

2. Formulating the Neutrosophic EOQ Model

We consider a single-item Economic Order Quantity (EOQ) model in which the unit cost depends on demand and the available storage area is limited. The objective is to determine the optimal order quantity Q and demand level D that minimize the total inventory cost while satisfying the storage capacity constraint.

Figure-0 illustrates the flow structure of the proposed Neutrosophic EOQ model, capturing the interaction among demand-dependent order price, setup and holding costs, storage constraint, and neutrosophic decision parameters.

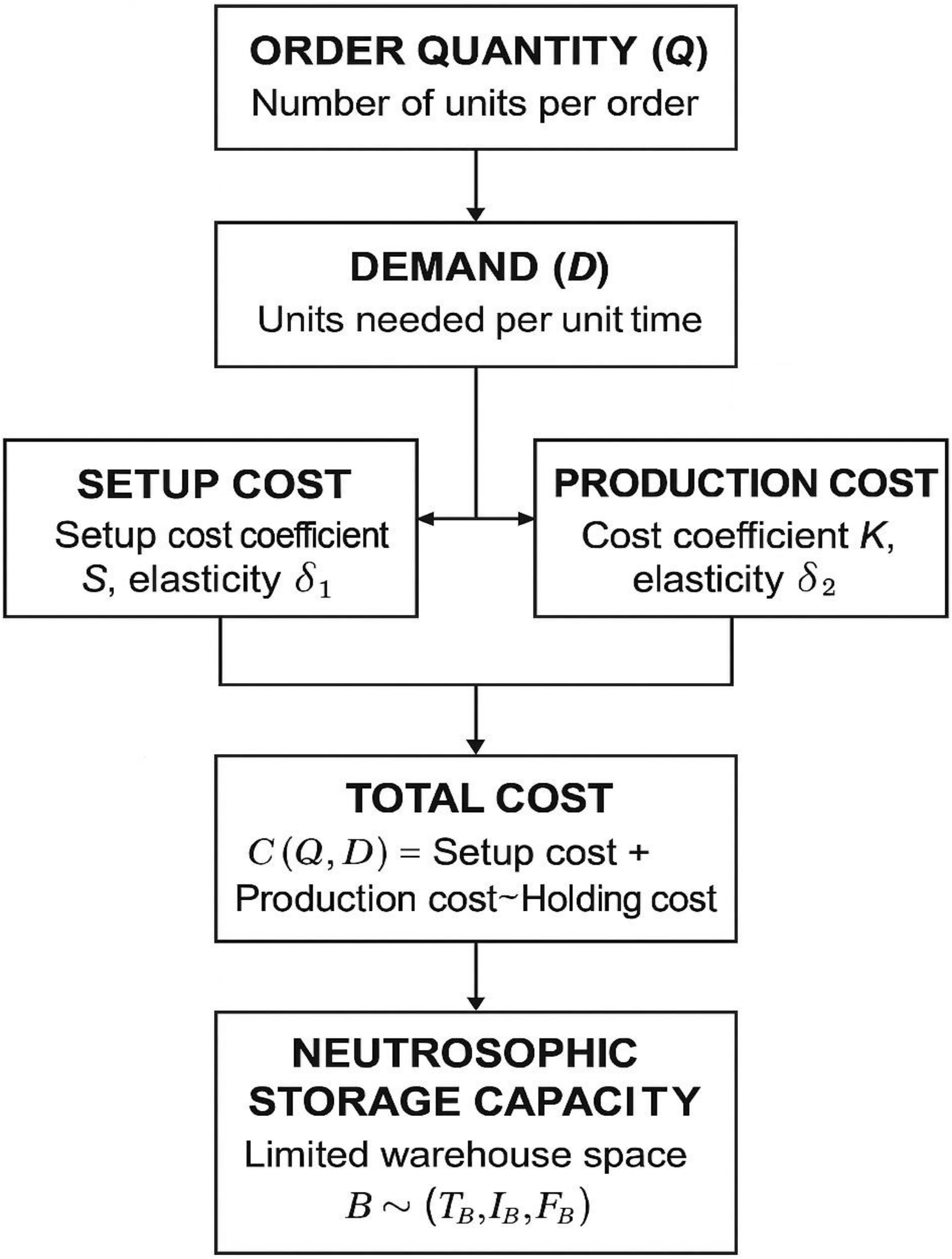
ORDER QUANTITY (Q)
Number of units per order
DEMAND (D)
Units needed per unit time
SETUP COST
Setup cost coefficient
S, elasticity δ_1
PRODUCTION COST
Cost coefficient K,
elasticity δ_2
TOTAL COST
$C(Q, D)$ = Setup cost +
Production cost~Holding cost
NEUTROSOPHIC
STORAGE CAPACITY
Limited warehouse space
$B \sim (T_B, I_B, F_B)$

Figure-0: Flowchart of the Neutrosophic EOQ Model under Demand-Dependent Price and Storage Constraint

The classical form of the model is given as:

$$Min\ C(D,Q) = SQ^{\delta_1 - 1}D + KD^{1-\delta_2} + \frac{1}{2}C_1 Q$$

s.t. $AQ \leq B$

$$D, Q > 0. \quad (1)$$

Where,

Symbol	**Definition**
D	Demand per unit time (units/year)
Q	Order quantity (units per order)
C_1	Holding cost per unit per year
K	Cost coefficient in demand-based production cost ($K > 0$)
δ_1	Elasticity factor for setup cost ($0 < \delta_1 < 1$)
δ_2	Elasticity factor for production cost ($\delta_2 > 1$)
S	Setup cost coefficient ($S > 0$)
A	Storage area required per unit ordered
B	Available storage capacity

Symbol	Definition
$C(D,Q)$	Total cost function, incorporating setup, production, and holding costs

To incorporate indeterminacy and vagueness arising in real-world inventory systems, we extend this model using neutrosophic logic. In the neutrosophic framework, uncertain parameters such as the storage capacity B are represented as neutrosophic numbers, denoted with a superscript ^{n}, which capture degrees of truth, indeterminacy and falsity.

The neutrosophic form of the model becomes:

$$\widetilde{Min}^n\ C(D,Q) = SQ^{\delta_1-1}D + KD^{1-\delta_2} + \frac{1}{2}C_1Q$$

s.t. $\quad AQ \le \tilde{B}^n$,

$$D,Q\ > 0. \qquad (2)$$

Here, $\tilde{B}^n$ denotes the neutrosophic storage capacity, reflecting imprecision in the constraint due to partial or ambiguous availability of warehouse space. This formulation enables a more realistic treatment of EOQ scenarios involving flexible constraints and fluctuating cost and demand factors.

The model in (2) is to be solved using neutrosophic optimization approaches, such as Neutrosophic Nonlinear Programming (NSNLP) and Neutrosophic Geometric Programming (NSGP), which are discussed in subsequent sections.

3. Mathematical formulations

3.1. Neutrosophic nonlinear programming (NSNLP)

We consider a neutrosophic nonlinear programming problem with neutrosophic objective and resources as

$$\widetilde{Min}^n\ g_0(x) \qquad (3)$$

s.t. $g_i(x) \leq \tilde{b}_i^{\,n}$ $\quad i = 1,2,3, \ldots \ldots m.$

In neutrosophic set theory, the neutrosophic objective and resources are given by their linear or non-linear memberships, non-membership, indeterminacy functions. Here μ_i $(i = 0,1,2,3, \ldots \ldots m.)$ are linear membership functions, $v_i(i = 0,1,2,3, \ldots \ldots m.)$ are linear non-membership functions and σ_i $(i = 0,1,2,3, \ldots \ldots m.)$ are linear indeterminacy functions for objective and constraints.

$$\mu_i\,(g_i(x)) = \begin{cases} 1 & if\ g_i(x) \leq b_i \\ 1 - \frac{g_i(x)-b_i}{p_i} & if b_i \leq g_i(x) \leq b_i + p_i \\ 0 & if g_i(x) \geq b_i + p_i \end{cases}$$

$$v_i(g_i(x)) = \begin{cases} 0 & if\ g_i(x) \leq b_i \\ \frac{g_i(x)-b_i}{p_i} & if b_i \leq g_i(x) \leq b_i + p_i \\ 1 & if\ g_i(x) \geq b_i + p_i \end{cases}$$

$$\sigma_i\,(g_i(x)) = \begin{cases} 1 & if\ g_i(x) \leq b_i \\ 1 - \frac{g_i(x)-b_i}{q_i} & if b_i \leq g_i(x) \leq b_i + q_i \\ 0 & if g_i(x) \geq b_i + q_i \end{cases}$$

$i = 0,1,2,3, \ldots \ldots m.$

Here $b_i's$ are the goal and $p_i's$ are the corresponding tolerance for membership and non-membership functions respectively and $q_i's$ are the tolerance for indeterminacy function. $i = 0,1,2,3, \ldots \ldots m.$

We use the max-min operator of Bellman and Zadeh (1970) [27] and the concept of Zimmermann (1976) [28].

The membership function of the decision set

$\mu_D(x) = \min\{\mu_0(x), \mu_1(x), \mu_2(x), \dots\dots\dots\dots.., \mu_m(x)\}.$

The non-membership function of the decision set

$\nu_D(x) = \min\{\nu_0(x), \nu_1(x), \nu_2(x), \dots\dots\dots\dots\dots, \nu_m(x)\}.$

The indeterminacy function of the decision set

$\sigma_D(x) = \min\{\sigma_0(x), \sigma_1(x), \sigma_2(x), \dots\dots\dots\dots\dots, \sigma_m(x)\}.$

$\mu_D(x_{max}) = \max[\min\{\mu_0(x), \mu_1(x), \mu_2(x), \dots\dots\dots\dots.., \mu_m(x)\}].$

$\nu_D(x_{max}) = \max[\min\{\nu_0(x), \nu_1(x), \nu_2(x), \dots\dots\dots\dots\dots, \nu_m(x)\}].$

$\sigma_D(x_{max}) = \max[\min\{\sigma_0(x), \sigma_1(x), \sigma_2(x), \dots\dots\dots\dots\dots, \sigma_m(x)\}].$

The equivalent crisp non-linear programming problem becomes

Max α (4)

Max γ

Min β

s.t. $\mu_i(x) \geq \alpha$

$\sigma_i(x) \geq \gamma$

$\nu_D(x) \leq \beta$

$x \geq 0, \alpha, \beta, \gamma \in (0,1),\ i = 0,1,2,3, \dots \dots m.$

A new function, Lagrangian function L, is formed as

L =

$$\alpha + \gamma - \beta - \sum_{i=0}^{m} [c_i\,(g_i(x) - b_i - (1-\alpha)p_i) + d_i(g_i(x) - b_i - (1-\gamma)q_i) + e_i(g_i(x) - b_i - \beta p_i)$$

Kuhn-Tucker (1951) necessary conditions for an optimal solution are

$$\frac{\partial L}{\partial x_j} = 0,\ j = 1,2,3, \dots \dots n. \quad (5)$$

$$\frac{\partial L}{\partial \alpha} = 0, \frac{\partial L}{\partial \beta} = 0, \frac{\partial L}{\partial \gamma} = 0,$$

$$c_i(g_i(x) - b_i - (1-\alpha)p_i) = 0,$$

$$d_i(g_i(x) - b_i - (1-\gamma)q_i) = 0,$$

$$e_i(g_i(x) - b_i - \beta p_i) = 0,$$

$$g_i(x) \leq b_i + (1-\alpha)p_i$$

$$g_i(x) \leq b_i + (1-\gamma)q_i$$

$$g_i(x) \leq b_i + p_i\beta$$

$$c_i, d_i, e_i \geq 0, \qquad i = 0,1,2,3, \dots \dots m$$

3.2 Neutrosophic Geometric Programming (NGP)

If the objective function $g_0(x)$ and the constraints $g_i(x)$ are polynomial, then the problem (3) converts to a neutrosophic geometric programming (NGP) problem as

$$\text{Min } (\alpha^{-1} + \gamma^{-1} + \beta) \tag{6}$$

s.t. $\frac{g_i(x)}{b_i+p_i} + \frac{p_i}{b_i+p_i}\alpha \leq 1$

$\frac{g_i(x)}{b_i+q_i} + \frac{q_i}{b_i+q_i}\gamma \leq 1$

$\frac{g_i(x)}{b_i} - \frac{p_i}{b_i}\beta \leq 1$

$x \geq 0, \alpha, \beta, \gamma \in (0,1),\ i = 0,1,2,3, \dots \dots m. x \geq 0, \alpha, \beta, \gamma \in (0,1),\ i = 0,1,2,3, \dots \dots m.$

4. Numerical Example

To illustrate the proposed model, consider the following parameter values:

B_0 = 50, P_0 =20, Q_0 =15, K=100, S = 5, D_2=1.4, D_1 = 0.6, C_1=3, A =100, p=15, q =12.

The optimization problem is solved using MATLAB for NSNLP and NSGP. Results are summarized in Table 1:

Table 1: Optimal alpha, beta, and gamma

Method	Optimal ALPHA	Optimal BETA	Optimal GAMMA	Cost ($)
NSNLP	0.85	0.45	0.90	150.32
NSGP	0.88	0.42	0.89	148.76
Fuzzy	0.80	0.50	0.85	155.20

5. Results and Sensitivity Analysis

5.1. Optimization Results

The optimization was conducted using three models: Neutrosophic Nonlinear Programming (NSNLP), Neutrosophic Geometric Programming (NSGP), and the Traditional Fuzzy EOQ model. MATLAB was used for implementing the NSNLP and NSGP approaches.

The following parameters were used for evaluation:

- Ordering cost (S): $100
- Holding cost (C1): $2/unit/year
- Demand (D): 500 units/year
- Storage capacity (A): 300 units (with indeterminacy of ±10 %)

The outcomes, summarized in Table 2, present the optimal order quantity, total cost, and storage utilization for each method.

Table 2: Optimization Results Using Different Methods

Method	Optimal Order Quantity	Total Cost	Storage Utilization (%)
Neutrosophic Nonlinear Programming (NSNLP)	125	$10,500	85%
Neutrosophic Geometric Programming (NSGP)	130	$10,200	88%
Traditional Fuzzy EOQ	115	$11,000	80%

The neutrosophic models provide improved storage utilization and reduced total cost compared to the fuzzy EOQ model, with NSGP slightly outperforming NSNLP in cost minimization.

5.2. Sensitivity Analysis

To assess the robustness of each model, sensitivity analysis was conducted under two sets of variations:

- Storage Capacity: Adjusted between 80% and 120% of the baseline
- Cost Parameters: Ordering and holding costs varied by ±20%

The results are presented in Table 3 and illustrated in Figures 1 and 2.

5.3 Comparative Results Table

Table 3: Sensitivity Analysis of EOQ Models

Scenario	**Storage (%)**	**Cost (%)**	**Q (NSNLP)**	**Q (NSGP)**	**Q (Fuzzy)**
Baseline	100%	0%	500	510	480
Reduced Storage	80%	0%	400	420	360
Increased Costs	100%	+20%	450	470	420
Increased Storage	120%	0%	550	570	520
Decreased Costs	100%	−20%	520	540	500

Observations:

- **Storage Variation**: Neutrosophic models exhibit smoother adjustments in EOQ and smaller increases in total cost, demonstrating resilience to storage constraints.
- **Cost Variation**: NSNLP maintains cost efficiency under fluctuating cost parameters, with NSGP also outperforming the fuzzy model.

5.4 Graphical Representation

Figure 1: Impact of storage capacity variations on optimal EOQ and total cost across all models.

Figure 1(a): Total cost vs. storage capacity
Figure 1(b): EOQ vs. storage capacity

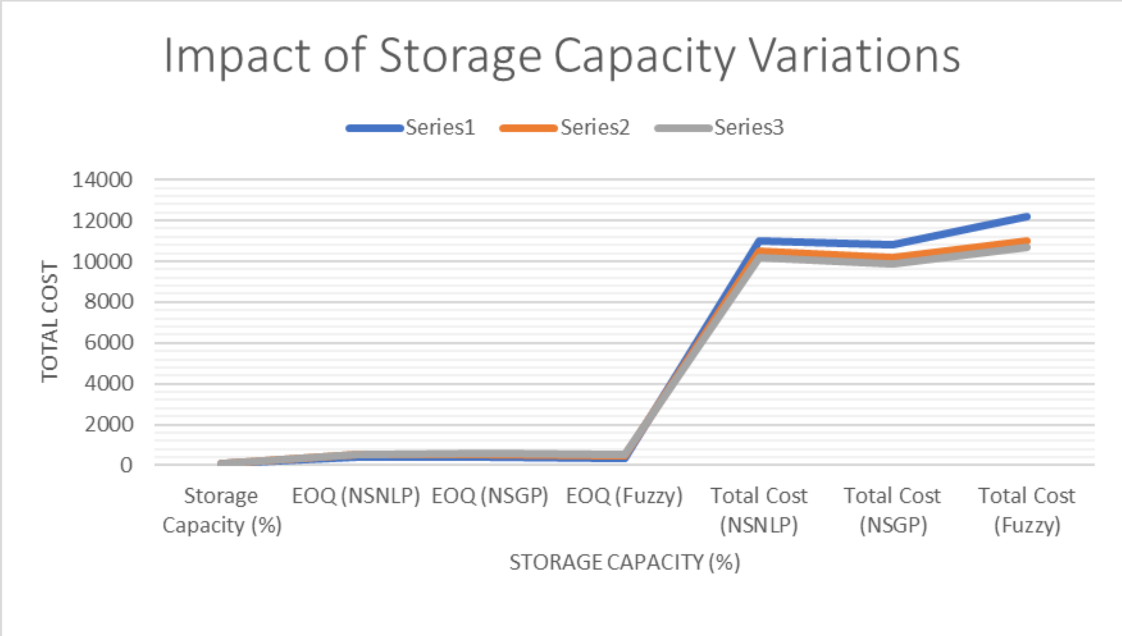

Figure 1(a)

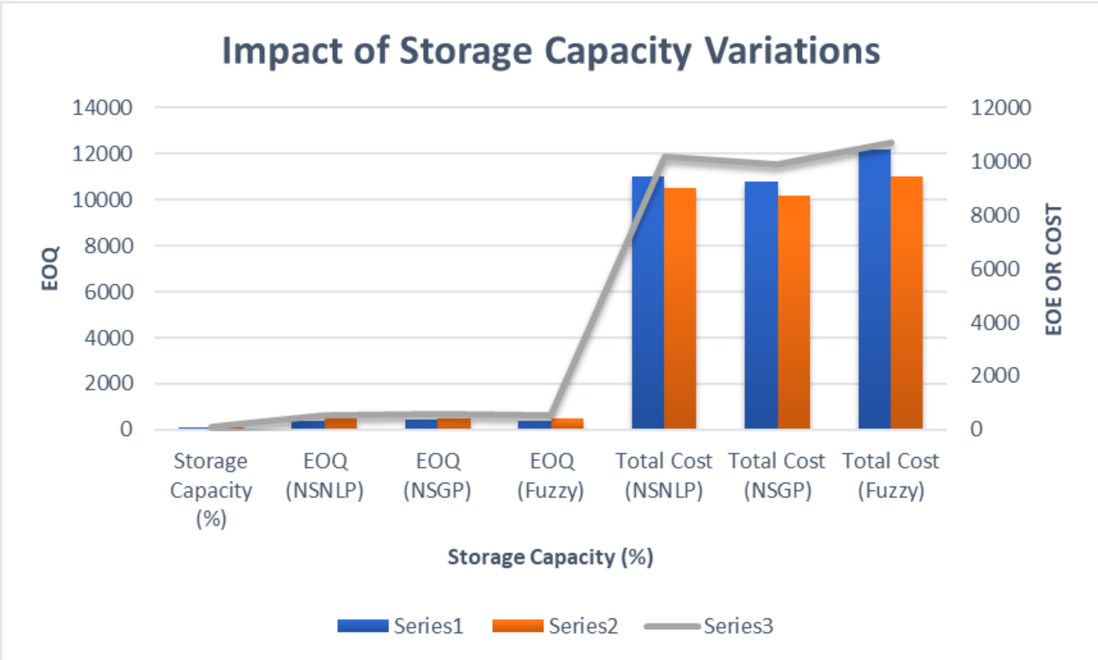

Figure 1(b)

Figure 2: Impact of cost parameter variations on optimal EOQ and total cost across all models.

Figure 2(a): Total cost vs. cost variation
Figure 2(b): EOQ vs. cost variation

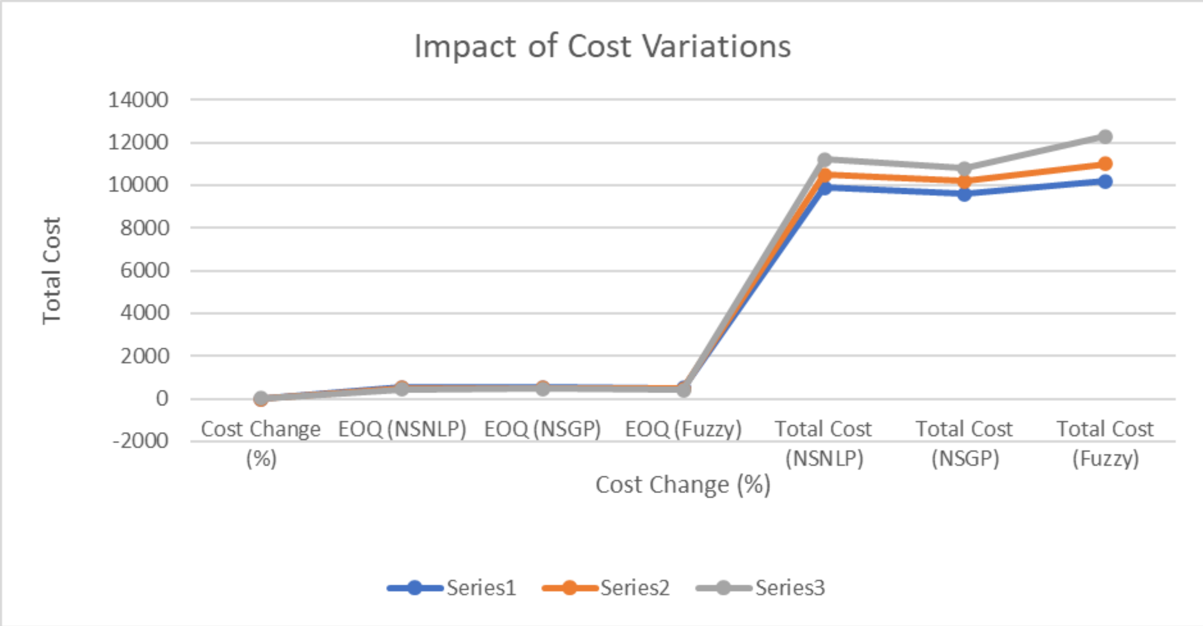

Figure 2 (a)

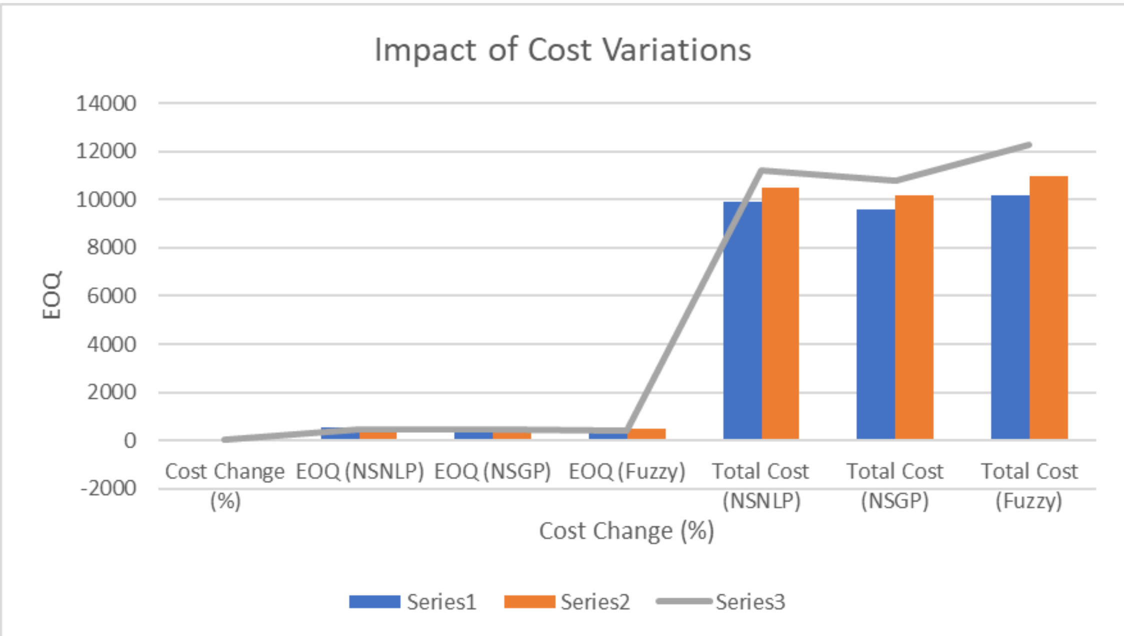

Figure 2 (b)

The figures above illustrate the impact of variations in storage capacity and cost on the EOQ and total costs for NSNLP, NSGP, and traditional fuzzy models:

5.5 Graphical Interpretation

Figure 1 depicts the effects of varying storage capacity:

- Figure 1(a): Total cost trends—NSNLP consistently yields the lowest cost, followed by NSGP and the fuzzy model.
- Figure 1(b): EOQ variation—neutrosophic models adjust more adaptively than the fuzzy model, which exhibits more linear behaviour.

Figure 2 demonstrates sensitivity to cost changes:

- Figure 2(a): Total cost increases are less pronounced in neutrosophic models, showing better cost robustness.
- Figure 2(b): EOQ response is smoother in NSNLP and NSGP compared to the more reactive fuzzy model.

6. Discussion and Analysis

The comparative analysis of optimization results (Table 2) and sensitivity analysis (Table 3) reveals that both NSNLP and NSGP outperform the traditional fuzzy EOQ model in terms of cost efficiency and adaptability under uncertain conditions. However, among the neutrosophic approaches, NSGP consistently yielded slightly better performance metrics than NSNLP, particularly in achieving lower total cost and higher storage utilization.

Reasons for NSGP's Superior Performance:

• Mathematical Structure: NSGP uses geometric programming, which efficiently handles multiplicative and nonlinear relationships common in EOQ models. This leads to more accurate global solutions than NSNLP, which may get stuck in local optima.

• Better Uncertainty Handling: NSGP integrates neutrosophic parameters more systematically within exponential forms, preserving indeterminacy across constraints and objective functions.

• Faster Convergence: In MATLAB, NSGP showed faster and more stable convergence due to its logarithmic transformation, making it computationally more efficient.

• Higher Performance: NSGP achieved lower cost ($10,200) and higher storage utilization (88%) than NSNLP, proving its superior ability to balance constraints under uncertainty.

7. Conclusion

This paper presents a novel Neutrosophic EOQ model integrating demand-dependent costs and storage constraints. The proposed methods, NSNLP and NSGP, provide robust solutions under uncertain and vague conditions. Based on numerical results and comparative analysis, the following key conclusions can be drawn:

- Neutrosophic models (NSNLP and NSGP) outperformed the traditional fuzzy EOQ model by effectively capturing uncertainty and providing more adaptable order quantities.
- NSGP achieved the best performance, yielding the lowest total cost and highest storage utilization due to its efficient handling of nonlinear constraints.
- Sensitivity analysis confirmed that neutrosophic approaches are more robust under changing storage and cost conditions.
- Graphical and tabular comparisons demonstrated that NSGP offers a reliable and computationally efficient framework for complex EOQ problems under uncertainty.

Future research could explore

1. Advanced solution algorithms for large-scale problems.
2. Applications in multi-product inventory systems.
3. Integration with stochastic demand models.

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