



Z-Connectedness and Z-Compactness in Neutrosophic Soft Topological Spaces and an MCDM Approach for Yarn Production Machine Selection Using Distance and Similarity Measures

Vijayalakshmi B¹, Madhunika S²

¹Assistant Professor, Department of Mathematics, Annamalai University, Annamalai Nagar - 608002, Tamilnadu, India; mathvijaya2006au@gmail.com

²Research Scholar, Department of Mathematics, Annamalai University, Annamalai Nagar - 608002, Tamilnadu, India; madhunika2020@gmail.com

*Correspondence: madhunika2020@gmail.com

Abstract. This study develops the concepts of Z-connectedness, Z-compactness, and Z-separated sets within neutrosophic soft topological spaces, establishing their essential properties and mutual relationships. The theoretical framework is supported by illustrative examples that clarify their significance. To demonstrate practical relevance, the notions are applied to a multi-criteria decision-making problem in yarn production machine selection, employing distance and similarity measures. The results highlight the dual impact of the proposed structures: advancing the abstract theory of neutrosophic soft topology while offering a robust tool for real-world decision-making under uncertainty.

Keywords: neutrosophic soft sets, neutrosophic soft topological spaces, Z-connected Spaces, Z-compact Spaces, Z-separated Sets.

1. Introduction

The seminal concept of fuzzy sets, introduced by Lotfi A. Zadeh in 1965 [23], established a powerful mathematical framework for modeling vagueness and ambiguity pervasive in real-world phenomena. Since then, fuzzy sets and fuzzy logic have found extensive applications across diverse domains including economics, sociology, and medical science where researchers

routinely confront imprecise or incomplete information. Classical fuzzy sets, however, rely solely on a membership function to quantify the degree of belongingness of an element, a task that often proves challenging in practice. In 1968, Chang [7] extended this idea into topology by formulating the theory of fuzzy topological spaces, thereby enriching both fuzzy mathematics and general topology.

Subsequent developments further broadened this landscape. In 1986, Atanassov [2] introduced intuitionistic fuzzy sets, characterized by independent degrees of membership and non-membership, which enabled a more refined treatment of uncertainty. This innovation was later incorporated into topology by Coker [8], who defined intuitionistic fuzzy topological spaces. Nevertheless, intuitionistic fuzzy sets are not fully equipped to accommodate contradictory or indeterminate dataa limitation frequently encountered in real-world, belief-based systems. To overcome this, Florentin Smarandache [20] advanced the theory of neutrosophic sets in 2005, offering a more general paradigm capable of handling imprecise, indeterminate, and inconsistent information simultaneously.

This theoretical progression stimulated further research. In 1999, Molodtsov [15] introduced soft set theory, a versatile mathematical tool for parameterized uncertainty, which has since demonstrated efficacy in decision-making, functional analysis, operational research, and related fields. Later, Salama and Alblowi [17] formalized neutrosophic topological spaces in 2012, while Shabir and Naz [19] defined soft topological spaces in 2011. Building upon these foundations, Maji [12] developed the concept of neutrosophic soft sets in 2013, which was subsequently refined by Deli and Broumi [9] and explored in an algebraic context by Bera and Mahapatra [3]. Parallel to this, the notion of Z -open sets, originally introduced in classical topology by El-Magharabi and Mubarki [10] in 2011, was extended into the neutrosophic topological setting by Vadivel et al. [21].

In recent years, the investigation of connectedness, compactness, and separation axioms in neutrosophic soft topological spaces has gained momentum, owing both to its theoretical depth and its practical implications [6, 11, 12]. Motivated by this line of inquiry, the present work introduces and studies the notions of Z -connectedness, Z -compactness, and Z -separated sets in neutrosophic soft topological spaces. Their fundamental properties are rigorously established and clarified through illustrative examples, thereby contributing to the continued enrichment of neutrosophic soft topological theory and its potential applications.

2. Preliminaries

This section offers a summary of essential definitions refers to *neutrosophic sets*, *soft sets*, and *neutrosophic soft sets* to ensure thorough understanding.

Definition 2.1. [18]

Vijayalakshmi B, Madhunika S, Z -Connectedness and Z -Compactness in Neutrosophic Soft Topological Spaces and an MCDM Approach for Yarn Production Machine Selection Using Distance and Similarity Measures

Let \mathbb{W} be an underlying universe. A neutrosophic set (in short, NS) D is an object having the form $D = \{\langle w, \mu_D(w), \sigma_D(w), \nu_D(w) \rangle : w \in \mathbb{W}\}$ where $\mu_D \rightarrow [0, 1]$ denote the degree of membership function, $\sigma_D \rightarrow [0, 1]$ denote the degree of indeterminacy function and $\nu_D \rightarrow [0, 1]$ denote the degree of non-membership function respectively of each element $w \in \mathbb{W}$ to the set D and $0 \leq \mu_D(w) + \sigma_D(w) + \nu_D(w) \leq 3$ for each $w \in \mathbb{W}$.

Definition 2.2. [15]

Assume that \mathbb{W} is the underlying universe & let ϱ is a parameter set. Let $\mathcal{P}(\mathbb{W})$ represent the collection of all neutrosophic sets within \mathbb{W} . A pair (D, ϱ) is known as the soft set (shortly, SS) over \mathbb{W} , where D is a mapping $D : \varrho \rightarrow \mathcal{P}(\mathbb{W})$. In other terms, a soft set can be viewed as a collection of subsets of the set \mathbb{W} , each associated with a specific parameter.

Definition 2.3. [9]

Assume that \mathbb{W} is the underlying universe & let ϱ is a parameter set. Let $\mathcal{P}(\mathbb{W})$ represent the collection of all neutrosophic sets within \mathbb{W} . Then a neutrosophic soft set (S, ϱ) over \mathbb{W} (shortly, NSS) is characterized by $(S, \varrho) = \{(\varphi, \langle \varepsilon, \mu_{S(\varphi)}(\varepsilon), \sigma_{S(\varphi)}(\varepsilon), \nu_{S(\varphi)}(\varepsilon) \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}$, where

$\mu_{S(\varphi)}(\varepsilon), \sigma_{S(\varphi)}(\varepsilon), \nu_{S(\varphi)}(\varepsilon) \in [0, 1]$ are respectively called the *degree of membership function*, *the degree of indeterminacy function* and *the degree of non-membership function* of $S(\varphi)$. As the maximum value for each of μ, σ, ν is 1.

The inequality $0 \leq \mu_{S(\varphi)}(\varepsilon) + \sigma_{S(\varphi)}(\varepsilon) + \nu_{S(\varphi)}(\varepsilon) \leq 3$ naturally holds.

Definition 2.4. [[12], [4]]

Assume that \mathbb{W} is an underlying universe & NS sets (S, ϱ) & (D, ϱ) are in the form

$$(S, \varrho) = \{(\varphi, \langle \varepsilon, \mu_{S(\varphi)}(\varepsilon), \sigma_{S(\varphi)}(\varepsilon), \nu_{S(\varphi)}(\varepsilon) \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\} \text{ \&}$$

$$(D, \varrho) = \{(\varphi, \langle \varepsilon, \mu_{D(\varphi)}(\varepsilon), \sigma_{D(\varphi)}(\varepsilon), \nu_{D(\varphi)}(\varepsilon) \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}, \text{ then}$$

$$(1) 0_{(\mathbb{W}, \varrho)} = \{(\varphi, \langle \varepsilon, 0, 0, 1 \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\} \text{ and } 1_{(\mathbb{W}, \varrho)} = \{(\varphi, \langle \varepsilon, 1, 1, 0 \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}.$$

$$(2) (S, \varrho) \subseteq (D, \varrho) \text{ iff } \mu_{S(\varphi)}(\varepsilon) \leq \mu_{D(\varphi)}(\varepsilon), \sigma_{S(\varphi)}(\varepsilon) \leq \sigma_{D(\varphi)}(\varepsilon) \text{ and } \nu_{S(\varphi)}(\varepsilon) \geq \nu_{D(\varphi)}(\varepsilon) : \varepsilon \in \mathbb{W} : \varphi \in \varrho.$$

$$(3) (S, \varrho) = (D, \varrho) \text{ iff } (S, \varrho) \subseteq (D, \varrho) \text{ and } (D, \varrho) \subseteq (S, \varrho).$$

$$(4) (S, \varrho)^c = \{(\varphi, \langle \varepsilon, \nu_{S(\varphi)}(\varepsilon), 1 - \sigma_{S(\varphi)}(\varepsilon), \mu_{S(\varphi)}(\varepsilon) \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}.$$

$$(5) (S, \varrho) \cup (D, \varrho) = \{(\varphi, \langle \varepsilon, \max(\mu_{S(\varphi)}(\varepsilon), \mu_{D(\varphi)}(\varepsilon)), \max(\sigma_{S(\varphi)}(\varepsilon), \sigma_{D(\varphi)}(\varepsilon)), \min(\nu_{S(\varphi)}(\varepsilon), \nu_{D(\varphi)}(\varepsilon)) \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}.$$

$$(6) (S, \varrho) \cap (D, \varrho) = \{(\varphi, \langle \varepsilon, \min(\mu_{S(\varphi)}(\varepsilon), \mu_{D(\varphi)}(\varepsilon)), \min(\sigma_{S(\varphi)}(\varepsilon), \sigma_{D(\varphi)}(\varepsilon)), \max(\nu_{S(\varphi)}(\varepsilon), \nu_{D(\varphi)}(\varepsilon)) \rangle : \varepsilon \in \mathbb{W}) : \varphi \in \varrho\}.$$

Definition 2.5. [4]

A neutrosophic soft topology (in short, NSt) on an underlying universe \mathbb{W} is a collection of τ of NS subsets (S, ϱ) of \mathbb{W} where ϱ be the parameters set, satisfying

$$(1) 0_{(\mathbb{W}, \varrho)}, 1_{(\mathbb{W}, \varrho)} \in \tau.$$

- (2) $[(S, \varrho) \cap (D, \varrho)] \in \tau$ for any $(S, \varrho), (D, \varrho) \in \tau$.
- (3) $\bigcup_{k \in K} (S, \varrho)_k \in \tau$, for every $(S, \varrho_k) : k \in K \subseteq \tau$.

Then $(\mathbb{W}, \tau, \varrho)$ is known as *neutrosophic soft topological space* (shortly, NSTs) and the elements of τ are known as *neutrosophic soft open sets* (shortly, NSOS) in \mathbb{W} . A NSS (S, ϱ) is called the *neutrosophic soft closed set* (in short, NSCS) if its complement $(S, \varrho)^c$ is NSOS.

Definition 2.6. [4]

Let $(\mathbb{W}, \tau, \varrho)$ act as a NSTs on \mathbb{W} & let (S, ϱ) is a NSS on \mathbb{W} . The *neutrosophic soft interior* of (S, ϱ) (in brief, $\text{NSint}(S, \varrho)$) and the *neutrosophic soft closure* of (S, ϱ) (in brief, $\text{NScl}(S, \varrho)$) are represented as

$$\text{NSint}(S, \varrho) = \bigcup \{(D, \varrho) : (D, \varrho) \subseteq (S, \varrho) \text{ and } (D, \varrho) \text{ is a NSOS in } \mathbb{W}\}.$$

$$\text{NScl}(S, \varrho) = \bigcap \{(D, \varrho) : (D, \varrho) \supseteq (S, \varrho) \text{ and } (D, \varrho) \text{ is a NSCS in } \mathbb{W}\}.$$

Definition 2.7. [4]

Suppose $(\mathbb{W}, \tau, \varrho)$ act as a NSTs on \mathbb{W} & let (S, ϱ) is a NSS on \mathbb{W} . Then (S, ϱ) is called the NS

- (i) regular-open set (in short, NSROS) if $(S, \varrho) = \text{NSint}(\text{NScl}(S, \varrho))$.
- (ii) pre-open set (briefly, NSPOS) if $(S, \varrho) \subseteq \text{NSint}(\text{NScl}(S, \varrho))$.
- (iii) semi-open set (briefly, NSSOS) if $(S, \varrho) \subseteq \text{NScl}(\text{NSint}(S, \varrho))$.
- (iv) α -open set (shortly, NS α OS) if $(S, \varrho) \subseteq \text{NSint}(\text{NScl}(\text{NSint}(S, \varrho)))$.
- (v) β -open set (shortly, NS β OS) if $(S, \varrho) \subseteq \text{NScl}(\text{NSint}(\text{NScl}(S, \varrho)))$.

The complement of a NSROS (resp. NSPOS, NSSOS, NS α OS, NS β OS) is called the neutrosophic soft regular (resp. pre, semi, α , β) closed set (shortly, NSRCS (resp. NSPCS, NSSCS, NS α CS, NS β CS)) in \mathbb{W} .

The family of all NSROS (resp. NSRCS, NSPOS, NSPCS, NSSOS, NSSCS, NS α OS, NS α CS, NS β OS, NS β CS) of \mathbb{W} is represented by NSROS(\mathbb{W}) (resp. NSRCS(\mathbb{W}), NSPOS(\mathbb{W}), NSPCS(\mathbb{W}), NSSOS(\mathbb{W}), NSSCS(\mathbb{W}), NS α OS(\mathbb{W}), NS α CS(\mathbb{W}), NS β OS(\mathbb{W}), NS β CS(\mathbb{W})).

Definition 2.8. [1]

Let (D, ϱ) be a NSTs. Then

- (i) neutrosophic soft δ -interior of (D, ϱ) (in short, $\text{NS}\delta\text{int}(D, \varrho)$) is defined by

$$\text{NS}\delta\text{int}(D, \varrho) = \bigcup \{(S, \varrho) : (S, \varrho) \subseteq (D, \varrho) \text{ and } (S, \varrho) \text{ is a NSROS in } \mathbb{W}\}$$
- (ii) neutrosophic soft δ -closure of (D, ϱ) (in short, $\text{NS}\delta\text{cl}(D, \varrho)$) is defined by

$$\text{NS}\delta\text{cl}(D, \varrho) = \bigcap \{(S, \varrho) : (S, \varrho) \supseteq (D, \varrho) \text{ \& } (S, \varrho) \text{ is a NSRCS in } \mathbb{W}\}$$

Definition 2.9. [1]

Vijayalakshmi B, Madhunika S, Z-Connectedness and Z-Compactness in Neutrosophic Soft Topological Spaces and an MCDM Approach for Yarn Production Machine Selection Using Distance and Similarity Measures

A NSS (D, ϱ) is referred as the neutrosophic soft δ -open set (shortly, NS δ OS) if $(D, \varrho) = \text{NS}\delta\text{int}(D, \varrho)$.

The complement of NS δ OS is called NS δ CS.

Definition 2.10. [16]

A NSS (D, ϱ) is called the neutrosophic soft

- (1) δ -semiopen set (in short, NS δ SOS) if $(D, \varrho) \subseteq \text{NScl}(\text{NS}\delta\text{int}(D, \varrho))$.
- (2) e-open set (briefly, NSeOS) if $(D, \varrho) \subseteq \text{NScl}(\text{NS}\delta\text{int}(D, \varrho)) \cup \text{NSint}(\text{NS}\delta\text{cl}(D, \varrho))$.

The complement of NS δ SOS and NSeOS is called NS δ SCS and NSeCS.

Throughout this paper, Let $(\mathbb{W}, \tau, \varrho)$ be any NSTs. Let (S, ϱ) & (D, ϱ) be a neutrosophic soft sets in NSTs.

3. Neutrosophic soft Z connected spaces

Definition 3.1. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs is neutrosophic soft (resp. δ , δ S, P & Z) disconnected (briefly, NSDCon (resp. NS δ DCon, NS δ SDCon, NSPDCon and NSZDCon)) if there exists NSOS (resp. NS δ OS, NS δ SOS, NSPOS and NSZOS) $(P, \varrho), (B, \varrho)$ in $\mathbb{W}, (P, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (B, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ such that $(P, \varrho) \cup (B, \varrho) = 1_{(\mathbb{W}, \varrho)}$ and $(P, \varrho) \cap (B, \varrho) = 0_{(\mathbb{W}, \varrho)}$. If \mathbb{W} is not NSDCon (resp. NS δ DCon, NS δ SDCon, NSPDCon and NSZDCon) then it is said to be neutrosophic soft (resp. δ , δ S, P & Z) connected (briefly, NSCon (resp. NS δ Con, NS δ SCon, NSPCon & NSZCon)).

Example 3.1. Let $\mathbb{W} = \{w_1, w_2, w_3\}, \varrho = \{e_1, e_2\}$ and NS sets $(S_1, \varrho), (S_2, \varrho), (S_3, \varrho), (S_4, \varrho)$ and (S_5, ϱ) in \mathbb{W} are defined as

$$(S_1, e_1) = \langle \langle (\frac{\mu_{w_1}}{0.4}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}), (\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.4}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}) \rangle \rangle$$

$$(S_1, e_2) = \langle \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.6}), (\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}) \rangle \rangle$$

$$(S_2, e_1) = \langle \langle (\frac{\mu_{w_1}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.6}), (\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.5}), (\frac{\mu_{w_3}}{0.6}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}) \rangle \rangle$$

$$(S_2, e_2) = \langle \langle (\frac{\mu_{w_1}}{0.4}, \frac{\sigma_{w_1}}{0.6}, \frac{\nu_{w_1}}{0.6}), (\frac{\mu_{w_2}}{0.3}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}), (\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.7}, \frac{\nu_{w_3}}{0.4}) \rangle \rangle$$

$$(S_3, e_1) = \langle \langle (\frac{\mu_{w_1}}{0.3}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.3}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.3}, \frac{\nu_{w_3}}{0.8}) \rangle \rangle$$

$$(S_3, e_2) = \langle \langle (\frac{\mu_{w_1}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.1}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}) \rangle \rangle$$

$$(S_4, e_1) = \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.2}, \frac{\nu_{w_3}}{0.7}) \rangle$$

$$(S_4, e_2) = \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.4}, \frac{\nu_{w_3}}{0.9}) \rangle$$

$$(S_5, e_1) = \langle (\frac{\mu_{w_1}}{0.5}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}), (\frac{\mu_{w_2}}{0.6}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.6}), (\frac{\mu_{w_3}}{0.5}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.4}) \rangle$$

$$(S_5, e_2) = \langle (\frac{\mu_{w_1}}{0.3}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.2}), (\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.6}), (\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.4}) \rangle$$

Then, we have

$\tau = \{0_{(\mathbb{W}, \varrho)}, 1_{(\mathbb{W}, \varrho)}, (S_1, \varrho), (S_2, \varrho), (S_3, \varrho)\}$ is a NSTs in \mathbb{W} , then

- (i) Let (S_3, ϱ) and (S_4, ϱ) are NSZOS(resp. NSPOS). Then \mathbb{W} is NSZCon(resp NSPCon).
- (ii) Let (S_2, ϱ) and (S_5, ϱ) are NS δ SOS. Then \mathbb{W} is NS δ SCon.
- (iii) Let (S_1, ϱ) and (S_2, ϱ) are NSOS. Then \mathbb{W} is NSCon.
- (iv) Let (S_1, ϱ) and $1_{(\mathbb{W}, \varrho)}$ are NSOS. Then \mathbb{W} is NS δ Con.

Example 3.2. In example 3.1 Let

$$(S_6, e_1) = \langle (\frac{\mu_{w_1}}{0}, \frac{\sigma_{w_1}}{0}, \frac{\nu_{w_1}}{1}), (\frac{\mu_{w_2}}{0}, \frac{\sigma_{w_2}}{0}, \frac{\nu_{w_2}}{1}), (\frac{\mu_{w_3}}{1}, \frac{\sigma_{w_3}}{0}, \frac{\nu_{w_3}}{0}) \rangle$$

$$(S_6, e_2) = \langle (\frac{\mu_{w_1}}{0}, \frac{\sigma_{w_1}}{1}, \frac{\nu_{w_1}}{1}), (\frac{\mu_{w_2}}{1}, \frac{\sigma_{w_2}}{0}, \frac{\nu_{w_2}}{0}), (\frac{\mu_{w_3}}{0}, \frac{\sigma_{w_3}}{0}, \frac{\nu_{w_3}}{1}) \rangle$$

$$(S_7, e_1) = \langle (\frac{\mu_{w_1}}{1}, \frac{\sigma_{w_1}}{1}, \frac{\nu_{w_1}}{0}), (\frac{\mu_{w_2}}{1}, \frac{\sigma_{w_2}}{1}, \frac{\nu_{w_2}}{0}), (\frac{\mu_{w_3}}{0}, \frac{\sigma_{w_3}}{1}, \frac{\nu_{w_3}}{1}) \rangle$$

$$(S_7, e_2) = \langle (\frac{\mu_{w_1}}{1}, \frac{\sigma_{w_1}}{0}, \frac{\nu_{w_1}}{0}), (\frac{\mu_{w_2}}{0}, \frac{\sigma_{w_2}}{1}, \frac{\nu_{w_2}}{1}), (\frac{\mu_{w_3}}{1}, \frac{\sigma_{w_3}}{1}, \frac{\nu_{w_3}}{0}) \rangle$$

Then (S_6, ϱ) and (S_7, ϱ) are NSZOS'S. Then \mathbb{W} is NSZDCon.

Definition 3.2. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. Let (T, ϱ) be a NSS of \mathbb{W} .

- (a) If there exists NSOS (resp. NS δ OS, NS δ SOS, NSPOS and NSZOS) (G, ϱ) and (S, ϱ) in \mathbb{W} satisfying the following properties, then (T, ϱ) is called neutrosophic soft (resp. δ , δ S, P & Z) C_i -disconnected (briefly, NSC_iDCon (resp. $NS\delta C_iDCon$, $NS\delta SC_iDCon$, $NSPC_iDCon$ and $NSZC_iDCon$)) ($i = 1, 2, 3, 4$)

$$C_1 : (T, \varrho) \subseteq (G, \varrho) \cup (S, \varrho), (G, \varrho) \cap (S, \varrho) \subseteq (T, \varrho)^c, (T, \varrho) \cap (G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (T, \varrho) \cap (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}.$$

$$C_2 : (T, \varrho) \subseteq (G, \varrho) \cup (S, \varrho), (T, \varrho) \cap (G, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}, (T, \varrho) \cap (G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (T, \varrho) \cap (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}.$$

$$C_3 : (T, \varrho) \subseteq (G, \varrho) \cup (S, \varrho), (G, \varrho) \cap (S, \varrho) \subseteq (T, \varrho)^c, (G, \varrho) \not\subseteq (T, \varrho)^c, (S, \varrho) \not\subseteq (T, \varrho)^c.$$

$$C_4 : (T, \varrho) \subseteq (G, \varrho) \cup (S, \varrho), (T, \varrho) \cap (G, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}, (G, \varrho) \not\subseteq (T, \varrho)^c, (S, \varrho) \not\subseteq (T, \varrho)^c.$$

(b) (T, ϱ) is said to be neutrosophic soft (resp. $\delta, \delta S, P$ & Z) C_i -connected (briefly, NSC_iCon (resp. $NS\delta C_iCon, NS\delta SC_iCon, NSPC_iCon$ and $NSZC_iCon$)), ($i = 1, 2, 3, 4$) if (M, ϱ) is not NSC_iDCon (resp. $NS\delta C_iDCon, NS\delta SC_iDCon, NSPC_iDCon$ and $NSZC_iDCon$), ($i = 1, 2, 3, 4$). Obviously, we can obtain the following implications between several types of NSC_iCon (resp. $NS\delta C_iCon, NS\delta SC_iCon, NSPC_iCon$ and $NSZC_iCon$), ($i = 1, 2, 3, 4$).

- (1) NSC_1Con (resp. $NS\delta C_1Con, NS\delta SC_1Con, NSPC_1Con$ and $NSZC_1Con$) \Rightarrow NSC_2Con (resp. $NS\delta C_2Con, NS\delta SC_2Con, NSPC_2Con$ and $NSZC_2Con$).
- (2) NSC_1Con (resp. $NS\delta C_1Con, NS\delta SC_1Con, NSPC_1Con$ and $NSZC_1Con$) \Rightarrow NSC_3Con (resp. $NS\delta C_3Con, NS\delta SC_3Con, NSPC_3Con$ and $NSZC_3Con$).
- (3) NSC_3Con (resp. $NS\delta C_3Con, NS\delta SC_3Con, NSPC_3Con$ and $NSZC_3Con$) \Rightarrow NSC_4Con (resp. $NS\delta C_4Con, NS\delta SC_4Con, NSPC_4Con$ and $NSZC_4Con$).
- (4) NSC_1Con (resp. $NS\delta C_1Con, NS\delta SC_1Con, NSPC_1Con$ and $NSZC_1Con$) \Rightarrow NSC_4Con (resp. $NS\delta C_4Con, NS\delta SC_4Con, NSPC_4Con$ and $NSZC_4Con$).

Example 3.3. In example 3.1, Let

$$(D, e_1) = \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}) \rangle$$

$$(D, e_2) = \langle (\frac{\mu_{w_1}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.9}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}) \rangle$$

If (S_1, ϱ) and (S_3, ϱ) are NSZOS'S, then (D, ϱ) is

- (i) $NSZC_2Con$ but not $NSZC_1Con$.
- (ii) $NSZC_3Con$ but not $NSZC_1Con$.
- (iii) $NSZC_4Con$ but not $NSZC_1Con$.

Example 3.4. Let $\mathbb{W} = \{w_1, w_2, w_3\}, \varrho = \{e_1, e_2\}$ and NS sets $(M_1, \varrho), (M_2, \varrho)$ and (M_3, ϱ) in \mathbb{W} are defined as

$$(M_1, e_1) = \langle (\frac{\mu_{w_1}}{1}, \frac{\sigma_{w_1}}{0}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0}, \frac{\nu_{w_2}}{0.6}), (\frac{\mu_{w_3}}{0}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{1}) \rangle$$

$$(M_1, e_2) = \langle (\frac{\mu_{w_1}}{0.3}, \frac{\sigma_{w_1}}{0}, \frac{\nu_{w_1}}{0.5}), (\frac{\mu_{w_2}}{0}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}), (\frac{\mu_{w_3}}{0}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.7}) \rangle$$

$$(M_2, e_1) = \langle (\frac{\mu_{w_1}}{0}, \frac{\sigma_{w_1}}{1}, \frac{\nu_{w_1}}{1}), (\frac{\mu_{w_2}}{0}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{1}), (\frac{\mu_{w_3}}{1}, \frac{\sigma_{w_3}}{0}, \frac{\nu_{w_3}}{0.6}) \rangle$$

$$(M_2, e_2) = \langle (\frac{\mu_{w_1}}{0}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{1}), (\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0}, \frac{\nu_{w_2}}{1}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0}, \frac{\nu_{w_3}}{1}) \rangle$$

$$(M_3, e_1) = \langle (\frac{\mu_{w_1}}{1}, \frac{\sigma_{w_1}}{1}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.6}), (\frac{\mu_{w_3}}{1}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.6}) \rangle$$

$$(M_3, e_2) = \langle (\frac{\mu_{w_1}}{0.3}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.5}), (\frac{\mu_{w_2}}{0.5}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}) \rangle$$

$$\text{Let, } (D, e_1) = \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.9}) \rangle$$

$$(D, e_2) = \langle (\frac{\mu_{w_1}}{0.1}, \frac{\sigma_{w_1}}{0.3}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.9}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.5}, \frac{\nu_{w_3}}{0.8}) \rangle$$

If (M_1, ϱ) and (M_2, ϱ) are NSZOS's, then (D, ϱ) is NSZC₄Con but not NSZC₃Con.

Definition 3.3. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs is neutrosophic soft (resp. $\delta, \delta S, P$ & Z) C_5 -disconnected (briefly, NSC_5DCon (resp. $NS\delta C_5DCon, NS\delta SC_5DCon, NSPC_5DCon$ and $NSZC_5DCon$)) if there exists NSS (S, ϱ) in \mathbb{W} which is both NSOS (resp. $NS\delta OS, NS\delta SOS, NSPOS$ and NSZOS) and NSCS (resp. $NS\delta CS, NS\delta SCS, NSPCS$ and NSZCS) in \mathbb{W} , such that $(S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (S, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. If \mathbb{W} is not NSC_5DCon (resp. $NS\delta C_5DCon, NS\delta SC_5DCon, NSPC_5DCon$ and $NSZC_5DCon$) then it is said to be neutrosophic soft (resp. $\delta, \delta S, P$ & Z) C_5 -connected (briefly, NSC_5Con (resp. $NS\delta C_5Con, NS\delta SC_5Con, NSPC_5Con$ and $NSZC_5Con$)).

Example 3.5. In example 3.1, Let

- (1) (S_1, ϱ) is NSZC₅Dcon (resp. NS δ SC₅DCon).
- (2) $(P, e_1) = \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.5}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.4}, \frac{\sigma_{w_2}}{0.4}, \frac{\nu_{w_2}}{0.8}), (\frac{\mu_{w_3}}{0.3}, \frac{\sigma_{w_3}}{0.2}, \frac{\nu_{w_3}}{0.7}) \rangle$

$$(P, e_2) = \langle (\frac{\mu_{w_1}}{0.2}, \frac{\sigma_{w_1}}{0.4}, \frac{\nu_{w_1}}{0.7}), (\frac{\mu_{w_2}}{0.2}, \frac{\sigma_{w_2}}{0.5}, \frac{\nu_{w_2}}{0.7}), (\frac{\mu_{w_3}}{0.2}, \frac{\sigma_{w_3}}{0.4}, \frac{\nu_{w_3}}{0.9}) \rangle \text{ is NSPC}_5\text{DCon.}$$

Theorem 3.1. $NSZC_5DCon$ (resp. $NSC_5DCon, NS\delta C_5DCon, NS\delta SC_5DCon$ and $NSPC_5DCon$) -ness implies $NSZCon$ (resp. $NSCon, NS\delta Con, NS\delta SCon$ and NSPCon)-ness.

Proof

Suppose that there exists non-empty NSZOS's (S, ϱ) and (G, ϱ) such that $(S, \varrho) \cup (G, \varrho) = 1_{(\mathbb{W}, \varrho)}$ and $(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$. In other words, $(G, \varrho)^c = (S, \varrho)$. Hence, (S, ϱ) is NSZCS which implies \mathbb{W} is $NSZC_5DCon$.

Other cases are similar.

But the converse may not be true as shown by the following example.

Example 3.6. In example 3.1, let (S_1, ϱ) and (S_2, ϱ) are NSZOS's. Then \mathbb{W} is NSZCon but not NSZC₅DCon.

Theorem 3.2. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSZ-irr (resp. NS-irr, $NS\delta - irr, NS\delta S - irr$ and NSP-irr) surjection, \mathbb{W} be a NSZCon (resp. NSCon, $NS\delta Con, NS\delta SCon$ and NSPCon). Then \mathbb{T} is NSZCon (resp. NSCon, $NS\delta Con, NS\delta SCon$ and $NSPCCon$).

Proof. Assume that \mathbb{T} is not NSZCon, then there exists nonempty NSZOS's $(A, \varrho) \& (C, \varrho)$ in $\mathbb{T} \ni (A, \varrho) \cup (C, \varrho) = 1_{(\mathbb{W}, \varrho)} \& (A, \varrho) \cap (C, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Since \mathcal{G} is NSZ-irr mapping, $(S, \varrho) = \mathcal{G}^{-1}(A, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (G, \varrho) = \mathcal{G}^{-1}(C, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$, which are NSZOS's in \mathbb{W} and $\mathcal{G}^{-1}(A, \varrho) \cup \mathcal{G}^{-1}(C, \varrho) = \mathcal{G}^{-1}(1_{(\mathbb{W}, \varrho)}) = 1_{(\mathbb{W}, \varrho)}$, which implies $(S, \varrho) \cup (G, \varrho) = 1_{(\mathbb{W}, \varrho)}$. Also $\mathcal{G}^{-1}(A, \varrho) \cap \mathcal{G}^{-1}(C, \varrho) = \mathcal{G}^{-1}(0_{(\mathbb{W}, \varrho)}) = 0_{(\mathbb{W}, \varrho)}$, which implies $(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Thus \mathbb{W} is NSZDCon, which is a contradiction to our hypothesis. Hence \mathbb{W} is NSZCon.

The same method can be used to prove the theorems other cases.

Theorem 3.3. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts is $NSZC_5Con$ (resp. $NSC_5Con, NS\delta C_5Con, N\delta SC_5Con, NSP C_5Con$) if and only if there exists no nonempty NSZOS (resp. NSOS, $NS\delta OS, NS\delta SOS$ and NSPOS) (G, ϱ) and (S, ϱ) in $\mathbb{W} \ni (G, \varrho) = (S, \varrho)^c$.

Proof Suppose that (G, ϱ) and (S, ϱ) are NSZOS's in \mathbb{W} such that $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (S, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and $(G, \varrho) = (S, \varrho)^c$. Since $(G, \varrho) = (S, \varrho)^c, (S, \varrho)^c$ is a NSZOS's and (S, ϱ) is NSZCS and $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ implies $(S, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. But this is a contradiction to the fact that \mathbb{W} is $NSZC_5Con$.

Conversely, let (G, ϱ) be a both NSZOS and (S, ϱ) is NSZCS in \mathbb{W} such that $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (G, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Now take $(G, \varrho)^c = (S, \varrho)$ is a NSZOS and $(G, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$ which implies $(G, \varrho)^c = (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ which is a contradiction. Hence \mathbb{W} is $NSZC_5Con$.

The same method can be used to prove the theorems other cases.

Theorem 3.4. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts is NSZCon (resp. NSCon, $NS\delta Con, NS\delta SCon, NSP-Con$) iff there exists no non-zero NSZOS (resp. NSOS, $NS\delta OS, NS\delta SOS, NSPOS$) (G, ϱ) and (S, ϱ) in \mathbb{W} , such that $(G, \varrho) = (S, \varrho)^c$.

Proof Necessity: Let (G, ϱ) and (S, ϱ) be two NSZOS's in \mathbb{W} such that $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ and $(G, \varrho) = (S, \varrho)^c$. Therefore $(S, \varrho)^c$ is a NSZCS. Since $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (S, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. This implies (S, ϱ) is a proper NSS which is both NSZOS and NSZCS in \mathbb{W} . Hence \mathbb{W} is not a NSZCon. But this is a contradiction to our hypothesis. Thus, there exist no non-zero NSZOS's (G, ϱ) and (S, ϱ) in \mathbb{W} , such that $(G, \varrho) = (S, \varrho)^c$.

Sufficiency: Let (G, ϱ) be both NSZOS and NSZCS, \mathbb{W} such that $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (G, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Now let $(S, \varrho) = (G, \varrho)^c$. Then (S, ϱ) is a NSZOS and $(S, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. This implies $(G, \varrho)^c = (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$, which is a contradiction to our hypothesis. Therefore \mathbb{W} is NSZCon.

The same method can be used to prove the theorems other cases.

Theorem 3.5. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts is NSZCon (resp. NSCon, $NS\delta Con, NS\delta SCon$ and NSPCon) iff there exists no non-zero NSS (B, ϱ) and (S, ϱ) in \mathbb{W} such that $(B, \varrho) = (S, \varrho)^c, (S, \varrho) = (NSZcl(B, \varrho))^c$ (resp. $(S, \varrho) = (NScl(B, \varrho))^c, (S, \varrho) =$

$(NS\delta cl(B, \varrho))^c, (S, \varrho) = (NS\delta Scl(B, \varrho))^c$ and $(S, \varrho) = (NSPcl(B, \varrho))^c$ and $(B, \varrho) = (NSZcl(S, \varrho))^c$ (resp. $(B, \varrho) = (NScl(S, \varrho))^c, (B, \varrho) = (NS\delta cl(S, \varrho))^c, (B, \varrho) = (NS\delta Scl(S, \varrho))^c$ and $(B, \varrho) = (NSPcl(S, \varrho))^c$).

Proof Necessity: Let (B, ϱ) and (S, ϱ) be two NSS in \mathbb{W} such that $(B, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ and $(B, \varrho) = (S, \varrho)^c, (S, \varrho) = (NSZcl(B, \varrho))^c$ and $(B, \varrho) = (NSZcl(S, \varrho))^c$. Since $(NSZcl(B, \varrho))^c$ and $(NSZcl(S, \varrho))^c$ are NSZOS's in \mathbb{W} , (B, ϱ) and (S, ϱ) are NSZOS in \mathbb{W} .

This implies \mathbb{W} is not a NSZCon, which is a contradiction. Therefore, there exists no non-zero NSZOS (B, ϱ) and (S, ϱ) in \mathbb{W} . Such that $(B, \varrho) = (S, \varrho)^c, (S, \varrho) = (NSZcl(B, \varrho))^c$ & $(B, \varrho) = (NSZcl(S, \varrho))^c$.

Sufficiency: Let (B, ϱ) be both NSZOS and NSZCS in \mathbb{W} such that $(B, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (B, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Now by taking $(S, \varrho) = (B, \varrho)^c$, We obtain a contradiction to our hypothesis. Hence, \mathbb{W} is NSZCon space.

The same method can be used to prove the theorems other cases.

Definition 3.4. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts is neutrosophic soft (resp. $\delta, \delta S, P\&Z$) strongly connected (briefly, $NSstCon$ (resp $NS\delta stCon, NS\delta SstCon, NSPstCon$ and NSZstCon) (S, ϱ) and (G, ϱ) in \mathbb{W} such taht $\mu_{(S, \varrho)} + \mu_{(G, \varrho)} \geq 1_{(\mathbb{W}, \varrho)}$.

In otherwords, a NSts \mathbb{W} is NSstCon (resp. $NS\delta stCon, NS\delta SstCon, NSPstCon$ and NSZstCon) if there exist no non empty NSCS(resp $NS\delta CS, NS\delta SCS, NSPCS$ and NSZCS) (S, ϱ) and (G, ϱ) in \mathbb{W} such that $(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$.

Theorem 3.6. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts is NSZStCon (resp. NSstCon, $NS\delta stCon, NS\delta SstCon$ and NSPstCon), if there exists non empty NSZOS (resp. NSOS, $NS\delta OS, NS\delta SOS, NSPOS$) (S, ϱ) and (G, ϱ) in \mathbb{W} , $(S, \varrho) \neq 1_{(\mathbb{W}, \varrho)} \neq (G, \varrho)$ such that $\mu_{(S, \varrho)} + \mu_{(G, \varrho)} \geq 1_{(\mathbb{W}, \varrho)}$.

Proof Let (S, ϱ) and (G, ϱ) be NSZOS's in \mathbb{W} such that $(S, \varrho) \neq 1_{(\mathbb{W}, \varrho)} \neq (G, \varrho)$ and $\mu_{(S, \varrho)} + \mu_{(G, \varrho)} \geq 1_{(\mathbb{W}, \varrho)}$. If we take $(P, \varrho) = (S, \varrho)^c$ and $(F, \varrho) = (G, \varrho)^c$, then (P, ϱ) and (F, ϱ) become NSZCS's in \mathbb{W} and $(P, \varrho) \neq 0_{(\mathbb{W}, \varrho)} \neq (F, \varrho), \mu_{(P, \varrho)} + \mu_{(F, \varrho)} \leq 1_{(\mathbb{W}, \varrho)}$, a contradiction.

Conversely, use a similar technique as above.

Other cases are similar.

Theorem 3.7. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSZ-irr (resp. NS-irr, $NS\delta - irr, NS\delta S - irr$ and NSP-irr) surjection, \mathbb{W} be a NSZstCon (resp. NSstCon, $NS\delta stCon, NS\delta SstCon$ and, NSPstCon). Then \mathbb{T} is also NSZStCon (resp. NSstCon, $NS\delta stCon, NS\delta SstCon$ and, NSPstCon).

Proof Assume that \mathbb{T} is not NSZstCon, then there exists nonempty NSZCS's (G, ϱ) and (S, ϱ) in \mathbb{T} such that $(G, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ and $(G, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Since \mathcal{G} is

NSZ-irr mapping, $(K, \varrho) = \mathcal{G}^{-1}(L) \neq 0_{(\mathbb{W}, \varrho)}$, $(D, \varrho) = \mathcal{G}^{-1}(S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$, which are NSZCS's in \mathbb{W} and $\mathcal{G}^{-1}(G, \varrho) \cap \mathcal{G}^{-1}(S, \varrho) = \mathcal{G}^{-1}(0_{(\mathbb{W}, \varrho)}) = 0_{(\mathbb{W}, \varrho)}$, which implies $(K, \varrho) \cap (D, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Thus \mathbb{W} is not a NZstCon, which is a contradiction to our hypothesis. Hence \mathbb{T} is NSZstCon.

Other cases are similar.

Remark 3.1. NSZstCon (resp. NSstCon, *NS δ stCon*, *NS δ SstCon* and NSPstCon) and *NSZC₅Con* (resp. *NSC₅Con*, *NS δ C₅Con*, *NS δ SC₅Con* and *NSPC₅Con*) are independent.

4. Neutrosophic soft Z separated sets

Definition 4.1. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. If (S, ϱ) & (B, ϱ) are non-zero NSS in \mathbb{W} . Then (S, ϱ) & (B, ϱ) are said to be

- (i) Neutrosophic soft (resp. $\delta, \delta S, P$ & Z) weakly separated (briefly, NSWSep (resp. NS δ WSep, NS δ SWSep, NSPWSep & NSZWSep)) if $NScl(S, \varrho) \subseteq (B, \varrho)^c$ (resp. $NS\delta cl(S, \varrho) \subseteq (B, \varrho)^c, NS\delta Scl(S, \varrho) \subseteq (B, \varrho)^c, NSPcl(S, \varrho) \subseteq (B, \varrho)^c$ & $NSZcl(B, \varrho) \subseteq (B, \varrho)^c$).
- (ii) Neutrosophic soft (resp. $\delta, \delta S, P$ & Z) separated (briefly, NSSep (resp. NS δ Sep, NS δ SSep, NSPSep & NSZSep) if $NScl(S, \varrho) \cap (B, \varrho) = (S, \varrho) \cap NScl(B, \varrho) = 0_{(\mathbb{W}, \varrho)}$ (resp. $NS\delta cl(S, \varrho) \cap (B, \varrho) = (B, \varrho) \cap NS\delta cl(B, \varrho) = 0_{(\mathbb{W}, \varrho)}$, $NS\delta Scl(S, \varrho) \cap (B, \varrho) = (S, \varrho) \cap NS\delta Scl(B, \varrho) = 0_{(\mathbb{W}, \varrho)}$, $NSPcl(S, \varrho) \cap (B, \varrho) = (S, \varrho) \cap NSPcl(B, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and $NSZcl(S, \varrho) \cap (B, \varrho) = (S, \varrho) \cap NSZcl(B, \varrho) = 0_{(\mathbb{W}, \varrho)}$).

Remark 4.1. Any two disjoint non-empty NSZCS (resp. NSCS, *NS δ CS*, *NS δ SCS*, *NSPCS*) are NSZSep (resp. NSSep, *NS δ Sep*, *NS δ SSep* and *NSPSep*).

Proof Suppose (S, ϱ) and (B, ϱ) are disjoint non-empty NSZCS's. Then $NSZcl(S, \varrho) \cap (B, \varrho) = (S, \varrho) \cap NSZcl(B, \varrho) = (S, \varrho) \cap (B, \varrho) = 0_{(\mathbb{W}, \varrho)}$. This shows that (S, ϱ) and (B, ϱ) are NSZSep.

The same method can be used to prove the theorems other cases.

Theorem 4.1. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. If (S, ϱ) and (B, ϱ) are non-zero NSS in \mathbb{W} .

- (i) If (S, ϱ) and (G, ϱ) are NSZSep (resp. NSSep, *NS δ Sep*, *NS δ SSep* and NSPSep) and $(A, \varrho) \subseteq (S, \varrho), (C, \varrho) \subseteq (G, \varrho)$, then (A, ϱ) and (C, ϱ) are also NSZSep (resp. NSSep, *NS δ Sep*, *NS δ SSep* and NSPSep).
- (ii) If (S, ϱ) and (G, ϱ) are both NSZOS (resp. NSOS, *NS δ OS*, *NS δ SOS* and NSPOS) and if $(S, \varrho) = (S, \varrho) \cap (G, \varrho)^c$ and $(Q, \varrho) = (G, \varrho) \cap (S, \varrho)^c$, then (S, ϱ) and (Q, ϱ) are NSZSep (resp. NSSep, *NS δ Sep*, *NS δ SSep* and NSPSep).

Proof

- (i) Let (S, ϱ) and (G, ϱ) be NSZSep sets in NSTs \mathbb{W} . Then $NSZcl(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)} = (S, \varrho) \cap NSZcl(G, \varrho)$. Since $(A, \varrho) \subseteq (S, \varrho)$ and $(C, \varrho) \subseteq (G, \varrho)$, then $NSZcl(A, \varrho) \subseteq NSZcl(S, \varrho)$ and $NSZcl(C, \varrho) \subseteq NSZcl(G, \varrho)$. This implies that, $NSZcl(A, \varrho) \cap (C, \varrho) \subseteq NSZcl(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and hence $NSZcl(A, \varrho) \cap (C, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Similarly $NSZcl(C, \varrho) \cap (A, \varrho) \subseteq NSZcl(G, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and hence $NSZcl(C, \varrho) \cap (A, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Therefore (A, ϱ) and (C, ϱ) are NSZSep.
- (ii) Let (S, ϱ) and (G, ϱ) both NSZO subsets in \mathbb{W} . Then $(S, \varrho)^c$ and $(G, \varrho)^c$ are NSZCS's. Since $(S, \varrho) \subseteq (G, \varrho)^c$, then $NSZcl(S, \varrho) \subseteq NSZcl(G, \varrho)^c = (G, \varrho)^c$ and so $NSZcl(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Since $(Q, \varrho) \subseteq (G, \varrho)$, then $NSZcl(S, \varrho) \cap (Q, \varrho) \subseteq NSZcl(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Thus, $NSZcl(S, \varrho) \cap (Q, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Similarly, $NSZcl(Q, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Hence (S, ϱ) and (Q, ϱ) are NSZSep.

The same method can be used to prove the theorems other cases.

Theorem 4.2. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. If (A, ϱ) and (C, ϱ) are non-zero NS subsets in \mathbb{W} are NSZSep (resp. NSSep, $NS\delta Sep$, $NS\delta SSep$ and NSP Sep) if and only if there exist (G, ϱ) and (S, ϱ) in NSZOS (resp. NSOS, $NS\delta OS$, $NS\delta SOS$, $NSPOS$) in $\mathbb{W} \ni (A, \varrho) \subseteq (G, \varrho), (C, \varrho) \subseteq (S, \varrho)$ and $(A, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)} \& (C, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$.

Proof Let (A, ϱ) and (C, ϱ) be NSZSep. Then $(A, \varrho) \cap NSZcl(C, \varrho) = 0_{(\mathbb{W}, \varrho)} = NSZcl(A, \varrho) \cap (C, \varrho)$. Take $(S, \varrho) = (NSZcl(A, \varrho))^c$ and $(G, \varrho) = (NSZcl(C, \varrho))^c$. Then (G, ϱ) and (S, ϱ) are NSZOS's $\ni (A, \varrho) \subseteq (G, \varrho), (C, \varrho) \subseteq (S, \varrho) \& (A, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and $(C, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$.

Conversely let (G, ϱ) and (S, ϱ) be NSZOS's $\ni (A, \varrho) \subseteq (G, \varrho), (C, \varrho) \subseteq (S, \varrho) \& (A, \varrho) \cap (S, \varrho) = 0_{(\mathbb{W}, \varrho)}, (C, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Then $(A, \varrho) \subseteq (S, \varrho)^c$ and $(C, \varrho) \subseteq (G, \varrho)^c$ and $(S, \varrho)^c$ and $(G, \varrho)^c$ are NSZCS. This implies, $NSZcl(A, \varrho) \subseteq NSZcl(S, \varrho)^c = (S, \varrho)^c \subseteq (C, \varrho)^c$ and $NSZcl(C, \varrho) \subseteq NSZcl(G, \varrho)^c = (G, \varrho)^c \subseteq (A, \varrho)$. That is, $NSZcl(A, \varrho) \subseteq (C, \varrho)^c$ and $NSZcl(C, \varrho) \subseteq (A, \varrho)^c$. Therefore $(A, \varrho) \cap NSZcl(C, \varrho) = 0_{(\mathbb{W}, \varrho)} = NSZcl(A, \varrho) \cap (C, \varrho)$. Hence (A, ϱ) and (C, ϱ) are NSZSep.

Other cases are similar.

Proposition 4.1. Each two NSZSep (resp. NSSep, $NS\delta Sep$, $NS\delta SSep$ and NSP Sep) sets are always disjoint.

Proof Let (S, ϱ) and (G, ϱ) be NSZSep. Then $(S, \varrho) \cap NSZcl(G, \varrho) = 0_{(\mathbb{W}, \varrho)} = NSZcl(S, \varrho) \cap (G, \varrho)$. Now, $(S, \varrho) \cap (G, \varrho) \subseteq (S, \varrho) \cap NSZcl(G, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Therefore $(S, \varrho) \cap (G, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and hence (S, ϱ) and (G, ϱ) are disjoint.

The same method can be used to prove the theorems other cases.

Theorem 4.3. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. Then \mathbb{W} is NSZCon (resp. $NSCon, NS\delta Con, NS\delta SCon$ and NSPCon) iff $1_{(\mathbb{W}, \varrho)} \neq (H, \varrho) \cup (J, \varrho)$, where (H, ϱ) and (J, ϱ) are NSZSep (resp. NSSep, $NS\delta Sep, NS\delta S Sep$ and NSP Sep) sets.

Proof Assume that, \mathbb{T} is a NSZCon space. Suppose $1_{(\mathbb{W}, \varrho)} = (H, \varrho) \cup (J, \varrho)$, where (H, ϱ) and (J, ϱ) are NSZSep sets. Then $NSZcl(H, \varrho) \cap (J, \varrho) = (H, \varrho) \cap NSZcl(J, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Since $(H, \varrho) \subseteq NSZcl(H, \varrho)$, we have $(H, \varrho) \cap (J, \varrho) \subseteq NSZcl(H, \varrho) \cap (J, \varrho) = 0_{(\mathbb{W}, \varrho)}$. Therefore $NSZcl(H, \varrho) \subseteq (J, \varrho)^c = (H, \varrho)$ and $NSZcl(J, \varrho) \subseteq (H, \varrho)^c = (J, \varrho)$. Hence $(H, \varrho) = NSZcl(H, \varrho)$ and $(J, \varrho) = (H, \varrho)^c$ are disjoint NSZOS's. Thus $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ such that $(H, \varrho) \cup (J, \varrho) = 1_{(\mathbb{W}, \varrho)}$ and $(H, \varrho) \cap (J, \varrho) = 0_{(\mathbb{W}, \varrho)}$, (H, ϱ) and (J, ϱ) are NSZOS's. That is \mathbb{T} is not NSZCon, which is a contradiction to \mathbb{W} is a NSZCon. Hence $1_{(\mathbb{W}, \varrho)}$ is not the union of any two NSZSep sets.

Conversely, assume that $1_{(\mathbb{W}, \varrho)}$ is not the union of any two NSZSep sets. Suppose \mathbb{W} is not NSZCon. Then $1_{(\mathbb{W}, \varrho)} = (H, \varrho) \cup (J, \varrho)$, where $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ such that $(H, \varrho) \cap (J, \varrho) = 0_{(\mathbb{W}, \varrho)}$, (H, ϱ) and (J, ϱ) are NSZOS's in \mathbb{W} . Since $(H, \varrho) \subseteq (J, \varrho)^c$ and $(J, \varrho) \subseteq (H, \varrho)^c, NSZcl(H, \varrho) \cap (J, \varrho) \subseteq (J, \varrho)^c \cap (J, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and $(H, \varrho) \cap NSZcl(J, \varrho) \subseteq (H, \varrho) \cap (H, \varrho)^c = 0_{(\mathbb{W}, \varrho)}$. That is (H, ϱ) and (J, ϱ) are NSZSep sets. This is a contradiction. Therefore \mathbb{W} is NSZCon.

Other cases are similar.

Definition 4.2. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. Let (S, ϱ) be a NSS of \mathbb{W} . Then Neutrosophic soft

- (i) δ (resp. $\delta S, P\&Z$) regular open set (briefly, $NS\delta ROS$ (resp. $NS\delta SROS, NSPROS$ & $NSZROS$)) if $(S, \varrho) = NS\delta int(NS\delta cl(S, \varrho))$ (resp. $(S, \varrho) = NS\delta Sint(NS\delta Scl(S, \varrho)), (S, \varrho) = NSPint(NSPcl(S, \varrho))$ & $(S, \varrho) = NSZint(NSZcl(S, \varrho))$).
- (ii) δ (resp. $\delta S, P\&Z$) regular closed set (briefly, $NS\delta RCS$ (resp. $NS\delta SRCS, NSPRCS\&NSZRCS$)) if $(S, \varrho) = NS\delta cl(NS\delta int(S, \varrho))$ (resp. $(S, \varrho) = NS\delta Scl(NS\delta Sint(S, \varrho)), (S, \varrho) = NSPcl(NSPint(S, \varrho))$ & $(S, \varrho) = NSZcl(NSZint(S, \varrho))$).
- (iii) The complement of $NS\delta ROS$ (resp. $NS\delta SROS, NSPROS\&NSZROS$) is $NS\delta RCS$ (resp. $NS\delta SRCS, NSPRCS$ & $NSZRCS$).

Proposition 4.2. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs.

- (i) Every NSZROS (resp. $NS\delta ROS, NS\delta SROS$ and NSPROS) is NSZOS (resp. $NS\delta OS, NS\delta SOS$ and NSPOS).
- (ii) Every NSRCS (resp. $NS\delta RCS, NS\delta SRCS$ and NSPRCS) is NSZCS (resp. $NS\delta CS, NS\delta SCS$ and NSPCS).

Definition 4.3. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts. Then \mathbb{W} is neutrosophic soft (resp. $\delta, \delta S, P\&Z$) super disconnected (briefly, NSsuperDCon (resp. $NS\delta superDCon, NS\delta SsuperDCon, NSPsuperDCon$ and $NSZsuperDCon$)) if there exists a NSROS (resp. $NS\delta ROS, NS\delta SROS, NSPROS\&NSZROS$)(S, ϱ) in $\mathbb{W} \ni (S, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ and $(S, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. A NSts. $(\mathbb{W}, \tau, \varrho)$ is called neutrosophic soft (resp. $\delta, \delta S, P\&Z$) super connected (briefly, NSsuperCon (resp. $NS\delta superCon, NS\delta SsuperCon, NSPsuperCon$ & $NSZsuperCon$)) if \mathbb{W} is not NS superDCon (resp. $NS\delta superDCon, NS\delta SsuperDCon, NSPsuperDCon$ and $NSZsuperDCon$).

Theorem 4.4. Let $(\mathbb{W}, \tau, \varrho)$ be a NSts, the statements that follows are equivalent:

- (i) \mathbb{W} is NSZsuperCon (resp. NSsuperCon, $NS\delta superCon, NS\delta SsuperCon$ and NSPsuperCon).
- (ii) For each NSZOS (resp. NSOS, $NS\delta OS, NS\delta SOS$ and NSPOS) $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ in \mathbb{W} , we have $NSZcl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}$ (resp. $NScl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}, NS\delta cl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}, NS\delta Scl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}$ and $NSPcl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}$).
- (iii) For each NSZCS (resp. NSCS, $NS\delta CS, NS\delta SCS$ and $NSPCS$)($H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$ in \mathbb{T} , we have $NSZint(H, \varrho) = 0_{(\mathbb{W}, \varrho)}$ (resp. $NSint(H, \varrho) = 0_{(\mathbb{W}, \varrho)}, NS\delta int(H, \varrho) = 0_{(\mathbb{W}, \varrho)}, NS\delta Sint(H, \varrho) = 0_{(\mathbb{W}, \varrho)}$ and $NSPint(H, \varrho) = 0_{(\mathbb{W}, \varrho)}$).
- (iv) There exists no NSZo (resp. NSo, $NS\delta o, NS\delta So$ and $NSPo$) subsets (H, ϱ) and (J, ϱ) in \mathbb{W} , such that $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ and $(H, \varrho) \subseteq (J, \varrho)^c$.
- (v) There exists no NSZO (resp. NSO, $NS\delta O, NS\delta SO$ and $NSPO$) subsets (H, ϱ) and (J, ϱ) in \mathbb{W} , such that $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (J, \varrho) = (NSZcl(H, \varrho))^c$ (resp. $(J, \varrho) = (NScl(H, \varrho))^c, (J, \varrho) = (NS\delta cl(H, \varrho))^c, (J, \varrho) = (NS\delta Scl(H, \varrho))^c$ and $(J, \varrho) = (NSPcl(H, \varrho))^c$) and $(H, \varrho) = (NSZcl(J, \varrho))^c$ (resp. $(H, \varrho) = (NScl(J, \varrho))^c, (H, \varrho) = (NS\delta cl(J, \varrho))^c, (H, \varrho) = (NS\delta Scl(J, \varrho))^c$ and $(H, \varrho) = (NSPcl(J, \varrho))^c$).
- (vi) There exists no NSZCS (resp. NSCS, $NS\delta CS, NS\delta SCS$ and $NSPCS$) subsets (H, ϱ) and (J, ϱ) in \mathbb{W} such that $(H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}, (J, \varrho) \neq 1_{(\mathbb{W}, \varrho)}, (J, \varrho) = (NSZcl(H, \varrho))^c$ (resp. $(J, \varrho) = (NScl(H, \varrho))^c, (J, \varrho) = (NS\delta cl(H, \varrho))^c, (J, \varrho) = (NS\delta Scl(H, \varrho))^c$ and $(J, \varrho) = (NSPcl(H, \varrho))^c$) and $(H, \varrho) = (NSZcl(J, \varrho))^c$ (resp. $(H, \varrho) = (NScl(J, \varrho))^c, (H, \varrho) = (NS\delta cl(J, \varrho))^c, (H, \varrho) = (NS\delta Scl(J, \varrho))^c$ and $(H, \varrho) = (NSPcl(J, \varrho))^c$).

Proof

(i) \Rightarrow (ii) Assume that there exists a NSZOS $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)} \ni NSZcl(H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Now take $(J, \varrho) = NSZint(NSZcl(H, \varrho))$. Then (J, ϱ) is proper $NSZROS$ in \mathbb{W} which contradicts that \mathbb{W} is NSZsuperCon -ness.

(ii) \Rightarrow (iii) Let $(H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$ be a NSZCS in \mathbb{W} . If $(J, \varrho) = (H, \varrho)^c$, then (J, ϱ) is NSZOS in \mathbb{W} and $(J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$. Hence $NSZcl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}, (NSZcl(J, \varrho))^c = 0_{(\mathbb{W}, \varrho)} \Rightarrow NSZint(J, \varrho)^c = 0_{(\mathbb{W}, \varrho)} \Rightarrow NSZint(H, \varrho) = 0_{(\mathbb{W}, \varrho)}$.

(iii) \Rightarrow (iv) Let (H, ϱ) and (J, ϱ) be NSZOS's in \mathbb{W} such that $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)} \neq (J, \varrho)$ and $(H, \varrho) \subseteq (J, \varrho)^c$. Since $(J, \varrho)^c$ is NSZCS in \mathbb{W} and $(J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ implies $(J, \varrho)^c \neq 1_{(\mathbb{W}, \varrho)}$, we obtain $NSZint(J, \varrho)^c = 0_{(\mathbb{W}, \varrho)}$. But, from $(H, \varrho) \subseteq (J, \varrho)^c, 0_{(\mathbb{W}, \varrho)} \neq (H, \varrho) = NSZint(H, \varrho) \subseteq NSZint(J, \varrho)^c = 0_{(\mathbb{W}, \varrho)}$, which is a contradiction.

(iv) \Rightarrow (i) Let $0_{(\mathbb{W}, \varrho)} \neq (H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$ be NZSROS in \mathbb{W} . If we take $(J, \varrho) = (NSZcl(H, \varrho))^c$, we get $(J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$. Otherwise, we have $(J, \varrho) \neq 0_{(\mathbb{W}, \varrho)}$ implies $(NSZcl(H, \varrho))^c = 0_{(\mathbb{W}, \varrho)}$. That implies $NSZcl(H, \varrho) = 1_{(\mathbb{W}, \varrho)}$. That shows $(H, \varrho) = NSZint(NSZcl(H, \varrho)) = NSZint(1_{(\mathbb{W}, \varrho)}) = 1_{(\mathbb{W}, \varrho)}$. But this is to a contradiction to $(H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Further, $(H, \varrho) \subseteq (J, \varrho)^c$, this is also a contradiction.

(i) \Rightarrow (v) Let (H, ϱ) and (J, ϱ) be NSZOS's in \mathbb{W} such that $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)} \neq (J, \varrho)$ and $(J, \varrho) = (NSZcl(H, \varrho))^c, (H, \varrho) = (NSZint(J, \varrho))^c$. Now $NSZint(NSZcl(H, \varrho)) = NSZint(J, \varrho)^c = (NSZcl(J, \varrho))^c = (H, \varrho)$ and $(H, \varrho) \neq 0_{(\mathbb{W}, \varrho)}, (H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Suppose not, if $(H, \varrho) = 1_{(\mathbb{W}, \varrho)}$, then $1_{(\mathbb{W}, \varrho)} = (NSZcl(J, \varrho))^c$ implies $0_{(\mathbb{W}, \varrho)} = NSZcl(J, \varrho) \Rightarrow (J, \varrho) = 0_{(\mathbb{W}, \varrho)}$. This is a contradiction.

(v) \Rightarrow (i) Let (H, ϱ) be NSZOS in \mathbb{W} such that $(H, \varrho) = NSZint(NSZcl(H, \varrho)) = 0_{(\mathbb{W}, \varrho)} \neq (H, \varrho) \neq 1_{(\mathbb{W}, \varrho)}$. Now $(J, \varrho) = (NSZcl(H, \varrho))^c$ and $(NSZcl(J, \varrho))^c = (NSZcl(NSZcl(H, \varrho))^c)^c = NSZint(NSZcl(H, \varrho)) = (H, \varrho)$. This is a contradiction.

(v) \Rightarrow (vi) Let (H, ϱ) and (J, ϱ) be NSZCS in \mathbb{W} such that $(H, \varrho) \neq 1_{(\mathbb{W}, \varrho)} \neq (J, \varrho), (J, \varrho) = (NSZint(H, \varrho))^c, (H, \varrho) = (NSZint(J, \varrho))^c$. Taking $(P, \varrho) \neq 0_{(\mathbb{W}, \varrho)} \neq (F, \varrho), (NSZcl(P, \varrho))^c = (NSZcl(H, \varrho))^c = (NSZint(H, \varrho))^c = NSZint(H, \varrho) = (J, \varrho)^c = (F, \varrho)$ and similarly $(NSZcl(F, \varrho))^c = (P, \varrho)$. But this is a contradiction.

(vi) \Rightarrow (v) Similar as in above.

5. Neutrosophic soft Z compact spaces

Definition 5.1. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. A collection (S, ϱ) of NSZOS in \mathbb{W} is called a neutrosophic soft Z open cover (briefly, NSZOCov) of a subset (S, ϱ) of \mathbb{W} if $(S, \varrho) \subseteq \cup\{(G, \varrho) : (G, \varrho) \in (S, \varrho)\}$.

Definition 5.2. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs, then \mathbb{W} is said to be neutrosophic soft Z compact (briefly, NSZComp) if every NSZOCov of \mathbb{W} has a finite subcover.

Definition 5.3. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. A NSS (S, ϱ) of \mathbb{W} is said to be NSZComp relative to \mathbb{W} if every NSZOCov of \mathbb{W} has a finite subcover.

Theorem 5.1. Let $(\mathbb{W}, \tau, \varrho)$ be a NSTs. Every

- (i) NSComp space is $NS\delta Comp$.
- (ii) $NS\delta SComp$ is a NSComp.
- (iii) NSPComp is a NSComp.

- (iv) NSZComp is a $NS\delta SComp$.
- (v) NSZComp is a NSPComp.

Proof (v) Let \mathbb{W} be NSZComp. Suppose \mathbb{W} is not NSPComp. Then there exists a NSPOCov (S, ϱ) of \mathbb{W} has no finite subcover. Since every NSPOS is NSZOS, then we have NSZOCov (S, ϱ) of \mathbb{W} , which has no finite subcover. This is a contradiction to \mathbb{W} is NSZComp. Hence \mathbb{W} is NSPComp.

Other cases are true.

Theorem 5.2. A NSZC (resp. NSC, $NS\delta C$, $NS\delta SC$ and NSPC) subset of a NSZComp (resp. NSComp, $NS\delta Comp$, $NS\delta SComp$ and NSPComp) \mathbb{W} is NSZComp (resp. NSComp, $NS\delta Comp$, $NS\delta SComp$ and NSPComp) relative to \mathbb{W} .

Proof Let (S, ϱ) be a NSZC subset of a NSZComp \mathbb{W} . Then $(S, \varrho)^c$ is NSZOS in \mathbb{W} . Let $(G, \varrho) = \{(S, \varrho)_i : i \in I\}$ be a NSZOCov of (S, ϱ) . Then $(G, \varrho) \cup (S, \varrho)^c$ is a NSZOCov of \mathbb{W} . Since \mathbb{W} is NSZComp, it has a finite subcover say $\{(Z_1, \varrho), (Z_2, \varrho), \dots, (Z_n, \varrho)(S, \varrho)^c\}$. Then $\{(Z_1, \varrho), (Z_2, \varrho), \dots, (Z_n, \varrho)\}$ is a finite NSZOCov. Thus (S, ϱ) is NSZComp relative to \mathbb{W} .

Other cases are similar.

Theorem 5.3. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSZCts (resp. $NS\delta Cts$, $NS\delta SCts$ and NSPCts) surjection and \mathbb{W} be NSZComp (resp. $NS\delta Comp$, $NS\delta SComp$ and NSPComp). Then \mathbb{T} is NSComp.

Proof Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSZCts surjection and \mathbb{W} be NSZComp. Let (S, ϱ) be a NSZOCov for \mathbb{T} . Since \mathcal{G} is NSZCts, $\{\mathcal{G}^{-1}(S, \varrho)\}$ is a NSZOCov of \mathbb{W} . Since \mathbb{W} is NSZComp, $\{\mathcal{G}^{-1}(S, \varrho)\}$ contains a finite subcover, namely $\{\mathcal{G}^{-1}(S_1, \varrho), \mathcal{G}^{-1}(S_2, \varrho), \dots, \mathcal{G}^{-1}(S_n, \varrho)\}$. Since \mathcal{G} is surjection, $\{(S_1, \varrho), (S_2, \varrho), \dots, (S_n, \varrho)\}$ is a finite subcover for \mathbb{T} . Thus \mathbb{T} is NSComp.

Other cases are similar.

Theorem 5.4. Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSZO (resp. $NS\delta O$, $NS\delta SO$ and NSPO) function and \mathbb{T} be NSZComp (resp. $NS\delta Comp$, $NS\delta SComp$ and NSPComp). Then \mathbb{T} is NSComp.

Proof Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \sigma, \varrho)$ be a NSZO function and \mathbb{T} be NSZComp. Let (S, ϱ) be a NSZOCov for \mathbb{W} . Since \mathcal{G} is NSZO, $\{\mathcal{G}(S, \varrho)\}$ is a NSZOCov of \mathbb{T} . Since \mathbb{T} is NSZComp, $\{\mathcal{G}(S, \varrho)\}$ contains a finite sub NSZOCov, namely $\{\mathcal{G}(S_1, \varrho), \mathcal{G}(S_2, \varrho), \dots, \mathcal{G}(S_n, \varrho)\}$.

Then $\{(S_1, \varrho), (S_2, \varrho), \dots, (S_n, \varrho)\}$ is a finite subcover for \mathbb{T} . Thus \mathbb{T} is NSComp.

Theorem 5.5. The image of a NSZComp (resp. $NS\delta Comp$, $NS\delta SComp$ and NSPComp) under a NSZCts (resp. $NS\delta Cts$, $NS\delta SCts$ and NSPCts) map is NSComp.

proof Let $\mathcal{G} : (\mathbb{W}, \tau, \varrho) \rightarrow (\mathbb{T}, \tau, \varrho)$ be a NSZCts map from a NSZComp \mathbb{W} onto \mathbb{T} . Let $\{(S, \varrho)_i : i \in I\}$ be a NSZOCov of \mathbb{T} . Since \mathcal{G} is NSZCts, $\{\mathcal{G}^{-1}((S, \varrho)_i : i \in I)\}$ is a NSZOCov of \mathbb{W} . As \mathbb{W} is NSZComp, the NSZOCov $\{\mathcal{G}^{-1}((S, \varrho)_i : i \in I)\}$ of \mathbb{T} has a finite subcover $\{\mathcal{G}^{-1}((S, \varrho)_i) : i = 1, 2, 3, \dots, n\}$. Therefore $(G, \varrho) = \bigcup_{\{i \in I\}} \mathcal{G}^{-1}((S, \varrho)_i)$. Then $\mathcal{G}(G, \varrho) = \bigcup_{\{i \in I\}} (S, \varrho)_i$, that is $(R, \varrho) = \bigcup_{\{i \in I\}} (S, \varrho)$. Thus $\{(S_1, \varrho), (S_2, \varrho), \dots, (S_n, \varrho)\}$ is a finite subcover of $\{(S, \varrho)_i : i \in I\}$ for \mathbb{T} . Hence \mathbb{T} is NSComp.

Other cases are similar.

6. Application

In this section, distance and similarity measures are applied through examples to select the most appropriate yarn production machine.

Example 6.1. A textile company specializing in yarn production plans to upgrade its manufacturing process to meet increasing market demand and improve product quality. Two advanced yarn production machines, Machine A and Machine B, have been shortlisted for purchase. To select the most suitable machine, the production management team evaluates both machines based on the following five key performance parameters: Production Speed, Yarn Strength, Energy Efficiency, Maintenance Requirements, Product Uniformity

Let $\mathbb{W} = \{w_1, w_2, w_3\} = \{Excellent, Good, Average\}$ = represent the universal set. Let $\varrho = \{e_1, e_2, e_3, e_4, e_5\} = \{\text{Production Speed, Yarn Strength, Energy Efficiency, Maintenance Requirements, Product Uniformity}\}$ be the set of evaluation parameters.

We define the Neutrosophic Soft Set (NSS) that gives, for each parameter, the degree of association, degree of indeterminacy, and degree of non-association between each machine and the performance rating set. Using this NSS model, the company determines which machine best meets the production requirements and offers the highest overall value.

The neutrosophic soft set (A, ϱ) represents the manufacturer's required performance specifications for yarn production.

$$(A, e_1) = \{\langle(m_1, 0.3, 0.5, 0.6)\rangle, \langle(m_2, 0.4, 0.5, 0.7)\rangle, \langle(m_3, 0.2, 0.5, 0.8)\rangle\}$$

$$(A, e_2) = \{\langle(m_1, 0.1, 0.5, 0.8)\rangle, \langle(m_2, 0.2, 0.5, 0.8)\rangle, \langle(m_3, 0.3, 0.5, 0.6)\rangle\}$$

$$(A, e_3) = \{\langle(m_1, 0.2, 0.5, 0.7)\rangle, \langle(m_2, 0.3, 0.5, 0.6)\rangle, \langle(m_3, 0.1, 0.5, 0.6)\rangle\}$$

$$(A, e_4) = \{\langle(m_1, 0.2, 0.5, 0.9)\rangle, \langle(m_2, 0.1, 0.5, 0.6)\rangle, \langle(m_3, 0.4, 0.5, 0.7)\rangle\}$$

$$(A, e_5) = \{\langle(m_1, 0.4, 0.5, 0.5)\rangle, \langle(m_2, 0.1, 0.5, 0.6)\rangle, \langle(m_3, 0.5, 0.5, 0.5)\rangle\}$$

The neutrosophic soft sets (B, ϱ) and (C, ϱ) represent the two shortlisted yarn production machines and their corresponding performance evaluations, respectively.

$$(B, e_1) = \{\langle(m_1, 0.2, 0.5, 0.8)\rangle, \langle(m_2, 0.5, 0.4, 0.3)\rangle, \langle(m_3, 0.7, 0.5, 0.1)\rangle\}$$

$$(B, e_2) = \{\langle(m_1, 0.1, 0.4, 0.9)\rangle, \langle(m_2, 0.7, 0.3, 0.4)\rangle, \langle(m_3, 0.8, 0.5, 0.5)\rangle\}$$

$$(B, e_3) = \{\langle(m_1, 0.2, 0.8, 0.9)\rangle, \langle(m_2, 0.3, 0.7, 0.7)\rangle, \langle(m_3, 0.4, 0.5, 0.9)\rangle\}$$

$$(B, e_4) = \{\langle(m_1, 0.3, 0.2, 0.8)\rangle, \langle(m_2, 0.4, 0.6, 0.6)\rangle, \langle(m_3, 0.5, 0.5, 0.9)\rangle\}$$

$$(B, e_5) = \{\langle(m_1, 0.4, 0.7, 0.7)\rangle, \langle(m_2, 0.3, 0.8, 0.8)\rangle, \langle(m_3, 0.9, 0.2, 0.7)\rangle\}$$

$$(C, e_1) = \{\langle(m_1, 0.1, 0.2, 0.3)\rangle, \langle(m_2, 0.4, 0.5, 0.6)\rangle, \langle(m_3, 0.7, 0.8, 0.9)\rangle\}$$

$$(C, e_2) = \{\langle(m_1, 0.2, 0.5, 0.8)\rangle, \langle(m_2, 0.1, 0.4, 0.7)\rangle, \langle(m_3, 0.3, 0.6, 0.9)\rangle\}$$

$$(C, e_3) = \{\langle(m_1, 0.5, 0.4, 0.6)\rangle, \langle(m_2, 0.2, 0.1, 0.3)\rangle, \langle(m_3, 0.8, 0.7, 0.9)\rangle\}$$

$$(C, e_4) = \{\langle(m_1, 0.4, 0.1, 0.7)\rangle, \langle(m_2, 0.5, 0.2, 0.8)\rangle, \langle(m_3, 0.6, 0.3, 0.9)\rangle\}$$

$$(C, e_5) = \{\langle(m_1, 0.4, 0.2, 0.8)\rangle, \langle(m_2, 0.4, 0.5, 0.8)\rangle, \langle(m_3, 0.7, 0.8, 0.8)\rangle\}$$

Distance measures are as follows:

(i) Hamming distance

$$d_h((A, \varrho), (B, \varrho)) = 2.7667, \quad d_h((A, \varrho), (C, \varrho)) = 3.0001.$$

Hence, $d_h((A, \varrho), (B, \varrho)) < d_h((A, \varrho), (C, \varrho))$.

(ii) Normalized hamming distance

$$d_{nh}((A, \varrho), (B, \varrho)) = 0.1844, \quad d_{nh}((A, \varrho), (C, \varrho)) = 0.2000.$$

Hence, $d_{nh}((A, \varrho), (B, \varrho)) < d_{nh}((A, \varrho), (C, \varrho))$.

(iii) Euclidean distance

$$d_e((A, \varrho), (B, \varrho)) = 0.9575, \quad d_e((A, \varrho), (C, \varrho)) = 0.9660.$$

$$\text{Hence, } d_e((A, \varrho), (B, \varrho)) < d_e((A, \varrho), (C, \varrho)).$$

(iv) Normalized Euclidean distance

$$d_{ne}((A, \varrho), (B, \varrho)) = 0.2472, \quad d_{ne}((A, \varrho), (C, \varrho)) = 0.2494.$$

$$\text{Hence, } d_{ne}((A, \varrho), (B, \varrho)) < d_{ne}((A, \varrho), (C, \varrho)).$$

In all the considered distance measures, the distance between (A, ϱ) and (B, ϱ) is consistently less than the distance between (A, ϱ) and (C, ϱ) , indicating that (B, ϱ) is closer to (A, ϱ) than (C, ϱ) .

Hence, the 1st machine is deemed to be the more suitable option.

Similarity measures are as follows:

(i) Similarity measure based on the Hamming distance

$$S_h((A, \varrho), (B, \varrho)) = 0.2655, \quad S_h((A, \varrho), (C, \varrho)) = 0.2499.$$

$$S_h((A, \varrho), (B, \varrho)) > S_h((A, \varrho), (C, \varrho)).$$

(ii) Similarity measure based on the Normalized Hamming distance

$$S_{nh}((A, \varrho), (B, \varrho)) = 0.8443, \quad S_{nh}((A, \varrho), (C, \varrho)) = 0.8333.$$

$$S_{nh}((A, \varrho), (B, \varrho)) > S_{nh}((A, \varrho), (C, \varrho)).$$

(iii) Similarity measure based on the Euclidean distance

$$S_e((A, \varrho), (B, \varrho)) = 0.5109, \quad S_e((A, \varrho), (C, \varrho)) = 0.5086.$$

$$S_e((A, \varrho), (B, \varrho)) > S_e((A, \varrho), (C, \varrho)).$$

(iv) Similarity measure based on the Normalized Euclidean distance

$$S_{ne}((A, \varrho), (B, \varrho)) = 0.8018, \quad S_{ne}((A, \varrho), (C, \varrho)) = 0.8004.$$

$$S_{ne}((A, \varrho), (B, \varrho)) > S_{ne}((A, \varrho), (C, \varrho)).$$

In all the considered similarity measures, the similarity between (A, ϱ) and (B, ϱ) is consistently greater than the similarity between (A, ϱ) and (C, ϱ) indicating that (B, ϱ) is more similar to (A, ϱ) than (C, ϱ) .

Hence, the 1st machine is deemed to be the more suitable option.

7. Conclusions

In this study, we have introduced and explored the concepts of Z -connectedness, Z -compactness, and Z -separated sets in neutrosophic soft topological spaces, establishing their fundamental properties with supporting examples. The integration of these topological notions with distance and similarity measures has been effectively applied to a multi-criteria decision-making problem for yarn production machine selection. This approach demonstrates

the potential of neutrosophic soft topology as a robust mathematical tool to handle uncertainty and complexity in real-world decision-making scenarios. Future research may extend these concepts to other applications and develop more comprehensive theoretical frameworks within neutrosophic soft set theory.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Acikgoz, A. and Esenbel, F. (2019). Neutrosophic soft δ -topology and neutrosophic soft compactness, *AIP Conference Proceedings*, Vol. 2183, p. 030002.
2. Atanassov, K. (1986). Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 20, pp. 87–96.
3. Bera, T. and Mahapatra, N.K. (2016). On neutrosophic soft function, *Annals of Fuzzy Mathematics and Informatics*, Vol. 12, No. 1, pp. 101–119.
4. Bera, T. and Mahapatra, N.K. (2017). Introduction to neutrosophic soft topological space, *Opsearch*, Vol. 54, pp. 841–861.
5. Broumi, S. and Smarandache, F. (2013). Intuitionistic neutrosophic soft set, *Journal of Information and Computing Science*, Vol. 8, No. 2, pp. 130–140.
6. Broumi, S., Smarandache, F., Mondal, K. and Deli, M. (2017). Separation axioms in neutrosophic soft topological spaces, *Progress in Science and Engineering Research Journal*, Vol. 5, No. 3, pp. 161–168.
7. Chang, C.L. (1968). Fuzzy topological spaces, *Journal of Mathematical Analysis and Applications*, Vol. 24, No. 1, pp. 182–190.
8. Coker, D. (1997). An introduction of intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 88, pp. 81–89.
9. Deli, I. and Broumi, S. (2015). Neutrosophic soft relations and some properties, *Annals of Fuzzy Mathematics and Informatics*, Vol. 9, No. 1, pp. 169–182.
10. El-Magharabi, A.I. and Mubarki, A.M. (2011). Z open sets and Z-continuity in topological spaces, *International Journal of Mathematics Archive*, Vol. 2, No. 10, pp. 1819–1827.
11. Fatimah, H. and Broumi, S. (2019). Connectedness and compactness in neutrosophic soft topological spaces, *Neutrosophic Sets and Systems*, Vol. 24, pp. 35–43.
12. Maji, P.K. (2013). Neutrosophic soft set, *Annals of Fuzzy Mathematics and Informatics*, Vol. 5, No. 1, pp. 157–168.
13. Maji, P.K., Roy, A.R. and Biswas, R. (2013). Neutrosophic soft sets and their application in decision making, *Journal of Intelligent and Fuzzy Systems*, Vol. 25, No. 1, pp. 245–252.
14. Mahanta, J. and Das, P.K. (2014). On soft topological spaces via semiopen and semiclosed soft sets, *Kyungpook Mathematical Journal*, Vol. 54, pp. 221–236.
15. Molodtsov, D. (1999). Soft set theory: first results, *Computers and Mathematics with Applications*, Vol. 37, pp. 19–31.
16. Revathi, P., Chitirakala, K. and Vadivel, A. (submitted). Neutrosophic soft e-open sets in Neutrosophic Soft Topological Spaces.
17. Salama, A.A. and Alblowi, S.A. (2012). Neutrosophic set and neutrosophic topological spaces, *IOSR Journal of Mathematics*, Vol. 3, No. 4, pp. 31–35.
18. Salama, A.A. and Smarandache, F. (2015). *Neutrosophic crisp set theory*, Educational Publisher, Columbus, Ohio, USA.

19. Shabir, M. and Naz, M. (2011). On soft topological spaces, *Computers and Mathematics with Applications*, Vol. 61, pp. 1786–1799.
20. Smarandache, F. (2005). Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, *International Journal of Pure and Applied Mathematics*, Vol. 24, pp. 287–297.
21. Vadivel, A., Moogambigai, N. and Tamilselvan, S. (2021). Z-open sets in a neutrosophic topological spaces, Vol. 12, pp. 357–362.
22. Venkateswara Rao, V. and Srinivasa Rao, Y. (2017). Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology, *International Journal of ChemTech Research*, Vol. 10, No. 10, pp. 449–458.
23. Zadeh, L.A. (1965). Fuzzy sets, *Information and Control*, Vol. 8, pp. 338–353.

Received: June 28, 2025. Accepted: Sep 10, 2025