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Enhancing Cryptocurrency Prediction: A Fusion of Machine Learning and Neutrosophic Programming

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Abstract. Fluctuations in cryptocurrency markets present a significant problem to the accuracy of forecasting trends and prices in this field. This paper proposes a new method of improving cryptocurrency forecasting by applying machine learning algorithms to a hybrid model. The framework integrates, a neural network model, with auto regressive integrated moving average (ARIMA) and trigonometric, Box-Cox, ARMA, Trend, Seasonal (TBATS) to capture the intricate relationship and dynamics in the data. Because most aspects affecting the cryptocurrency's price are uncertain, we propose that fuzzy parameters be used to reflect this uncertainty in the market. Furthermore, we apply neutrosophic programming to optimize predictions where the indeterminacy of the data is considered. The hybrid model thus incorporates short-term market volatility and long-term market trends, making the model rigid and accurate. Here, we compare this approach's performance with other forecasting models using actual cryptocurrency data. The results indicate that the hybrid model developed achieves better predictive accuracy and is more flexible than the conventional models. To sum up, this research offers significant knowledge of applying the newest machine learning methods to enhance cryptocurrency prediction and improve its efficiency for investors, traders, and financial institutions.

keywords: Hybrid modeling, Machine learning, ARIMA, TBATS, Optimization methods, Neural networks, Time series forecasting, Neutrosophic programming

1. **Introduction**

Digital currencies are among the most active and constantly evolving segments of modern financial systems [35]. Two significant problems complicating the forecasting process are high variability and sensitivity to macroeconomic and social factors and decentralized structures [27]. Classical time series forecasting approaches, although beneficial in some cases, fail to capture cryptocurrencies' high-order dependencies and non-linearity characteristics. Therefore, there is an increasing interest in developing new approaches for solving this problem that would take advantage of the richness of this domain [21]. This paper aims to meet these challenges by proposing a new modelling approach that blends machine learning algorithms with traditional statistical methods. The proposed framework is an integration between recent methods, including Neural Network Model and the more conventional ones, such as ARIMA or Auto-Regressive Integrated Moving Average and TBATS or Trigonometric, Box-Cox, ARMA, Trend, and Seasonal. Using both short-term and long-term data to derive the projections makes the hybrid model rich and accurate in both the short and long run.

Also, optimization integrations make it possible to adjust the model's parameters and thus improve prediction abilities. It is assessed using actual data of cryptocurrencies and compared with conventional forecasting models for cryptocurrency prices. The findings support the hypothesis of the higher efficiency of the hybrid model and show its ability to become an important tool for stock investors, traders and financial companies.

The cryptocurrency market has recently expanded into a significant sector of financial services, attracting investors, scholars, and policymakers [36]. Currently, there are dozens of functional cryptocurrencies, and the market has many unique features, including high volatility, relatively fast evolution, and numerous interdependent factors determining fluctuations [10]. Due to the critical market cap, advanced technology, adoption rate, and huge community, Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), and Solana (SOL) are in these categories. Due to its encompassing nature, this type of currency is also ideal for this study, and seen as such, these four currencies were selected for this research [39].

[51] presented a threat that combined both the ARIMA model with ANN due to the weakness within the data series of linear and nonlinear modeling to analyze the real datasets more accurately. [20] proposed the AR-ANN hybrid model for time series forecasting and reported that the incorporation of annexe linear and nonlinear components gave improved results for the stream flow of the Colorado River. [14] used a hybrid ARIMA–ANN in modeling water quality, and it indicated higher accuracy than individual models in identifying both linear and nonlinear time series models. [22] provided a great ARIMA-ANN hybrid model that can accomplish better performance than other hybrid and standalone models by using linear and nonlinear models simultaneously. [47] developed a combined model involving both linear and nonlinear components reflected through ARIMA and ANN, respectively, and showed enhanced prediction capability in using sunspot data, Canadian lynx data, and IBM stock data.

[37] introduced a Neutrosophic Compromise Programming Algorithm (NCPA) for handling multi-objective transportation exercise that performs better than the traditional fuzzy and other methods. [48] presented neutrosophic numbers (NN) and their operations, founded an NN linear programming (NNLP) for the optimization under indeterminacy with implications to solving production planning problems and evaluating solution range. [12] established that by integrating the ARIMA-BP hybrid model, more forecasting accuracy is achieved than when

the ARIMA and BP neural networks models are used individually in estimating the nonlinear stock price indices. [5] developed a lexicographic goal programming framework to evaluate and optimize India's progress toward SDGs, focusing on GDP, sustainability, and employment across key economic sectors. [1] proposed the new NL models using trapezoidal neutrosophic numbers and illustrated through numerical cases that NL LP is more effective and simpler than the other related methods in dealing with the uncertainty information in decision-making. [4] introduced a new model for MO-NTP with fuzzy parameters and solved it by employing a developed approach, neutrosophic compromise programming approach (NCPA); the efficiency of the proposed approach was compared to other approaches. [2] introduced neutrosophic optimization models of MOPPs with an emphasis on satisfying decision-makers. Its models, when implemented in relation to a pharmaceutical supply chain, illustrated the overall enhancement of efficiency and flexibility of their models.

[31] developed a forecasting and optimization texhniques to achives the India's sustainable development goals agenda 2030. [13] established a neutrosophic programming model for a sustainable biomass supply chain with multiple objectives. Their approach minimizes cost, emission, employment time, and transfer time, and it has a high degree of accuracy and is robust to uncertainty.

[29] introduced an improved return rate prediction method called the RRP-DLBFP (Return Rate Predictive using Deep Learning for Blockchain Financial Products) using LSTM for analyzing predictions. Their findings show that LSTM models are sufficiently good to capture the temporal structure of financial data in the blockchain.

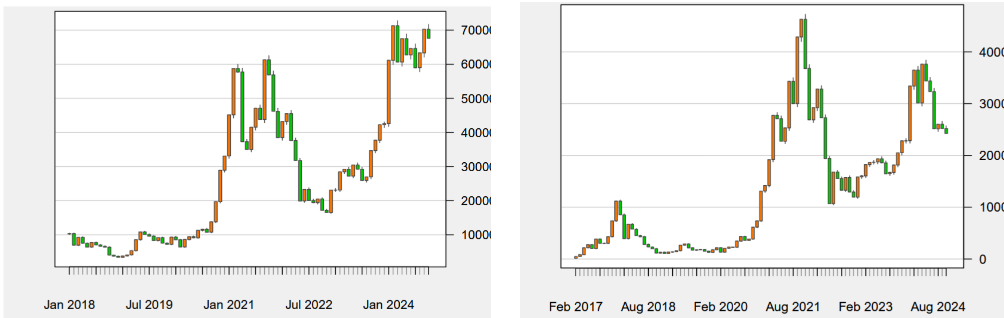
[8] designed a risk evaluation model of applying machine learning algorithms combined with the hyperparameter tuning approach. In order to identify and prioritize risk features, the authors used a recursive feature elimination technique, while the identified important risk attributes were further assessed by medical practitioners. [23] discussed state-of-the-art multi-class forecasting developments from 2018 to 2023 in terms of hybrid modeling, profitability analysis, and technical viewpoint. They pointed out that the shift in their study involved combination and blended systems, and other methods including LSTM networks and SVMs for making prospective financial trends and prices. [55] introduced Tribonacci sequence spaces; neutrosophic norms were applied to study their topological properties. [53] proposed Nörlund ideal convergent sequence spaces and studied their properties in neutrosophic normed spaces. [15] proposed a waste management model in a neutrosophic hesitant fuzzy context by minimizing profit, carbon footprint, and workload fluctuation, with Pareto-optimal solutions.

[9] focused on the possible application of Artificial Neural Networks (ANNs) for predicting cryptocurrencies' prices. They explained that ANNs learning from the data makes them efficient in modeling large data sets that contain nonlinear relationships. [6] developed a multi-objective

problem model in healthcare using neutrosophic goal and fermatean fuzzy programming. [19] described the primary contribution of this paper that has analyzed the cryptocurrency volatility using the ARMA-GARCH-VaR as a model has determined the risk level of different cryptocurrencies including the highest level in Ethereum which is important for market risks and provides useful information to investors and policymakers in the management of digital assets. [26] analyzed bitcoin prediction literature dealing with statistical, neural, and hybrid techniques and employed LSTMs and reinforcement learning when addressing market volatility and proposed a new direction for further research – a hybrid or AutoML technique.
[54] developed a new mixed machine learning model known as Bi-LSTM-GRU-BERT-VADER (BLGBV) to enhance the efficiency of cryptocurrency price prediction. For the price forecast, and for which historical data permitted accurate predictions, a Bi-LSTM and a GRU model were employed. [46] introduced neutrosophic I convergent difference sequence spaces using a modulus function and explored their topology. [32] introduced the Intuitionistic Fuzzy Chebyshev Goal Programming (IFCGP) model to solve multi-level multi-objective linear fractional optimization problems in neutrosophic conditions with less variability. [56] proposed $\alpha\beta$-Rough-I-statistical convergence of order η and analyzed its behavior in neutrosophic normed spaces. [28] utilized neutrosophic programming to maximize the Canadian SDGs 2030 objectives that include GDP, employment, carbon footprint, and electricity usage. In GDP and employment, Latin America was advancing; however, emissions and consumption were beyond objectives, indicating a similar challenge of managing growth and sustainability. In this work, in order to forecast the prices of BTC, ETH, BNB, and SOL in the period of 36 months, a hybrid model combining fuzzy logic and machine learning is suggested. In order to enhance the predictions and cope up with the vagueness and indefiniteness in the data, neutrosophic programming will then be applied. As this volatile market continues to gain traction and burgeon with more studies on consolidated forecasts aiming at validating the effectiveness of the complicated modelling techniques, this work will seek to contribute to this mass-growing literature of Cryptocurrency forecast and compare the results generated by our Hybrid model with the standard fuzzy goal programming approach.

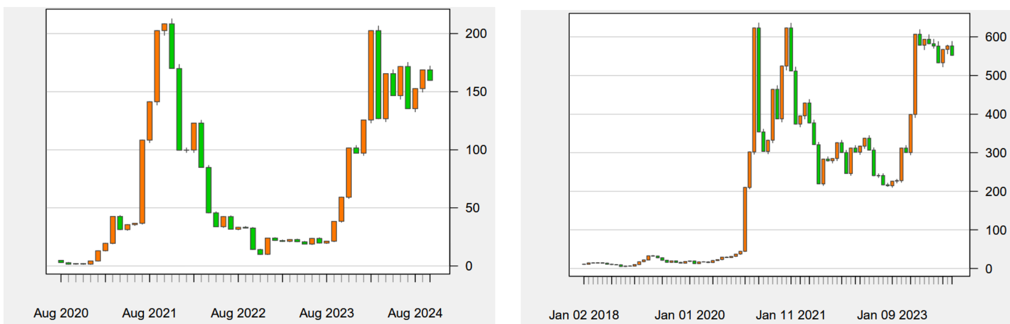
2. Overview of Major Cryptocurrencies: Bitcoin, Ethereum, Solana, and BNB

Cryptocurrencies have revolutionized the digital economy, each offering unique features and use cases. Bitcoin, the pioneer of decentralized digital currencies, introduced the concept of blockchain technology to facilitate peer-to-peer (P2P) transactions without intermediaries. As a decentralized currency, Bitcoin relies on its blockchain to enable secure and transparent transactions, making it a benchmark for the crypto market [38]. Ethereum, on the other hand,



(A) Bitcoin plot (B) Ethereum coin plot.

FIGURE 1. (a) Graphical representation of Bitcoin blockchain. (b) Graphical representation of Ethereum blockchain



(A) Solana coin plot (B) BNB coin plot.

FIGURE 2. (a) Graphical representation of Solana blockchain. (b) Graphical representation of BNB blockchain

expanded the scope of blockchain technology by introducing smart contracts, which allow for programmable agreements between users. Its native token, Ether (ETH), powers transactions and operations within the Ethereum network, fostering the growth of decentralized applications (dApps) and the decentralized finance (DeFi) ecosystem [30]. In contrast, Solana stands out as a high-performance blockchain designed for scalability and speed. It utilizes a unique Proof-of-History (PoH) consensus mechanism, enabling extremely high transaction throughput and low latency. This makes Solana an attractive platform for dApps and DeFi projects that require fast and efficient processing [44]. Meanwhile, BNB (Binance Coin) serves as the native token of the Binance ecosystem. Initially created to reduce trading fees on the Binance exchange, BNB has evolved to play a central role in the Binance Smart Chain (BSC), supporting dApps, token transfers, and low-cost transactions. Its integration with BSC has further solidified its utility in the broader crypto space [25].

3. Cryptocurrency model

Cryptocurrency model is a distributed an electronic money framework that depends on a cryptographic number conversation code system called the blockchain so as to affirm, confirm and record exchanges. It eliminates the involvement of the third party through which two parties are usually required to transact business [43]. Being based on cryptographic techniques for decentralisation, openness and for having tamper–proof functionality, the Cryptocurrencies helps in performing many financial operations like payment, investments, smart contracts in decentralized global interconnected network [36].

3.1. *Mathematical model for the cryptocurrency*

The mathematical model in trading of cryptocurrencies is meant to determine the distribution of resources and the highest returns in Solana (SOL), Binance Coin (BNB), Ethereum (ETH) and Bitcoin among others. Below are the objective functions and constraints defining the model,

$$Z_1 = \sum_{j=1}^{10} \left(\frac{\text{SOL}}{X_e}\right)_j x_j. \tag{3.1}$$

$$Z_2 = \sum_{j=1}^{10} \left(\frac{\text{BNB}}{X_e}\right)_j x_j. \tag{3.2}$$

$$Z_2 = \sum_{j=1}^{10} \left(\frac{\text{ETH}}{X_e}\right)_j x_j. \tag{3.3}$$

$$Z_4 = \sum_{j=1}^{10} x_j e_j \tag{3.4}$$

subject to the Constraints:

$$\sum_{j=1}^{10} (\text{SOL})_j \, x_j^e \leq (\text{SOL})_g \tag{3.5}$$

$$\sum_{j=1}^{10} (\text{BNB})_j \, x_j^e \leq (\text{BNB})_g \tag{3.6}$$

$$\sum_{j=1}^{10} (\text{ETH})_j \, x_j^e \leq (\text{ETH})_g \tag{3.7}$$

$$\sum_{j=1}^{10} x_j^e \leq e_g \tag{3.8}$$

$$e_j \leq x_j^e \leq e_{gj}, \quad \forall j = 1, 2, \ldots, 10. \tag{3.9}$$

Here, the following symbols Z_1 optimize the per-capita Solana coin, and Z_2 optimize the per-capita BNB coin and Z_3 is ethereum and Z_4 is optimize the bitcoin. The first constraint is the restriction due to the overall solana coin. and so on. The optimization of per-capita cryptocurrency output concerning workforce participation serves as each objective function to establish the efficient Bitcoin-equivalent value generation through SOL, BNB, ETH. Here, the following symbols are used:

- Objective function Z_1 optimizes the solana coin across the j-th month.
- Objective function Z_2 optimizes the BNB coin across the j-th month.
- Objective function Z_3 optimizes the ethereum across the j-th month.
- Objective function Z_4 optimizes the bitcoin across the j-th month.
- The first constraint places restrictions due to the overall solana coin.
- The second constraint places restrictions due to the overall BNB coin.
- The third constraint places restrictions due to the overall ethereum coin.
- The fourth constraint places restrictions due to the overall bitcoin coin.

4. Methodology

In this section, we discuss both forecasting and optimization techniques for analyzing the cryptocurrency market. We employed a hybrid modeling approach that integrates machine learning methods and Neutrosophic modeling to optimize the future predictive prices of cryptocurrency data.

4.1. *Hybrid modeling*

Hybrid modeling for time series analysis is a technique that combines multiple forecasting models to leverage their respective strengths and minimize their individual weaknesses. The idea is that no single model is universally optimal for all types of time series data. By combining models, hybrid modeling aims to improve accuracy, robustness, and reliability in forecasting [16]. Hybrid models integrate different forecasting approaches, such as statistical, machine learning, or deep learning models such as auto-regressive integrated moving average (ARIMA), trigonometric Box-Cox arma trend seasonal (TBATS) and neural network model.

Hybrid modeling captures both linear and non-linear patterns and reduces forecasting errors compared to standalone models. It integrate any combination of models, from statistical to machine learning [7]. Let y_t represent the observed time series, and suppose we use n individual models $(M_1, M_2 ... M_n)$ to generate forecasts. The hybrid forecast $\hat{y}_t^{hybrid}$ can be constructed as:

$$\hat{y}_t^{hybrid} = \sum_{i=1}^{n} w_i \hat{y}_{t,i}, \tag{4.1}$$

where, $\hat{y}_{t,i}$ Forecast from model M_i and w_i weight assigned to the model M_i.

4.1.1. *ARIMA (Auto-Regressive Integrated Moving Average)*

The ARIMA model is a popular method for analyzing and forecasting time series data. The `auto.arima` function in R automatically selects the best ARIMA model by optimizing the parameters (p, d, q) [33]. The components of ARIMA are follows

- **Autoregressive**: Captures dependencies between the current observation and its lagged values.

$$\phi(B)y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p}, \tag{4.2}$$

  where, B is the backshift operator, $\phi^k y_t = y_{t-k}$ and $\phi(B)$ is the AR polynomial.
- **Differencing (I)**: Makes the series stationary by removing trends.

$$y_t^{(d)} = (1-B)^d y_t, \tag{4.3}$$

- **Moving Average (MA)**: Models the relationship between the current observation and past forecast errors.

$$\theta(B)\epsilon_t = \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + ... + \theta_p\epsilon_{t-p}, \tag{4.4}$$

The combined model of ARIMA equation is:

$$\phi(B)(1-B)^d y_t = \theta(B)\epsilon_t. \tag{4.5}$$

4.1.2. *TBATS (Trigonometric, Box-Cox, ARMA, Trend, Seasonal)*

The TBATS model is designed to handle time series with complex seasonality, such as multiple or non-integer seasonal patterns [45].The components of TBATS model is given

- **Box-Cox Transformation**: The Box-Cox Transformation is a valuable preprocessing step that simplifies the analysis of time series or other datasets with non-constant variance and non-normality. Reduces heteroscedasticity (non-constant variance) by transforming the data so that the variance becomes approximately constant across time i.e. it stabilizes variance [24].
  The Box-Cox Transformation is defined as:

$$y_t^{(\lambda)} = \begin{cases} \frac{y_t^\lambda - 1}{\lambda}, & \lambda \neq 0, \\ \ln(y_t), & \lambda = 0. \end{cases} \tag{4.6}$$

  where, y_t is the original data $y_t > 0$, λ be the transformation parameter and $y_t^{(\lambda)}$ transformed data. λ controls the nature of the transformation:
  (1) $\lambda = 1$: No transformation (identity function).
  (2) $\lambda = 0$: Logarithmic transformation ($\ln(y_t)$).
  (3) $\lambda < 1$: Compresses large values more than small ones (e.g., square root, cube root).

(4) $\lambda > 1$: Expands large values.

- **Trend Component**: Includes a level l_t which captures the smoothed level of the time series at time t also models the underlying trend (upward, downward, or stable movement) and trend b_t represents the rate of change or slope of the trend at time t also adds flexibility to the trend component by allowing it to adjust dynamically over time [11].

$$\begin{aligned} l_t &= \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}), \\ b_t &= \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}. \end{aligned} \tag{4.7}$$

- **Seasonal Component**: Modeled using Fourier terms for flexibility which captures periodic variations in the data. TBATS supports complex seasonal patterns, including multiple and non-integer seasonality, making it suitable for a wide variety of time series [49].

$$s_t = \sum_{k=1}^{K} \gamma_k \sin\left(\frac{2\pi kt}{m}\right) + \delta_k \cos\left(\frac{2\pi kt}{m}\right). \tag{4.8}$$

  Where, m is the seasonal period, and K is the number of harmonics.

- **ARMA Errors**: Represents the random noise or deviations from the predicted values and captures residual dependencies using an ARMA model.

$$\epsilon_t = \phi(B)\epsilon_{t-1} + \theta(B)\nu_t. \tag{4.9}$$

  where ν_t is white noise.

The TBATS model breaks down the time series into interpretable components and its equation given as

$$y_t^{(\lambda)} = l_t + b_t + s_t + \epsilon_t. \tag{4.10}$$

4.1.3. *Neural Network Model(NNETAR)*

The neural network model is a type of non-linear time series forecasting model that combines principles of autoregression with artificial neural networks (ANNs). It is particularly effective at capturing non-linear dependencies and complex patterns in time series data, which traditional statistical methods like ARIMA might fail to address [41]. The components of neural network model is as follows

- **Auto-Regressive Component**: The model uses lagged values of the time series as predictors and for a given time series y_t, the input at time t is a vector of past values [17].

$$X_t = \{y_{t-1}, y_{t-2}, \ldots, y_{t-p}\}.$$

- **Neural Network Structure**: A feed forward neural network with at least one hidden layer is used to model the relationship between the lagged inputs and the current output also this allows the model to capture complex, non-linear patterns in the data [40].
  (1) **Hidden layer**: The input vector X_t is passed through a hidden layer with H neurons. Each neuron applies a non-linear activation function (e.g., sigmoid, ReLU) to a weighted sum of the inputs:
  $$z_j = \sigma\left(\sum_{i=1}^{p} w_{ji} y_{t-i} + b_j\right) \quad \text{for } j = 1, 2, \ldots, H. \tag{4.11}$$
  where, w_{ji} weight connecting input y_{t-i} to hidden neuron j, b_j term for neuron j and σ be the activation function e.g., sigmoid: $\sigma(x) = \frac{1}{1+e^{-x}}$.
  (2) **Output layer**: The outputs of the hidden layer are combined to produce the forecasted value $\hat{y}_t$
  $$\hat{y}_t = \sum_{j=1}^{H} v_j z_j + c, \tag{4.12}$$
  where, ν_j be the weight connecting hidden neuron $j$to the output layer and c be the bias term for the output layer.
- **Forcaste**: After training the model on historical data, it can be used to forecast future values by recursively predicting the next value using previously forecasted values.

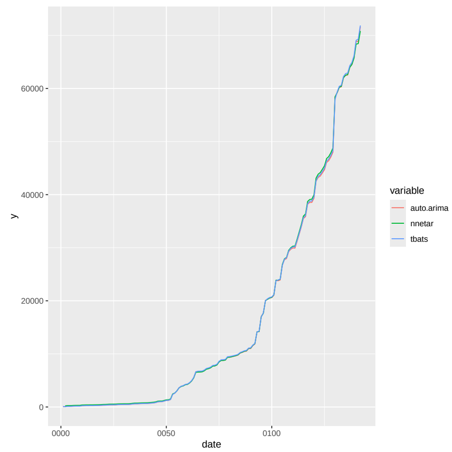
4.2. *Modeling and forecasting of Bitcoin price*

Hybrid modeling for Bitcoin price forecasting involves combining machine learning (ML) techniques with traditional statistical models to enhance predictive accuracy. Given Bitcoin's highly volatile nature, a robust approach typically integrates deep learning, time series models, and Bayesian methods. Table (1) presents the forecasted prices of the Bitcoin for the next 36

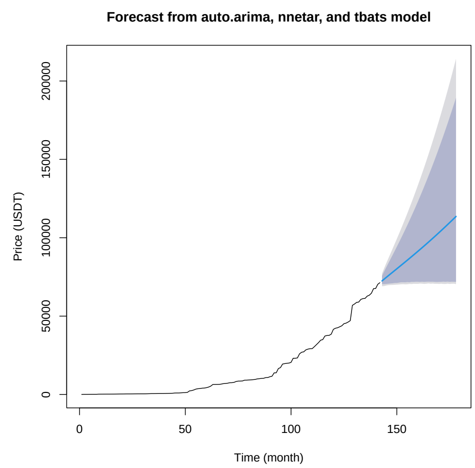
Table 1. Forecasted Value of Bitcoin for the Next 36 Months from December 2024 using Hybrid Modeling of Machine Learning

Time	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
143	72465.28	70427.57	75650.07	69086.67	77074.31
144	73579.54	70419.56	78438.77	69161.19	80479.91
145	74686.94	70559.30	81094.47	69425.67	83670.84
...	...	...	...	...	...
177	112263.72	71782.06	185097.33	70644.37	208969.76
178	113580.87	72045.74	189322.69	70560.98	214197.97

months. Using hybrid modeling, we extended the available actual price data, which covers 143 months since the coin's launch. The forecast plot depicts these projections, with shaded regions indicating the 80% and 95% confidence intervals, reflecting the widening range of potential future values.



(A) Model plot.



(B) Forecast plot.

FIGURE 3. (a) Model Fitting for Bitcoin: Fitted values of ARIMA, TBATS, and neural network models in the hybrid framework.(b) 36-Month Bitcoin Forecast: Hybrid model projection showing an increasing trend with shaded region 80% and 90% confidence interval.

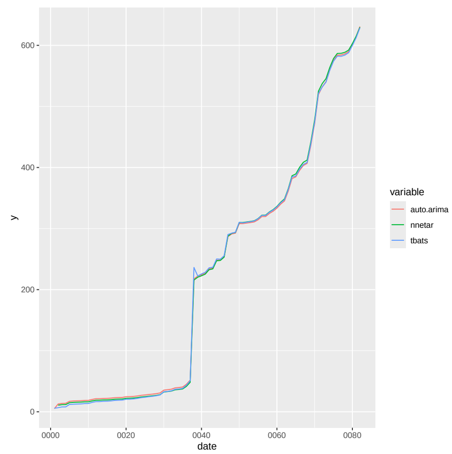
4.3. *Modeling and forecasting of BNB price*

Hybrid modeling combines different statistical, econometric, and machine learning techniques to improve forecasting accuracy. For BNB coin price prediction, we have integrated machine learning with time series models. Table (2) presents the forecasted prices of the BNB

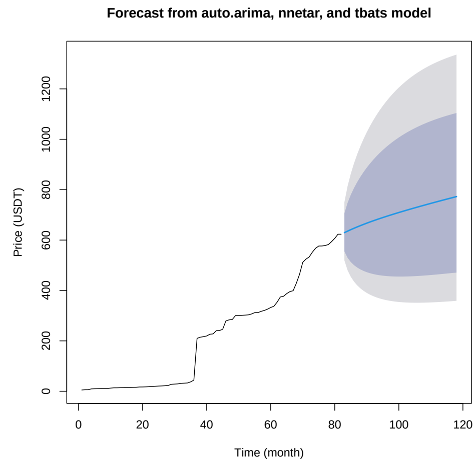
TABLE 2. Forecasted Value of Binance Coin (BNB) for the Next 36 Months from December 2024 using Hybrid Modeling of Machine Learning

Time	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
83	629.48	555.28	706.22	519.31	750.32
84	635.51	529.57	747.74	480.00	814.17
85	641.30	512.84	779.69	454.26	863.27
...	...	...	...	...	...
117	769.29	469.87	1100.93	357.97	1331.84
118	772.57	471.18	1104.31	358.93	1336.00

coin for the next 36 months. Using hybrid modeling, we extended the available actual price data, which covers 82 months since the coin's launch. The forecast plot illustrates these projections, with the shaded regions representing the 80% and 95% confidence intervals, indicating the range of possible future values.



(A) Model plot.



(B) Forecast plot.

FIGURE 4. (a) Model Fitting for BNB: Fitted values of ARIMA, TBATS, and neural network models in the hybrid framework.(b) 36-Month BNB Forecast: Hybrid model projection showing an increasing trend with shaded region 80% and 90% confidence interval.

4.4. *Modeling and forecasting of Ethereum price*

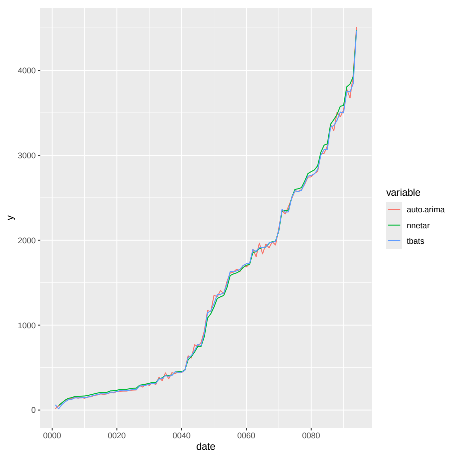
Hybrid modeling and forecasting of Ethereum (ETH) using machine learning involves combining multiple models to capture different aspects of its price movements, volatility, and market trends. Given your background in Bayesian modeling and time series forecasting, you can explore various hybrid approaches that integrate statistical models with machine learning techniques. Table (3) presents the forecasted prices of the Etherium coin for the next 36 months. Using hybrid modeling, we extended the available actual price data, which covers 82 months since the coin's launch. The forecast plot depicts these projections, with shaded regions indicating the 80% and 95% confidence intervals, reflecting the widening range of potential future values.

4.5. *Modeling and forecasting of Solana price*

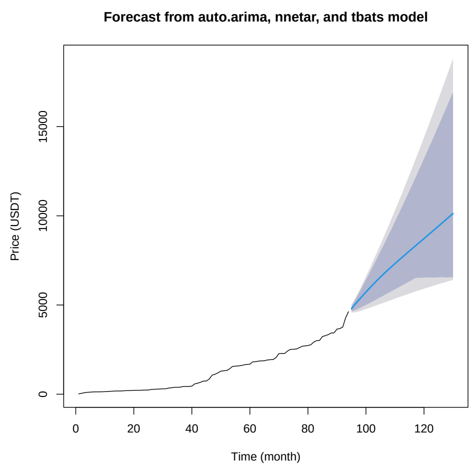
Hybrid modeling for forecasting Solana (SOL) prices using machine learning involves combining multiple models to improve prediction accuracy. This approach leverages the strengths of different models, such as statistical, machine learning (ML), and deep learning techniques.

Table 3. Forecasted Value of Ethereum (ETH) for the Next 36 Months from December 2024 using Hybrid Modeling of Machine Learning

Time	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
95	4791.95	4645.86	4921.45	4574.04	4995.50
96	4991.37	4698.47	5203.71	4585.37	5281.90
97	5182.07	4767.17	5500.43	4620.62	5600.45
...	...	...	...	...	...
129	9988.33	6566.32	16555.78	6358.24	18361.75
130	10130.56	6567.19	16937.90	6403.95	18816.39



(a) Model plot.



(b) Forecast plot.

Figure 5. (a) Model Fitting for Ethereum: Fitted values of ARIMA, TBATS, and neural network models in the hybrid framework.(b) 36-Month Ethereum Forecast: Hybrid model projection showing an increasing trend with shaded region 80% and 90% confidence interval.

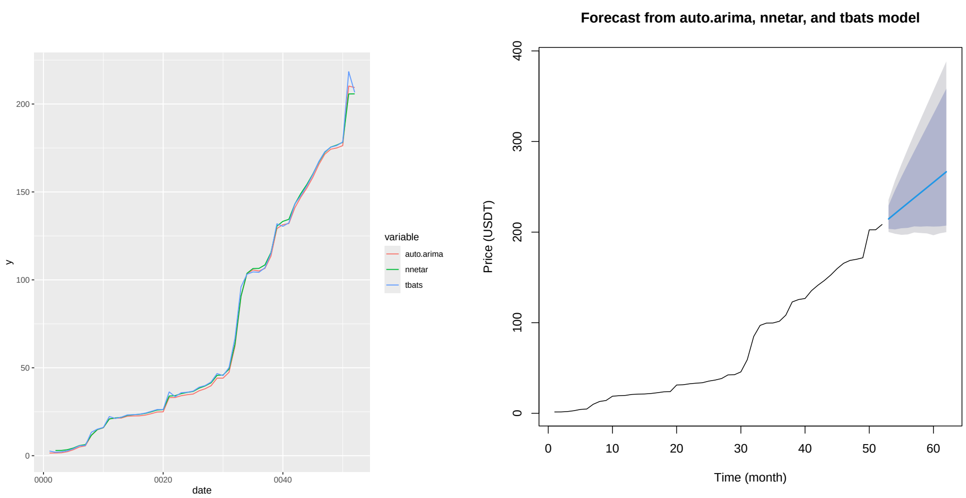
Table (4) presents the forecasted prices of the BNB coin for the next 36 months. Using hybrid modeling, we extended the available actual price data, which covers 82 months since the coin's launch. The forecast plot depicts these projections, with shaded regions indicating the 80% and 95% confidence intervals, reflecting the widening range of potential future values.

4.6. *Performance Metrics of Models*

The table displays the accuracy metrics of hybrid machine learning models applied to four cryptocurrencies Bitcoin, BNB, Ethereum, and Solana. Each cryptocurrency is evaluated using three models auto.arima, neural network, and tbats. The metrics used to assess model performance include ME (Mean Error), which measures the average error between predicted

Table 4. Forecasted Value of Solana (SOL) for the Next 36 Months from December 2024 using Hybrid Modeling of Machine Learning

Time	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
53	214.52	203.50	229.30	199.35	235.56
54	220.38	202.94	245.99	198.59	256.54
55	226.19	203.80	260.90	197.92	274.70
...	...	...	...	...	...
87	428.39	208.16	748.06	202.65	820.20
88	435.60	207.73	765.59	200.75	839.19



(A) Model plot. (B) Forecast plot.

Figure 6. (a) Model fitting for Solana: Fitted values of ARIMA, TBATS, and neural network models in the hybrid framework.(b) 36-Month Solana Forecast: Hybrid model projection showing an increasing trend with shaded region 80% and 90% confidence interval.

and actual values, with a value closer to zero indicating better accuracy. RMSE (Root Mean Squared Error) represents the square root of the average squared errors, where lower values indicate better model performance. MAE (Mean Absolute Error) quantifies the average absolute difference between predicted and actual values, with lower values being preferable. MPE (Mean Percentage Error) reflects the average percentage error, where a value closer to zero is desirable. MAPE (Mean Absolute Percentage Error) measures the average absolute percentage error, with lower values signifying higher accuracy. Lastly, MASE (Mean Absolute Scaled Error) compares the model's accuracy to a naive forecast, where values less than 1 indicate the model outperforms the naive forecast.

TABLE 5. Performance Metrics of Hybrid Model Across Cryptocurrencies

Crypto	**Model**	**ME**	**RMSE**	**MAE**	**MPE**	**MAPE**	**MASE**
Bitcoin	auto.arima	124.1092	948.8838	383.9357	0.52996	3.886719	0.759271
	neural network	-5.0972	922.4600	447.3368	-14.5813	17.30695	0.884653
	tbats	35.2428	926.5848	401.0618	-1.5997	5.926597	0.793139
BNB	auto.arima	-0.00003	19.5852	8.6717	-17.9036	20.89222	1.136194
	neural network	-0.0158	19.5397	7.7926	-11.3514	14.33592	1.021017
	tbats	1.0184	19.3625	7.4674	-5.4046	8.769035	0.978398
Ethereum	auto.arima	7.2419	61.4555	39.4665	-0.1801	3.91642	0.795655
	neural network	-0.2739	68.9823	45.3018	-2.6759	5.846399	0.913294
	tbats	9.3028	69.4141	39.4940	-1.9606	7.426422	0.796209
Solana	auto.arima	0.9548	5.8343	3.1085	2.7283	6.419634	0.766192
	neural network	-0.0086	5.5993	3.4076	-3.7522	9.624483	0.839929
	tbats	-0.3477	5.6019	3.3867	-3.3018	8.796377	0.834770

The Table (5) presents the performance metrics of three hybrid machine learning models auto.arima, neural network, and tbats applied to four cryptocurrencies: Bitcoin, BNB, Ethereum, and Solana. Each model's accuracy is assessed using multiple error metrics, including Mean Error (ME), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE), and Mean Absolute Scaled Error (MASE). For Bitcoin, auto.arima exhibits the lowest RMSE (948.88) and MAPE (3.89), indicating better predictive accuracy than the other models. However, neural network shows a much higher MAPE (17.31), suggesting less reliable predictions, while tbats performs moderately with a lower ME. In the case of BNB, tbats achieves the best performance with the lowest MAE (7.47), MAPE (8.77), and MASE (0.98), demonstrating improved accuracy compared to auto.arima and neural network. Ethereum follows a similar trend, where auto.arima delivers the lowest RMSE (61.46), MAPE (3.92), and MASE (0.80), making it the most effective model for this cryptocurrency. On the other hand, neural network exhibits the highest RMSE (68.98) and MAPE (5.85), indicating weaker performance. For Solana, auto.arima again emerges as the most accurate model with the lowest RMSE (5.83), MAE (3.11), and MASE (0.77), suggesting its superiority in forecasting Solana prices. While neural network and tbats yield similar RMSE values (5.60), tbats achieves a slightly lower MAPE (8.80) compared to neural network (9.62). Overall, auto.arima demonstrates the most consistent predictive performance across cryptocurrencies, particularly for Bitcoin, Ethereum, and

Solana, while tbats performs best for BNB. We assume that the forecasted values present in table are fuzzy in nature. We defuzzified the fuzzy values to crisp values using the defuzzification formula in 5.2. As a result, we obtain the crisp value for each blockchain.

Table 6. Forecasted Values of Selected Cryptocurrencies in USD

Cryptocurrency	**Value ($)**
Solana	612.36
BNB	940.03
Ethereum	14067.51
Bitcoin	157444.33

5. **Fuzzy Set (FS)**

A fuzzy set X within a universe W [52] , where each element is denoted by w, is characterized by a membership function $\mu_X(w)$ that maps elements from W to the interval $[0, 1]$. This function assigns to each element $w \in W$ a membership degree $\mu_X(w) \in [0, 1]$, indicating the extent of w's membership in the fuzzy set X. Specifically, $\mu_X(w) = 0$ signifies that w is not a member of X, $\mu_X(w) = 1$ indicates full membership, and values between 0 and 1 represent partial membership. An extension of a fuzzy number specified by the following five parameters is called a pentagonal fuzzy number (PFN): (a, b, c, d, e). A pentagonal fuzzy number's piecewise linear membership function, $\mu(x)$, usually looks like this:

$$\mu(x) = \begin{cases} 0 & \text{if } x \leq a \text{ or } x \geq e, \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b, \\ 1 & \text{if } b \leq x \leq c, \\ \frac{e-x}{e-d} & \text{if } d \leq x \leq e, \\ 0 & \text{if } x \notin [a, e]. \end{cases} \tag{5.1}$$

The process of converting a fuzzy number into a precise or crisp value is known as defuzzification [34] for a pentagonal fuzzy number, the de-fuzzified value is determine by the formula,

$$D = \frac{a + b + 5c + d + e}{9}. \tag{5.2}$$

6. **Neutrosophic Sets: A Generalization Framework**

As introduced by [42], neutrosophic sets extend classical, fuzzy, and intuitionistic fuzzy sets by incorporating three distinct membership degrees for each element: truth (T), uncertainty (I), and falsity (F). Let X represent a universal set such that $x \in X$. A neutrosophic set B

within X is characterized by three membership functions: $T_B(x)$, $I_B(x)$, and $F_B(x)$, and is represented as:

$$B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}, \tag{6.1}$$

where $T_B(x)$, $I_B(x)$, and $F_B(x)$ are real subsets within the range $]0^-, 1^+[$. These membership functions are defined as $T_B(y) : Y \to]0^-, 1^+[$, $I_B(y) : Y \to]0^-, 1^+[$, and $F_B(x) : Y \to]0^-, 1^+[$. Notably, the sum of $T_B(x)$, $T_B(x)$, and $F_B(x)$ is unrestricted, leading to:

$$0^- \leq \sup T_B(x) + I_B(x) + \sup F_B(x) \leq 3^+. \tag{6.2}$$

For practical purposes, a single-valued neutrosophic set (SVNS) B over X is expressed as:

$$B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}, \tag{6.3}$$

where $T_B(x), I_B(x), F_B(x) \in [0, 1]$ and $0 \leq T_B(x) + I_B(x) + F_B(x) \leq 3$, for all $x \in X$.

7. Types of Neutrosophic Optimization Models

Neutrosophic optimization models can be categorized into two primary types, **maximization-based models**, which aim to optimize objectives by maximizing desired outcomes while handling indeterminacy and uncertainty, and **minimization-based models**, which focus on reducing costs, risks, or inefficiencies while considering neutrosophic parameters. These models leverage neutrosophic logic to handle imprecise, inconsistent, and incomplete information, making them suitable for complex decision-making scenarios in uncertain environments.

1. Maximization-Based Neutrosophic Optimization

In maximization-based neutrosophic programming, the upper and lower bounds are defined as follows:

- Truth membership:

$$U_k^T = U_k,\ L_k^T = L_k, \tag{7.1}$$

- Uncertainty membership:

$$U_k^I = U_k^T,\ L_k^I = L_k^T + s_k, \tag{7.2}$$

- Falsity membership:

$$U_k^F = L_k^T + t_k,\ L_k^F = L_k^T, \tag{7.3}$$

where $k = 1, 2, \ldots, n$, and s_k and t_k represent predefined tolerance parameters for uncertainty and falsity, respectively. The membership functions for truth, uncertainty, and falsity are expressed as:

$$\mu_k^T = \begin{cases} 0 & \text{if } Z_k < L_k^T, \\ \frac{Z_k - L_k^T}{U_k^T - L_k^T} & \text{if } L_k^T \leq Z_k \leq U_k^T, \\ 1 & \text{if } Z_k > U_k^T. \end{cases} \tag{7.4}$$

$$\sigma_k^I = \begin{cases} 0 & \text{if } Z_k < L_k^I, \\ \frac{Z_k - L_k^I}{U_k^I - L_k^I} & \text{if } L_k^I \leq Z_k \leq U_k^I, \\ 1 & \text{if } Z_k > U_k^I. \end{cases} \tag{7.5}$$

$$\gamma_k^F = \begin{cases} 0 & \text{if } Z_k < L_k^F, \\ \frac{Z_k - L_k^F}{U_k^F - L_k^F} & \text{if } L_k^F \leq Z_k \leq U_k^F, \\ 1 & \text{if } Z_k > U_k^F. \end{cases} \tag{7.6}$$

2. Minimization-Based Neutrosophic Optimization

For minimization-based neutrosophic programming, the bounds are:

- Truth membership:

$$U_k^T = U_k, \ L_k^T = L_k, \tag{7.7}$$

- Uncertainty membership:

$$U_k^I = L_k^T + s_k, \ L_k^I = L_k^T, \tag{7.8}$$

- Falsity membership:

$$U_k^F = U_k^T, \ L_k^F = L_k^T + t_k, \tag{7.9}$$

Simplified Neutrosophic Model for Crypto-Currencies

$$\text{Maximize } \alpha + \beta - \gamma \tag{7.10}$$

Subject to:

$$Z_1 = \sum_{j=1}^{10} \left(\frac{\text{SOL}}{X_e}\right)_j x_j. \tag{7.11}$$

$$Z_2 = \sum_{j=1}^{10} \left(\frac{\text{BNB}}{X_e}\right)_j x_j. \tag{7.12}$$

$$Z_2 = \sum_{j=1}^{10} \left(\frac{\text{ETH}}{X_e}\right)_j x_j. \tag{7.13}$$

$$Z_4 = \sum_{j=1}^{10} x_j e_j \tag{7.14}$$

Constraints:

$$\sum_{j=1}^{10} (\text{SOL})_j \, x_j^e \leq (\text{SOL})_g \tag{7.15}$$

$$\sum_{j=1}^{10} (\text{BNB})_j \, x_j^e \leq (\text{BNB})_g \tag{7.16}$$

$$\sum_{j=1}^{10} (\text{ETH})_j \, x_j^e \leq (\text{ETH})_g \tag{7.17}$$

$$\sum_{j=1}^{10} x_j^e \leq e_g \tag{7.18}$$

$$e_j \leq x_j^e \leq e_{gj}, \quad \forall j = 1, 2, \ldots, 10 \tag{7.19}$$

Membership Function Constraints

$$Z_k + (U_k^T - L_k^T)\alpha \leq U_k^T \tag{7.20}$$

$$Z_k + (U_k^I - L_k^I)\beta \leq U_k^I \tag{7.21}$$

$$Z_k - (U_k^F - L_k^F)\gamma \leq L_k^F \tag{7.22}$$

Additional constraints:

$$\alpha + \beta + \gamma \leq 3, \quad \alpha \geq \beta, \quad \alpha \geq \gamma \tag{7.23}$$

$$\alpha, \beta, \gamma \in (0, 1)$$

8. **Case study**

The cryptocurrency market has grown to be a prominent financial industry in recent years, drawing interest from investors, scholars, and decision-makers alike. There are dozens of cryptocurrencies available, and the market is known for its extreme volatility, quick development, and intricate web of interrelated factors that affect price changes. Because of their significant market capitalization, cutting-edge technology, and active community, Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), and Solana (SOL) stand out among these cryptocurrencies. The values of these coins monthly in 2024 are given the following table:

Monthly Data of Major Blockchains

Table (7) presents the month-wise data of four major blockchains: SOL, BNB, ETH, and BTC, along with the maximum BTC price observed during each month. This data has been collected and verified from various reliable sources. The Table (7) provides valuable insights into the monthly price trends of these blockchains, serving as a foundation for further analysis and forecasting studies. Now, the final model is formulated as,

Table 7. Monthly Data for 2024 in USD

Month	**SOL**	**BNB**	**ETH**	**BTC**	**Max BTC**
January	168.75	576.60	2519.14	70281.80	77309.98
February	152.63	567.51	2603.48	63339.20	69673.12
March	135.34	532.90	2513.56	58978.60	64876.46
April	171.68	576.40	3231.84	64626.00	71088.60
May	146.56	582.30	3437.84	62754.30	69029.73
June	165.58	593.80	3762.66	67530.10	74283.11
July	126.74	578.41	3014.41	60666.60	66733.26
August	202.51	606.89	3647.09	71332.00	78465.20
September	125.59	399.10	3339.26	61169.30	67286.23
October	97.04	300.50	2283.14	42580.50	46838.55

Data sources: https://www.binance.com/en-INBinance, https://in.investing.com/Investing.com, and https://finance.yahoo.com/Yahoo Finance.

Data sources: Binance, Investing.com, and Yahoo Finance.

Table 8. Cryptocurrency Forecasted vs Current Values

Cryptocurrency	**Current Value (USD)**	**Forecasted Value (USD)**
Solana	159.70	612.36
BNB	551.98	940.03
Ethereum	2423.64	14067.51
Bitcoin	67623.30	157444.33

Objective functions of selected cryptocurrency model:

$$Z_1 = 0.002401 \cdot x_1 + 0.002410 \cdot x_2 + 0.002295 \cdot x_3 + 0.002657 \cdot x_4 + 0.002335 \cdot x_5 + 0.002452 \cdot x_6 + 0.002089 \cdot x_7 + 0.002839 \cdot x_8 + 0.002053 \cdot x_9 + 0.002279 \cdot x_{10},$$

$$Z_2 = 0.008204 \cdot x_1 + 0.008960 \cdot x_2 + 0.009035 \cdot x_3 + 0.008919 \cdot x_4 + 0.009279 \cdot x_5 + 0.008793 \cdot x_6 + 0.009534 \cdot x_7 + 0.008508 \cdot x_8 + 0.006525 \cdot x_9 + 0.007057 \cdot x_{10},$$

$$Z_3 = 0.035843 \cdot x_1 + 0.041104 \cdot x_2 + 0.042618 \cdot x_3 + 0.050008 \cdot x_4 + 0.054783 \cdot x_5 + 0.055718 \cdot x_6 + 0.049688 \cdot x_7 + 0.051128 \cdot x_8 + 0.054590 \cdot x_9 + 0.053619 \cdot x_{10},$$

$$Z_4 = 1.0 \cdot x_1 + 1.0 \cdot x_2 + 1.0 \cdot x_3 + 1.0 \cdot x_4 + 1.0 \cdot x_5 + 1.0 \cdot x_6 + 1.0 \cdot x_7 + 1.0 \cdot x_8 + 1.0 \cdot x_9 + 1.0 \cdot x_{10}.$$

Constraints of selected cryptocurrency model:

$$Z_1 = 0.002401 \cdot x_1 + 0.002410 \cdot x_2 + 0.002295 \cdot x_3 + 0.002657 \cdot x_4 + 0.002335 \cdot x_5 + 0.002452 \cdot x_6 + 0.002089 \cdot x_7 + 0.002839 \cdot x_8 + 0.002053 \cdot x_9 + 0.002279 \cdot x_{10} \leq 612.36,$$

$$Z_2 = 0.008204 \cdot x_1 + 0.008960 \cdot x_2 + 0.009035 \cdot x_3 + 0.008919 \cdot x_4 + 0.009279 \cdot x_5 + 0.008793 \cdot x_6 + 0.009534 \cdot x_7 + 0.008508 \cdot x_8 + 0.006525 \cdot x_9 + 0.007057 \cdot x_{10} \leq 940.03,$$

$$Z_3 = 0.035843 \cdot x_1 + 0.041104 \cdot x_2 + 0.042618 \cdot x_3 + 0.050008 \cdot x_4 + 0.054783 \cdot x_5 + 0.055718 \cdot x_6 + 0.049688 \cdot x_7 + 0.051128 \cdot x_8 + 0.054590 \cdot x_9 + 0.053619 \cdot x_{10} \leq 14067.51,$$

$$Z_4 = 1.0 \cdot x_1 + 1.0 \cdot x_2 + 1.0 \cdot x_3 + 1.0 \cdot x_4 + 1.0 \cdot x_5 + 1.0 \cdot x_6 + 1.0 \cdot x_7 + 1.0 \cdot x_8 + 1.0 \cdot x_9 + 1.0 \cdot x_{10} \leq 157444.33.$$

$$70281.8 \le x_1 \le 77309.98,$$

$$63339.2 \le x_2 \le 69673.12,$$

$$58978.6 \le x_3 \le 64876.46,$$

$$64626.0 \le x_4 \le 71088.6,$$

$$62754.3 \le x_5 \le 69029.73,$$

$$67530.1 \le x_6 \le 74283.11,$$

$$60666.6 \le x_7 \le 66733.26,$$

$$71332.0 \le x_8 \le 78465.2,$$

$$61169.3 \le x_9 \le 67286.23,$$

$$42580.5 \le x_{10} \le 46838.55.$$

Where $x_1, x_2, x_3, \ldots, x_{10}$ represent the number of employees in each sector, and Z_i for $i = 1, 2, 3, 4$ are the different objective goals: The above cryptocurrency model is solved by LINGO 16.0

9. **Results**

The optimal values of all four objective on solving each individually, the payoff matrix is given as follows, For the first objective, i.e., maximization of Solana, the upper and lower

Table 9. Optimal Solutions for Objective Function

Objectives	**Z1**	**Z2**	**Z3**	**Z4**
Max Z1	310.75	940.03	6081.83	117072.00
Max Z2	294.77	940.03	6929.27	134901.00
Max Z3	292.41	940.03	7264.66	133509.30
Max Z4	310.75	939.55	6929.24	134901.00

bounds with the Equations (7.1,7.2,7.3) are as follows:

$$U_1^T = 310.75, \quad L_1^T = 292.41, \tag{9.1}$$

$$U_1^I = 310.75, \quad L_1^I = 292.41 + s_1, \tag{9.2}$$

$$U_1^F = 292.41 + t_1, \quad L_1^F = 292.41. \tag{9.3}$$

Using the Equations (7.4,7.5,7.6), membership functions for objective 1 can be constructed as follows:

$$\mu_1^T = \begin{cases} 0, & \text{if } Z_1 < 292.41 \\ \frac{Z_1 - 292.41}{310.75 - 292.41}, & \text{if } 292.41 \leq Z_1 \leq 310.75 \\ 1, & \text{if } Z_1 > 310.75. \end{cases} \tag{9.4}$$

$$\sigma_1^I = \begin{cases} 0, & \text{if } Z_1 < 292.41 + s_1 \\ \frac{Z_1 - (292.41 + s_1)}{310.75 - 292.41 - s_1}, & \text{if } 292.41 + s_1 \leq Z_1 \leq 310.75 \\ 1, & \text{if } Z_1 > 310.75. \end{cases} \tag{9.5}$$

$$\gamma_1^F = \begin{cases} 0, & \text{if } Z_1 < 292.41 + s_1 \\ \frac{292.41 + t_1 - Z_1}{292.41 - t_1 - 292.41}, & \text{if } 292.41 \leq Z_1 \leq 292.41 + t_1 \\ 1, & \text{if } Z_1 > 292.41 + t_1. \end{cases} \tag{9.6}$$

For the second objective, i.e., maximization of BNB, the upper and lower bounds with the Equations (7.1,7.2,7.3) are as follows:

$$V_2^T = 940.30, \quad L_2^T = 940, \tag{9.7}$$

$$V_2^I = 940.30, \quad L_2^I = 940 + s_2, \tag{9.8}$$

$$V_2^F = 940 + t_2, \quad L_2^F = 940. \tag{9.9}$$

Using the Equations (7.4,7.5,7.6), membership functions for objective 2 can be constructed as follows:

$$\mu_2^T = \begin{cases} 0 & \text{if } Z_2 < 940 \\ \frac{Z_2 - 940}{940.30 - 940} & \text{if } 940 \leq Z_2 \leq 940.30 \\ 1 & \text{if } Z_2 > 940.30 \end{cases} \tag{9.10}$$

$$\sigma_2^I = \begin{cases} 0 & \text{if } Z_2 < 940 + s_2 \\ \frac{Z_2 - (940 + s_2)}{940.30 - 940 - s_2} & \text{if } 940 + s_2 \leq Z_2 \leq 940.30 \\ 1 & \text{if } Z_2 > 940.30 \end{cases} \tag{9.11}$$

$$\gamma_2^F = \begin{cases} 0 & \text{if } Z_2 < 940 + s_2 \\ \frac{940 + t_2 - Z_2}{940 + t_2 - 940} & \text{if } 940 \leq Z_2 \leq 940 + t_2 \\ 1 & \text{if } Z_2 > 940 + t_2 \end{cases} \tag{9.12}$$

For the third objective, i.e., maximization of Ethereum, the upper and lower bounds with the Equations (7.1,7.2,7.3) are as follows:

$$V_3^T = 7264.66, \quad L_3^T = 6081.83, \tag{9.13}$$

$$V_3^I = 7264.66, \quad L_3^I = 6081.83 + s_3, \tag{9.14}$$

$$V_3^F = 6081.83 + t_3, \quad L_3^F = 6081.83. \tag{9.15}$$

Using the Equations (7.4,7.5,7.6), membership functions for objective 3 can be constructed as follows:

$$\mu_3^T = \begin{cases} 0 & \text{if } Z_3 < 6081.83 \\ \frac{Z_3 - 6081.83}{7264.66 - 6081.83} & \text{if } 6081.83 \leq Z_3 \leq 7264.66 \\ 1 & \text{if } Z_3 > 7264.66. \end{cases} \tag{9.16}$$

$$\sigma_3^I = \begin{cases} 0 & \text{if } Z_3 < 6081.83 + s_3 \\ \frac{Z_3 - (6081.83 + s_3)}{7264.66 - 6081.83 - s_3} & \text{if } 6081.83 + s_3 \leq Z_3 \leq 7264.66 \\ 1 & \text{if } Z_3 > 7264.66. \end{cases} \tag{9.17}$$

$$\gamma_3^F = \begin{cases} 0 & \text{if } Z_3 < 6081.83 + s_3 \\ \frac{6081.83 + t_3 - Z_3}{6081.83 + t_3 - 6081.83} & \text{if } 6081.83 \leq Z_3 \leq 6081.83 + t_3 \\ 1 & \text{if } Z_3 > 6081.83 + t_3. \end{cases} \tag{9.18}$$

For the fourth objective, i.e., maximization of Bitcoin, the upper and lower bounds with the Equations (7.1,7.2,7.3) are as follows:

$$V_4^T = 134901, \quad L_4^T = 117072, \tag{9.19}$$

$$V_4^I = 134901, \quad L_4^I = 117072 + s_4, \tag{9.20}$$

$$V_4^F = 117072 + t_4, \quad L_4^F = 117072. \tag{9.21}$$

Using the Equations (7.4,7.5,7.6), membership functions for objective 4 can be constructed as follows:

$$\mu_4^T = \begin{cases} 0 & \text{if } Z_4 < 117072 \\ \frac{Z_4 - 117072}{134901 - 117072} & \text{if } 117072 \leq Z_4 \leq 134901 \\ 1 & \text{if } Z_4 > 134901. \end{cases} \tag{9.22}$$

$$\sigma_4^I = \begin{cases} 0 & \text{if } Z_4 < 117072 + s_4 \\ \frac{Z_4 - (117072 + s_4)}{134901 - 117072 - s_4} & \text{if } 117072 + s_4 \leq Z_4 \leq 134901 \\ 1 & \text{if } Z_4 > 134901. \end{cases} \tag{9.23}$$

$$\gamma_4^F = \begin{cases} 0 & \text{if } Z_4 < 117072 + s_4 \\ \frac{117072+t_4-Z_4}{117072+t_4-117072} & \text{if } 117072 \leq Z_4 \leq 117072 + t_4 \\ 1 & \text{if } Z_4 > 117072 + t_4. \end{cases} \tag{9.24}$$

Finally, the neutrosophic programming model be constructed as follows,

$$\begin{aligned}
\max_{\alpha,\beta,\gamma} \quad & \alpha + \beta - \gamma, \\
\text{subject to} \quad & Z_1 = 0.002401 \cdot x_1 + 0.002410 \cdot x_2 + 0.002295 \cdot x_3 + 0.002657 \cdot x_4 + 0.002335 \cdot x_5 \\
& \quad + 0.002452 \cdot x_6 + 0.002089 \cdot x_7 + 0.002839 \cdot x_8 + 0.002053 \cdot x_9 + 0.002279 \cdot x_{10}, \\
& Z_2 = 0.008204 \cdot x_1 + 0.008960 \cdot x_2 + 0.009035 \cdot x_3 + 0.008919 \cdot x_4 + 0.009279 \cdot x_5 \\
& \quad + 0.008793 \cdot x_6 + 0.009534 \cdot x_7 + 0.008508 \cdot x_8 + 0.006525 \cdot x_9 + 0.007057 \cdot x_{10}, \\
& Z_3 = 0.035843 \cdot x_1 + 0.041104 \cdot x_2 + 0.042618 \cdot x_3 + 0.050008 \cdot x_4 + 0.054783 \cdot x_5 \\
& \quad + 0.055718 \cdot x_6 + 0.049688 \cdot x_7 + 0.051128 \cdot x_8 + 0.054590 \cdot x_9 + 0.053619 \cdot x_{10}, \\
& Z_4 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}, \\
& Z_1 \leq 612.36, \\
& Z_2 \leq 940.03, \\
& Z_3 \leq 14067.51, \\
& Z_4 \leq 157444.33, \\
& 70281.8 \leq x_1 \leq 77309.98, \\
& 63339.2 \leq x_2 \leq 69673.12, \\
& 58978.6 \leq x_3 \leq 64876.46, \\
& 64626.0 \leq x_4 \leq 71088.6, \\
& 62754.3 \leq x_5 \leq 69029.73, \\
& 67530.1 \leq x_6 \leq 74283.11, \\
& 60666.6 \leq x_7 \leq 66733.26, \\
& 71332.0 \leq x_8 \leq 78465.2, \\
& 61169.3 \leq x_9 \leq 67286.23, \\
& 42580.5 \leq x_{10} \leq 46838.55,
\end{aligned}$$

Continue...

$$
\begin{aligned}
&Z_1 - (310.75 - 292.41) \cdot \alpha \leq 310.75,\\
&Z_2 - (940.30 - 940) \cdot \alpha \leq 940.30,\\
&Z_3 - (7264.66 - 6081.83) \cdot \alpha \leq 7264.66,\\
&Z_4 - (134901 - 117072) \cdot \alpha \leq 134901,\\
&Z_1 - (310.75 - 292.41 - s_1) \cdot \beta \geq 292.41 + s_1,\\
&Z_2 - (940.30 - 940 - s_2) \cdot \beta \leq 940 + s_2,\\
&Z_3 - (7264.66 + 6081.83 - s_3) \cdot \beta \leq 6081.83 + s_3,\\
&Z_4 - (134901 - 117702 - s_4) \cdot \beta \leq 117072 + s_4,\\
&Z_1 - (292.41 - t_1 - 292.41) \cdot \gamma \leq 292.41 + t_1,\\
&Z_2 - (940 + t_2 - 940) \cdot \gamma \leq 940 + t_2,\\
&Z_3 - (6081.83 + t_3 - 6081.83) \cdot \gamma \leq 6081.83 + t_3,\\
&Z_4 - (117072 + t_4 - 117072) \cdot \gamma \leq 117072 + t_4,\\
&\alpha \geq \beta,\\
&\alpha \geq \gamma,\\
&\alpha + \beta + \gamma \leq 3.
\end{aligned}
$$

The above blockchain model is solved using optimization software Lingo 16.0. The optimal solution for the Neutrosophic programming are given in Table 10

Table 10. Optimal Values of Selected Cryptocurrencies in USD

Cryptocurrency	**Value ($)**
Solana	293.03
BNB	903.45
Ethereum	5334.30
Bitcoin	104911.50

10. **Results Analysis and Conclusion**

The forecasted values of the four significant blockchains with hybrid modelling using machine learning are shown in Table (1). As we see that the forecasted value of Bitcoin by hybrid modelling using machine learning is $113580.87 in Table (1) and at other levels are different values and also for the rest of coins, and then we let these numbers as a fuzzy numbers, then we defuzzified these values and getting the value of Bitcoin is $157444.33 which is shown in Table (8) and for the rest coins. We will use these values as our goal for the next 36 months. We use neutrosophic programming to achieve all four central coins values up to the next 36, and we get the compromise solution by neutrosophic programming, which is presented in Table (10). Using neutrosophic programming, the current study aimed to predict four leading cryptocurrencies, which include Solana, BNB, Ethereum, and Bitcoin, and compared those predictions to target values derived from the hybrid modeling approach of machine learning. The first objective was to show that neutrosophic programming could provide reliable forecasting while trying to meet the goals set by the targets of the hybrid model.

In this case, Solana, with an initial value of 159.703, was estimated to be valued at 293.0273 with the use of neutrosophic programming. As shown, it increases, but not at the desired rate of the hybrid model, which predicted 612.36. As for BNB, it was at 612.36, and it was expected that the rate would improve to 903.4527 and get nearer to the projected value of 940.03. As for Ethereum, which started with a value of 2423.64, an upward trend was observed; it was forecasted as 5334.299, but it needed more steps to reach the target of 14067.51. Last but not least, let us analyze the case of Bitcoin, which was initially set at 67623.3 and was expected to make a big leap and touch 104911.5, which makes good progress towards the target value of 157444.33.

These results corroborate the potential of neutrosophic programming when it comes to analyzing cryptocurrency markets and the expected substantial increase in this market. The forecast values fit reality market conditions well, further demonstrating the effectiveness of the approach. They are easier to measure and mark the targets of the hybrid model as more realistic, which indicates that they are efficient when it comes to strategic planning and control.

The findings of this research show that by using neutrosophic programming alongside multiple machine learning models, one can overcome the difficulties and contingencies characteristic of cryptocurrency prediction. Altogether, these approaches form a perfect system where accurate and inspiring estimates are reached simultaneously. These results confirm that it is necessary to use sophisticated models for assessing the potential of cryptocurrencies in the context of an always-changing financial environment.

11. Investment in the Crypto Market

Cryptocurrency has attracted intensive market interest since volatility and substantial profit potential have driven its growth. The high volatility in cryptocurrencies carries many dangers for financial investors. The significant cryptocurrencies experience profound price variations within short periods. Cryptocurrencies register fast price variations because of market speculation, regulatory changes, macroeconomic factors, and innovations.

The cryptocurrency market functions without official regulatory oversight around the clock, so it faces amplified price volatility because there is no centralized authority. The uncertain market value of cryptocurrencies increases due to external social media trends, investor perceptions, and the absence of inherent asset worth.

Cryptocurrency investment demands complete knowledge about risk management tactics, including portfolio spreading, automatic loss prevention tools, and extended asset holding plans. Confident investors exploit high price swings to achieve maximal profit from a short-term investment. In contrast, other investors opt to sustain asset ownership over time in anticipation of the evolution of blockchain technology.

Retail investors and institutions show interest in cryptocurrencies because they view them as varied assets that add to diversified investment collections. Due to their unclear regulatory framework and digital wallet security risks, investors face significant hurdles because of payment disk systems. Before entering the cryptocurrency market, investors must thoroughly examine the opportunities and determine their risk aversion level while developing risk reduction approaches.

12. **Future scope of the study**

According to the findings of this study, the process of combining neutrosophic programming with hybrid machine learning models for the forecasting of cryptocurrency has a lot of prospects. Nevertheless, more efforts could be devoted in the future to refine and develop this strategy still further. Subsequent research could extend to include more variables like macroeconomic factors, underlying social mood, and geopolitical variables into the model to achieve higher levels of accuracy and resilience of the model.

The last area of development is related to the application of more sophisticated deep learning methods, specifically, transformer-based models to enhance the forecasting performance of hybrid models. Such approaches may be combined with neutrosophic frameworks in order to treat a rather high level of uncertainty and volatility in cryptocurrency markets.

At the same, expanding the dataset of this study to other types of cryptocurrencies and analyzing the behavior of these assets under different economic conditions may enlighten further. The use of real-time data streams and dynamic models that may be updated on a daily basis might also help to close the gap between theoretical predictions and practical applications.

Last, future studies could concern the identification of standardized strategies to reach the defined target values, and utilizing optimization tools that would specify suggested investment and risk management plans for the stakeholders. They would also greatly enhance the future development of cryptocurrency forecasting as a science and its application in daily financial decisions.

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