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Structural equilibrium in universities: quality, finance and sustainability from an empirical perspective: An integrative analysis with neutrosophic cognitive maps

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Abstract: This study explores the structural relationships between institutional financial health and academic quality in Chilean higher education institutions (HEIs) by applying multivariate tools such as the dynamic HJ-Biplot and fixed-effects regression models for panel data, complemented by neutrosophic cognitive maps (NCMs). Using data from the 2016–2023 period, differentiated institutional trajectories are analyzed based on their membership in free tuition policies, university type (universities belonging to the Council of University Chancellors (CRUCH) and private universities), and financial structure. The results show that variables such as student retention, physical infrastructure, and academic training significantly influence key financial indicators such as liquidity, leverage, and operating margin. NCMs reveal degrees of indeterminacy in these relationships, especially in contexts of high uncertainty such as private universities not affiliated with free tuition. Their integration into this study represents a novel methodological contribution, enabling the modeling of causal ambiguity and structural instability within the academic-financial nexus of higher education institutions. Specific patterns of vulnerability are identified in these institutions, and the stabilizing role of fiscal contributions is confirmed. Research shows that quality and financial sustainability are interdependent dimensions, whose coordination, analyzed from a neutrosophic perspective, is key to strengthening educational well-being and the resilience of the university system in contexts of change.

Keywords: Multivariate Analysis; Financial Health; Educational Quality; Higher Education; Institutional Sustainability; HJ- Dynamic Biplot, Panel Models; Neutrosophic Cognitive Maps.

1. Introduction

The Chilean university system is characterized by its institutional diversity and a mixed financing model that combines public and private resources. In this context, Higher Education Institutions (HEIs) face growing challenges in maintaining financial sustainability without compromising academic quality. While public universities enjoy certain guarantees of survival—such as fiscal support and access to core funds—many of them suffer chronic operational deficits and low profitability [1]. Private universities, meanwhile, subject to market dynamics, have been vulnerable to bankruptcy and closure, generating critical consequences for their student communities and highlighting weaknesses in the system's regulatory mechanisms [2] (Superintendency of Higher Education [SES], 2023).

In recent decades, various reforms have attempted to strengthen the system's governance, including the creation of the SES (National Service for Quality Assurance), the national quality assurance framework, and the implementation of policies such as free tuition. However, questions remain about how the financial and academic dimensions are articulated in Chilean universities. Internationally, studies have highlighted the importance of having solid financial structures to guarantee quality education, especially in contexts of crisis or transformation [2-5]. The literature also suggests that financial management should not be understood as an end in itself, but as a means to sustain educational projects consistent with institutional objectives [6].

However, in the Chilean case, the relationship between financial health and academic quality remains underexplored and conceptually fragmented. Some research argues that financial indicators do not always correlate with the levels of educational quality perceived by students and academic communities [7]. Others argue that there is a disconnect between financial logic—dominated by criteria of efficiency and profitability—and academic logic, focused on the development of knowledge and comprehensive training [8]. This tension takes on special relevance in a scenario of growing demand for transparency, accountability, and institutional quality.

Within this framework, this study seeks to provide empirical evidence on the links between financial health and academic quality in the Chilean university system through the use of multivariate analysis tools, regression models with longitudinal data from the 2016–2023 period, and neutrosophic cognitive maps (NCMs). The objective is twofold: first, to characterize the financial profiles of universities; second, to evaluate the impact of key academic variables—such as student retention, teaching staff, and infrastructure—on financial health indicators, considering the uncertainty inherent in these relationships. NCMs, based on the principles of neutrosophy (Smarandache, 1998), allow for the modeling of causal interactions between financial and academic variables, assigning truth (T), falsity (F), and indeterminacy (I) values to capture ambiguities, especially in universities with high variability, such as private universities not affiliated with free tuition. This is expected to contribute to the design of more integrated public policies that consider both the economic sustainability and educational mission of HEIs, enriching the understanding of their structural balance from a neutrosophic perspective.

1.1. Financial health and risk profiles in Chilean HEIs

Recent rapid social, economic, and health changes—such as the COVID-19 pandemic—have highlighted the need for resilient and financially sustainable higher education institutions (HEIs). In this context, analyzing the financial health of HEIs becomes relevant not only as a management tool but also as a key input for safeguarding the continuity and quality of the educational services they provide (OECD, 2021; World Bank, 2022).

Unlike private companies, universities' central mission is the development of knowledge, comprehensive education, and commitment to the community. Therefore, even if they do not pursue profit maximization, they require financial stability to sustain their infrastructure, academic staff, research activities, and training programs [9]. From this perspective, two key concepts emerge: financial sustainability, understood as the ability to cover operating costs and invest in future development; and financial health, which refers to the institutional capacity to cope with economic stress and meet its financial obligations [2].

Since 2023, the Superintendency of Higher Education (SES) has developed a methodology to assess the financial risk of Chilean HEIs based on indicators such as leverage, liquidity, gross margin, operating cash flow, and fixed asset ratio. The analysis covers the period 2012–2022 and includes universities, professional institutes (PIs), and technical training centers (TCCs). Its main finding is that financial health profiles are not static, and that the proportion of institutions and enrollment within each profile varies from year to year [2].

1.1.1. Institutional risk profiles

Universities: Generally, they have a more solid financial profile. 72% of universities remained stable during the period analyzed, and only 5.8% of their enrollment corresponds to institutions at financial risk. However, some institutions have high levels of debt with a low proportion of fixed assets, making them financially fragile. Universities in this situation (15.5% of the total) depend on liquidity without having equity to support their operations, which generates a structural exposure to leverage [8]. For example, Professional Institutes (PI) exhibit financial behavior similar to that of universities, but with slightly higher debt levels. 18.8% have a risky profile, affecting 2% of enrolled students. The most stable institutions tend to have tighter cost structures and higher income-to-enrollment ratios. Technical Training Centers (CFTs): These are the group with the highest proportion of financial risk. 22.7% of CFTs have this profile, representing 4.5% of enrollment. This group tends to display high levels of debt without sufficient financial backing, which creates greater vulnerability to fluctuations in enrollment or regulatory changes. Furthermore, between 2015 and 2022, the number of CFTs with a risky profile doubled, although this increase was attenuated after the implementation of the free tuition policy starting in 2016 [10].

1.1.2. Longitudinal perspective

The longitudinal analysis shows that most universities and private universities maintained stable financial profiles between 2012 and 2022. In contrast, the CFTs exhibited greater volatility. Free tuition appears to have played a stabilizing role, especially in institutions with a higher concentration of vulnerable students, by reducing their exposure to financial risk [5].

In summary, the financial health of Chilean HEIs is determined by a combination of structural and management factors. While the majority of the system is in a reasonably solid position, some segments remain highly vulnerable. Linking financial indicators with academic variables allows for a more comprehensive approach to quality assurance, in line with the recommendations of international organizations [3-4].

1.2. Sustainability and quality in Chilean HEIs: An integrative look at financial and academic profiles.

Although the financial health of higher education institutions (HEIs) has traditionally been assessed through accounting and asset indicators, in recent years there has been a growing recognition that academic and educational management variables play a decisive role in institutional sustainability [3, 9]. This approach seeks to overcome a purely economic view of university performance, recognizing that quality and financial stability are built on a relationship of mutual influence.

In the Chilean case, the report prepared by the Superintendency of Higher Education (SES, 2023) has been criticized for overemphasizing financial indicators as a measure of institutional health. Several academics have warned that this approach runs the risk of obscuring key academic factors, such as teaching quality, student retention, or scientific productivity, which not only constitute essential dimensions of the university's mission but also impact financial stability.[8]

From a critical perspective, [8] argue that there is a disconnect between academic and financial logic in many Chilean HEIs, which generates tensions in quality assurance models. This phenomenon has been particularly observed in state and regional universities, which, despite having high social valuation and good quality indicators, exhibit persistent financial weaknesses, associated with their lower capacity to generate their own income or dependence on public funding [1].

In turn, Moraga and Rossi [7] argue that the relationship between financial structure and perception of institutional quality is neither linear nor automatic. In their study of traditional Chilean universities, the authors found that, although there are economic indicators that correlate with perceived quality, when analyzed together the effect is diluted, which suggests the need for more robust multivariate models to understand this complex relationship.

In this sense, [11] propose that university quality should be understood as the result of a coherence between financial strategies, academic policies, and institutional management mechanisms. From this perspective, a financially healthy but academically weak HEI, or vice versa, cannot be considered quality. The key lies in the articulation between both dimensions, mediated by strategic management that responds to the characteristics and mission of each institution.

Among the academic variables most frequently associated with financial health are:

- Student retention, which directly impacts the stability of tuition revenue flow and the efficient use of resources.
- The teaching load (students per full-time equivalent), which allows estimating the efficiency in the allocation of academic staff.
- Infrastructure per student, as an indicator of institutional investment in teaching-learning conditions.
- The level of academic training, especially the proportion of professors with postgraduate or doctoral degrees, can influence institutional prestige and the ability to obtain competitive funding, but also increase operating costs.
- The degree of dependence on state funding, which can act as a stabilizing factor or as a source of budgetary rigidity, depending on the context.

The findings of this study—using fixed-effects regression models—support this perspective: it is observed that student retention, the provision of adequate infrastructure, and access to fiscal contributions are correlated with lower exposure to financial risk. Conversely, in some institutional segments, such as universities with no tuition fees, maintaining a highly qualified faculty can represent a financial burden that reduces liquidity and operating margins.

These results are consistent with international studies that have documented how the most resilient university systems are those that successfully integrate academic and financial decisions into a coherent institutional strategy [3-4]. Rather than opposing quality and sustainability, these approaches propose viewing them as interdependent elements, whose alignment is essential for strengthening the higher education system.

2. Materials and Methods

This study uses a quantitative, non-experimental approach, with a longitudinal design and an exploratory-correlational approach. Its purpose is to examine the links between financial health and selected academic variables in Chilean higher education institutions (HEIs) based on an analysis of secondary data reported between 2016 and 2023, integrating a neutrosophic approach using neutrosophic cognitive maps (NCMs) to model uncertainty in these relationships.

The research adopts a longitudinal design based on time series analysis, which allows observing the evolution of institutions over time and detecting differentiated trajectories according to their financial and academic behavior. This approach is relevant in studies on institutional sustainability, as it allows identifying patterns of change, persistence or instability over time [12]. NCMs, based on the principles of neutrosophy (Smarandache, 1998), complement this design by representing the causal relationships between academic variables (such as student retention and infrastructure) and financial variables (such as leverage and liquidity), assigning truth (T), falsity (F) and indeterminacy (I) values to capture ambiguities in the interactions.

This is a non-experimental quantitative study, since there is no manipulation of independent variables. Instead, data obtained from official sources are analyzed and statistical relationships between them are modeled. Furthermore, it is framed within an explanatory-correlational level, as it seeks to generate empirical knowledge in a field that has received little attention in the Chilean context and to detect significant associations between financial and academic indicators [13]. The use of MCN enriches this approach by allowing a more robust interpretation of the relationships, highlighting the indeterminacy in contexts of high institutional heterogeneity, such as private universities not affiliated with free education. The data used come from the Superintendency of Higher Education (SES) and the Undersecretariat of Higher Education, agencies that annually publish financial, academic, and administrative information on Chilean HEIs. Universities belonging to and not belonging to the CRUCH were included.

2.1. Analysis methods

Biplot methods [14 -18] and regression models with fixed effects (PanelOLS).

2.1.1. HJ-Dynamic Biplot

Dynamic Biplot [17, 18] was used, a multivariate technique that allows to simultaneously represent the institutions (rows) and the variables (columns) in a two-dimensional factorial space, maximizing the quality of the representation, from the singular value decomposition (SVD) [14].

dynamic HJ- Biplot proposed by Egido and Galindo- Villardón [18], extends the classical HJ- Biplot proposed by Galindo- Villardón [15], by incorporating a temporal dimension, which allows to visualize the evolution of individuals or institutions over time. This technique is especially useful in longitudinal studies, since it captures not only the relative positions in each period, but also the trajectory and trends followed by the observations [17].

The dynamic component of the Biplot allowed for observation of institutional trajectories over time, highlighting shifts in their financial. The length and direction of the vectors represent the variance and correlation explained by each indicator, while the relative position of the institutions facilitates comparisons between levels of performance and stability.

2.1.2. Fixed effects regression models (PanelOLS)

To identify significant associations between academic and financial health variables, five fixed effects regression models for panel data (PanelOLS) were fitted, an appropriate technique when it is desired to control for unobservable heterogeneity between entities (institutions) and for common period effects [19].

The base model is composed of the form

$$y_{it} = \alpha_i + \beta X_{it} + \lambda_t + \epsilon_{it}$$

where:

- $y_i t$is the value of the financial indicator for the institution $i$in the year$t$
- $\alpha_i$represents unobservable effects specific to each institution (constant over time).
- $X_{it}$is the vector of academic variables.
- $\lambda_t$captures common temporary effects (such as regulatory or economic changes)
- $\epsilon_{it}$is the error term

This approach improves the robustness of estimates by considering that each institution has structural characteristics that are constant over time and can influence the results. Six key financial

health indicators were analyzed as dependent variables: leverage, adjusted leverage , current liquidity, operating cash flow, gross margin, and fixed asset ratio.

Independent variables included academic dimensions such as first- and second-year retention, teaching load, level of academic training (postgraduate and doctoral), infrastructure per student, and dependence on fiscal funding.

The analyses were conducted by differentiating between institutional segments: private universities, CRUCH, those affiliated with and those not affiliated with free tuition, in order to capture possible specific patterns by type of institution.

2.2. Neutrosophic Cognitive Maps

Definition 1: ([20,21,22]) Let X be a universe of discourse . A *Neutrosophic Set* (NS) is characterized by three membership functions , , $u_A(x), r_A(x), v_A(x) : X \to]_A^-0, 1^+[$ that satisfy the condition $_A^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for everyone $x \in X$. $u_A(x), r_A(x) \text{and} v_A(x)$ are the truthfulness , indeterminacy , and falsity membership functions of x in A, respectively , and their images are subsets standard or non- standard of $]_A^-0, 1^+[$.

Definition 2: ([20,21,22]) Let X be a universe of discourse . A *Univalued Neutrosophic Set* (SVNS) A over X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \qquad (1)$$

Where $u_A, r_A, v_A : X \to [0,1]$, satisfies the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for everyone $x \in X$. $u_A(x), r_A(x) \text{and} v_A(x)$ denote the truthfulness , indeterminacy , and falsity membership functions of x in A , respectively . For convenience , a *Univalent Neutrosophic Number* (NNUN) will express itself as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Others definitions important are related to graphics.

Definition 3: ([20,21,22, 26, 27]) A *neutrosophic graph* is a graph that contains at least one edge indeterminate , which is represented by dotted lines .

Definition 4: ([20,21,22, 26, 27]) A *graph directed neutrosophic* is a graphic directed that contains at least one edge indeterminate , which is represented by dotted lines .

Definition 5: ([20,21,22]) A *Neutrosophic Cognitive Map* (NCM) is a graph directed neutrosophic, whose nodes represent concepts and whose edges represent relations causal between the edges .

Yeah$C_1, C_2, \ldots, C_k$ There are k nodes, each of whichC_i $(i = 1, 2, \ldots, k)$ can represent themselves by means of a vector $(x_1, x_2, \ldots, x_k)$where $x_i \in \{0, 1, I\}$. $x_i = 0$means the nodeC_i this in a state activated, $x_i = 1$means that the nodeC_i this in a state disabled and$x_i = I$ means that the nodeC_i this in a state indeterminate, in a time specific or in a situation specific.

YeahC_m andC_n They are two nodes of the NCM, one edge addressed fromC_m toC_n It is called connection and represents the causality of C_ma C_n. Each node in the NCM is associated with a weight within the set $\{-1, 0, 1, I\}$. Ifα_{mn} denotes the weight of the edge $C_m C_n$,$\alpha_{mn} \in \{-1, 0, 1, I\}$ So we have the following:

$\alpha_{mn} = 0$YeahC_m does not affect C_n,

$\alpha_{mn} = 1$If there is an increase (decrease) inC_m produces an increase (decrease) in C_n,

$\alpha_{mn} = -1$If there is an increase (decrease) inC_m produces a decrease (increase) in C_n,

$\alpha_{mn} = I$If the effect ofC_m inC_n is indeterminate.

Definition 6: ([20,21,22]) An NCM that has edges with weights in$\{-1, 0, 1, I\}$ It is called Simple Neutrosophic *Cognitive Map* .

Definition 7: ([20,21,22]) If$C_1, C_2, \dots, C_k$ are the nodes of an NCM. The neutrosophic matrix N(E) is defined as $N(E) = (\alpha_{mn})$, whereα_{mn} denotes the weight of the edge addressed $C_m C_n$, such that $\alpha_{mn} \in \{-1, 0, 1, I\}$. $N(E)$is called *the neutrosophic adjacency matrix* of the NCM.

Definition 8: ([20,21,22]) Let$C_1, C_2, \dots, C_k$ Let be the nodes of an NCM. Let $A = (a_1, a_2, \dots, a_k)$, where $a_m \in \{-1, 0, 1, I\}$. A is called *the neutrosophic state vector instant* and represents the position of the state node on- off -indeterminate at a given moment .

$a_m = 0$YeahC_m this disabled (does not have effect),

$a_m = 1$YeahC_m is activated (has an effect),

$a_m = I$YeahC_m is indeterminate (it cannot be determine his effect).

Definition 9: ([20,21,22]) Let$C_1, C_2, \dots, C_k$ Let, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…, $\overrightarrow{C_mC_n}$be the nodes of an NCM.$\overrightarrow{C_1C_2}$ be the edges of the NCM, then the edges constitute a *cycle addressed* .

The NCM is called *cyclic* Yeah presents a cycle directed . It is called *acyclic* if it does not present a cycle directed .

Definition 10: ([20,21,22]) An NCM with cycles is said to be has feedback . When exists feedback in the NCM is said to be a *system dynamic* .

Definition 11: ([20,21,22]) Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_{k-1}C_k}$ be a cycle. WhenC_m is activated and its causality flows by the edges of the cycle and then it is the cause ofC_m Per se same, then he system dynamic circulates. This is true for each node . $C_m$with $m = 1, 2, \dots, k$. The equilibrium state of this dynamic system is called *pattern hidden* .

Definition 12: ([20,21,22]) If the equilibrium state of a system dynamic is a state unique , then it is called *fixed point* .

An example of a fixed point is when a system dynamic begins to be activated by [number] C_l. If the NCM is assumed to lie on [number] C_l and [number C_k], i.e. the state remains as [$(1, 0, \dots, 0, 1)$number], then this neutrosophic state vector is called *fixed point* .

Definition 13: ([20,21,22]) If the NCM is established with a repeating neutrosophic state vector in the form:

$A_1 \to A_2 \to \cdots \to A_m \to A_1$, then the equilibrium is called *a cycle limit* from the NCM.

Method for determining patterns hidden

Leave$C_1, C_2, \dots, C_k$ Let be the nodes of the NCM with feedback. Suppose E is the adjacency matrix associated . A pattern is found hidden when C_l is activated and a vector input is used. $A_1 = (1, 0, 0, \dots, 0)$ It is provided . The data must go through the neutrosophic $N(E)$matrix, which is obtained by multiplyingA_1 by the matrix $N(E)$.

Leave $A_1N(E) = (\alpha_1, \alpha_2, \dots, \alpha_k)$ with the replacement α_m threshold operation for 1 yes $\alpha_m > p$ andα_m for 0 yes$\alpha_m < p$ (p is an integer positive suitable) andα_m is replaced by I if it is not an integer . The concept resulting vector is updated;C_1 is included in the updated vector transforming the first coordinate of the resulting vector in 1.

Yeah$A_1N(E) \to A_2$ It is assumed, $A_2N(E)$considered and the same procedure is repeated until a limit cycle or a fixed point is reached.

Definition 14: ([23,24, 25]) A *neutrosophic number* N is defined as a number as go on :

$N = d + I$ (2)

Where d is called part determined and i is called part indeterminate .

Giventhat $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2$Ithey are two neutrosophic numbers , some Operations between them are defined as follows manner :

$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$ (Addition);

$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

3. Results

This section presents the findings obtained from the multivariate analysis of financial health and its relationship with academic variables in Chilean higher education institutions (HEIs), using two complementary approaches: the visualization of trajectories through dynamic HJ- Biplots and the adjustment of regression models with fixed effects (PanelOLS). The combination of both methods allows capturing not only the temporal evolution of institutions in terms of financial risk, but also establishing statistically significant associations between structural academic variables and the main indicators of financial sustainability. The results are presented differentiating institutional types (CRUCH, private, affiliated and non-affiliated with free education), which allows identifying specific patterns and relevant disparities in the system.

3.1. Results obtained with the dynamic Biplot

Figure 1 shows the financial and financial ealth indicators of the CRUCH universities, highlighting their trajectories from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are marked in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A 60% inertia filter was used for the visualization, omitting institutions that vary less than this percentage around the data mean (coordinate origin). The observed variance absorption is close to 60%, ensuring a good representation of the data. Quite varied trajectories are observed, but universities with potential financial risk and financial risk with high debt ratios are evident in both Leverage and Financial Leverage. This indicates the indebtedness of Higher Education Institutions (HEIs) as the main cause of risk. Furthermore, greater volatility is evident in these institutions, with the University of Aysén standing out as the one with the highest risk and greatest variability, along with the Alberto Hurtado University, which, despite having potential financial risk, shows a trajectory toward high debt indicators.

Regarding the variables, high correlations are observed between the Gross Margin and the operational cash flow and, in the CRUCH institutions, it is observed that the total debt ratios present moderate correlations with the debt ratios with the financial system, not relating to a large extent, as would be expected, the equity commitment of the institutions with the commitment of the HEIs with the financial market and the dependence on financial institutions.

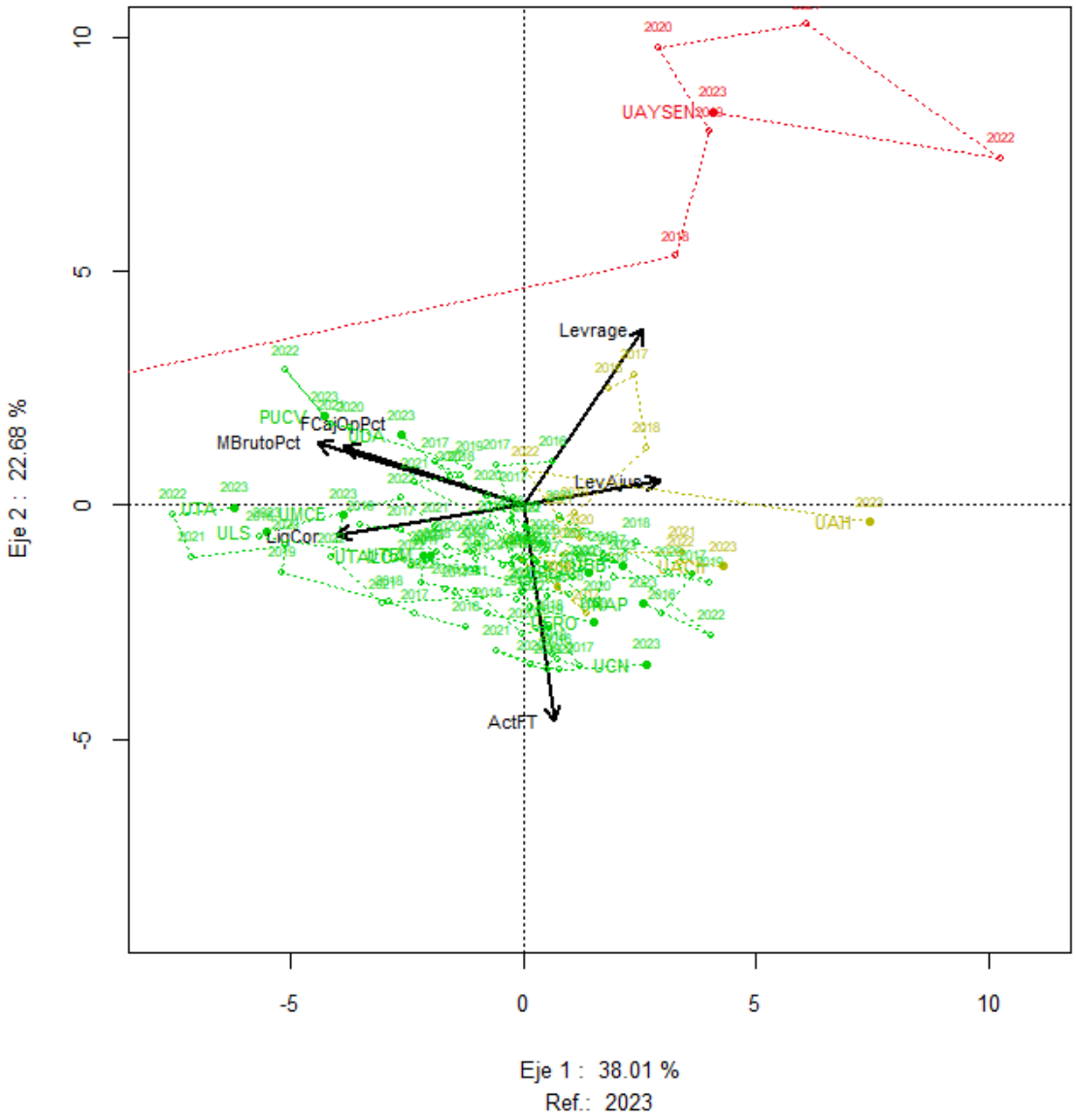

Figure 1. Dynamics of CRUCH Universities.

Figure 2 shows the financial and financial health indicators of private universities, highlighting their trajectories from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are marked in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A 50% variance filter was used for the visualization, omitting institutions that vary less than this percentage around the information mean (coordinate origin). The observed variance absorption rate is close to 58%, ensuring a highly consistent representation of the study data.

Regarding private universities, a strong positive correlation is observed between Leverage and Adjusted Leverage, demonstrating strong relationships between equity commitment and commitment to the financial system of private universities. Furthermore, it is evident that Gabriela Mistral University, having a high financial risk, presents the greatest variability in its performance and high debt ratios. Meanwhile, Finis University Terrae, classified by the SES as having potential financial risk, shows low mobility in its trajectory over the years of analysis, but even with high values in proportion to fixed assets, it presents low Gross Margin and Operating Cash Flow values, framing its potentially risky positioning.

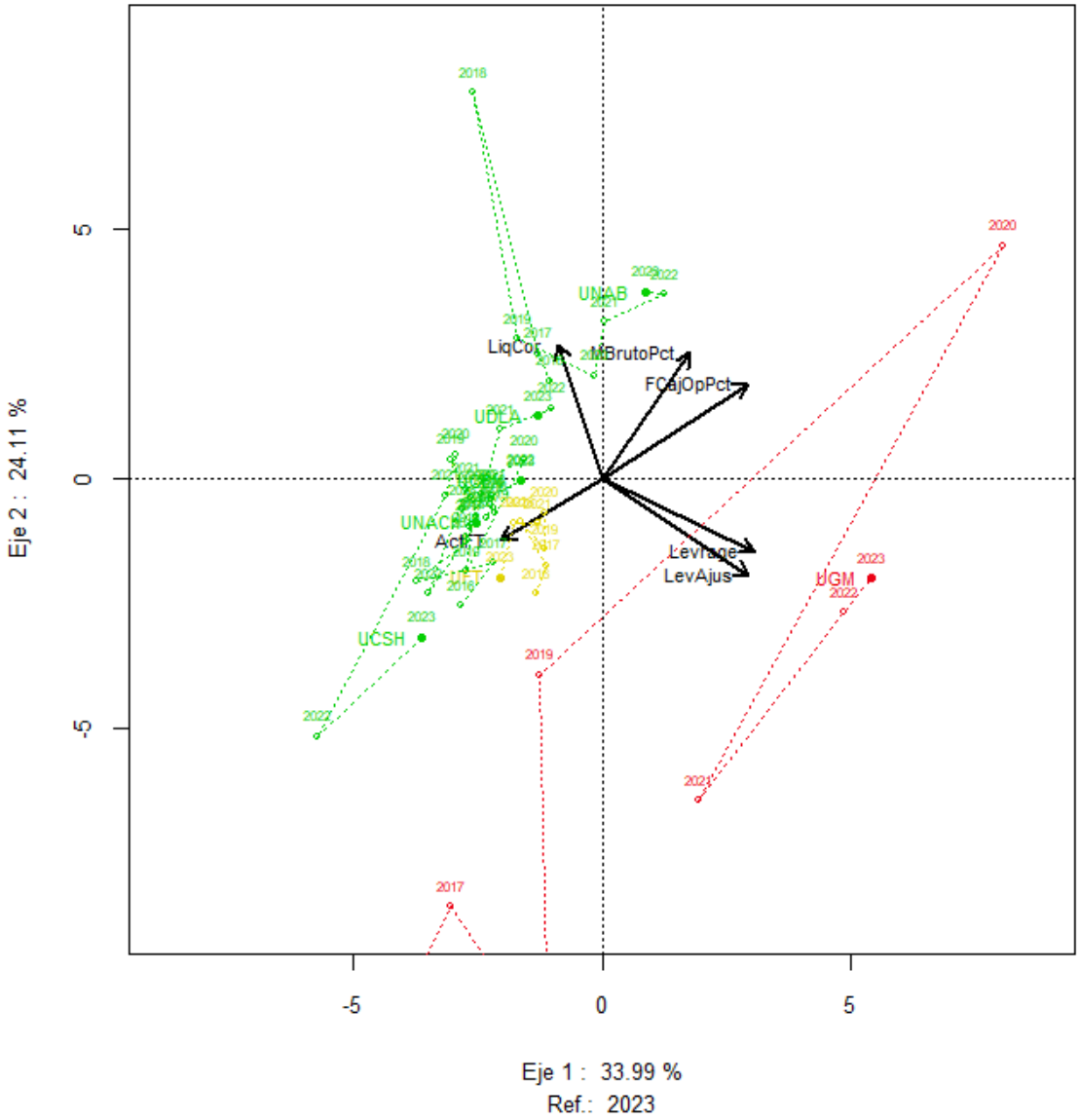

Figure 2. Dynamics of Private Universities.

Figure 3 shows the financial and financial health indicators of universities that offer free tuition, highlighting their performance from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are highlighted in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A 60% variance filter was used for the visualization, omitting institutions that vary less than this percentage around the data mean (coordinate origin). The observed variance absorption rate is close to 59%, ensuring a highly consistent representation of the study data.

This new segmentation again reveals high variability in the performance of the University of Aysén and Alberto Hurtado University in terms of Financial Risk and Potential Risk, respectively. Once again, these universities are framed within high levels of debt (Leverage and Adjusted Leverage) among institutions offering tuition-free tuition, in addition to low proportions of fixed assets and current liquidity.

In this segment, debt with the financial system shows moderate correlations with total debt, but the former is framed within much lower variability, suggesting that universities with positive

trajectories in this direction indicate greater risk even though they visually show less movement within the reference framework.

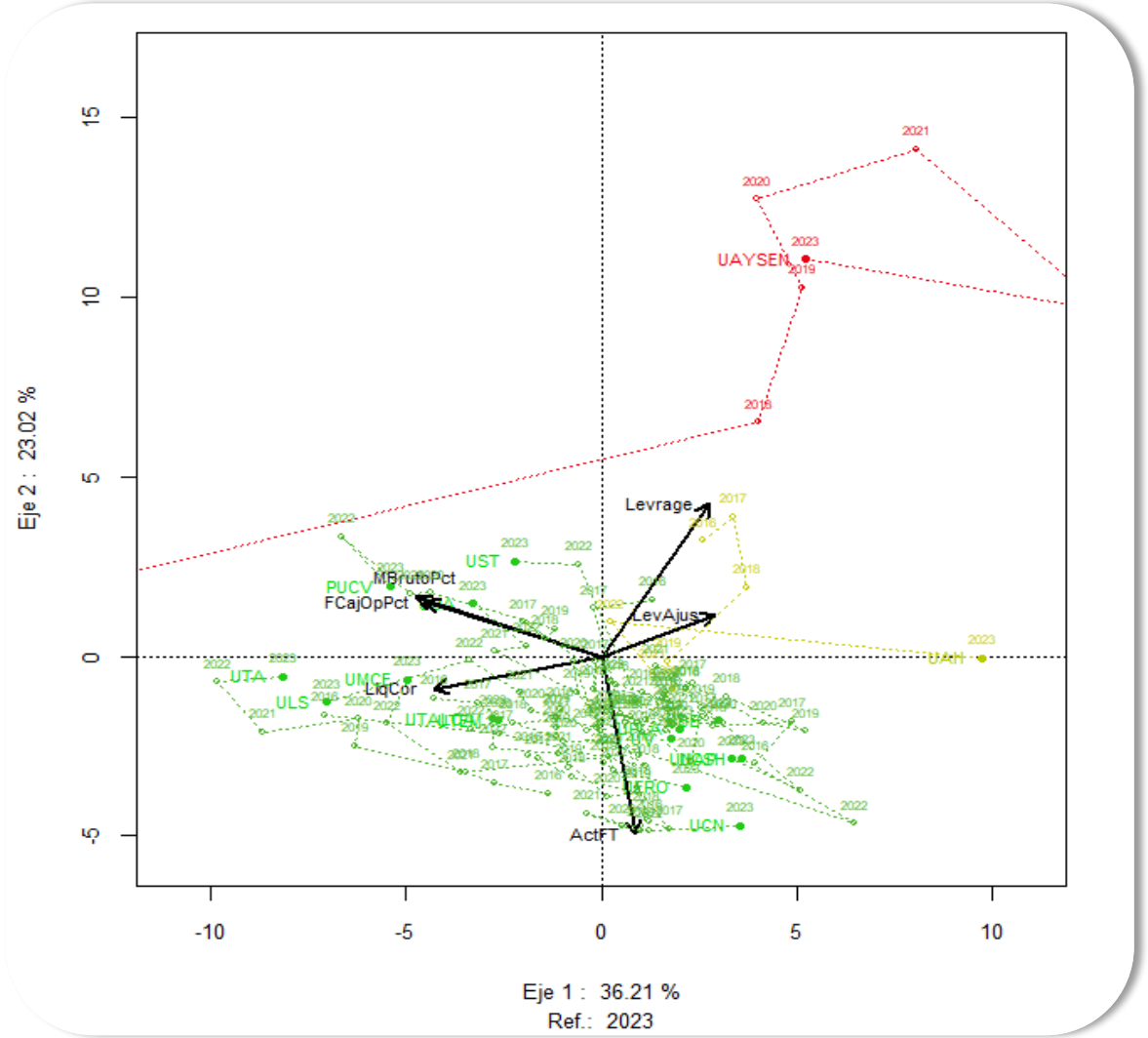

Figure 3. Dynamics of free institutions

Figure 4 shows the financial and financial health indicators of universities not affiliated with free tuition, highlighting their trajectories from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are marked in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A variance filter was not used for the visualization given the small number of cases compared to the other representations, omitting institutions that vary less than this percentage around the information mean (coordinate origin). The observed variance absorption rate is close to 61%, ensuring a good quality representation of the study data.

Volatile trajectories are observed in most institutions in this segment, especially at Gabriela Mistral University and Universidad del Desarrollo, classified as Financial Risk and Potential Risk, respectively, according to the SES. Both institutions present trajectories around high levels of debt, highlighting that Leverage and Financial Leverage present an almost perfect linear correlation. This indicates that in the case of universities not affiliated with free tuition, there is a high dependence on the financial system, a structural risk, since if there are credit restrictions, rate increases, or bank liquidity demands, their sustainability could be quickly compromised. Without a "fiscal cushion" like free tuition, they lack stable state revenues to cushion the leverage.

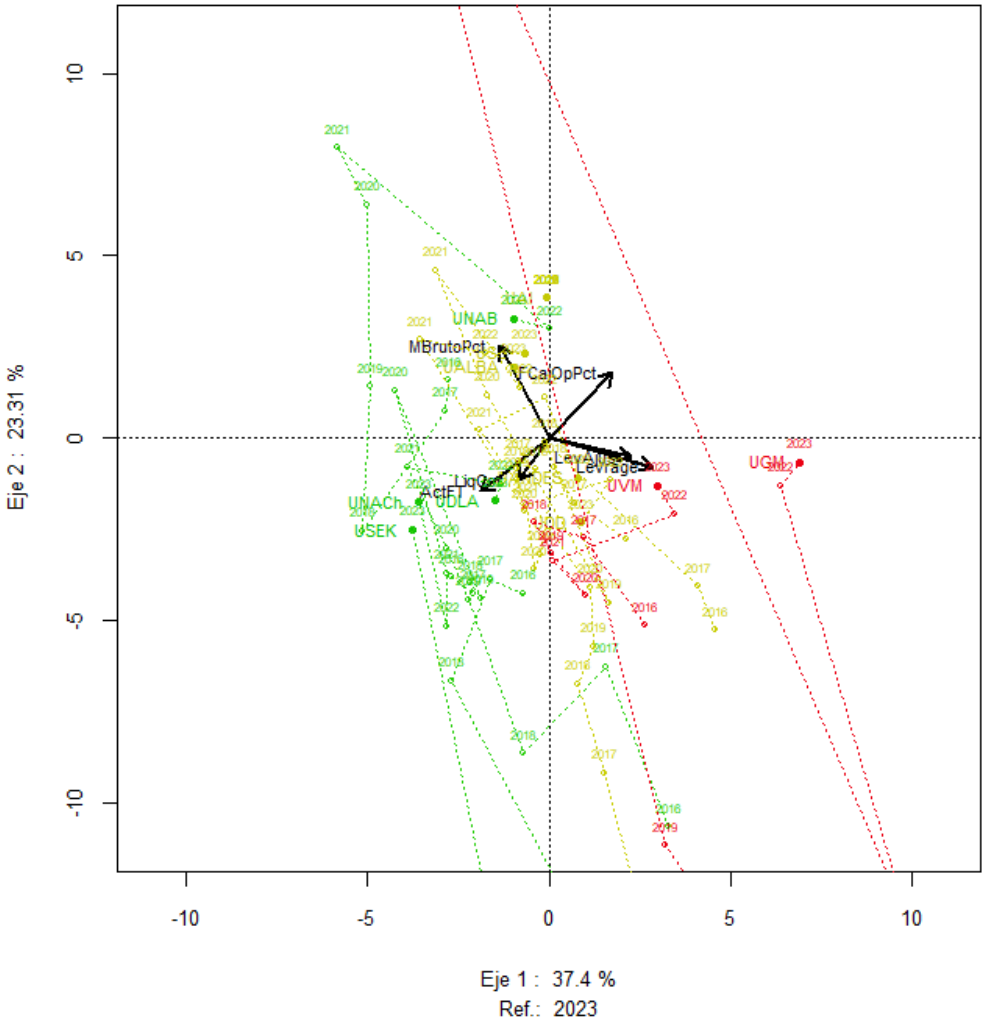

Figure 4. Dynamics of institutions not affiliated with free education

3.2. Fixed effects regression model

Below are the results of the fixed-effects regression models for each of the financial indicators in the analysis, highlighting the comparison between the general model and the segmentations for private universities, CRUCH universities, tuition-free universities, and non-tuition-free universities.

Table 1 shows that higher student retention (first and second years) is associated with higher overall debt, especially at tuition-free institutions and private universities. Greater infrastructure per student and higher fiscal contributions, both as a percentage of income and assets, are consistently associated with lower leverage, confirming their protective role. At CRUCH universities, it is notable that having a higher proportion of faculty with postgraduate degrees is also associated with lower debt, suggesting greater institutional strength. Finally, universities without tuition-free programs have stronger relationships between retention and debt, reflecting possible structural fragility when financing operations without state support.

Table 1. Comparative fixed effects regression model for Leverage.

Leverage	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	1,785	2,6961	7.08	3,0119	5,4028
1st_year_retention	**1.6527	-1.6846	**1.4578	**1.2108	-2.9758
2nd_year_retention	**1.2695	**4.9329	0.1767	**0.7608	**6.8045
NEM	-0.5649	-0.7236	-0.7285	-0.2821	-1.3678
Proof	0.0037	0.0051	0.001	0.0008	0.0104
Duration_Index	-0.0543	**-0.2551	0.0515	0.0081	**-0.4936
Students ___JCED	**0.0018	**0.0018	**-0.0116	-0.0003	**0.002
Students ___JCE	**-0.0361	**-0.042	**0.0485	-0.0131	**-0.0675
%_ JCE_with_Doctorate	0.8253	0.1218	0.0351	0.9724	2,9656
%_ JCE_Postgraduate	0.4662	0.1027	**-1.9154	-0.8957	-0.0667
m²_built_per_student	**-0.0903	-0.0309	**-0.1696	**-0.1372	-0.0403
% _of_tax_contributions_of_income	-0.7058	0.0758	**-1.1711	-0.47	-3.6562
% _of_tax_contributions_of_total_assets	**-0.934	-1,029	**-0.9035	**-0.8992	-0.5616

Table 2 shows, from the Adjusted Leverage model (financial debt/equity), that structural variables such as the percentage of tax contributions over income are negatively and significantly associated in almost all models, especially in universities without tuition fees, confirming that the lack of state funding leads to greater dependence on bank debt. It is also observed that a higher proportion of faculty with doctorates is associated with greater financial debt in the general and GRT models, which may reflect institutional investment to raise academic standards. Academic efficiency variables, such as the number of students per JCE, have negative and significant effects, reinforcing the idea that institutions with a higher optimized academic load tend to incur less debt from banks. Finally, the positive result of the admissions test variable in several segments suggests that a better student intake profile could be associated with financial investment strategies in quality.

Table 2. Comparative fixed effects regression model for Adjusted Leverage

LevAjus	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	-1.8019	-1.4236	-1.1878	-1.7308	0.631
1st_year_retention	0.3516	-0.8866	0.1995	0.2124	-2.2856
2nd_year_retention	0.171	0.1548	0.2317	0.2089	1,1889
NEM	-0.0423	-0.2239	0.049	0.1263	-0.365
Proof	**0.0038	**0.008	0.001	**0.0016	**0.0086
Duration_Index	-0.0043	-0.0435	0.0145	0.0084	-0.1276

Students ___JCED	**0.0007	**0.0008	0.0007	0.0001	0.0007
Students ___JCE	**-0.0094	**-0.0196	0.0096	0.0003	**-0.037
%_ JCE_with_Doctorate	**1.0537	1,0571	0.8215	**0.9985	2,9707
%_ JCE_Postgraduate	-0.1572	-0.2168	-0.1202	-0.3587	-0.71
m²_built_per_student	-0.0068	-0.0136	-0.0028	-0.0071	-0.0209
% _of_tax_contributions_of_income	**-0.4543	-0.4765	**-0.4037	**-0.304	**-2.3554
% _of_tax_contributions_of_total_assets	0.2062	0.3882	0.0616	**0.2137	0.5096

Table 3 shows that a higher NEM score at entry is positively associated with operational cash flow across all segments, which can be interpreted as a sign that students with better prior performance contribute to financial operational stability. The number of students per JCE is also positively associated in the general and CRUCH models, indicating that greater teaching efficiency could improve net financial flow. It is noteworthy that a higher percentage of faculty with postgraduate degrees is associated with lower cash flow in the general, CRUCH, and GRT models, possibly due to the higher structural costs associated with this academic body. Furthermore, the negative constant in several models indicates that even with all predictors at low values, cash flow tends to be negative, revealing a fragile financial foundation in several institutions, especially those without tuition.

Table 3. Comparative fixed effects regression model for operating cash flow.

FCajOpPct	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	**-1.0027	**-1.1169	-0.7382	-0.7913	**-1.5489
1st_year_retention	-0.0765	-0.3103	**-0.166	-0.0829	-0.3046
2nd_year_retention	0.0201	0.0471	-0.0065	0.0195	-0.0482
NEM	**0.2106	**0.1722	**0.2524	**0.2154	**0.2339
Proof	-0.0002	**0.0007	-0.0006	-0.0005	0.0009
Duration_Index	-0.0103	-0.0105	-0.0064	-0.011	-0.0097
Students ___JCED	-0.00006867	-0.00002712	**-0.0013	-0.0002	-0.00001592
Students ___JCE	**0.002	0.0006	**0.0092	0.0032	0.0008
%_ JCE_with_Doctorate	-0.0453	-0.2206	0.2454	0.0563	-0.2891
%_ JCE_Postgraduate	**-0.1704	-0.0243	**-0.748	**-0.3289	0.0314
m²_built_per_student	0.0018	0.0006	-0.0009	0.0009	0.0035
% _of_tax_contributions_of_income	0.0353	0.0732	0.0642	0.0406	0.0716
% _of_tax_contributions_of_total_assets	0.0022	-0.0593	0.0859	0.0263	-0.0688

Table 4 shows that higher student retention (first year) is associated with decreased liquidity in the general, CRUCH, and GRT models. This could reflect the fact that institutions must cover more operational costs to maintain academic continuity, putting pressure on their short-term cash flows. In contrast, institutions with greater academic efficiency (more students per JCE) tend to have greater liquidity, possibly due to better use of teaching resources. It is also noteworthy that the increase in square meters per student, as well as fiscal contributions, are positively related to liquidity in the general, CRUCH, and GRT models, reinforcing the role of public contributions as short-term financial buffers. Finally, the percentage of JCE with doctorates has a negative effect in the general and private models, suggesting that institutions with more academics with doctorates face greater financial commitments, reducing operational liquidity.

Table 4. Comparative fixed effects regression model for current liquidity.

LiqCor	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	1,1144	-3.9212	-2.6732	-1.0854	6.9954
1st_year_retention	**-11,255	1,625	**-12,846	**-11,518	-0.5152
2nd_year_retention	**-3.3769	-2.7773	-1.6536	-2.1534	-1.7957
NEM	1,4551	0.5065	1,4996	1,1133	-0.6704
Proof	0.0013	0.0041	-0.0066	0.0009	-0.002
Duration_Index	-0.1872	0.0798	-0.0653	-0.0012	0.1149
Students ___JCED	-0.0006	-0.0001	0.0207	0.0031	0.0003
Students ___JCE	**0.0856	-0.0089	**0.1823	**0.1227	-0.0147
%_ JCE_with_Doctorate	**-5.4569	**-5.4352	-4.8808	-4.4086	-1.1354
%_ JCE_Postgraduate	-0.0048	**2,8811	1,1966	-0.7507	1,8873
m²_built_per_student	**0.4241	-0.0096	**0.7999	**0.7156	-0.0612
%_of_tax_contributions_of_income	**-3.9994	-1.5788	**-3.5566	**-3.9464	-0.3687
%_of_tax_contributions_of_total_assets	**4,1729	1,1109	**3.3634	**4.4752	0.7432

Table 5 shows that the gross operating margin is negatively affected by higher student retention rates in both the first and second years (general, CRUCH, and GRT segments), which could indicate that student retention generates higher costs that are not always offset by additional revenue. In contrast, institutions with more faculty with postgraduate or doctoral degrees tend to show lower operating margins, possibly reflecting higher structural academic costs. It is also noteworthy that a high NEM is associated with better margins only in GRT, but the opposite is true for non-GRT segments. An increase in square meters per student has a positive effect on CRUCH, but a negative effect on non-GRT segments, which could be associated with different infrastructure investment strategies. Finally, it is confirmed that higher tax contributions (on income or assets) contribute positively to the gross margin in all segments, reinforcing their role as an operational financial support for HEIs.

Table 5. Comparative fixed effects regression model for gross margin.

MBrutoPct	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	0.0973	-0.0106	-1.1521	**-0.9689	**4,4914
1st_year_retention	-0.165	-0.629	**-0.4303	**-0.3154	-0.8097
2nd_year_retention	**-0.3292	**1.1017	-0.0605	-0.1172	**1.4256
NEM	0.0584	0.0281	0.2171	**0.208	**-0.6398
Proof	0.0001	-0.0002	0.0001	0.0003	-0.0006
Duration_Index	-0.0094	-0.0241	0.0009	-0.0095	**-0.052
Students ___JCED	0.00003726	0.00002124	0.0006	0.0001	0.000009572
Students ___JCE	**0.0058	0.0037	**0.0082	0.0012	0.0045
%_ JCE_with_Doctorate	**-0.8294	**-1.5615	0.0004	-0.2157	-0.6533
%_ JCE_Postgraduate	**0.303	0.2515	-0.0525	0.0311	-0.2094
m²_built_per_student	-0.0031	-0.0074	**0.0079	0.0046	**-0.0144
% _of_tax_contributions_of_income	**-0.3691	**-0.6935	**-0.1282	**-0.1337	**-1.7695
% _of_tax_contributions_of_total_assets	**0.2469	**0.4967	0.0837	**0.1211	**0.7788

Table **6** suggests that higher first-year student retention is associated with a higher proportion of fixed assets at CRUCH universities and those with free tuition, which could reflect a link between retention and infrastructural expansion. Infrastructure per student (m²) has a significant positive impact on several segments, which is expected given that this indicator is part of fixed assets. Furthermore, universities with students who remain in school longer (higher duration index) tend to have a lower proportion of fixed assets, possibly due to prioritizing operating expenses over infrastructure investment. In the GRT model, it is observed that the fiscal contribution as a proportion of assets is also positively related to the proportion of fixed assets, while in private and non-GRT universities, the fiscal contribution to revenue has opposing effects. This suggests structural differences in how institutions capitalize state resources on infrastructure.

Table 6. Comparative fixed effects regression model for fixed asset ratio.

ActFT	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	0.7381	2,1466	-0.8544	**1.2536	2,1183
1st_year_retention	0.2304	0.6559	**0.3074	**0.3467	0.0281
2nd_year_retention	0.1264	0.4647	-0.0227	0.0225	0.1963
NEM	-0.0293	-0.3424	0.2126	-0.1384	-0.2058
Proof	-0.00008854	-0.0001	-0.0003	-0.0004	-0.0007
Duration_Index	**-0.0368	**-0.0568	0.0004	-0.0024	**-0.097

Students ___JCED	**-0.0003	**-0.0003	-0.0013	-0.0002	-0.0002
Students ___JCE	-0.0013	-0.0036	0.0061	**0.0055	-0.0111
%_ JCE_with_Doctorate	0.0691	0.946	**-0.7201	-0.1886	0.0381
%_ JCE_Postgraduate	-0.2013	-0.345	0.2578	-0.0129	0.0996
m²_built_per_student	**0.0116	-0.0069	**0.0219	**0.0204	-0.0056
% _of_tax_contributions_of_income	0.0523	**0.5296	-0.0678	**-0.1602	**1.9979
% _of_tax_contributions_of_total_assets	0.0587	-0.3772	0.0602	**0.1293	-0.4191

The main objective of this study was to explore the links between financial health and academic quality indicators in Chilean higher education institutions (HEIs) using a mixed approach that combines multivariate visualization (HJ- Biplots) and panel data econometric models (PanelOLS). The evidence obtained allows us to answer the research question affirmatively: there are significant structural relationships between certain academic variables and the financial health of HEIs, although these relationships are not homogeneous, linear, or necessarily universal to all types of institutions.

3.3. Relationship between financial health and academic performance

It was found that the financial health of HEIs does not depend exclusively on accounting indicators such as leverage or cash flow, but is strongly influenced by key academic variables. Among those that showed the most consistent effects were student retention, infrastructure per student, faculty development, and government support.

Student retention (first and second years) influences several financial indicators: it increases debt and reduces gross margin and operating liquidity, especially in private and non-free universities (Tables 1, 3, and 4). This reflects the fact that, without sufficient public support, prolonged student retention can put a strain on operating resources.

Infrastructure per student (m^2) acts as a protective factor in terms of liquidity and fixed asset ratio (Tables 4 and 6), confirming that HEIs with sustained investments in physical capacities have greater financial resilience.

The education of academic staff has mixed effects. A higher proportion of professors with postgraduate or doctoral degrees is associated with higher perceptions of quality and prestige, but also with higher operating costs that negatively affect cash flow and gross margin (Tables 3 and 5). This tension reinforces the need for strategic planning that links academic quality with financial sustainability.

State support, measured as a percentage of tax contributions relative to income or assets, is a decisive factor. Universities with free tuition show more stable trajectories in the HJ- Biplots (Figures 3 and 1) and better financial results in the econometric models (Tables 1, 4, and 5), suggesting that the free tuition policy has had an effective stabilizing effect for institutions with greater structural challenges.

3.4. Institutional segmentation and differentiated trajectories

Dynamic HJ- Biplot visualization [18] confirmed that not all HEIs follow the same financial trajectory, even under similar structural conditions. The most vulnerable segments were private

universities not affiliated with free tuition, which presented the most volatile trajectories and an almost total dependence on the financial system (Figures 2 and 4), especially in cases such as Gabriela Mistral University and the Universidad del Desarrollo. Their structural risk is accentuated by the lack of direct fiscal support, which makes them highly sensitive to regulatory changes or credit contraction.

For their part, the CRUCH universities, while more stable, also identified risk factors, such as the University of Aysén and the Alberto Hurtado University, whose trajectories shifted toward critical areas in the biplots (Figures 1 and 3), indicating persistent financial imbalances even with public support. This behavior suggests that a free tuition policy alone does not guarantee financial health unless accompanied by operational efficiency, asset planning, and a sustainable academic structure.

3.5. Quality as a balance between academic and financial aspects

One of the central contributions of the study is that academic quality cannot be understood in isolation from financial health, but neither can they be considered equivalent. There are financially sound institutions with low academic standards, as well as high-quality educational institutions with financial risk profiles. This tension confirms what has been pointed out [11], [3]: university quality is built when there is coherence between the academic and economic dimensions, mediated by effective strategic management.

3.6. Implications for management and public policy

The findings of this study have important implications at the institutional and public policy levels. At the institutional level, HEIs must strengthen their integrated financial and academic analysis capabilities, developing planning models that link academic decision-making (such as increasing retention or faculty) with their financial effects. At the public policy level, a more holistic institutional assessment model is needed, one that does not reduce quality to enrollment or accreditation indicators, but rather incorporates variables such as leverage, liquidity, equity, and the financial environment in which universities operate.

Finally, there is a growing urgency to monitor private institutions not affiliated with free education, which exhibit structural risk profiles and could compromise the educational continuity of their students in crisis contexts.

In short, the study confirms that there is a deep, albeit complex, link between financial health and academic quality in Chilean HEIs. These links are dynamic, mediated by the type of institution and its structural conditions, and require multivariate and longitudinal analytical approaches such as those used here. Strengthening these links through public policy and institutional management is not only a technical task, but a condition for ensuring a sustainable, equitable, and high-quality higher education system.

3.7. Analysis with Neutrosophic Cognitive Maps

To complement the analyses multivariate (HJ- Dynamic Biplot) and econometric (PanelOLS), a neutrosophic approach was applied by maps Neutrosophic cognitive (MCN), based in the principles of neutrosophy [20, 21, 22]. MCNs model the relationships causal relationships between academic variables (retention in first and second year , infrastructure by student , level of education teaching) and financial (leverage, liquidity current , cash flow operational , margin gross , asset ratio fixed), assigning truth (T), falsity (F) and indeterminacy (I) values in he range [0,1], according to Definitions 5-13 of subsection 2.2. This approach allows capturing uncertainty inherent to interactions, especially in segments institutional with high variability, such as universities private not assigned to free education.

3.7.1. Construction of the Neutrosophic Cognitive Map

The MCN nodes were defined as the significant variables identified in the regression models (Tables 1-6) and the dynamic HJ -Biplot trajectories (Figures 1-4). The nodes included :

Academic variables: Retention in the first year (R1), retention in second year (R2), infrastructure by student (m^2/E), percentage of teachers with a doctorate (% JCE_Doc), percentage of teachers with a postgraduate degree (% JCE_Pos).

Financial variables: Leverage (Lev), liquidity current (LiqCor) , cash flow operational (FCajOp), margin gross (MBross) , asset ratio fixed (ActFT).

Contextual variable: Percentage of contributions prosecutors about he income (% AportFis).

The arches directed between nodes were built using the coefficients significant from Tables 1-6, where the magnitude and the sign of the coefficients determined the values of T (causality positive), F (causality negative or null) and I (indeterminacy). Indeterminacy was estimated considering the variability of the trajectories in HJ-Biplots and heterogeneity between segments institutional . The MCNs were implemented using customized software to neutrosophic environments (by example , FCMExpert modified) and were validated with experts in higher education to ensure the coherence of relations causal .

3.7.2. MCN Results

For the assignment of neutrosophic weights (T, F, I) in the causal relationships, a methodological triangulation process was followed that integrated the quantitative findings of the fixed effects regression models (PanelOLS) and the observation of the trajectories in the dynamic HJ- Biplots , together with expert validation.

- **Truth (T): A high T value was assigned when the** PanelOLS regression coefficients were statistically significant and the sign of the causal relationship (positive or negative) was consistent with the expected direction. The magnitude of the regression coefficient also influenced the strength of T; larger coefficients in absolute value indicated greater truth. For example, a strong positive or negative correlation in the regression tables (Tables 1–6) led to a high T value for that causal relationship.
- **Falsification (F):** An F value was assigned when the causal relationship was not statistically significant in the regression models, or when the sign of the coefficient contradicted the expected direction, suggesting an absence or cancellation of the effect. Coefficients close to zero or with a high probability (p- value) indicated greater falsification.
- **Indeterminacy (I):** This value captured the ambiguity and uncertainty inherent in interactions, particularly relevant in contexts of high institutional heterogeneity. Indeterminacy was estimated by considering the variability of the trajectories observed in the dynamic HJ- Biplots (Figures 1-4). More volatile trajectories, with erratic changes in the position of institutions over time, indicated a greater degree of indeterminacy in the causal relationships associated with that institutional segment. Furthermore, the inconsistency of the same effect across different segments (e.g., a significant coefficient

in one model but not in another) also contributed to a higher I value. The judgment of higher education experts was also crucial in refining these values, especially in cases where the data did not offer a clear conclusion.

This approach allowed the transformation of quantitative information and visual variability into neutrosophic values that more fully reflect the complexity of the relationships between academic and financial variables in Chilean HEIs.

The MCN was structured as a system dynamic with feedback (Definition 10, subsection 2.2), reflecting cycles causal relationships between academic and financial variables . The main ones are presented below relations causes modeled , with their neutrosophic weights:

1. **Retention student (R1, R2) → Leverage (Lev)** : In CRUCH universities and with free tuition , greater retention in first and second year is associated with an increase in debt (Table 1, coefficients 1.6527 and 1.2695, general model). The MCN assigned T=0.7, F=0.1, I=0.2, reflecting a causality positive strong , but with indeterminacy due to variability in the trajectories (by example , University of Aysén, Figure 1). In universities not affiliated with free tuition , this relationship is more intense (coefficient 6.8045 for R2, Table 1), with T=0.8, F=0.05, I=0.15, indicating greater dependence on debt without the fiscal support .
2. **Infrastructure by student (m^2/E) → Liquidity current (LiqCor)** : The infrastructure by student has an effect positive significant in liquidity in the general, CRUCH and GRT models (Table 4, coefficients 0.4241, 0.7999, 0.7156). The MCN assigned T=0.65, F=0.1, I=0.25, highlighting a causality positive, but with indeterminacy in universities not affiliated with free tuition , where he effect is not significant (coefficient -0.0612, Table 4) and the trajectories are volatile (Figure 4, Gabriela Mistral University).
3. **Percentage of contributions fiscal (% AportFis) → Leverage (Lev) and Liquidity (LiqCor)** : The contributions prosecutors reduce significantly he indebtedness (Table 1, coefficients -0.7058, -1.1711, -0.8992) and increase liquidity (Table 4, coefficients 4.1729, 3.3634, 4.4752) in the general, CRUCH and GRT models . The MCN assigned T=0.8, F = 0.05, I=0.15 for the % AportFis relationship → Lev (causality) negative) and T=0.75, F=0.1, I=0.15 for % AportFis → LiqCor , reflecting a role clear stabilizer , but with indeterminacy in universities not affiliated with free tuition , where contributions prosecutors are minimal (Figure 4).
4. **Training teacher (% JCE_Doc , % JCE_Pos) → Cash flow operational (FCajOp) and Margin gross (MBrute)** : A higher proportion of teachers with a doctorate or postgraduate degree is associated with lower cash flows and margins gross (Tables 3 and 5, coefficients negatives as -0.1704 for % JCE_Pos in FCajOp , general model). The MCN assigned T=0.3, F=0.5, I=0.2, indicating a causality negative moderate , with indeterminacy due to costs structural high and variability in the trajectories (by example , Alberto Hurtado University, Figure 1).

Table 7. Neutrosophic adjacency matrix N(E) for the relationships causal relationships between academic and financial variables

Node origin → Node destination	R1 → Lev	R2 → Lev	m²/E → LiqCor	% FisContribution → Lev	% FisContribution → LiqCor	% JCE_Doc → FCajOp	% JCE_Pos → MBruto
T (Truth)	0.7	0.8	0.65	0.8	0.75	0.3	0.3
F (Falsehood)	0.1	0.05	0.1	0.05	0.1	0.5	0.5
I (Indeterminacy)	0.2	0.15	0.25	0.15	0.15	0.2	0.2

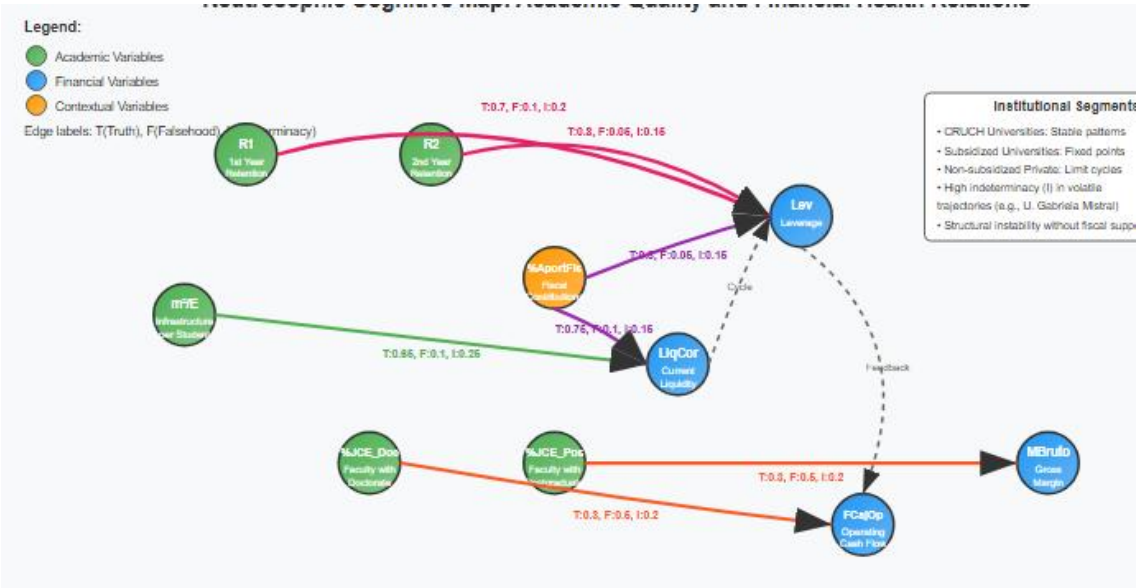

Figure 5: Representation graph of the obtained Neutrosophic Cognitive Map.

3.7.3. Analysis dynamic and patterns hidden

The MCN was analyzed as a system dynamic cyclic (Definition 9, subsection 2.2), using the neutrosophic adjacency matrix (Definition 7) and the state vector instantaneous (Definition 8). The activation of key nodes , such as R1 (retention) was simulated in the first year) and % AportFis (contributions prosecutors), to identify patterns hidden (Definition 11). The results showed:

- **Fixed point:** In CRUCH universities and with free tuition, the activation of % AportFis stabilizes the Lev and LiqCor nodes, reaching a state of equilibrium with T=0.8 for relationships negative with him indebtedness and T=0.7 for relationships positive with liquidity, confirming he protective role of financing state.

- **Cycle limit:** In universities not affiliated with free tuition, the activation of R1 generates a cycle dynamic between Lev, LiqCor and MBruto , with high indeterminacy (I = 0.3-0.4), reflecting instability structural observed in the trajectories volatiles from Gabriela Mistral University and the University of Development (Figure 4).

3.7.4. Implications of the neutrosophic approach

The use of MCN allowed capturing complexity and uncertainty in the relationships between quality academic and health financial, which are not completely linear neither homogeneous, as observed in the trajectories differentiated from the HJ-Biplots (Figures 1-4). The assignment of indeterminacy (I) values highlighted the ambiguity in segments such as universities not affiliated with free tuition , where the lack of contributions prosecutors and the high system dependency financial generate instability (Figure 4). This approach enriches the results of the models PanelOLS by providing a representation graphic and dynamic interactions causal, facilitating the identification of critical points for management institutional and the policy design public .

The results obtained through maps Neutrosophic cognitive (MCN) in subsection 3.7 enrich the understanding of the relationships structural between quality academic and health financial in Chilean higher education institutions (HEIs) , by incorporating uncertainty as a key dimension in he analysis . The MCNs, based in the principles of neutrosophy [20, 21, 22], allowed model interactions causal relationships between academic variables (retention student , infrastructure by student , training teaching) and financial (leverage, liquidity current , cash flow operational , margin gross , asset ratio fixed), assigning truth (T), falsity (F) and indeterminacy (I) values . This approach complements the results of the dynamic HJ -Biplot (Figures 1-4) and the effects regression models fixed (Tables 1-6), highlighting the ambiguity inherent in high contexts heterogeneity institutional .

Relationship modeling such as retention student → leverage (T=0.7-0.8, I= 0.15-0.2) reveals that , although retention in first and second anus increases he indebtedness , especially in universities not affiliated with free tuition (Table 1, coefficient 6.8045), the indeterminacy reflects variations institutional , such as trajectories volatiles from Gabriela Mistral University (Figure 4). This finding stresses that continuity student, although it is an indicator of quality academic , can trigger pressures financial in absence of robust fiscal support , confirming the observations in subsection 3.4 on the vulnerability of universities private not assigned to free education .

Similarly, the relationship infrastructure by student → liquidity current (T=0.65, I=0.25) showed a protective effect on CRUCH universities and with free tuition (Table 4, coefficients 0.7999, 0.7156), but with high indeterminacy in universities not affiliated with free tuition , where he impact is not significant (Table 4, coefficient -0.0612). This suggests that investments in infrastructure , although strengthen financial resilience in supported contexts state , generate uncertainty in institutions system dependent financial , as can be seen in the trajectories of the University of Development (Figure 4).

The role contribution stabilizer prosecutors (% AportFis → leverage and liquidity , T=0.75-0.8, I=0.15) corroborates the results of Tables 1 and 4, highlighting that the financing state reduces the indebtedness and improves liquidity in CRUCH universities and free tuition (Figures 1 and 3). However, the uncertainty in universities not affiliated with free tuition reflects the lack of a " fiscal cushion ", which exposes them to risks structural , as evidenced in the HJ-Biplots (Figure 4).

The analysis dynamic MCN, with fixed points in free universities and cycles boundaries In the non- attached ones, stability stands out structural of the first against the instability of the latter. These results reinforce the conclusions of subsection 3.5, which advocate for a comprehensive approach to quality university , where coherence between decisions academic and financial support is crucial. The incorporation of MCNs contributes a perspective novel in capturing unaddressed uncertainty by the methods statistics traditional , aligning with the recommendations of [11] and [3] on the need for models that integrate both dimensions .

In terms of implications , the MCNs highlight the importance of designing policies public policies that mitigate uncertainty financial in universities private schools not affiliated with free education , promoting financing strategies that balance quality academic and sustainability . At the level institutional , the MCNs offer a Visual and dynamic tool for HEIs to identify critical points in their relationships causal , facilitating a planning strategic further informed . This neutrosophic approach , by complementing the analyses multivariate and econometric , positions the study as a contribution methodological innovative for futures jobs in the field of Chilean higher education.

4. Applications

The results of this research allow to significantly advance the understanding of the structural links between financial health and academic quality in Chilean higher education institutions (HEIs). The empirical evidence obtained through dynamic HJ- Biplots [18], fixed effects regression models (PanelOLS), and neutrosophic cognitive maps (NCM) not only corroborates the existence of significant interrelationships, but also reveals a structural complexity that varies according to the type of institution, its access to state funding and its academic orientation, capturing the uncertainty inherent in these interactions.

First, the analysis shows that financial health cannot be separated from institutional academic functioning, as previously warned [3], when pointing out that world-class universities combine academic excellence with financial sustainability. This premise is empirically confirmed by observing that indicators such as student retention, available infrastructure, faculty quality, and state support have direct and significant impacts on key financial health variables such as leverage, cash flow, and gross margin (Tables 1 to 6). NCMs, based on neutrosophy [20, 21, 22], enrich this interpretation by modeling these relationships with neutrosophic weights (T, F, I), highlighting, for example, the positive causality between student retention and leverage (T=0.7-0.8, I=0.15-0.2, Table 2) and the indeterminacy in universities not affiliated with free tuition due to volatile trajectories (Figure 4). In this way, quality ceases to be an abstract category and is linked to the capacity of HEIs to sustain viable, stable, and consistent educational projects with their mission.

The evidence also calls into question an excessively economic approach to university management. As [8] warn, the exclusive emphasis on accounting indicators can obscure strategic risks if the academic logic and training commitments that characterize public or regional universities are not considered. This study shows that universities with high social valuation—such as some of the CRUCH—can be at financial risk if they do not have adequate financing structures, as observed in the trajectories of the University of Aysén and the Alberto Hurtado University (Figure 1). The MCNs reinforce this observation by assigning high indeterminacy (I=0.2) to relationships such as retention → leverage in CRUCH (Table 2), which suggests the need for a thorough review of the

Chilean financing model, especially with regard to structural support for institutions with relevant public roles.

On the other hand, the results show that private universities not affiliated with free tuition present the most critical profiles, with highly volatile trajectories and a high dependence on the financial system (Figures 2 and 4). The absence of a "fiscal cushion" and exposure to credit cycles makes them fragile institutions, whose equilibrium can easily be altered by regulatory, demographic, or economic changes. The MCNs highlight this vulnerability by assigning high indeterminacy (I = 0.25-0.3) to relationships such as infrastructure → current liquidity in this segment (Table 2), reflecting the structural instability of institutions such as Gabriela Mistral University (Figure 4). This situation reinforces the OECD's 2021 and World Bank 2022 approaches regarding the structural vulnerability of excessively commercialized university systems with little preventive regulation.

The study also sheds light on the relationship between academic quality and financial health. Although many HEIs with better infrastructure and more qualified faculty display more stable financial indicators, in other cases, these same variables are associated with higher operating costs that reduce liquidity and operating margins (Tables 3 and 5). NCMs capture this tension by assigning high falsity (F=0.5) and moderate indeterminacy (I=0.2) to relationships such as teacher training → cash flow and gross margin (Table 2), indicating that quality without structural financing can become a source of instability. This poses a challenge for quality assurance models, which must integrate economic criteria into their standards without sacrificing academic merit.

In this context, the relevance of the approach of [6] is reinforced. They propose evaluating institutional quality as an interdependent system, where academic, financial, and governance decisions are articulated within a common strategic framework. The results obtained through the regression models and the MCN confirm that public policies must adopt a more comprehensive perspective, one that overcomes the dichotomy between quality and sustainability, and that recognizes the diversity of the Chilean university system in its different structural realities. The incorporation of MCN, by modeling uncertainty in causal relationships, adds a dynamic dimension that facilitates the identification of critical points for institutional management, especially in contexts of high heterogeneity such as universities not affiliated with free tuition.

In methodological terms, this study also contributes by employing a longitudinal design with multivariate techniques, allowing to observe not only static correlations, but also trajectories and trends over time. The integration of MCN complements dynamic HJ- Biplots and PanelOLS models by providing a graphical and dynamic representation of causal interactions, following recommendations by authors such as [19] and [15, 16]. This neutrosophic approach allows to capture ambiguities that traditional statistical methods do not address, such as the high indeterminacy in universities not affiliated with free tuition (Table 2), strengthening the robustness of the analysis.

Finally, it should be noted that, while the findings offer a systematic and empirically supported perspective, they also reveal the lack of national studies that integrate financial and academic data in a cross-sectional manner. The bibliographic gap in this field—mentioned in the introduction—remains prevalent, and this paper seeks to pave the way for future research that delves into other dimensions, such as institutional governance, pedagogical models, or the use of technology, in relation to financial health and educational quality, potentially incorporating neutrosophic approaches to model uncertainty in these contexts.

5. Conclusions

This exploratory study advanced our empirical understanding of the links between financial health and academic quality in the Chilean university system, a relationship frequently intuited in political and academic discourse but scarcely investigated from a quantitative and comparative perspective.

First, statistically significant relationships are found between certain academic variables—such as student retention, faculty training, and infrastructure—and key financial health indicators, such as leverage, current assets, and gross margin (Tables 1–6). Neutrosophic cognitive maps (NCMs), based on neutrosophic theory, enrich this analysis by modeling these relationships with neutrosophic weights (T, F, I), highlighting, for example, the positive causality between student retention and leverage (T=0.7–0.8, I=0.15–0.2, Table 2) and the uncertainty in universities not affiliated with free tuition due to their high dependence on the financial system (Figure 4). These associations are not uniform across the system but vary according to the type of institution, its funding structure, and its affiliation with policies such as free tuition.

The results also demonstrate that good financial health is not necessarily synonymous with high academic quality, and vice versa. Some institutions have stable financial indicators but weak academic standards, as well as prestigious universities that face structural risks due to a lack of state support or poor financial management, such as the University of Aysén and the Gabriela Mistral University (Figures 1 and 4). The MCNs reveal that these tensions are accentuated in contexts of high uncertainty, such as in universities not affiliated with free education, where relationships such as infrastructure → liquidity show high indeterminacy (I = 0.25, Table 2). This complexity demonstrates that university quality requires a comprehensive approach that considers not only learning and scientific production , but also the institutional conditions that make it possible.

In this context, one of the central contributions of the study lies in the application of methodological tools that allow the visualization of institutional trajectories (HJ- Dynamic Biplots) and the modeling of complex relationships (PanelOLS), complemented by the MCN, which captures the uncertainty in causal interactions (Table 2). This combination provides a robust empirical basis for decision-making at both the public policy and institutional management levels, especially when identifying critical points in universities with high volatility, such as those not affiliated with free tuition.

Furthermore, the findings reinforce the value of public funding—particularly indirect fiscal contributions and free tuition—as stabilizing mechanisms for universities with a greater public focus, especially those located in regions or with a high proportion of vulnerable students. The MCNs confirm this role by assigning low indeterminacy (I = 0.15) to relationships such as fiscal contributions → liquidity in CRUCH universities and those with free tuition (Table 2, Figures 1 and 3). However, it is noted that state funding, by itself, does not guarantee stability if it is not accompanied by strategic planning aimed at efficiency, investment in assets, and academic strengthening.

Finally, the study highlights the need to design more comprehensive assessment frameworks that integrate financial and academic indicators into quality assurance systems, in line with international recommendations. NCMs provide an innovative perspective by modeling uncertainty in these interactions, facilitating the identification of structural risks and opportunities for integrated management. This implies overcoming the separation between financial management and educational quality, assuming that both dimensions are interdependent and essential for the sustainability of the system. In particular, the use of neutrosophic cognitive maps enhances the analytical scope by introducing uncertainty as a formal dimension, offering a dynamic tool for strategic planning in heterogeneous institutional contexts.

References

[1] F. Barriga, R. Gálvez, and B. Sáez (2021), *Financing of State Universities in Chile: Relevant Indicators and Financial Health of State Universities, 1st ed. Santiago, Chile: Sol y* Metuech Foundation, pp. 1–56. [Online]. Available: https://fundacionsol.cl/blog/sindicatos-12/post/financiamiento-de-las-universidades-del-estado-en-chile-6812

[2] Superintendency of Higher Education (2023), *Financial Health in Chilean Higher Education,* 1st ed. Santiago, Chile: Ediciones SES, pp. 1–67. [Online]. Available: https://www.sesuperior.cl/wp-content/uploads/2023/11/SES_Salud-Financiera_2023.pdf

[3] J. Salmi (2009), *The Challenge of Establishing World-Class Universities,* 1st ed. Washington, D.C.: The World Bank, pp. 1–136. doi :10.1596/978-0-8213-7865-6

[4] OECD (2021), *The State of Higher Education: One Year into The COVID-19 Pandemic.* [Online]. Available: https://www.oecd.org/education/state-of-higher-education-2021.htm

[5] World Bank (2022), *Education Finance Watch 2022.* [Online]. Available: https://documents.worldbank.org/en/publication/documents-reports/documentdetail/472681646593733236

[6] E. Estermann , A. Nokkala , and M. Steinel (2011), *University Autonomy in Europe II: The Scorecard* , 1st ed. Brussels , Belgium : European University Association , pp. 1–84. [Online]. Available: https://eua.eu/downloads/publications/university%20autonomy%20in%20europe%20ii%20-%20the%20scorecard.pdf

[7] H. Moraga and M. Rossi (2013), "Financial analysis of traditional Chilean universities and its relationship with their perception of academic quality," *Horizontes Empresariales* , vol. 12, pp. 63–77. [Online]. Available: https://revistas.ubiobio.cl/index.php/HHEE/article/view/2406/2108

[8] V. Salas and C. Yévenes (2024), *Comments on the report "Financial Health" in Chilean Higher Education.* Santiago, Chile: University of Santiago de Chile, pp. 1–24. [Online]. Available: https://www.economia.usach.cl/index.php/investigacion/documentos-de-trabajo/item/467

[9] T. Estermann , T. Nokkala , and M. Steinel (2009), *University Autonomy in Europe I: The Scorecard* , 1st ed. Brussels , Belgium : European University Association , pp. 1–48. [Online]. Available: https://www.eua.eu/publications/reports/university-autonomy-in-europe-i-exploratory-study.html

[10] N. Barrientos (2024), "University System in Chile and its Financing Mechanisms," *Economic Observatory,* vol. 1, no. 184, pp. 2–4. doi : 10.11565/ oe.v 1i184.534

[11] A. Figueroa Rubalcava, G. Medina, M. Del Carmen, and V. Marfileño Eugenia (2008), "The Teaching Role at the University," *Electronic Journal of Educational Research,* vol. 10, no. 1, pp. 66–78. [Online]. Available: http://redie.uabc.mx/vol10no1/contenido-figueroaetal.html

[12] S. Menard (2008), *Handbook of Longitudinal Research: Design, Measurement, and Analysis,* 1st ed. USA: Academic Press , pp. 13–56.

[13] R. Hernández, C. Fernández, and P. Baptista (2014), *Research Methodology,* 6th ed. UK: McGraw-Hill Education, pp. 15–56.

[14] K. Gabriel (1971), " The biplot-graphic display of matrices with application to principal component analysis ," *Biometrika* , vol. 58, pp. 453–467.

[15] P. Galindo- Villardón (1986), "An alternative to simultaneous representation: HJ- Biplot ," *Questiio* , vol. 10, no. 1, pp. 13–23.

[16] R. Cascante- Yarlequé , P. Galindo- Villardón , F. Guevara- Viejó , J.L. Vicente- Villardón , and P. Vicente-Galindo (2025), "HJ-BIPLOT: A Theoretical and Empirical Analysis of the Biplot Methods of the Biplot Analysis," in Systematic Review of Its 38 Years of History , Using Text Mining and LLMs ," *Mathematics* , vol. 13, no. 1913. doi : 10.3390/math13121913

[17] J. Egido (2015), *Dynamic Biplot,* PhD Thesis, University of Salamanca. [Online]. Available: http://hdl.handle.net/10366/125245

[18] A. Egido , P. Galindo- Villardón , JL Vicente- Villardón , and A. Hernández (2015), "Dynamic Biplot . Application to the Study of the Stability of the Sensory Profile of Virgin Olive Oil ," *Br. J. Appl . Sci . Technol .* , vol. 11, no. 3, pp. 1–13.

[19] J.M. Wooldridge (2010), *Econometric Analysis of Cross Section and Panel Data,* 2nd ed. Cambridge, MA: MIT Press .

[20] WBV Kandasamy and F. Smarandache (2004), *Analysis of the social aspects of migrant workers with HIV/AIDS. Using fuzzy theory and neutrosophic cognitive maps, with special reference to rural Tamil Nadu in India.* Craiova: Infinity Study.

[21] S. Pramanik and S. Chackrabarti (2013), "A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps," *IJIRSET,* vol. 2, pp. 6387–6394.

[22] K. Mondal and S. Pramanik (2014), "A study of the problems of hijras in West Bengal based on neutrosophic cognitive maps," *NSS* , vol. 5, pp. 21–26.

[23] RP Alava , JM Murillo, RB Zambrano, MIZ Vélez, and M. Leyva- Vazquez (2018), "PEST analysis based on neutrosophic cognitive maps: A case study for the food industry," *NSS* , vol. 21, pp. 84–92.

[24] AJP Palacios, J. Estupiñán-Ricardo, IAC Piza, and MEE Herrería (2021), "Phenomenological hermeneutic method and neutrosophic cognitive maps in the causal analysis of transgressions against the homeless," *NSS* , vol. 44, pp. 147–156.

[25] M.S. Wajid, H. Terashima-Marin, P.N. Paul-Rad, and M.A. Wajid (2022), "Violence detection approach based on cloud data and neutrosophic cognitive maps," *J. Cloud Comput .* , vol. 11, pp. 1–18.

[26] SH Alsubhi , EI Papageorgiou , PP Pérez, GSS Mahdi , and LA Acuña (2021), "Triangular neutrosophic cognitive map for multi-stage sequential decision-making problems," *Int . J. Fuzzy Syst .* , vol. 23, pp. 657–679.

[27] B. Chithra and R. Nedunchezhian (2022), "Dynamic Neutrosophic Cognitive Map with Improved Cuckoo Search Algorithm (DNCM-ICSA) and Ensemble Classifier for Rheumatoid Arthritis (RA)," *J. King Saud Univ., Comput . Inf .* , vol. 34, no. 6, pp. 3236–3246.

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