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A Model of Organizational Dynamics of Phonological Knowledge in Preschool Children Based on Bratianu's Theory and the Neutrosophic AHP Technique

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Abstract. Phonological knowledge is a component of linguistics that allows children to understand the relationship between sounds and words. It is a skill that should be stimulated and enhanced from the age of 3, as children begin the process of reading and writing at 6. If this component is not yet acquired, it can generate difficulties in the school stage, where learning demands are more challenging and higher cognitive processes are required in terms of decoding phonemes, graphemes, words, and sentences. In this paper, we propose a model that allows us to study the dynamics of phonological knowledge in preschool children. To do so, we base ourselves on the theory proposed by Professor Constantin Bratianu, where the dynamics of organizational knowledge are modeled with equations equivalent to the thermodynamic energy flow. He uses the well-known multicriteria decision-making method Analytic Hierarchy Process (AHP) to numerically obtain the coefficients of its equations. In this paper, we propose extending this model by replacing the classic AHP with the Neutrosophic AHP. This preserves the linguistic scale proposed in the original model, and additionally includes the uncertainty and indeterminacy that are part of decision-making, increasing accuracy and explicitly modeled with the help of Neutrosophic AHP.

Keywords: Phonological awareness, Literacy, Preschool children, Organizational Knowledge Dynamics (OKD), Knowledge Dynamic, Analytic Hierarchy Process (AHP), Neutrosophic Analytic Hierarchy Process (NAHP).

1. Introduction

Phonological awareness is the ability preschoolers acquire to analyze and reflect on the different sounds of their native language. This process involves deliberately manipulating the sounds of language to differentiate between words, syllables, and phonemes.

Children at this age need constant language stimulation so they can naturally develop the levels of phonological awareness they need to begin the process of reading and writing. They must acquire phonological awareness skills at this stage, as this will allow them to recognize syllables, identify sounds, count words, manipulate phonemes, blend sounds, and recognize rhymes.

Given the importance of phonological awareness in the reading and writing process and language development in general, it is essential to address it from three perspectives.

- Parental: With the direct participation of parents during interactive communication processes within the home, where the child feels involved, communicates from his or her perspective, and interacts efficiently and effectively with the family. In this regard, parental responsibility is fundamental.

- Early childhood education: Where children interact with others their age and also receive ongoing language stimulation from professionals specialized in this area, which strengthens their overall development and language in particular. At this stage, early childhood teachers pave the way for later literacy learning.
- Therapeutic: Some children require specialized therapeutic help, involving professionals in the field of Speech Therapy or Language Therapy, who assess phonological knowledge by applying specific tests that determine the level of this linguistic component in children.

In a study conducted by some authors on phonological awareness and reading learning, they mention that the development of phonological awareness has a very high correlation with learning to read and write. According to these authors, a deficiency in this component predicts future difficulties in school learning during the early years. Hence, this is the importance of children developing phonological skills before the age of 6.

Furthermore, they state that the difficulties observed in speech are replicated in reading and writing; that is, if the child does not speak well, he or she will not write well either. Lack of phonological awareness is a factor that prevents children from correctly understanding sounds and smaller phonological units such as phonemes, syllables, words, and rhymes.

For many children, mastery of these phonetic skills determines success or failure in learning to read. However, it is important to note that reading and writing difficulties are not necessarily entirely due to a lack of phonological awareness. Other factors could also influence this process; for example, Attention Deficit Disorder (ADD), Attention Deficit Hyperactivity Disorder (ADHD), and Dyslexia, etc. These and other learning disabilities (LD) could also significantly impact the reading and writing process.

Children with this condition not only present difficulties in phonology, but also in other aspects: semantic, morpho-syntactical, and pragmatic. Their poor attention span impacts their learning to read and write. For this reason, these children in particular are at a serious disadvantage compared to their peers. For them, recognizing the relationship between sounds, phonemes, graphemes, rhymes, and syllables represents a greater effort that can often be frustrating if these previous diagnoses are unknown.

Some researchers mention that a well-developed phonological awareness (PA), as a metalinguistic skill, allows children to reflect on and intentionally manipulate language units, which will then translate into improved cognitive processes for learning to read and write. The same authors state that this skill is a continuous process that encompasses several stages: from rhyme recognition, through syllable segmentation and intrasyllabic units, to phonemes.

Therefore, they argue that the preschool stage is fundamental for stimulating the phonetic-phonological component. Children who are constantly interacting with their family environment and peers are more likely to acquire good levels of phonological awareness, and language awareness in particular. In principle, it is the responsibility of parents to provide their children with appropriate environments where they can develop all the communicative skills they require according to their age.

It has been proven that phonological skills training significantly improves performance on phonological awareness tasks, and reading and writing acquisition. Therefore, timely speech-language therapy or intervention is vitally important for children and their parents.

The goal should always be to prevent any difficulties that may arise during the school years related to learning to read and write. When a speech-language pathologist observes a deficit in language development, they immediately suggest early therapeutic intervention to help children and their families reduce the problems that could arise as a result of a language disorder.

In this paper, we propose to apply a method to assess the language knowledge dynamics within a generic group of children. Specifically, we are inspired by Professor Constantin Bratianu's theory, which uses the flow of energy within the framework of thermodynamics as a metaphor to represent the dynamic flow of knowledge within an organization [1].

Professor Bratianu argues that metaphor is a way of linking a more familiar event to represent a lesser-known one. In the case of thermodynamics, its logic can serve as a basis for representing the flow of knowledge, where knowledge dynamics is a very complex and multidimensional process [2]. In the case of organizational knowledge dynamics (OKD), we have different components which are: knowledge creation, knowledge acquisition, knowledge loss, knowledge sharing, knowledge storage and retrieval, knowledge diffusion, conversion from one form of knowledge into another form of knowledge, and organizational learning [3].

The model we offer in this article is based on the well-known Analytic Hierarchy Process (AHP) as Bratianu does to find the coefficients of the equations that model the dynamics of knowledge [3, 4]. AHP is a very popular method designed by T. L. Saaty for decision-making in complex contexts, where mathematical and psychological results are used [5, 6]. Specifically, the importance of each evaluation criterion relative to the others, each sub-criterion of each criterion relative to the others, and so on, is compared on a scale of 1 to 9. Finally, the alternatives are represented in a tree of different levels. The results obtained allow the assignment of a coefficient that represents the importance or weight of each criterion, sub-criterion, and alternative. So, Bratianu uses these weights to complete his equations for the dynamics of knowledge flow in the organization.

In this paper, we extend this method, where the classical AHP method is replaced by a Neutrosophic AHP technique in Bratianu's model [7]. The Neutrosophic AHP is used to replace the numbers between 1 and 9 of the original method by Single-Valued Trapezoidal Neutrosophic Numbers that incorporate into the model the uncertainty and indeterminacy that are part of the Neutrosophy theory [8, 9]. Decision-makers also maintain the ease with which they can use a linguistic scale that represents what they wish to express.

The article is divided into a Preliminaries section that reviews the fundamental theoretical elements, such as the basic notions of Bratianu's theory and the Neutrosophic AHP. This section is followed by another that explains the components of the model and illustrates it with an example. The paper finishes with conclusions.

2. Preliminaries

In this section, we review the basic notions about Neutrosophic AHP and then Bratianu's model.

2.1 On Neutrosophic AHP

Neutrosophic AHP technique, like crisp AHP and fuzzy AHP variants, combines fundamentals of psychology and mathematics [10, 11, 12]. It is based on a hierarchical tree-like structure, where the main node represents the goal to be achieved. From there, the different descending levels contain the criteria and subcriteria necessary to evaluate a set of alternatives located at the base of the tree. Thomas L. Saaty originally developed this methodology in the 1970s. Elements located at the same hierarchical level are compared with each other in pairs, using a rating scale defined by Saaty himself.

In the Neutrosophic AHP approach, traditional numerical values are replaced by neutrosophic trapezoidal numbers to incorporate the uncertainty and indeterminacy inherent in the decision-making process. The key concepts of this technique are presented below.

Definition 1: ([13]) The *Neutrosophic set* N is composed by three membership functions, viz., the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, such that U is the Universe of Discourse and for all $x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Let us note that, $T_A(x), I_A(x), F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ so, $T_A(x), I_A(x), F_A(x)$ may be subintervals of $[0, 1]$.

Definition 2: ([14]) The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$, and $F_A: U \to [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The Single-Valued Neutrosophic Number (SVNN) is denoted by $N = (t, i, f)$, where $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 3: ([15]) $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is called a *single-valued trapezoidal neutrosophic number* or a *neutrosophic set on* $\mathbb{R}$, where truthfulness, indeterminacy, and falsity membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & a_2 \le x \le a_3 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

We have $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3, a_4 \in \mathbb{R}$, and $a_1 \le a_2 \le a_3 \le a_4$.

Definition 4: ([15]) Given two single-valued trapezoidal neutrosophic numbers $\tilde{a} = \langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, and $\tilde{b} = \langle(b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$, and also $\lambda \neq 0$ is a real number, we have the following definitions of operators:

1. Addition: $\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$
2. Subtraction: $\tilde{a} - \tilde{b} = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$
3. Inversion: $\tilde{a}^{-1} = \langle({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, where $a_1, a_2, a_3, a_4 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, & \lambda > 0 \\ \langle(\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, & \lambda < 0 \end{cases}$$

5. Division of two trapezoidal neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle\left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_4 > 0 \text{ and } b_4 > 0 \\ \langle\left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_4 < 0 \text{ and } b_4 > 0 \\ \langle\left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_4 < 0 \text{ and } b_4 < 0 \end{cases}$$

6. Multiplication of two trapezoidal neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle(a_1b_1, a_2b_2, a_3b_3, a_4b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_4 > 0 \text{ and } b_4 > 0 \\ \langle(a_1b_4, a_2b_3, a_3b_2, a_4b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_4 < 0 \text{ and } b_4 > 0 \\ \langle(a_4b_4, a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_4 < 0 \text{ and } b_4 < 0 \end{cases}$$

Such that $\wedge$ is a t-norm and $\vee$ is a t-conorm.

The integration of AHP with neutrosophic set theory provides a more adaptable framework for handling uncertainty in decision-making processes. Indeterminacy is recognized as a critical factor to be considered in real-world organizational decisions.

Table 1 outlines the scale used for conducting pairwise comparisons of criteria, sub-criteria, and alternatives. It presents Saaty's original scale translated into a neutrosophic trapezoidal format.

Table 1. Saaty's original scale adapted into a neutrosophic trapezoidal representation [7, 8].

Saaty scale	Definition	Neutrosophic Trapezoidal Scale
1	Equally influential	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
7	Very strongly influential	$\tilde{7} = \langle(6,7,8,9); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9,9,9,9); 1.00, 1.00, 1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$ $\tilde{4} = \langle(3,4,5,6); 0.60, 0.35, 0.40\rangle$ $\tilde{6} = \langle(5,6,7,8); 0.70, 0.25, 0.30\rangle$ $\tilde{8} = \langle(7,8,9,9); 0.85, 0.10, 0.15\rangle$

Equation 4 defines the pairwise comparison matrix within the neutrosophic framework.

$$\widetilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \quad (4)$$

$\widetilde{A}$ satisfies the condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, according to the inversion operator defined in Definition 4.

Two indices were introduced to transform a neutrosophic trapezoidal number into a crisp numerical value [16]. These indices—score and accuracy—are mathematically defined in Equations 5 and 6, respectively.

$$\mathcal{S}(\tilde{a}) = \frac{1}{12}[a_1 + a_2 + a_3 + a_4](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (5)$$

$$H(\tilde{a}) = \frac{1}{12}[a_1 + a_2 + a_3 + a_4](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (6)$$

In the proposed method, it is only necessary to determine the weights or relative importance of each criterion. Nevertheless, in the AHP technique, there is the consistency index (C.I.) of the matrix A, which is calculated by the maximum eigenvalue, and the Consistency Ratio (C.R.) such that if it is less than or equal to 0.1 it is considered that the matrix consistency is acceptable.

2.2 Bratianu's model of knowledge dynamics

Professor Constantin Bratianu's theory of knowledge dynamics offers a lens on how knowledge behaves and transforms—especially within organizations. Rather than relying on traditional Newtonian metaphors like "stocks and flows," Bratianu introduces a thermodynamics-inspired framework that treats knowledge as a field, much like energy.

The core concepts of Bratianu's theory are the following:

1. Knowledge as Energy: Bratianu uses the energy metaphor to describe knowledge. Just as energy transforms and flows, so also does knowledge—across rational, emotional, and spiritual dimensions.
2. Multifield Knowledge Spectrum: He identifies three fundamental "fields" of knowledge:
    - Rational: Logical, analytical knowledge,
    - Emotional: Affective, experience-based knowledge,
    - Spiritual: Values-driven, purpose-oriented knowledge.
3. Irreversible Transformation: Drawing from thermodynamics, Bratianu argues that knowledge transformation is irreversible —once knowledge shifts from one form to another (say, from emotional to rational), it cannot fully revert.
4. Organizational Application: This theory helps explain how organizations convert potential

knowledge into operational knowledge through what he calls "organizational integrators"—people or processes that catalyze transformation.

Bratianu's approach is especially useful in:

1. Knowledge management: Offering a richer, more dynamic understanding of how knowledge evolves,
2. Leadership and strategy: Helping leaders recognize the emotional and spiritual dimensions of decision-making,
3. Innovation: Encouraging organizations to embrace non-linear, creative knowledge flows.

Professor Constantin Bratianu's work on Organizational Knowledge Dynamics using the Analytic Hierarchy Process (AHP) is a fusion of systems thinking and decision science to better understand how knowledge evolves within organizations.

Bratianu models organizational knowledge dynamics using a dynamic equilibrium equation over a time interval (ΔT), which includes:

- ΔK: Total variation in organizational knowledge,
- ΔC: Variation in knowledge creation,
- ΔA: Variation in knowledge acquisition,
- ΔL: Variation in knowledge loss.

Equation 7 is as follows:

$$\Delta K = F_C(\Delta C) + F_A(\Delta A) - F_L(\Delta L) \qquad (7)$$

Each of these components contributes differently to the overall knowledge balance, so Bratianu uses AHP —a structured decision-making method developed by Thomas Saaty —to assign weighting factors to them, which are F_c, F_A, and F_L.

So, the AHP model is structured into three levels:

1. Goal Level: Increase the level of organizational knowledge.
2. Strategy Level:

   - S_1: Increase knowledge creation,
   - S_2: Increase knowledge acquisition,
   - S_3: Reduces knowledge loss.

3. Activity Level:

   - A_1: Hire valuable human resources,
   - A_2: Develop training programs,
   - A_3: Motivate employees effectively,
   - A_4: Purchase knowledge resources (books, software, etc.).

   See Figure 1.

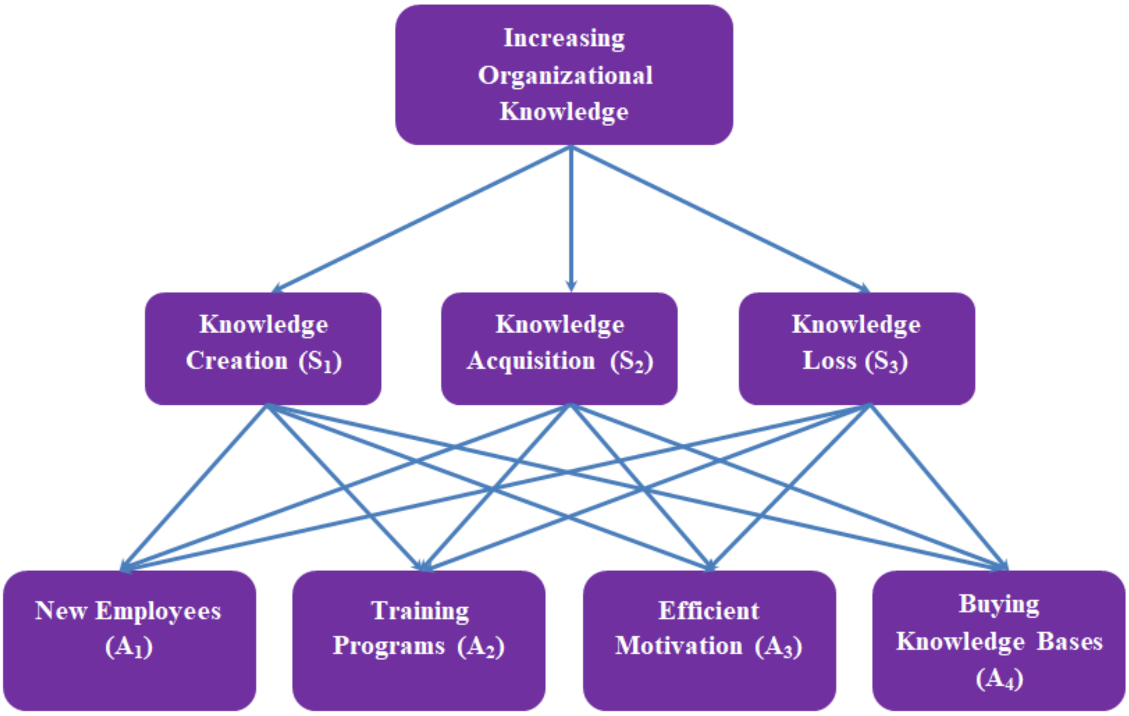

Figure 1. AHP hierarchical tree about Bratianu's model. Source [3].

Employees are asked to make pairwise comparisons between strategies and activities. These comparisons generate priority vectors (via eigenvalue calculations), which are then used to determine the relative importance of each component in the knowledge dynamics equation.

Additionally, the dynamic equilibrium equation for a generic component is expressed by Equation 8, where S symbolizes the strategy.

$$\Delta S = w_1(\Delta A_1) + w_2(\Delta A_2) + w_3(\Delta A_3) + w_4(\Delta A_4) \quad (8)$$

ΔS represents the variation of the generic knowledge-related component—namely, knowledge creation, acquisition, and loss—during the time interval ΔT. ΔA_i denotes the change in activity i over the same time interval ΔT. W_i corresponds to the weighting factor associated with activity i.

This model helps organizations:

- Quantify abstract knowledge processes,
- Prioritize strategic initiatives,
- Align knowledge management with broader business goals.

3. Design of the proposed model

In this section, we present our proposed design of a model to measure phonological knowledge in preschool children.

Two variables were considered in the analysis: chronological age and phonological knowledge.

To assess the percentage of the children's phonological knowledge viz., phonological, syllabic, and phonetic, the Phonological Awareness Assessment Test (PAAT) was administered. This test assesses phonological awareness at the two aforementioned levels, considering the initial, medial, and final positions of syllables and sounds (phonemes) in each word. The original test is adapted to the neutrosophic framework with the idea of incorporating indeterminacy and uncertainty into the assessments.

The test consists of six sections that are divided into three activities for both the syllabic and phonemic levels.

1. Identification: Ability to recognize syllables and phonemes in a word through the presentation of five images,

2. Addition: Ability to add a syllable or phoneme to a word using representative tokens,
3. Omission: Ability to remove a syllable or phoneme from a word and form a new one using five images that must be named.

Below is a table that summarizes the two levels assessed: syllabic and phonemic in the three activities carried out.

Table 2. Levels of phonological knowledge

Level	Identification	Addition	Omission
Syllabic	1. Wolf 2. Die 3. Meat 4. Sheep 5. Bottle	11. Elbow 12. Salty 13. Sunday 14. Folded 15. Silky	21. House 22. Shirt 23. Sack 24. Toad 25. Worm
Phonemic	6. Nail 7. Pencil 8. Cup 9. Cheese 10. Ship	16. Sun 17. Goal 18. Magnifying glass 19. Bookseller 20. Peeled	26. Seal 27. Skirt 28. Sofa 29. Glasses 30. Flan
Total	I: 10	A: 10	O: 10

For response analysis, the results were assessed by adding the scores obtained at both the syllabic and phonemic levels for the three activities proposed in the test: identification, addition, and omission; assigning a point $\langle(0.5,1,1,1.5);1,0,0\rangle$ for each correct answer and a point $\langle(-0.5,0,0,0.5);0,0,1\rangle$ for each error. The maximum score is $\langle(15,30,30,45);1,0,0\rangle$ points. If the child does not understand the activity, it must be explained up to three times. After that, if the child persists in making a mistake, the error is rescinded $\langle(-0.5,0,0,0.5);0,0,1\rangle$ and the child continues with the next activity.

Thus, if there are m children, the evaluation of the ith child is calculated by the formula:

$$\tilde{a}_i = \oplus_{j=1}^{30} a_{ij} \qquad (9)$$

Where a_{ij} is the evaluation of the ith child relative to the jth test, while $\oplus$ is the addition operator from Definition 4. This value becomes a single crisp value with the help of formula 5 of the score function:

$$b_i = \mathcal{S}(\tilde{a}_i) \qquad (10)$$

The total knowledge of the group of children, assuming that n children are studied, is calculated by the arithmetic mean, that is:

$$K = \frac{\sum_{i=1}^{m} b_i}{m} \qquad (11)$$

For threshold tone hearing assessment, each child must undergo threshold tone audiometry using an audiometer. This is a subjective hearing test in which children must voluntarily respond to the stimulus emitted. Two pathways were evaluated: an air pathway with supra-aural headphones, and a bone pathway with a bone vibrator using pure tones and a voice test (recorded voice).

Pure tones are used; in the air conduction, in a range of 125 Hz and 8000 kHz or 12 kHz in frequency and intensity from -10dB to 120dB, and in the bone conduction, in a range of 250 Hz and 4 kHz in intensity.

To this end, we adapted Bratianu's model to phonological knowledge in preschool children. To do so, we maintained the same goal and strategies for improving knowledge, but changed the alternatives, which are shown in Table 2.

Table 3. Alternatives to improve collective phonological knowledge in preschool children

Notation	Alternative	Brief description
A_1	Guided phonological songs	Structured musical activities that emphasize rhyme, alliteration, and rhythm.
A_2	Sound segmentation and matching games	Dynamics where children divide and combine phonemes into simple words.
A_3	Dialogic reading with phonological emphasis	Reading aloud where the adult guides and reinforces the identification of sounds.
A_4	Puppets with articulatory exaggeration	Theatrical activities where puppets emphasize key sounds with movements.

The AHP hierarchical tree looks like Figure 2.

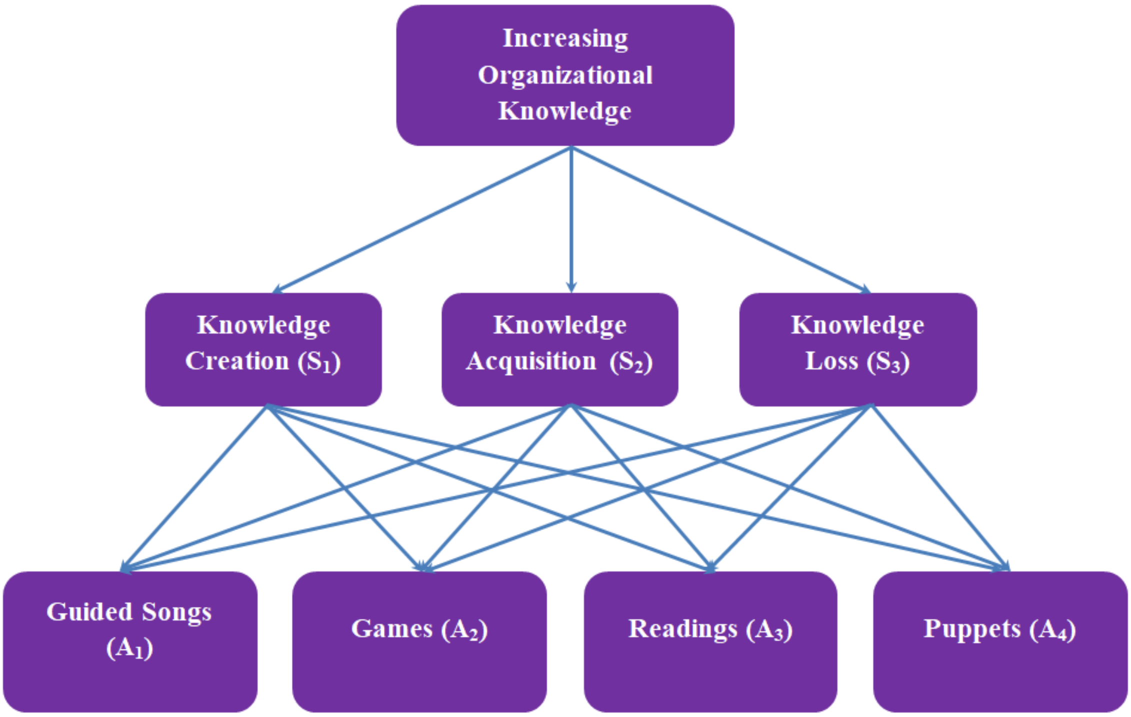

Figure 2. AHP hierarchical tree about the proposed model.

Suppose there are *n* experts who evaluate how to improve the phonological knowledge of approximately 5-year-old children concerning strategies S_1, S_2, and S_3 and the new alternatives A_1, A_2, A_3, and A_4, which appear in Table 1.

Each of them is asked the following questionnaire:

1. a. If you wanted to improve the performance of a group of preschool children in their phonological skills, say which strategy do you prefer either “Increase knowledge creation” (S1) or “Increase knowledge acquisition” (S2) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
2. a. If you wanted to improve the performance of a group of preschool children in their phonological skills, say which strategy do you prefer either “Increase knowledge creation” (S1) or “Reduce knowledge loss” (S3) (They can be equally preferred).

b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.

3. a. If you wanted to improve the performance of a group of preschool children in their phonological skills, say which strategy do you prefer either "Increase knowledge acquisition" (S2) or "Reduce knowledge loss" (S3) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.

The second part of the questionnaire contains comparisons between alternatives for each of the strategies.

4. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Guided phonological songs" (A_1) or "Sound segmentation and joining games" (A_2) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
5. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Guided phonological songs" (A_1) or "Dialogic reading with phonological emphasis" (A_3) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
6. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Guided phonological songs" (A_1) or "Puppets with articulatory exaggeration" (A_4) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
7. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Segmentation and sound joining games" (A_2) or "Dialogic reading with phonological emphasis" (A_3) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
8. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Games of segmentation and union of sounds" (A_2) or "Puppets with articulatory exaggeration" (A_4) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
9. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Dialogic reading with phonological emphasis" (A_3) or "Puppets with articulatory exaggeration" (A_4) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.

Finally, questions 4-9 are repeated by setting "Increase knowledge acquisition" (S_2) instead of S_1, and then replacing "Reduce knowledge loss" (S_3) instead of S_1. Therefore, there will be 12 more questions in the questionnaire.

The values of the numbers from 1 to 9 are replaced by the Neutrosophic Trapezoidal Scale, according to Table 1.

The following procedure is followed using the procedure of the Aggregation of Individual Judgments (AIJ) ([17, 18]):

1. The values of each of the expert's responses are added with the help of Equation 12:

$$\tilde{X}_{l_1} = \oplus_{k_1}^{n_1} \tilde{x}_{k_1 l_1}$$
$$\tilde{X}_{l_2} = \oplus_{k_2}^{n_2} \tilde{x}_{k_2 l_2} \quad (12)$$

Where $\tilde{x}_{k_1 l_1}$ is the Single-Valued Trapezoidal Neutrosophic Number corresponding to the answers related to the first option of the two possible for each question. $\tilde{x}_{k_2 l_2}$ is the same but corresponding to the second option.

2. $\tilde{X}_{l_1}$ is replaced by $\tilde{Y}_{l_1} = \mathcal{S}(\tilde{X}_{l_1})$, to convert it into crisp values. Similarly, we have $\tilde{Y}_{l_2} = \mathcal{S}(\tilde{X}_{l_2})$.
3. Preference matrices are formed.
   With the results of questions 1 to 3, the preference matrix with respect to the strategies is formed.
   With the results of questions 4-9, comparisons between alternatives are formed, establishing strategy S_1.
   With the results of questions 10-15, comparisons between alternatives are formed, establishing strategy S_2.
   With the results of questions 16-21, comparisons between alternatives are formed, establishing strategy S_3.

This is done using the following method:

If for the aggregate values of one of the matrices that is $\tilde{Y}_{l_1} > \tilde{Y}_{l_2}$, then the comparison matrix is formed as $z_{pq} = \tilde{Y}_{l_1}$ and $z_{qp} = \frac{1}{z_{pq}}$, where $p < q$.

If $\tilde{Y}_{l_2} > \tilde{Y}_{l_1}$, then the comparison matrix is formed as $z_{qp} = \tilde{Y}_{l_2}$ and $z_{pq} = \frac{1}{z_{qp}}$.

If $\tilde{Y}_{l_2} = \tilde{Y}_{l_1}$, then the comparison matrix is formed as $z_{pq} = z_{qp} = 1$.

Note that the elements of the main diagonal are always equal to 1 and in the case that no expert has chosen one of the options, it is evaluated as $\tilde{0} = \langle(0,0,0,0); 0,1,1\rangle$.

4. The classical AHP method is applied to these results.
5. The priority vectors are used as coefficients of Equations 7 and 8.

To show the effectiveness of the proposed method we illustrate with an example:

Example 1: Let us suppose there is a group of 5-year-old Ecuadorian children and we want to determine the dynamics of their phonological knowledge, where total Knowledge is taken into account. Knowledge Creation is the knowledge that arises within the group spontaneously, mainly as a result of interaction between the teacher and the children and the children among themselves; Knowledge Acquisition is the knowledge acquired internally by the child's teacher or externally outside the group, through support from each child's family, an outside teacher, among others; and finally, the Loss of Knowledge, as a result of a child no longer in the group, a teacher who for some reason left, among others.

Given, there are three experts denoted by e_1, e_2, e_3, each one whom answers the questionnaire described above and the results are the following ones summarized in Tables 4-6:

Table 4. Survey results for expert 1.

Question	Favorite	Assessment
1	S_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
2	S_1	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
3	S_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
4	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$

Question	Favorite	Assessment
5	A3	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
6	A1	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
7	A3	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
8	A4	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
9	A4	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
10	A2	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
11	A3	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
12	Both	$\tilde{1} = \langle(1,1,1,1);0.50,0.50,0.50\rangle$
13	A2	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
14	A4	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
15	A3	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
16	A1	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
17	A1	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
18	A1	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
19	A2	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
20	Both	$\tilde{1} = \langle(1,1,1,1);0.50,0.50,0.50\rangle$
21	A3	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$

Table 5. Survey results for expert 2.

Question	Favorite	Assessment
1	S2	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
2	S3	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
3	S2	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
4	A2	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
5	A3	$\tilde{4} = \langle(3,4,5,6);0.60,0.35,0.40\rangle$
6	Both	$\tilde{1} = \langle(1,1,1,1);0.50,0.50,0.50\rangle$
7	A3	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
8	A2	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
9	A3	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
10	A2	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
11	A1	$\tilde{4} = \langle(3,4,5,6);0.60,0.35,0.40\rangle$
12	A1	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
13	A2	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$
14	A2	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
15	A4	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
16	A2	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
17	A1	$\tilde{4} = \langle(3,4,5,6);0.60,0.35,0.40\rangle$
18	Both	$\tilde{1} = \langle(1,1,1,1);0.50,0.50,0.50\rangle$
19	A2	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
20	A4	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
21	A4	$\tilde{5} = \langle(4,5,6,7);0.80,0.15,0.20\rangle$

Table 6. Survey results for expert 3.

Question	Favorite	Assessment
1	Both	$\tilde{1} = \langle(1,1,1,1);0.50,0.50,0.50\rangle$
2	S3	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
3	S2	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
4	Both	$\tilde{1} = \langle(1,1,1,1);0.50,0.50,0.50\rangle$
5	A1	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
6	A1	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
7	A2	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
8	A4	$\tilde{3} = \langle(2,3,4,5);0.30,0.75,0.70\rangle$
9	A4	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$
10	A1	$\tilde{2} = \langle(1,2,3,4);0.40,0.65,0.60\rangle$

Question	Favorite	Assessment
11	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
12	A1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
13	A3	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
14	A2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
15	A3	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
16	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
17	A1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
18	A1	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
19	A3	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
20	A2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
21	A4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$

The aggregated results for all experts have the following results as shown in Table 7.

Table 7. Results of the survey aggregation for all experts for each of the preferences.

Question	Aggregated results when the first option is preferred	Aggregated results when the second option is preferred
1	$\langle(3.0, 4.0, 5.0, 6.0); 0.3, 0.75, 0.75\rangle$	$\langle(5.0, 6.0, 7.0, 8.0);\ 0.5, 0.5, 0.5\rangle$
2	$\langle(4, 5, 6,7); 0.80, 0.15, 0.20\rangle$	$\langle(3.0, 5.0, 7.0, 9.0);\ 0.3, 0.75, 0.75\rangle$
3	$\langle(4.0, 7.0, 10.0, 13.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(0,0,0,0); 0,1,1\rangle$
4	$\langle(3.0, 4.0, 5.0, 6.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(2.0, 3.0, 4.0, 5.0);\ 0.4, 0.65, 0.65\rangle$
5	$\langle(2, 3, 4,5); 0.30, 0.75, 0.70\rangle$	$\langle(4.0, 6.0, 8.0, 10.0);\ 0.4, 0.65, 0.65\rangle$
6	$\langle(4.0, 6.0, 8.0, 10.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 1,1,1); 0.50, 0.50, 0.50\rangle$
7	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$	$\langle(6.0, 8.0, 10.0, 12.0);\ 0.3, 0.75, 0.75\rangle$
8	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$	$\langle(3.0, 5.0, 7.0, 9.0);\ 0.3, 0.75, 0.75\rangle$
9	$\langle(4, 5, 6,7); 0.80, 0.15, 0.20\rangle$	$\langle(3.0, 5.0, 7.0, 9.0);\ 0.3, 0.75, 0.75\rangle$
10	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$	$\langle(3.0, 5.0, 7.0, 9.0);\ 0.3, 0.75, 0.75\rangle$
11	$\langle(4.0, 5.0, 6.0, 7.0);\ 0.5, 0.5, 0.5\rangle$	$\langle(5.0, 6.0, 7.0, 8.0);\ 0.5, 0.5, 0.5\rangle$
12	$\langle(4.0, 6.0, 8.0, 10.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 1,1,1); 0.50, 0.50, 0.50\rangle$
13	$\langle(6.0, 8.0, 10.0, 12.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
14	$\langle(4.0, 6.0, 8.0, 10.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
15	$\langle(6.0, 8.0, 10.0, 12.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
16	$\langle(3.0, 4.0, 5.0, 6.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(2.0, 3.0, 4.0, 5.0);\ 0.4, 0.65, 0.65\rangle$
17	$\langle(6.0, 9.0, 12.0, 15.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(0,0,0,0); 0,1,1\rangle$
18	$\langle(4.0, 6.0, 8.0, 10.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 1,1,1); 0.50, 0.50, 0.50\rangle$
19	$\langle(6.0, 8.0, 10.0, 12.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
20	$\langle(3.0, 4.0, 5.0, 6.0);\ 0.3, 0.75, 0.75\rangle$	$\langle(2.0, 3.0, 4.0, 5.0);\ 0.4, 0.65, 0.65\rangle$
21	$\langle(2, 3, 4,5); 0.30, 0.75, 0.70\rangle$	$\langle(5.0, 7.0, 9.0, 11.0);\ 0.4, 0.65, 0.65\rangle$

In Table 8 we convert the results in Table 7 to crisp values.

Table 8. Conversion of the results from Table 7 into crisp values. Note that the option with the highest value appears in red letters.

Question	Results when the first option is preferred	Results when the second option is preferred
1	1.1999999999999997	3.25
2	4.491666666666666	1.5999999999999996
3	2.2666666666666657	0.0
4	1.1999999999999997	1.2833333333333332
5	0.9916666666666664	2.5666666666666664
6	1.866666666666666	0.5
7	0.9583333333333331	2.3999999999999995
8	0.9583333333333331	1.5999999999999996

Question	Results when the first option is preferred	Results when the second option is preferred
9	4.491666666666666	1.5999999999999996
10	0.95833333333333331	1.5999999999999996
11	2.75	3.25
12	1.866666666666666	0.5
13	2.3999999999999995	0.95833333333333331
14	1.866666666666666	0.95833333333333331
15	2.3999999999999995	0.95833333333333331
16	1.1999999999999997	1.28333333333333332
17	2.7999999999999994	0.0
18	1.866666666666666	0.5
19	2.3999999999999995	0.95833333333333331
20	1.1999999999999997	1.28333333333333332
21	0.9916666666666664	2.93333333333333336

The preference matrices for each of the AHP levels are as follows, see Tables 9-12:

Table 9. Pair-wise matrix preferences between strategies.

Strategy	S_1	S_2	S_3
S_1	1	1/3.25	4.492
S_2	3.25	1	2.27
S_3	1/4.492	1/2.27	1

Table 10. Pair-wise matrix preferences of the alternatives given strategy S_1.

Alternative	A_1	A_2	A_3	A_4
A_1	1	1/1.283	1/2.57	1.87
A_2	1.283	1	1/2.4	1/1.6
A_3	2.57	2.4	1	4.492
A_4	1/1.87	1.6	1/4.492	1

Table 11. Pair-wise matrix preferences of the alternatives given strategy S_2.

Alternative	A_1	A_2	A_3	A_4
A_1	1	1/1.6	1/3.25	1.9
A_2	1.6	1	2.4	1.9
A_3	3.25	1/2.4	1	2.4
A_4	1/1.9	1/1.9	1/2.4	1

Table 12. Pair-wise matrix preferences of the alternatives given strategy S_3.

Alternative	A_1	A_2	A_3	A_4
A_1	1	1/1.283	2.8	1.9
A_2	1.283	1	2.4	1/1.28
A_3	1/2.8	1/2.4	1	1/2.93
A_4	1/1.9	1.28	2.93	1

The results of applying the AHP method and the priority vectors for the matrices shown in Tables 9-12 are as follows:

Table 13 contains the vector of priorities between the strategies.

Table 13. Vector of priorities for the pairwise comparison matrix S.

Strategies	Priorities
S_1	0.32610087521915954
S_2	0.5303680832323449
S_3	0.14353104154849564

In this matrix, C.R. = 0.3516554630297951 was obtained, which indicates insufficient consistency.

Table 14 contains the priority vectors of the alternatives concerning each of the strategies.

Table 14. Vector of priorities for the pairwise comparison matrices of alternatives for every strategy S_1, S_2, and S_3, respectively.

Alternative	Priority Vector corresponding to S_1	Priority Vector corresponding to S_2	Priority Vector corresponding to S_3
A_1	0.18660691978792374	0.18467616437284845	0.32976329453713604
A_2	0.17370741401018375	0.3715342150844241	0.28748786884937794
A_3	0.48692234557507535	0.3119371145799139	0.10675771232862649
A_4	0.15276332062681713	0.13185250596281362	0.27599112428485956

Here we obtain C.R. = 0.07408881902313505, which is acceptable consistency for the alternatives concerning S_1, C.R. = 0.1165789677683065 which is insufficient consistency for the alternatives concerning S_2, and C.R. = 0.045569823859665876 which is acceptable consistency concerning S_3.

We should note that some results are not consistent, but this is not taken into account within the Bratianu method, however, if the decision-makers wish, they can repeat the evaluations in those comparisons with inconsistencies.

In this way, Equation 7 would be as follows:

$$\Delta K = 0.33\Delta C + 0.53\Delta A - 0.14\Delta L$$

So, the equations derived from Equation 8 are the following:

$$\Delta S_1 = \Delta C = 0.19\Delta A_1 + 0.17\Delta A_2 + 0.49\Delta A_3 + 0.15\Delta A_4,$$

$$\Delta S_2 = \Delta A = 0.19\Delta A_1 + 0.37\Delta A_2 + 0.31\Delta A_3 + 0.13\Delta A_4,$$

$$\Delta S_3 = \Delta L = 0.33\Delta A_1 + 0.29\Delta A_2 + 0.11\Delta A_3 + 0.27\Delta A_4.$$

According to the equations above to increase knowledge acquisition is the most influential aspect of knowledge dynamics with over 53% influence, followed by increasing knowledge creation with 33% influence.

On the other hand, the dynamic changes within the "increase knowledge creation" highlight the influence that the alternative of dialogic reading with phonological emphasis can have. Regarding the strategy of increasing knowledge acquisition, the best way to achieve it is by applying the alternative"Sound segmentation and joining games", with 37% followed by "dialogic reading with phonological emphasis" with 31%. To avoid the loss of knowledge, the most influential is to apply the alternative "Guided phonological songs" with 33% of importance, followed by "Sound segmentation and joining games" with 29% of weight and "Puppets with articulatory exaggeration" with 27%; however, in this last strategy there is no alternative that prevails as much over the others, as it happens with the two previous strategies.

The last four equations can be used to study the dynamics of knowledge, based on measurements that can be made experimentally over time, using equations 9-11.

4. Conclusion

This paper proposes a model for representing phonological knowledge in preschool children. It is important at this stage for children to articulate and express themselves correctly in their native language, and to be able to understand the people around them, of all ages. This knowledge is critical for good cognitive development, social interaction, and psychological development. In particular, we draw on the model of Professor C. Bratianu, who studied and represented the dynamics of knowledge inspired by the energy flow that occurs in thermodynamics. In the proposed model, we offer a way to measure phonological skills using the Single-Valued Trapezoidal Neutrosophic Numbers and we also propose to apply the Neutrosophic AHP technique instead of the classical AHP in the calculations of the dynamic equations within the Bratianu model. The Neutrosophic AHP and the Single-Valued Trapezoidal Neutrosophic Numbers in general, allow capturing the indeterminacy, uncertainty, and vagueness that exist in evaluations based on subjective opinions, which is a limitation of the crisp technique. Respondents are asked for their opinions on a scale based on natural language, as in the classic AHP model. In summary, it can be said that the proposed model extends Bratianu's model to the field of neutrosophy, and also adapts to the specific problem of studying the dynamics of phonological knowledge in children of approximately 5 years of age, which constitutes a contribution to the study of this phenomenon.

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