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A Linear Mathematical Model of the Vocational Training Problem in a Company Using Neutrosophic Logic, Hyperfunctions, and SuperHyperFunction

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Abstract:
A mathematical model is a simplified representation of a practical, real-life system or a proposed idea for an executable system. Operations research methods have the ability to express the concepts of efficiency and scarcity in a well-defined mathematical model for a given situation. Scientific methods are used to solve complex problems in managing large systems in factories, institutions, and companies, enabling them to make optimal scientific decisions for their operation. Mathematical models consist of an objective function, through which we search for the maximum or minimum value, and constraints. If the objective function and the constraints are all linear relationships, we obtain a linear mathematical model. Linear programming is one of the most important methods of operations research due to its widespread use in most areas of life. When constructing a mathematical model, we rely on data collected by experts on the issue under study. This data is affected by the surrounding conditions. In light of the constant changes and instability of these conditions, this data is uncertain and subject to change depending on the circumstances. To develop a mathematical model that accommodates all possible circumstances, this research reformulates the vocational training problem within a company using the concepts of neutrosophic logic, along with the definitions of hyperfunctions and SuperHyperFunction.
Keywords: Linear programming; linear models; neutrosophics logic; neutrosophic linear models; hyperfunction; the issue of corporate vocational training.

1.Introduction:

1.1. Neutrosophic Logic and Linear programming:

At the core of operations research lies the presence of problems that require informed decision-making—needs that become more critical as the complexity of the problem increases. The success of operations research depends not only on the decision-makers themselves, but also on the accuracy of the data collection process and how well the resulting model reflects reality.

To solve problems through quantitative analysis, analysts must first understand the numerical facts and relevant data, then formulate a mathematical model based on a thorough understanding of the situation. This model should accurately capture the objectives, constraints, and interdependencies involved in the problem.

For a mathematical model to remain valid under varying conditions, the input data must be adaptable to potential changes in the working environment and account for all possible scenarios. For this reason, in previous studies, we have reformulated various operations research methods using the principles of neutrosophic logic [1–12]. In particular, within the field of linear programming, we have redefined core concepts and addressed numerous problems through this lens [6–12]. Note that neutrosophic logic is known as a generalization of concepts such as fuzzy logic [34] and intuitionistic fuzzy logic [35].

1.2. Hyperfunction and SuperHyperfunction:

Functions are often used to model real-world concepts and are commonly applied in areas such as linear programming. A hyperfunction maps each element of a set to a subset of that set, using powerset values as codomain. Several researchers have explored operations research methods using the concept of hyperfunctions [13–16]. An n-SuperHyperfunction maps each subset of a set into a nested collection of subsets across n iterations. The concept of the SuperHyperfunction was defined by Smarandache. SuperHyperfunctions, like Hyperfunctions, have been the subject of various research studies [24–26]. SuperHyperfunctions are also known as a type of SuperHyperstructure [30–33]. Examples of SuperHyperstructures include Superhypergraphs [37-39,43], Superhyperuncertain Set [41,42], and Superhyperprobabilities [40].

Building on these earlier works, this study aims to develop a mathematical model that addresses the issue of vocational training within a company. Our approach integrates both neutrosophic logic and the concept of hyperfunctions to enhance the model's effectiveness.

1.3. Discussion:

Many real-life problems we face in our daily lives have been studied through the lens of operations research. A key characteristic that sets operations research apart from other quantitative sciences is its emphasis on formulating problems as mathematical models.

In this study, we present a new mathematical formulation to address vocational training within a company. This approach integrates concepts from neutrosophic logic, as well as the definitions of hyperfunctions and superhyperfunctions. By incorporating neutrosophic values, the proposed model provides a more robust and stable workflow for organizations that implement it.

2. Previous study: Classical Function

We provide an overview of the related (or previous) research. We then examine the following classical problem (i.e., not involving Neutrosophic or Fuzzy frameworks). Hereafter, all numerical values considered in this paper are assumed to be finite.

2.1. Classical Study of the Problem [17]:

Problem Text:

A machinery manufacturing company is conducting a vocational training program for mechanics, the goal of which is to secure its need for mechanics for the coming periods. The training program lasts one month. Each trainer in the program is responsible for training ten new recruits. Previous experience has shown that out of every ten new trainees, only seven are able to successfully complete the program, with the unsuccessful trainee leaving the company. The company uses the trainee mechanics to operate machinery, and the company's need for them over the next three months is estimated as follows:

January 100
February 150
March 200

At the beginning of April, the company needs 250 trained mechanics. Given that the number of trained mechanics available at the beginning of the year is 130, and the monthly wages are as follows:

The wage of each trainee mechanic is 400$ (for new mechanics during the training month), and the wage of each trained mechanic (working in teaching or operating machinery) is 700$ (the employment contract presumes that trained mechanics cannot be laid off). That is, a mechanic who successfully completes the program remains with the company, even if he or she is not assigned any work, and receives a wage of 500$. The required mathematical model is to be developed to enable you to hire and train mechanics at the lowest cost and meet the company's requirements.

Model formulation:
We first note that the trained mechanic each month may be in one of three states:
(1) Working in teaching
(2) Operating machinery
(3) Unemployed.
Since the number of trained mechanics operating machinery is specified for each month, the only unknown decision variables are the number of mechanics working in teaching and the number of mechanics unemployed during that month. Therefore, the variables to be specified are:
x_1: Number of trained mechanics teaching during January.
x_2: Number of trained mechanics unemployed during January.
x_3: Number of trained mechanics teaching during February.
x_4: Number of trained mechanics unemployed during February.
x_5: Number of trained mechanics teaching during March.
x_6: Number of trained mechanics unemployed during March.
Constraints require that a sufficient number of trained mechanics be available each month to operate machinery. This can be achieved by writing the following equation for each month:
Number of trained mechanics operating machinery + Number of teaching mechanics + Number of unemployed mechanics = Total number of mechanics available at the beginning of the month.
For example, for January, the constraint becomes:

$$100 + x_1 + x_2 = 130$$

For February, the number of trainees in January is $10x_1$, and the number of those who successfully complete the program to become trained mechanics is only $7x_1$.
Therefore, the total number of trained mechanics available is equal to the sum of the number of trained mechanics in January and the number of graduates from the training program. The entry for February is written as follows:

$$130 + 7x_1 = 150 + x_3 + x_4$$

The entry for March is as follows:

$$130 + 7x_1 + 7x_3 = 200 + x_5 + x_6$$

For April, the company needs 250 trained mechanics. In April, the entry is as follows:

$$130 + 7x_1 + 7x_3 + 7x_5 = 250$$

When writing the objective function, there is no need to include the cost of the mechanics operating the machines, as this is a fixed cost. The important costs are the cost of the training program, including trainees and instructors, and the cost of unemployed mechanics. Therefore, the objective function becomes:
We want to minimize the function Z:

$$Z = 400(10x_1 + 10x_3 + 10x_5) + 700(x_1 + x_3 + x_5) + 500(x_2 + x_4 + x_6)$$

From the above, we can write the following mathematical model:
Find:

$$MinZ = 4700x_1 + 500x_2 + 4700x_3 + 500x_4 + 4700x_5 + 500x_6$$

subject to:

$$x_1 + x_2 = 30$$
$$7x_1 - x_3 - x_4 = 20$$
$$7x_1 + 7x_3 - x_5 - x_6 = 70$$
$$7x_1 + 7x_3 + 7x_5 = 120$$
$$x_j \geq 0 \quad ; j = 1, \dots, 6$$

In the previous question, we note that all the data provided can be considered accurate and completely specific, except for the number of those who successfully pass the training course, which, according to what is stated in the text of the question, depends on previous experiences and the circumstances surrounding these circumstances. Therefore, it cannot be considered confirmed data and there is some uncertainty in it and it is subject to increase or decrease depending on the capabilities of the members and the surrounding circumstances. In the worst circumstances, it may be equal to zero, and in the best circumstances, it may be equal to ten. Since this number affects the objective function, which expresses the amounts that the company will pay to secure what it needs from trained mechanics, we suggest taking this value as neutrosophic values limited to the range [0,10] and then we build the appropriate mathematical model.

3. Current Study: Neutrosophic Logic and Linear Objective Function

We provide an overview of the current research, which refers to problems that involve the use of Neutrosophic logic and related approaches. We examine the following neutrosophic problem.

3.1. The Problem of Vocational Training in a Company Using Neutrosophic Logic Concepts:

Problem Text:

A machinery manufacturing company is implementing a vocational training program for mechanics, the goal of which is to secure its need for mechanics for the coming periods. The training program lasts one month. Each trainer in the program is responsible for training ten new trainees. Based on the trainees' capabilities, the number of trainees who can successfully complete the program is limited to [0.10] out of every ten trainees. The unsuccessful trainee leaves the company. The company benefits from trained mechanics in the operation of machinery. The company's estimated need for them over the next three months is as follows:

January 100
February 150
March 200

At the beginning of April, the company will need 250 trained mechanics. Given that the number of trained mechanics available at the beginning of the year was 130 and the monthly wages were as follows:

The wage of each trainee mechanic is determined by the mechanic's commitment to the program and falls within the field of $[350,400]$$ (new mechanics during the training month). The wage of each trained mechanic (working in teaching or operating machinery) is determined based on the mechanic's competence and falls within the field of $[650,700]$$ (assuming the employment contract prohibits the dismissal of trained mechanics). That is, a mechanic who successfully completes the program remains with the company even if he is not assigned any work and receives a wage of 500$. The required mathematical model is to be developed to enable the hiring and training of mechanics at the lowest cost and to meet the company's requirements.

Model formulation:

We first note that the trained mechanic each month may be in one of three states:

(1) Working in teaching
(2) Operating machinery
(3) Unemployed.

Since the number of trained mechanics operating machinery is specified for each month, the only unknown decision variables are the number of mechanics working in teaching and the number of mechanics unemployed during that month. Therefore, the variables to be specified are:

x_1: Number of trained mechanics teaching during January.
x_2: Number of trained mechanics unemployed during January.
x_3: Number of trained mechanics teaching during February.
x_4: Number of trained mechanics unemployed during February.

x_5: Number of trained mechanics teaching during March.
x_6: Number of trained mechanics unemployed during March.
Constraints require that a sufficient number of trained mechanics be available each month to operate machinery. This can be achieved by writing the following equation for each month:
Number of trained mechanics operating machinery + Number of teaching mechanics + Number of unemployed mechanics = Total number of mechanics available at the beginning of the month.
For example, for January, the constraint becomes:

$$100 + x_1 + x_2 = 130$$

For February, the number of trainees in January is $10x_1$, and the number of those who successfully complete the program to become trained mechanics is $[0,10]x_1$.
Therefore, the total number of trained mechanics available is equal to the sum of the number of trained mechanics in January and the number of graduates from the training program. The entry for February is written as follows:

$$130 + [0,10]x_1 = 150 + x_3 + x_4$$

The entry for March is as follows:

$$130 + [0,10]x_1 + [0,10]x_3 = 200 + x_5 + x_6$$

For April, the company needs 250 trained mechanics. In April, the entry is as follows:

$$130 + [0,10]x_1 + [0,10]x_3 + [0,10]x_5 = 250$$

When writing the objective function, there is no need to include the cost of the mechanics operating the machines, as this is a fixed cost. The important costs are the cost of the training program, including trainees and instructors, and the cost of unemployed mechanics. Therefore, the objective function becomes:
We want to minimize the function Z:

$$Z = [350,400](10x_1 + 10x_3 + 10x_5) + [650,700](x_1 + x_3 + x_5) + 500(x_2 + x_4 + x_6)$$

From the above, we can write the following mathematical model:
Find:

$$MinZ = [1000,1100]x_1 + 500x_2 + [1000,1100]x_3 + 500x_4 + [1000,1100]x_5 + 500x_6$$

subject to:

$$x_1 + x_2 = 30$$
$$[0,10]x_1 - x_3 - x_4 = 20$$
$$[0,10]x_1 + [0,10]x_3 - x_5 - x_6 = 70$$
$$[0,10]x_1 + [0,10]x_3 + [0,10]x_5 = 120$$
$$x_j \geq 0 \quad ; j = 1, \dots, 6$$

We get a linear neutrosophic model.

3.2. Definition of Linear Objective (Profit/Cost) Function

In this paper, we make use of the Linear Objective (Profit/Cost) Hyperfunction. We begin by presenting the formal definition of the Linear Objective (Profit/Cost) function as follows.
Definition (Linear Objective (Profit/Cost) Function) [18, 19]:
Let $X = (x_1, x_2, \dots, x_n)$ be the decision vector and $C = (c_1, c_2, \dots, c_n)$ the coefficient vector. A linear objective function is:

$$f_{obj}(x) = C^T X = \sum_{j=1}^{n} c_j x_j$$

which one seeks to maximize (profit) or minimize (cost).

4. Result of This Paper: Hyperfunction and SuperHyperFunction

This section presents the main results of this paper.

4.1 Linear Objective (Profit/Cost) Hyperfunction

The definition of the Linear Objective (Profit/Cost) Hyperfunction is provided below. A Linear Objective (Profit/Cost) Hyperfunction maps each decision vector to a set of possible objective values under coefficient uncertainty.

Definition (Hyperfunction) [9,10].
A Hyperfunction is a function where the domain remains a classical set X, but the codomain is extended to the powerset of X, denoted P(X). Formally, a Hyperfunction f is defined as:
f: X → P(X). For any x ∈ X, f(x) ⊆ X is a subset of X.
Definition (Linear Objective Hyperfunction). Let $S = R^n$ be the decision space, and let $C \subseteq R^n$ be a nonempty compact set of possible coefficient vectors. The Linear Objective Hyperfunction is the set-valued mapping:

$$H_{obj}: S \longrightarrow P(R)\ ,\ H_{obj}(x) = \{C^T X | c \in C\}$$

which assigns to each decision vector x the set of all possible objective values $C^T X$ as c varies over C. Since C is compact, for each x the image $H_{obj}(x)$ is a nonempty compact interval $[m(x), M(x)]$ where $m(x) = \ min_{c \in C} C^T X\ M(x) = \ max_{c \in C}\ C^T X$.

Remark: In a classical Linear Objective Function, each decision vector x produces exactly one scalar outcome

$$f_{obj}(x) = C^T X.$$

because the coefficient vectors are assumed known and fixed. By contrast, the Linear Objective Hyperfunction treats the coefficients as uncertain within a compact set C. Instead of a single value, each x maps to the entire interval

$$H_{obj}(x) = [\ min_{c \in C} C^T X,\ \ max_{c \in C}\ C^T X].$$

Intuitively:

- Deterministic vs. Uncertain: The classical function gives one "best guess" profit or cost, whereas the hyperfunction yields a range of possible outcomes reflecting coefficient variability.
- Point vs. Interval: A single point value can be overly optimistic or pessimistic if the true coefficients shift; the hyperfunction's interval captures both extremes.
- Risk Awareness: By exposing the best-case and worst-case objective values, the hyperfunction enables decision-makers to balance between optimistic and robust solutions, rather than relying on a single estimate.

In short, the Linear Objective Hyperfunction generalizes the standard linear objective by turning it into a set-valued map that naturally handles parameter uncertainty.
We consider solving the problem using the Linear Objective Hyperfunction. The problem statement and the computational method are provided below.
Problem Text:
A machinery manufacturing company is implementing a vocational training program for mechanics, the goal of which is to secure its need for mechanics for the coming periods. The training program lasts one month. Each trainer in the program is responsible for training ten new trainees. Based on the trainees' capabilities, the number of trainees who can successfully complete the program is limited to [0.10] out of every ten trainees. The unsuccessful trainee leaves the company. The company benefits from trained mechanics in the operation of machinery. The company's estimated need for them over the next three months is as follows:

January: 100 trained mechanics

February: 150 trained mechanics
March: 200 trained mechanics
Beginning of April: 250 trained mechanics

Given that the number of trained mechanics available at the beginning of the year was 130 and the monthly wages were as follows:

The wage of each trainee mechanic is determined by the mechanic's commitment to the program and falls within the field of $[350,400] (new mechanics during the training month). The wage of each trained mechanic (working in teaching or operating machinery) is determined based on the mechanic's competence and falls within the field of $[650,700] (assuming the employment contract prohibits the dismissal of trained mechanics). That is, a mechanic who successfully completes the program remains with the company even if he is not assigned any work and receives a wage of $500. The required mathematical model is to be developed to enable the hiring and training of mechanics at the lowest cost and to meet the company's requirements.

Model formulation:

To formulate the mathematical model of the company's vocational training problem using the previous definition:

$$S = \{(x_1, x_2, x_3, x_4, x_5, x_6) \in R^6 | x_1, x_2, x_3, x_4, x_5, x_6 \geq 0\}$$

The variables $x_1, x_2, x_3, x_4, x_5, x_6$ express:

x_1: Number of trained mechanics teaching during January.
x_2: Number of trained mechanics unemployed during January.
x_3: Number of trained mechanics teaching during February.
x_4: Number of trained mechanics unemployed during February.
x_5: Number of trained mechanics teaching during March.
x_6: Number of trained mechanics unemployed during March.

Since the wage given to a mechanic who teaches or operates machinery is related to his or her efficiency, and the same applies to those enrolled in the training course, which is related to the level of commitment, and the wage of an unemployed mechanic is the same for everyone, the cost vector falls into the integrated set:

$$C = [350,400] \times [650,700] \times [500,500] \subset R^3$$

We take the following variables:

y_1 : represents the total number of mechanics who are taking the training program in the months of January, February, and March, i.e.,

$$y_1 = 10x_1 + 10x_3 + 10x_5$$

y_2 : represents the total number of mechanics who are teaching in the months of January, February, and March, i.e.,

$$y_2 = x_1 + x_3 + x_5$$

y_3 : represents the total number of unemployed mechanics in the months of January, February, and March. That is:

$$y_3 = x_2 + x_4 + x_6$$

Then the set S is written as follows:

$$S = \{(y_1, y_2, y_3) \in R^3 | y_1, y_2, y_3 \geq 0\}$$

Then the hyperlinear objective function is:

$$H_{obj}(x) = \{C^T X | c \in C\} = \{c_1 y_1 + c_2 y_2 + c_3 y_3 | c_1 \in [350,400], c_2 \in [650,700], c_3 \in [500,500]\}$$

Which is calculated over the interval:

$$H_{obj}(x) = [350y_1 + 650y_2 + 500y_3, 400y_1 + 700y_2 + 500y_3]$$

Using the basic variables $x_1, x_2, x_3, x_4, x_5, x_6$ we write it as follows:

$$H_{obj}(x) = [4150(x_1 + x_3 + x_5) + 500(x_2 + x_4 + x_6), 4700(x_1 + x_3 + x_5) + 500(x_2 + x_4 + x_6)]$$

The following constraints must be satisfied:

$$x_1 + x_2 = 30$$
$$[0,10]x_1 - x_3 - x_4 = 20$$
$$[0,10]x_1 + [0,10]x_3 - x_5 - x_6 = 70$$
$$[0,10]x_1 + [0,10]x_3 + [0,10]x_5 = 120$$
$$x_j \geq 0 \quad ; j = 1, \dots, 6$$

A robust optimization formulation uses the hyperfunction's extremal values:

$$Z_{min}(x) = 4150(x_1 + x_3 + x_5) + 500(x_2 + x_4 + x_6) \;\; Z_{max}(x) = 4700(x_1 + x_3 + x_5) + 500(x_2 + x_4 + x_6)$$

$$\text{Thus, the robust} - \text{optimistic linear program is:}$$

$$min_{(x_1, x_2, x_3, x_4, x_5, x_6)}\big(Z_{min}(x),\ Z_{max}(x)\big)$$

Subject to:

$$x_1 + x_2 = 30$$
$$[0,10]x_1 - x_3 - x_4 = 20$$
$$[0,10]x_1 + [0,10]x_3 - x_5 - x_6 = 70$$
$$[0,10]x_1 + [0,10]x_3 + [0,10]x_5 = 120$$
$$x_j \geq 0 \quad ; j = 1, \dots, 6$$

4.2 Linear Objective (Profit/Cost) n-SuperHyperfunction

Linear Objective (Profit/Cost) n-SuperHyperfunction is a generalized function concept that extends the Linear Objective (Profit/Cost) Hyperfunction based on the idea of an n-SuperHyperfunction. We begin with its definition. Note that n here refers to an integer.

Definition (n-Superhyperfunction): An n-Superhyperfunction generalizes the concept of a Hyperfunction by using the n-th powerset $P_n(S)$ as the codomain. Formally, for $n \geq 2$, an n - Superhyperfunction f is defined as:

$$f: P_r(S) \longrightarrow P_n(S)$$

where $0 \leq r \leq n$, and $P_n(S)$ is the n-th powerset of S. This definition allows f to map subsets of S (from $P_r(S)$) to elements in the n -th powerset $P_n(S)$.

Definition (Linear Objective n-SuperHyperfunction): Let $S = \mathbb{R}^n$ be the decision space and $C \subseteq \mathbb{R}^n$ a nonempty compact set of coefficient vectors. Define for each$x \in S$:

$$H^1_{0bj} = \{c^T x | c \in C\} \subseteq \mathbb{R}$$

and recursively for $k = 2,3, \dots, n$:

$$H^k_{0bj} = P\{H^{k-1}_{0bj}(X))\}\backslash\{\emptyset\}$$

The family of all nonempty subsets of $H^{k-1}_{0bj}(X)$.

Then:

$$H^n_{0bj}: S \longrightarrow P^n(\mathbb{R}), x \longrightarrow H^n_{0bj}(x)$$

Is called the Linear Objective n-SuperHyperfunction.

As a concrete example of the Linear Objective SuperHyperfunction, the Problem Text and Model Formulation are presented below.

Problem Text: A machinery-manufacturing firm runs a one-month vocational training program for mechanics to meet its staffing needs over the next three months. Each trainer takes ten trainees, but both the trainees' completion rate and the wage rates are uncertain:

• Completion rate P lies anywhere in a compact interval $P = [p_{min}, p_{max}] \subset [0,10]$.

• Trainee wage c_1 lies in $[350,400]\$$; trained mechanic wage c_2 lies in $[650,700]\$$; idle trained wage c_3 is fixed at $500\$$.

At time zero, 130 trained mechanics are available. Projected staffing requirements are 100 , 150 , and 200 trained mechanics in January, February, and March, respectively, rising to 250 by April 1. Determine the numbers of trainees and assignments each month so that, under both sources of uncertainty, the worst-case cost is minimized while requirements are met.

Model formulation: Decision Variables ($x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$):

x_1, x_2: trainees hired and unemployed trained in January

x_3, x_4: trainees hired and unemployed trained in February

x_5, x_6: trainees hired and unemployed trained in March

Auxiliary Variables:

$y_1 = 10x_1 + 10x_3 + 10x_5$ (total trainees)

$y_2 = x_1 + x_3 + x_5$ (trained mechanics teaching/operating)

$y_3 = x_2 + x_4 + x_6$ (trained mechanics idle)

Parameter Sets:

$$C = \{ (c_1, c_2, c_3) \mid c_1 \in [350,400], c_2 \in [650,700], c_3 = 500 \}$$
$$P = [p_{min}, p_{max}] \subset [0,10]$$

Recursive Objective Mapping:

First-level (Hyperfunction):

$$H^1(x) = \{ c_1 \cdot p \cdot y_1 + c_2 \cdot y_2 + c_3 \cdot y_3 \mid (c_1, c_2, c_3) \in C, p \in P \} \subset \mathbb{R}$$

This yields an interval $[\underline{m}(x), M(x)]$.

Second-level (2-SuperHyperfunction):

$$H^2(x) = P(H^1(x)) \setminus \{\emptyset\} \text{ (Nonempty subsets of that interval)}$$

Subject to:

$$x_1 + x_2 = 30$$
$$p \cdot x_1 - x_3 - x_4 = 20$$
$$p \cdot x_1 + p \cdot x_3 - x_5 - x_6 = 70$$
$$p \cdot x_1 + p \cdot x_3 + p \cdot x_5 = 120$$
$$x_j \geq 0 \quad ; j = 1, \dots, 6$$

Robust-Optimistic Program:

Minimize the 2-SuperHyperfunction objective: $min_x \, H^2(x)$, subject to the above constraints.

This formulation captures two nested layers of uncertainty—wage coefficients and completion rates—by mapping decisions into the second iterated powerset of ℝ. To extend to a general n, define recursively:

$$H^k(x) = P(H^{k-1}(x)) \setminus \{\emptyset\}, for\ k = 2, \dots, n$$

And optimize over $H^n(x)$.

5.Conclusion and results:

In this research, we presented a study of the issue of vocational training in companies, which aims to meet companies' need for employees with good experience in the company's field of work. We presented a formulation of this issue as stated in the classic reference. Since all companies operate in unstable conditions, and the data in the issue are subject to change according to the surrounding conditions, making it uncertain, we used the concepts provided by neutrosophic logic to reformulate this model using neutrosophic values and the concept of a hyperfunction. We obtained neutrosophic linear mathematical models whose optimal solutions include a margin of flexibility, allowing them to adapt to all the circumstances the company may face.

Furthermore, as a future direction of this research, we hope that studies will advance using models based on Plithogenic Logic [21–23, 36] and Hyperneutrosophic Logic [27-29], both of which generalize Neutrosophic Logic.

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