Neutrosophic Sets and Systems, Vol. 97, 2026
University of New Mexico

A Cooperative Neutrosophic Sets with Dual Rough Set and Entropy Enhanced Multi-Similarity Measures Based Decision Making for Sustainable Transportation

[1]K. Senbagam [2]R. Ramesh*

[1]Assistant Professor, Department of Mathematics, PSG College of Arts & Science, Coimbatore – 641014, Tamilnadu, India. senbagam@psgcas.ac.in

[2]Assistant Professor, Department of Mathematics, K.S.R. College of Engineering, Tiruchengode - 637215, Tamilnadu, India, mathematicsramesh@gmail.com and Research Scholar, PSG College of Arts &Science, Coimbatore – 641014, Tamilnadu, India.

Abstract

Sustainable transportation helps address environmental, social and urban mobility challenges in modern cities and campuses. The present work highlights a cooperative decision-making framework that integrates Dual Rough Sets (RS), Real Interval Order Relations and Neutrosophic Sets (NS) within a Multi-Criteria Decision-Making (MCDM) approach. Shannon Entropy serves to determine objective weights and multiple similarity measures such as Jaccard Similarity, Hamming Distance and Cosine Similarity improve robustness in evaluation. The three NS's are Single-Valued (SVNS), Interval-Valued (IVNS) and Pythagorean (PNS), all of which are systematically applied and compared. The framework is validated by real-time data on sustainability criteria (for example, population, modes of transport, trip length and vehicle density) from twelve metropolitan Indian cities. Delhi is consistently recognized as the most sustainable city, followed by Bangalore and Ahmedabad, while the worst-performing cities were Bhopal and Indore. The stability of rankings across different neutrosophic environments is confirmed through sensitivity analysis of the metropolis. The framework represents a consistent and flexible decision-making tool for sustainable transportation planning in the face of uncertainty, considering the combination of cutting-edge similarity measures, composite entropy-based weighting and neutrosophic reasoning, it provides decision-making support for transparency and adaptive decision-making processes at both campus and urban levels.

Keywords: Neutrosophic Sets, Single Valued Neutrosophic Sets, Shannon entropy, Interval Valued Neutrosophic Sets and Pythagorean Neutrosophic Sets.

1. Introduction

Neutrosophic Sets (NS) were first proposed as an extension of intuitionistic fuzzy and classical fuzzy sets in managing situations in which data is inconsistent, incomplete or uncertain [1]. Rather than a one-dimensional approach based solely on truth(T) or falsity(F), NSs consider indeterminacy represented by three separate elements and demonstrate how each has a value between 0 and 1 [2],[3]. With this structure, it becomes simpler to create flexible models of circumstances where there is a lot of uncertainty [4]. Sustainable transportation means designing and running transportation systems that are good for the environment, useful for the economy and fair for all users [5]. Sustainable transportation is designed to lower greenhouse gas emissions, utilize less of our resources, create energy savings and make it possible for anyone to travel [6]. Among its advantages are keeping nature balanced, boosting public health and contributing to the economy by providing dependable and supportive transportation for all [7].

To improve the decision-making process in similar systems, Dual Rough Sets (RSs) and Entropy concepts are used [8], [9]. This theory was created to handle both vague and fine-grained data and classifies things by means of lower and upper approximations. It uses two kinds of approximations, namely positive and negative, to better understand uncertainties in determining whether samples belong in or are excluded from a decision class. Alternatively, the concept of entropy describes the level of uncertainty or disorder present in a system [10]. The use of entropy with RSs helps figure out which features matter most and improve the process of choosing them [11]. Multi-Similarity Measures (MSM) stands for the practice of using different similarity functions to figure out the similarity between objects in a decision-making context [12]. It is common for the main similarity measures used in the pastnot to capture the multi-dimensional natureof practical issues. Assessing alternatives become more reliable and accurate with the use of several similarity methods, including cosine similarity, Jaccard Index, and Euclidean Distance [13]. MSM helps decision-making by accepting multiple inputs, limiting large biases, and improving decision outcomes.

In spite of the promising outcomes achieved by methods that use RSs, entropy and similarity measures, they have several drawbacks [14]. Most models are limited in how they address indeterminacy and inconsistency together. In addition to this, they usually rely on a single way of measuring similarity, even though transportation systems are always evolving and uncertain. In addition, traditional RS methods make information disappear because their boundaries are very rigid and entropy measures could fail to express how important attributes are in complex cases [15]. This work advances a solution using Cooperative NSs with Dual RS and Entropy Enhanced Multi-Similarity Measures in a decision-making model for Sustainable Transportation. By using neutrosophic examination along with dual rough ideas and entropy-based weighting, the cooperative neutrosophic framework helps to efficiently make fairer, more reliable and better contextual evaluations of transportation possibilities. This way of integrating allows for decisions in transportation to be more specific, tolerant of uncertainty, based on data, evolving and sustainable.

1.1 Research Objectives

- To address these limitations, this research proposes a Cooperative NS-based decision-making framework that integrates Dual RS, Real Interval Order Relations (RIOR), Shannon Entropy and Multiple Similarity Measures.
- To provide a reliable and robust tool for handling uncertainty, improve decision-making through entropy-based weighting and compare Single-Valued (SVNS), Interval-Valued (IVNS) and Pythagorean Neutrosophic Sets (PNS) for sustainable transportation planning.

1.2 Research Motivation

Sustainable transportation has drawn substantial focus as a result of urbanization, reliance on automobiles and environmental issues. Proper planning here can help limit congestion and emissions and promote effective mobility for modern cities and campuses.

1.3 Significant Contributions

- Created a decision-making model based on Cooperative NSs using Dual RS and RIORs to accommodate uncertainty in the data.
- Used Shannon Entropy to objectively calculate weights for individual attributes to avoid subjectivity.
- Utilized Jaccard, Hamming and Cosine calculations to support robustness and reduce bias.
- Compared SVNS, IVNS and PNS for practical applications.
- Used the decision model on data collected from 12 Indian cities and used a sensitivity analysis to show confidence in the rankings.

2. Literature Review

In 2023, Naeem and Divyaz [16] provided a number of data metrics for m-polar NSs, like distance, similarity, correlation, divergence and dice measurements. Additionally, desirable attributes of these measures were showcased. The concepts of entropy, less and more fuzzy, λ-similarity and angle of similarity between two m-polar NSs were also discussed.

In 2024, Ulucav and Deli [17] created an MCDM technique for neutrosophic trapezoidal numbers with N values. For neutrosophic trapezoidal numbers with N values, some new generalized distance measurements were put out. Additionally, an entropy metric was suggested in order to determine the weight of criteria in a decision-making process. The TOPSIS-based entropy approach was then developed as an MCDM method under N-valued neutrosophic trapezoidal numbers.

In 2023, Yolcu et.al. [18] expanded the application of RS theory, Soft Set (SS) theory, and NSs theory in developing the idea of neutrosophic soft rough topology, which was founded on a novel neutrosophic soft RS approach. The idea of soft RSs that are neutrosophic has been presented. Numerous definitions, characteristics and cases have been developed on the neutrosophic soft RS.

In 2021, Das et.al. [19] expanded the scope of RS, soft set, and NSs theory by introducing the idea of neutrosophic soft set with roughness without employing a full soft set. On neutrosophic soft RSs, several explanations, characteristics, as well as instances have been examined. Additionally, using a neutrosophic soft set, equalities and dispensables were written on roughness.

In 2023, Martina and Deepa [20] established a multi-valued rough neutrosophic matrix and a multi-valued rough NS. A novel method for a multi-valued neutrosophic with a rough structure was presented using separation measures. By using the separation formula for the multi-valued rough NS, the proposed method made it easy to assess the alternatives.

In 2021, Rogulj et.al. [21] suggested an algorithm developed from the notion of rough NSs to address the strategic planning issue about historic pedestrian bridge restoration. Under a rough neutrosophic environment, a new cross entropy was developed that did not have the drawbacks of asymmetrical nature and unfamiliar existences. Furthermore, a rough neutrosophic VIKOR technique and a weighted rough neutrosophic symmetric cross entropy were suggested.

In 2022, Mohammad et.al. [22] introduced a few new linear Diophantine fuzzy set distances and similarity metrics. Then, based on similarity measurements for Linear Diophantine Fuzzy Sets (LDFSs), the Cosine and Cotangent functions, the exponential similarity measure, and the Jaccard similarity measure were proposed. The outcomes of applying the recently defined similarity metrics to the COVID-19 virus medical diagnosis problem have been discussed.

In 2023, Bhatia et.al., [23] highlighted the standardized parameters that demonstrated Pythagorean Fuzzy Sets (PFSs), introducing some new cosine similarity measurements. The suggested measures were adaptable and simple to apply to a range of decision-making scenarios. Additionally, a numerical example has been employed to confirm the validity of the suggested similarity metrics.

In 2022, Arora and Naithani et.al., [24] suggested a logarithmic function for Pythagorean fuzzy sets (PFSs) that is based on similarity in solving the issue. To determine whether careers were appropriate for candidates, a decision-making process was introduced. Furthermore, the validity and strength of the suggested similarity measures were assessed using numerical demonstration.

In 2025, Deli and Ulucay [25] examined a technique for the cases when neutrosophic values were used to express the input data. Consequently, weighted harmonic mean operators were proposed on N-valued neutrosophic trapezoidal numbers and two aggregations were named harmonic aggregation operators. Additionally, a method for comparing N-valued neutrosophic trapezoidal numbers was created by establishing a score function under these numbers.

3. Problem Statement

Even with recent developments in fuzzy and NS theories, today's decision-making models usually struggle to address the problems of being indeterminate, vague and multi-valued together. Currently, only a single similarity or entropy measure, a simple representation, and scarce integration of RSs reduce the effectiveness of the method. Besides, group or organizational decision-making in different neutrosophic models has not been adequately explored. Consequently, a combined model for decision-making using entropy, multiple measures of similarity, dual RSs and neutrosophic concepts is necessary to handle the uncertainties faced in planning and evaluating sustainable transportation.

3.1 Research Gaps

Many current decision-making models struggle with uncertainty, vagueness and inconsistency found in the real world. Classical fuzzy and RS models only look at T and F while ignoring indeterminacy, which limits the ability to represent uncertainty. Moreover, they often rely on a single similarity measure, which can affect the evaluation, bias it and decrease robustness. These models rarely use entropy, which can assist with objectively weighting attributes and seldom leverage entropy to add even more powerful feature selection capabilities.

4. Preliminaries

This section outlines the basic definitions of NSs and other related definitions that are needful for this research.

Definition 4.1 (NS)

An NS in a universal set X is categorized by a truth-Membership Function (MF), $T_A(x)$, an indeterminacy-MF$I_A(x)$ anda falsity-MF$F_A(x)$. These three functions $T_A(x), I_A(x), F_A(x)$ in X are real standard or non-standard subsets of$[0,1]$, such that $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$.Thus, it also satisfies the condition that $0 \leq sup\, T_A(x) + sup\, I_A(x) + sup\, F_A(x) \leq 3$.

Definition 4.2 (SVNS)

An SVNS is signified by $A = \{x, T_A(x), I_A(x), F_A(x) | x \in X\}$ where $T_A(x), I_A(x), F_A(x) \in [0,1]$ for each x in X. Then, the SVNS satisfies the statement that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 4.3 (IVNS)

Consider a non empty set X and let us construct an IVNS A of X as

$A = \{\langle x, [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U]\rangle : x \in X\}$, Where $[T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U] \subset [0,1]$ for each $x \in X$.

Definition 4.4 (PNS)

A PNS defined on a universal set$X$ is characterized by three MFs$(\mu_A, \beta_A, \sigma_A)$ such as truth $\mu_A: X \to [0,1]$, indeterminacy$\beta_A: X \to [0,1]$ and falsity$\sigma_A: X \to [0,1]$of an element $x \in X$ in the set A.This function $(\mu_A, \beta_A, \sigma_A)$ satisfies the condition that $0 \leq \mu_A(x)^2 + \beta_A(x)^2 + \sigma_A(x)^2 \leq 2$ for all $x \in X$.

Definition 4.5 (RSs)

For any finite set $X \neq \emptyset$, let $\mathcal{K}$ be an equivalency relation. Assigning two subsets to each $\eta \subseteq X$, where $\overline{\mathcal{K}}(\eta)$ and $\underline{\mathcal{K}}(\eta)$ represents the upper and lower approximations of η.

$\underline{\mathcal{K}}(\eta) = \cup\{A \in X/\mathcal{K} \ : \ A \subseteq \eta\}$ & $\overline{\mathcal{K}}(\eta) = \cup\{A \in X/\mathcal{K} \ : \ A \cap \eta \neq \emptyset\}$.

Definition 4.6 (Neutrosophic RSs)

Let $\mathcal{K}$ be an equivalence correspondence on X and let X be a non-null set. With MF of Truth (T), indeterminacy function (I) and MF of Falsity (F). Let A be NS in X. $\underline{N}(A)$ and $\overline{N}(A)$, which represent the lower and upper approximations of A in the approximation $(X, \mathcal{K})$, respectively, are defined as follows.

$$\underline{N}(A) = \{\langle x, T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x)\rangle / y \in \mathcal{K}, x \in X\}$$

$$\overline{N}(A) = \{\langle x, T_{\overline{N}(A)}(x), I_{\overline{N}(A)}(x), F_{\overline{N}(A)}(x)\rangle / y \in \mathcal{K}, x \in X\}$$

Here, $T_{\underline{N}(A)}(x) = \wedge_{y \in \mathcal{K}} T_A(y)$, $I_{\underline{N}(A)}(x) = \vee_{y \in \mathcal{K}} I_A(y)$, $F_{\underline{N}(A)}(x) = \vee_{y \in \mathcal{K}} F_A(y)$

$T_{\overline{N}(A)}(x) = \vee_{y \in \mathcal{K}} T_A(y)$, $I_{\overline{N}(A}(x) = \wedge_{y \in \mathcal{K}} \mathrm{I}_A(y)$, $F_{\overline{N}(A}(x) = \wedge_{y \in \mathcal{K}} \mathrm{F}_A(y)$.

Also, $0 \leq T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \leq 3$ and $0 \leq T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \leq 3$.

Definition 4.7 (Jaccard Similarity)

The Jaccard Similarity is a metric of similarity between two sets or vectors, defined as the ratio of the size of their intersection to the size of their union. It ranges from 0 to 1. For any two set A and B, $Jaccard\ Similarity = \frac{|A \cap B|}{|A \cup B|}$.

Definition 4.8 (Hamming Distance)

The Hamming Distance is a metric of dissimilarity between two vectors of equal length. It calculates how many positions the corresponding elements are different. For two binary vectors A and B of length n, $Hamming\ Distance = \sum_{i=1}^{n} \delta(A_i, B_i)$, where $\delta(A_i, B_i) = \begin{cases} 1, & if\ A_i \neq B_i \\ 0, & if\ A_i = B_i \end{cases}$.

Definition 4.9 (Cosine Similarity)

Cosine Similarity is a metric that computes the cosine of the angle between two non-zero vectors in a multidimensional space.

$$Cosine\ Similarity = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2} \cdot \sqrt{\sum_{i=1}^{n} B_i^2}}$$

Definition 4.10 (Shannon entropy)

Shannon entropy is a measure of uncertainty or randomness in a system. The Shannon entropy of a discrete r.v X with possible values$x_1, x_2, \dots, x_n$ and probabilities $p(x_1), p(x_2), \dots, p(x_n)$ is $H(X) = -\sum[p(x_i) \cdot \log p(x_i)]$ where the summation is over all possible values x_i of X.

5. Proposed Methodology

Assume that $B = \{B_1, B_2, \dots, B_n\}$ denotes the collection of different alternatives and $L = \{L_1, L_2, L_3, \dots, L_m\}$ represents the collection of attributes. Also $\omega = \{\omega_1, \omega_2, \omega_3, \dots, \omega_m\}$ be the corresponding weights for a NS where $\omega \geq 0$ and $\sum_{i=1}^{m} \omega_i = 1$. Consider $T = \{T_1, T_2, T_3, \dots, T_k\}$ as the decision makers.This paper introduces a novel decision-making model which is formulated by embedding both RS and RIOR theories into a NS-based MCDM framework. This research model employs Shannon entropy to obtain the attribute weights and also compared with three different types of NSs such as SVNS, IVNS and PNS. The stages for the suggested approach are as follows.

Step 1:

An organized decision matrix F= $[f_{ij}]_{n\times m}$ is formed where each entry f_{ij} is a neutrosophic value representing aggregated performance of alternative B_i on criterion L_j. Each entry encodes truth (T), indeterminacy (I) and falsity (F). Linguistic judgments are mapped to SVNS, IVNS or PNS using the tables in Section 6 (Tables 2–4).

Step 2:

Each Decision Makers (DMs) linguistic score is converted to the chosen neutrosophic format. Aggregate DMs component-wise using a weighted arithmetic mean to

obtain $\widehat{f_{ij}} = (\widehat{T_{ij}}, \widehat{I_{ij}}, \widehat{F_{ij}})$.

SVNS$_{ij} = (T_{ij}, I_{ij}, F_{ij})$, where $T_{ij} + I_{ij} + F_{ij} \leq 3$

$IVNS_{ij} = ([T_{ij}{}^L, T_{ij}{}^U], [I_{ij}{}^L, I_{ij}{}^U], [F_{ij}{}^L, F_{ij}{}^U])$

$PNS_{ij} = (T_{ij}, I_{ij}, F_{ij})$, where $T_{ij}{}^2 + I_{ij}{}^2 + F_{ij}{}^2 \leq 2$

Step 3:

To manage imprecision, compute lower($\underline{f}$) and upper ($\overline{f}$) approximations for each neutrosophic component across equivalence classes K: For SVNS

Truth membership: $\underline{T}(x) = \inf_{y\in\mathcal{K}} T(y), \overline{T}(x) = \sup_{y\in\mathcal{K}} T(y)$

Indeterminacy membership: $\underline{I}(x) = \inf_{y\in\mathcal{K}} I(y), \overline{I}(x) = \sup_{y\in\mathcal{K}} I(y)$

Falsity membership: $\underline{F}(x) = \inf_{y\in\mathcal{K}} F(y), \overline{F}(x) = \sup_{y\in\mathcal{K}} F(y)$

For IVNS, apply inf/sup to endpoints and use RIOR to order intervals consistently rather than collapsing them to midpoints.

Truth membership: $\underline{T}(x) = \left[\inf_{y\in\mathcal{K}} T^L(y), \inf_{y\in\mathcal{K}} T^U(y)\right], \overline{T}(x) = \left[\sup_{y\in\mathcal{K}} T^L(y), \sup_{y\in\mathcal{K}} T^U(y)\right]$

Indeterminacy membership: $\underline{I}(x) = \left[\inf_{y\in\mathcal{K}} I^L(y), \inf_{y\in\mathcal{K}} I^U(y)\right], \overline{I}(x) = \left[\sup_{y\in\mathcal{K}} I^L(y), \sup_{y\in\mathcal{K}} I^U(y)\right]$

Falsity membership: $\underline{F}(x) = \left[\inf_{y\in\mathcal{K}} F^L(y), \inf_{y\in\mathcal{K}} F^U(y)\right], \overline{F}(x) = \left[\sup_{y\in\mathcal{K}} F^L(y), \sup_{y\in\mathcal{K}} F^U(y)\right]$

For PNS:

Truth membership: $\underline{T}(x) = \inf_{y \in \mathcal{K}} T(y), \overline{T}(x) = \sup_{y \in \mathcal{K}} T(y)$

Indeterminacy membership: $\underline{I}(x) = \inf_{y \in \mathcal{K}} I(y), \overline{I}(x) = \sup_{y \in \mathcal{K}} I(y)$

Falsity membership: $\underline{F}(x) = \inf_{y \in \mathcal{K}} F(y), \overline{F}(x) = \sup_{y \in \mathcal{K}} F(y)$

Also, $\left(\underline{T}^2(x) + \underline{I}^2(x) + \underline{F}^2(x)\right) \leq 2$ and $\left(\overline{T}^2(x), \overline{I}^2(x), \overline{F}^2(x)\right) \leq 2$.

This produces neutrosophic RSs such as $\underline{f_{ij}} = \left(\underline{T}(x), \underline{I}(x), \underline{F}(x)\right), \overline{f}_{ij} = \left(\overline{T}(x), \overline{I}(x), \overline{F}(x)\right)$.

Step 4:

For each criterion C_j and for all alternatives B_i, calculate the normalized truth-membership as per the equation (1) which forms a probability distribution across alternatives for each criterion.

$$p_{ij} = \frac{T_{ij}}{\sum_{i=1}^{m} T_{ij}} \tag{1}$$

For IVNS, the midpoint formula is used as per the equation (2) for each truth value is an interval $\left[T_{ij}{}^{L}, T_{ij}{}^{U}\right]$.

$$T_{ij} = \frac{T_{ij}{}^{L} + T_{ij}{}^{U}}{2} \tag{2}$$

Step 5:

The aggregated neutrosophic decision matrix $\widehat{F^{agg}} = [\,\widehat{f_{ij}{}^{agg}}]$ is computed to aggregate DM opinions first, then apply rough approximations that ensures approximations reflect group consensus (3).

$$f_{ij}^{agg} = \left(\sum_{j=1}^{m} w_j T_{ij}, \sum_{j=1}^{m} w_j I_{ij}, \sum_{j=1}^{m} w_j F_{ij}\right) \tag{3}$$

Step 6:

After obtaining the aggregated neutrosophic decision matrix f_{ij}, the ideal neutrosophic solution $R^{+} = \left(T_j^{+}, I_j^{+}, F_j^{+}\right)$ is constructed using the equation (4) for each criterion C_j.

$$T_j^{+} = \max_i T_{ij}, I_j^{+} = \min_i I_{ij}, F_j^{+} = \min_i F_{ij} \tag{4}$$

Step 7:

To measure the closeness by the proposed cooperative neutrosophic sets such as SVNS, IVNS and PNS, three types of similarity or distance measures are obtained using Jaccard Similarity, Hamming Distance and Cosine Similarity. Jaccard Similarity measures overlap between sets or values. For neutrosophic values, Jaccard similarity is defined as per the equation (5).

$$S_J(B_i) = \frac{1}{n}\sum_{j=1}^{n}\left[\frac{\min\left(T_{ij},T_j^{+}\right)}{\max\left(T_{ij},T_j^{+}\right)} + \frac{\min\left(I_{ij},I_j^{+}\right)}{\max\left(I_{ij},I_j^{+}\right)} + \frac{\min\left(F_{ij},F_j^{+}\right)}{\max\left(F_{ij},F_j^{+}\right)}\right]/3 \tag{5}$$

Hamming distance measures the sum of absolute differences between corresponding neutrosophic values and its mathematical model is defined in the equation (6).

$$S_H(B_i) = \sum_{j=1}^{n}\left(\left|T_{ij} - T_j^{+}\right| + \left|I_{ij} - I_j^{+}\right| + \left|F_{ij} - F_j^{+}\right|\right) \tag{6}$$

Cosine Similarity evaluates the angle between two vectors and its mathematical expression is given in the equation (7).

$$S_C(B_i) = \frac{\sum_{j=1}^{n}\left(T_{ij}T_j^{+} + I_{ij}I_j^{+} + F_{ij}F_j^{+}\right)}{\sqrt{\sum_{j=1}^{n}\left(T_{ij}^2 + I_{ij}^2 + F_{ij}^2\right)} \cdot \sqrt{\sum_{j=1}^{n}\left(T_j^{+2} + I_j^{+2} + F_j^{+2}\right)}} \tag{7}$$

Step 8:

In this corresponding step the objective weights for each decision criterion C_j is calculated based on the truth-membership values using Shannon entropy as per the equation (8) to quantify the uncertainty for each criterion, where p_{ij} denoted the normalized truth-membership of alternative i under criterion j and $k = \frac{1}{In\ (m)}$ which is a normalization factor to ensure $E_j \in [0,1]$.

$$E_j = -\sum_{i=1}^{m} p_{ij} \cdot log\ (p_{ij}) \tag{8}$$

The divergence d_j is used to indicate the informativeness of each criterion by equation (9). A higher $d_j$means the criterion provides more discriminative information.

$$d_j = 1 - E_j \tag{9}$$

Finally, normalize the divergence values to get the objective weights for each criterion using the equation (10).

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \tag{10}$$

Step 9:

Ranking the alternative B_i with the criteria weight, greatest value is chosen as the superlative sustainable solution. Such alternative has maximum truth membership values and minimum I and F relative to others.

5.1 Rationale for method selection

Neutrosophic sets (SVNS/IVNS/PNS) explicitly model indeterminacy alongside truth/falsity essential for transport planning where data is incomplete. Dual RSs control granularity, handle boundary uncertainty and reduce false precision introduced by hard partitions. RIOR avoids spurious ordering of overlapping intervals in IVNS, preserving meaningful partial orders. Entropy objective criterion weighting that emphasizes discriminative criteria instead of subjective weighting. Multi-similarity ensemble reduces bias from relying on a single metric each measure captures different aspects of closeness.

5.2 Limitations of the method selection

The ultimate rankings are susceptible linguistic to neutrosophic mappings and whatever scaling choices (like midpoint for IVNS) we choose, Entropy assumes independence between criteria and does not capture interactions $(e.g., public\ transport\ share \times vehicle\ ownership)$. Rough approximations depend on equivalence relation choice (how you form K classes). Computing three $neutrosophic\ variations \times multiplicative\ similarity\ assays \times rough\ approximations$ is more computationally intensive than chains using one method. Entropy and similarity measures are only as accurate as the data and inputs that are provided any out of date or noisy city statics will introduce bias into the results.

6. Application of the Study

In recent years, the world has witnessed increasing challenges related to environmental degradation and urbanization. Sustainable transportation refers to transportation systems designed to minimize environmental impact, promote social equity and accessibility and enhance economic efficiency. The proposed model is developed to address the multi-dimensional decision-making challenge by integrating advanced mathematical tools such as NSs, RSs and entropy, which utilize MCDM to evaluate alternatives that support collaborative decision-making across diverse campus stakeholders. This proposed model helps decision-makers objectively assess and select the most sustainable transportation solution tailored to cities, while laying the groundwork for potential urban-scale implementation. Also, there is a comparison between three forms of NSs, namely SVNS, IVNS and PNS, within the proposed decision-making model. Alternatives represent the various cities that are possible to implement are Ahmedabad (B_1), Bangalore (B_2), Bhopal (B_3), Chennai (B_4), Delhi (B_5), Indore (B_6), Jaipur (B_7), Mumbai (B_8), Mysore (B_9), Pune (B_{10}), Rajkot (B_{11}) and Surat (B_{12}). The criteria are selected based on the sustainability of such DMs Population (L_1), Public Transport (L_2), Private Transport (L_3), Bicycling & Walking (L_4),Average Trip Length (L_5), Vehicles per 1000 (L_6) and Passenger Cars per 1000 (L_7). Table 1 shows the real-time data of selected cities which is taken for the study.

Table 1: Real time data collected

City		Population (2001 Census)	Public Transport	Private Transport	Bicycling & Walking	Average Trip Length Km	Vehicles per 1000	Passenger Cars per 1000
		L_1	L_2	L_3	L_4	L_5	L_6	L_7
Ahmedabad	B_1	4500000	30	38	32	5.4	371	55
Bangalore	B_2	8625000	36	39	25	9.6	283	50
Bhopal	B_3	1433000	28	19	53	3.1	189	24
Chennai	B_4	7014000	39	30	31	8.6	226	45
Delhi	B_5	13840000	48	19	33	10.2	355	117
Indore	B_6	1759000	16	37	47	5.6	257	27
Jaipur	B_7	2032000	17	39	44	5.4	359	55
Mumbai	B_8	17702000	52	15	33	11.9	54	24
Mysore	B_9	787000	26	23	51	2.5	380	40
Pune	B_{10}	4200000	12	54	33	6.1	335	48
Rajkot	B_{11}	1002000	13	38	49	3.7	403	33
Surat	B_{12}	2430000	13	31	55	5.3	492	55

The linguistic values for single, interval valued, and Pythagorean neutrosophic numbers are shown in the below tables 2 - 4.

Table 2: The linguistic values of SVNS

Linguistic Term	SVNS Values
Very Low (VL)	[0.1,0.2,0.8]
Low (L)	[0.2,0.2,0.7]
Medium Low (ML)	[0.35,0.15,0.6]
Medium (M)	[0.5,0.1,0.5]
Medium High (MH)	[0.65,0.1,0.35]
High (H)	[0.8,0.1,0.2]
Very High (VH)	[0.9,0.05,0.1]

Table 3: The linguistic values of IVNS

Linguistic Term	IVNS Values
VL	[0.0, 0.2; 0.0, 0.2; 0.8, 1.0]
L	[0.2, 0.4; 0.2, 0.4; 0.6, 0.8]
ML	[0.4, 0.6; 0.4, 0.6; 0.4, 0.6]
M	[0.5, 0.7; 0.5, 0.7; 0.3, 0.5]
MH	[0.6, 0.8; 0.6, 0.8; 0.2, 0.4]
H	[0.7, 0.9; 0.7, 0.9; 0.1, 0.3]
VH	[0.8, 1.0; 0.8, 1.0; 0.0, 0.2]

Table 4: The linguistic values of PNS

Linguistic Term	PNS Values
VL	[0.15,0.2,0.95]
L	[0.3,0.25,0.85]
ML	[0.45,0.2,0.75]
M	[0.6,0.15,0.6]
MH	[0.75,0.1,0.45]

H	[0.85,0.05,0.3]
VH	[0.95,0.02,0.15]

Consider three DMs T_1, T_2, T_3. Table 5 presents the linguistic terms that is gathered from the three DMs T_1, T_2, T_3 on the alternatives$\{B_1, B_2, \ldots, B_{12}\}$.

Table 5: Decisions of T_1, T_2, T_3

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
			T_1				
B_1	VL	L	VH	H	M	ML	ML
B_2	ML	ML	H	VH	VH	VH	MH
B_3	L	VL	H	VL	VH	VH	H
B_4	H	MH	H	M	MH	MH	MH
B_5	ML	L	ML	VL	VL	VH	M
B_6	H	VL	MH	H	L	M	MH
B_7	H	H	MH	VH	L	ML	M
B_8	ML	ML	M	VL	VL	MH	VL
B_9	L	MH	L	VH	VL	L	H
B_{10}	VH	ML	L	VL	MH	ML	VH
B_{11}	L	MH	M	MH	VH	MH	ML
B_{12}	VH	VL	L	H	ML	M	VH
			T_2				
B_1	ML	VH	L	M	M	ML	M
B_2	VH	H	H	H	M	VH	ML
B_3	M	VL	H	ML	VH	VL	MH
B_4	ML	ML	VH	L	H	H	ML
B_5	L	MH	VL	ML	M	MH	H
B_6	VL	H	L	L	VH	H	ML
B_7	ML	L	VL	M	VL	ML	M
B_8	H	L	M	VH	ML	H	MH
B_9	H	MH	L	MH	VH	H	VH
B_{10}	MH	M	VH	VL	VH	ML	ML
B_{11}	VL	MH	VL	ML	MH	M	H

B_{12}	H	L	VL	VL	MH	VH	H
T_3							
B_1	VH	M	M	ML	ML	M	VH
B_2	VL	L	M	MH	L	MH	H
B_3	ML	H	VH	H	ML	VH	ML
B_4	MH	H	ML	VL	VL	VL	M
B_5	L	L	M	VL	VH	ML	VH
B_6	L	H	VH	VH	VL	VL	H
B_7	MH	VL	VL	MH	VH	L	H
B_8	MH	H	VH	L	ML	H	M
B_9	MH	VL	L	ML	VL	H	M
B_{10}	MH	MH	MH	VL	ML	VH	M
B_{11}	VL	ML	MH	L	H	MH	L
B_{12}	ML	H	MH	L	H	VL	H

These linguistic values of three decision makers are converted into SVNS, IVNS and PNS by multiplying each linguistic value of the decision makers with $\omega = \{0.33, 0.32, 0.35\}$ whose sum is 1. Thus, the decision matrix is presented in the table 6-8.

Table 6: Decision matrix of SVNS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	[0.46,0.13,0.49]	[0.53,0.12,0.44]	[0.54,0.12,0.43]	[0.55,0.12,0.44]	[0.45,0.12,0.54]	[0.40,0.13,0.57]	[0.59,0.10,0.39]
B_2	[0.44,0.14,0.51]	[0.44,0.15,0.51]	[0.70,0.10,0.31]	[0.78,0.080,0.22]	[0.53,0.12,0.44]	[0.81,0.070,0.19]	[0.61,0.12,0.38]
B_3	[0.35,0.15,0.60]	[0.35,0.17,0.59]	[0.84,0.080,0.17]	[0.43,0.15,0.53]	[0.71,0.090,0.28]	[0.64,0.10,0.32]	[0.59,0.12,0.39]
B_4	[0.60,0.12,0.38]	[0.61,0.12,0.38]	[0.67,0.10,0.31]	[0.26,0.17,0.67]	[0.51,0.14,0.46]	[0.51,0.14,0.46]	[0.50,0.12,0.48]
B_5	[0.25,0.18,0.67]	[0.34,0.17,0.59]	[0.32,0.15,0.63]	[0.18,0.18,0.74]	[0.51,0.12,0.46]	[0.63,0.10,0.36]	[0.74,0.080,0.26]

B_6	[0.37,0.17,0.57]	[0.57,0.13,0.40]	[0.59,0.11,0.37]	[0.64,0.11,0.33]	[0.39,0.15,0.54]	[0.56,0.14,0.41]	[0.61,0.12,0.38]
B_7	[0.60,0.12,0.38]	[0.36,0.17,0.57]	[0.28,0.17,0.65]	[0.68,0.080,0.32]	[0.41,0.15,0.52]	[0.30,0.17,0.64]	[0.61,0.10,0.40]
B_8	[0.65,0.10,0.35]	[0.46,0.15,0.49]	[0.64,0.080,0.36]	[0.39,0.15,0.54]	[0.27,0.17,0.67]	[0.75,0.10,0.25]	[0.42,0.13,0.55]
B_9	[0.55,0.13,0.42]	[0.46,0.14,0.51]	[0.20,0.20,0.70]	[0.63,0.10,0.36]	[0.36,0.15,0.58]	[0.60,0.13,0.37]	[0.73,0.080,0.27]
B_{10}	[0.73,0.080,0.27]	[0.50,0.12,0.48]	[0.58,0.12,0.39]	[0.10,0.20,0.80]	[0.63,0.10,0.36]	[0.54,0.12,0.43]	[0.58,0.10,0.40]
B_{11}	[0.13,0.20,0.77]	[0.55,0.12,0.44]	[0.42,0.13,0.54]	[0.40,0.15,0.55]	[0.79,0.080,0.22]	[0.60,0.10,0.40]	[0.44,0.15,0.51]
B_{12}	[0.68,0.10,0.31]	[0.38,0.17,0.56]	[0.33,0.17,0.61]	[0.37,0.17,0.57]	[0.60,0.12,0.38]	[0.49,0.12,0.48]	[0.83,0.080,0.17]

Table 7: Decision matrix of IVNS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	[0.41,0.61;0.41,0.61; 0.39,0.59]	[0.50,0.70; 0.50,0.70; 0.30,0.50]	[0.50,0.70; 0.50,0.70; 0.30,0.50]	[0.53,0.73; 0.53,0.73; 0.27,0.47]	[0.47,0.67;0.47,0.67;0.34,0.54]	[0.44,0.64;0.44,0.64;0.37,0.57]	[0.57,0.77; 0.57,0.77;0.23,0.43]
B_2	[0.39,0.59;0.39,0.59; 0.41,0.61]	[0.43,0.63; 0.43,0.63; 0.37,0.57]	[0.63,0.83; 0.63,0.83; 0.17,0.37]	[0.70,0.90; 0.70,0.90; 0.10,0.30]	[0.49,0.69;0.49,0.69;0.31,0.51]	[0.73,0.93;0.73,0.93;0.070,0.27]	[0.57,0.77; 0.57,0.77;0.23,0.43]
B_3	[0.37,0.57;0.37,0.57; 0.43,0.63]	[0.25,0.45; 0.25,0.45; 0.56,0.76]	[0.74,0.94; 0.74,0.94; 0.070,0.27]	[0.37,0.57; 0.37,0.57; 0.43,0.63]	[0.66,0.86;0.66,0.86;0.14,0.34]	[0.54,0.74;0.54,0.74;0.26,0.46]	[0.56,0.76; 0.56,0.76;0.24,0.44]
B_4	[0.57,0.77;0.57,0.77; 0.23,0.43]	[0.57,0.77; 0.57,0.77; 0.23,0.43]	[0.63,0.83; 0.63,0.83; 0.17,0.37]	[0.23,0.43; 0.23,0.43; 0.57,0.77]	[0.42,0.62;0.42,	[0.42,0.62;0.42,	[0.50,0.70; 0.50,0.70;0.30,0.50]

					0.62;0.38,0.58]	0.62;0.38,0.58]	
B_5	[0.27,0.47;0.27,0.47;0.53,0.73]	[0.33,0.53;0.33,0.53;0.47,0.67]	[0.31,0.51;0.31,0.51;0.49,0.69]	[0.13,0.33;0.13,0.33;0.67,0.87]	[0.44,0.64;0.44,0.64;0.36,0.56]	[0.60,0.80;0.60,0.80;0.20,0.40]	[0.67,0.87;0.67,0.87;0.13,0.33]
B_6	[0.30,0.50;0.30,0.50;0.50,0.70]	[0.47,0.67;0.47,0.67;0.33,0.53]	[0.54,0.74;0.54,0.74;0.26,0.46]	[0.58,0.78;0.58,0.78;0.23,0.43]	[0.32,0.52;0.32,0.52;0.48,0.68]	[0.46,0.66;0.46,0.66;0.35,0.55]	[0.57,0.77;0.57,0.77;0.23,0.43]
B_7	[0.57,0.77;0.57,0.77;0.23,0.43]	[0.30,0.50;0.30,0.50;0.51,0.71]	[0.20,0.40;0.20,0.40;0.60,0.80]	[0.63,0.83;0.63,0.83;0.17,0.37]	[0.35,0.55;0.35,0.55;0.45,0.65]	[0.33,0.53;0.33,0.53;0.47,0.67]	[0.57,0.77;0.57,0.77;0.23,0.43]
B_8	[0.60,0.80;0.60,0.80;0.20,0.40]	[0.44,0.64;0.44,0.64;0.36,0.56]	[0.61,0.81;0.61,0.81;0.20,0.40]	[0.33,0.53;0.33,0.53;0.47,0.67]	[0.27,0.47;0.27,0.47;0.53,0.73]	[0.67,0.87;0.67,0.87;0.13,0.33]	[0.37,0.57;0.37,0.57;0.43,0.63]
B_9	[0.50,0.70;0.50,0.70;0.30,0.50]	[0.39,0.59;0.39,0.59;0.41,0.61]	[0.20,0.40;0.20,0.40;0.60,0.80]	[0.60,0.80;0.60,0.80;0.20,0.40]	[0.26,0.46;0.26,0.46;0.54,0.74]	[0.54,0.74;0.54,0.74;0.27,0.47]	[0.66,0.86;0.66,0.86;0.14,0.34]
B_{10}	[0.67,0.87;0.67,0.87;0.13,0.33]	[0.50,0.70;0.50,0.70;0.30,0.50]	[0.53,0.73;0.53,0.73;0.27,0.47]	[0,0.20;0,0.20;0.80,1]	[0.59,0.79;0.59,0.79;0.21,0.41]	[0.54,0.74;0.54,0.74;0.26,0.46]	[0.57,0.77;0.57,0.77;0.23,0.43]
B_{11}	[0.070,0.27;0.070,0.27;0.73,0.93]	[0.53,0.73;0.53,0.73;0.27,0.47]	[0.38,0.58;0.38,0.58;0.43,0.63]	[0.40,0.60;0.40,0.60;0.40,0.60]	[0.70,0.90;0.70,0.90;0.10,0.30]	[0.57,0.77;0.57,0.77;0.23,0.43]	[0.43,0.63;0.43,0.63;0.37,0.57]
B_{12}	[0.63,0.83;0.63,0.83;0.17,0.37]	[0.31,0.51;0.31,0.51;0.49,0.69]	[0.28,0.48;0.28,0.48;0.52,0.72]	[0.30,0.50;0.30,0.50;0.50,0.70]	[0.57,0.77;0.57,	[0.42,0.62;0.42,	[0.73,0.93;0.73,0.93;0.070,0.27]

					0.77;0.23,0.43]	0.62;0.38,0.58]	

Table 8: Decision matrix of PNS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	[0.53,0.14,0.61]	[0.61,0.14,0.54]	[0.62,0.14,0.53]	[0.63,0.13,0.55]	[0.55,0.17,0.65]	[0.50,0.18,0.70]	[0.67,0.12,0.49]
B_2	[0.51,0.14,0.63]	[0.53,0.17,0.64]	[0.76,0.090,0.41]	[0.85,0.060,0.30]	[0.61,0.14,0.54]	[0.88,0.050,0.26]	[0.69,0.11,0.49]
B_3	[0.45,0.20,0.74]	[0.40,0.15,0.72]	[0.89,0.040,0.25]	[0.49,0.15,0.66]	[0.78,0.080,0.36]	[0.69,0.080,0.41]	[0.68,0.12,0.51]
B_4	[0.69,0.12,0.50]	[0.69,0.11,0.49]	[0.74,0.090,0.41]	[0.35,0.20,0.80]	[0.57,0.12,0.58]	[0.57,0.12,0.58]	[0.60,0.15,0.60]
B_5	[0.35,0.23,0.82]	[0.44,0.20,0.72]	[0.41,0.18,0.76]	[0.25,0.20,0.89]	[0.57,0.12,0.56]	[0.71,0.11,0.46]	[0.80,0.070,0.35]
B_6	[0.43,0.17,0.70]	[0.62,0.10,0.51]	[0.68,0.12,0.47]	[0.71,0.10,0.42]	[0.46,0.16,0.66]	[0.61,0.10,0.53]	[0.69,0.11,0.49]
B_7	[0.69,0.12,0.50]	[0.43,0.17,0.70]	[0.35,0.17,0.79]	[0.77,0.090,0.40]	[0.48,0.15,0.64]	[0.40,0.22,0.79]	[0.69,0.12,0.50]
B_8	[0.73,0.10,0.45]	[0.54,0.16,0.62]	[0.72,0.10,0.44]	[0.46,0.16,0.66]	[0.35,0.20,0.82]	[0.82,0.070,0.35]	[0.50,0.15,0.67]
B_9	[0.63,0.13,0.53]	[0.54,0.14,0.63]	[0.30,0.25,0.85]	[0.71,0.11,0.46]	[0.41,0.14,0.69]	[0.67,0.12,0.48]	[0.79,0.080,0.36]
B_{10}	[0.82,0.070,0.35]	[0.60,0.15,0.60]	[0.67,0.12,0.49]	[0.15,0.20,0.95]	[0.71,0.11,0.46]	[0.63,0.14,0.54]	[0.67,0.12,0.50]
B_{11}	[0.20,0.22,0.92]	[0.65,0.14,0.56]	[0.51,0.15,0.66]	[0.50,0.18,0.69]	[0.85,0.060,0.30]	[0.70,0.12,0.50]	[0.53,0.17,0.64]
B_{12}	[0.74,0.090,0.41]	[0.44,0.16,0.69]	[0.41,0.18,0.74]	[0.43,0.17,0.70]	[0.69,0.12,0.50]	[0.55,0.13,0.58]	[0.88,0.040,0.25]

From the decision matrix of SVNS, IVNS and PNS, the normalised matrix is obtained in table 9, 10 and 11 respectively using the equation (1).

Table 9: Normalized matrix of SVNS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	[0.426,0.984,0.231]	[0.486,0.987,0.226]	[0.495,0.987,0.225]	[0.495,0.987,0.222]	[0.405,0.987,0.224]	[0.364,0.985,0.223]	[0.546,0.991,0.220]
B_2	[0.404,0.982,0.229]	[0.400,0.980,0.227]	[0.631,0.991,0.193]	[0.722,0.994,0.159]	[0.486,0.987,0.226]	[0.757,0.995,0.144]	[0.550,0.987,0.212]
B_3	[0.318,0.980,0.218]	[0.315,0.974,0.218]	[0.771,0.994,0.129]	[0.387,0.980,0.224]	[0.657,0.993,0.187]	[0.604,0.991,0.205]	[0.536,0.987,0.216]
B_4	[0.545,0.987,0.214]	[0.550,0.987,0.212]	[0.620,0.991,0.198]	[0.236,0.974,0.201]	[0.459,0.982,0.224]	[0.459,0.982,0.224]	[0.455,0.987,0.227]
B_5	[0.227,0.971,0.201]	[0.309,0.974,0.220]	[0.291,0.980,0.212]	[0.164,0.971,0.175]	[0.468,0.987,0.228]	[0.578,0.991,0.211]	[0.685,0.994,0.178]
B_6	[0.333,0.974,0.221]	[0.518,0.985,0.218]	[0.551,0.989,0.218]	[0.593,0.989,0.205]	[0.361,0.979,0.230]	[0.505,0.982,0.218]	[0.550,0.987,0.212]
B_7	[0.545,0.987,0.214]	[0.327,0.974,0.223]	[0.255,0.974,0.207]	[0.630,0.994,0.201]	[0.380,0.979,0.231]	[0.270,0.974,0.208]	[0.550,0.991,0.216]
B_8	[0.591,0.991,0.207]	[0.418,0.980,0.227]	[0.593,0.994,0.213]	[0.361,0.979,0.230]	[0.243,0.974,0.199]	[0.682,0.991,0.170]	[0.382,0.985,0.225]
B_9	[0.500,0.985,0.221]	[0.414,0.982,0.225]	[0.182,0.964,0.191]	[0.578,0.991,0.211]	[0.330,0.979,0.223]	[0.545,0.985,0.212]	[0.676,0.994,0.183]
B_{10}	[0.676,0.994,0.183]	[0.455,0.987,0.227]	[0.532,0.987,0.218]	[0.0910,0.964,0.145]	[0.578,0.991,0.211]	[0.495,0.987,0.225]	[0.537,0.991,0.222]
B_{11}	[0.118,0.964,0.161]	[0.495,0.987,0.222]	[0.385,0.984,0.228]	[0.364,0.980,0.225]	[0.725,0.994,0.157]	[0.545,0.991,0.218]	[0.400,0.980,0.227]
B_{12}	[0.624,0.991,0.196]	[0.342,0.974,0.222]	[0.297,0.974,0.214]	[0.333,0.974,0.221]	[0.545,0.987,0.214]	[0.450,0.987,0.229]	[0.769,0.994,0.131]

Table 10: Normalized matrix of IVNS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	[0.51,0.6889,0.2009]	[0.60,0.58,0.20]	[0.60,0.58,0.20]	[0.63,0.5401,0.1961]	[0.57,0.6181,0.2024]	[0.54,0.6544,0.2021]	[0.67,0.4841,0.1881]

B_2	[0.49,0.7109,0.1989]	[0.53,0.6661,0.2021]	[0.73,0.3941,0.1701]	[0.80,0.28,0.14]	[0.59,0.5929,0.2009]	[0.83,0.2281,0.1241]	[0.67,0.4841,0.1881]
B_3	[0.47,0.7321,0.1961]	[0.35,0.8425,0.1584]	[0.84,0.2104,0.1241]	[0.47,0.7321,0.1961]	[0.76,0.3464,0.1584]	[0.64,0.5264,0.1944]	[0.66,0.4984,0.1904]
B_4	[0.67,0.4841,0.1881]	[0.67,0.4841,0.1881]	[0.73,0.3941,0.1701]	[0.33,0.8581,0.1541]	[0.52,0.6776,0.2016]	[0.52,0.6776,0.2016]	[0.60,0.58,0.20]
B_5	[0.37,0.8261,0.1701]	[0.43,0.7721,0.1881]	[0.41,0.7909,0.1829]	[0.23,0.9241,0.1001]	[0.54,0.6544,0.2024]	[0.70,0.44,0.18]	[0.77,0.3301,0.1541]
B_6	[0.40,0.80,0.18]	[0.57,0.6181,0.2021]	[0.64,0.5264,0.1944]	[0.68,0.4696,0.1881]	[0.42,0.7816,0.1856]	[0.56,0.6304,0.2025]	[0.67,0.4841,0.1881]
B_7	[0.67,0.4841,0.1881]	[0.40,0.80,0.1769]	[0.30,0.88,0.14]	[0.73,0.3941,0.1701]	[0.45,0.7525,0.1925]	[0.43,0.7721,0.1881]	[0.67,0.4841,0.1881]
B_8	[0.70,0.44,0.18]	[0.54,0.6544,0.2024]	[0.71,0.4249,0.18]	[0.43,0.7721,0.1881]	[0.37,0.8261,0.1701]	[0.77,0.3301,0.1541]	[0.47,0.7321,0.1961]
B_9	[0.60,0.58,0.20]	[0.49,0.7109,0.1989]	[0.30,0.88,0.14]	[0.70,0.44,0.18]	[0.36,0.8344,0.1664]	[0.64,0.5264,0.1961]	[0.76,0.3464,0.1584]
B_{10}	[0.77,0.3301,0.1541]	[0.60,0.58,0.20]	[0.63,0.5401,0.1961]	[0.10,0.98,0]	[0.69,0.4549,0.1829]	[0.64,0.5264,0.1944]	[0.67,0.4841,0.1881]
B_{11}	[0.17,0.9541,0.05810]	[0.63,0.5401,0.1961]	[0.48,0.7216,0.1961]	[0.50,0.70,0.20]	[0.80,0.28,0.14]	[0.67,0.4841,0.1881]	[0.53,0.6661,0.2021]
B_{12}	[0.73,0.3941,0.1701]	[0.41,0.7909,0.1829]	[0.38,0.8176,0.1736]	[0.40,0.80,0.18]	[0.67,0.4841,0.1881]	[0.52,0.6776,0.2016]	[0.83,0.2281,0.1241]

Table 11: Normalized matrix of PNS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	[0.414,0.985,0.186]	[0.473,0.985,0.193]	[0.481,0.985,0.193]	[0.481,0.987,0.189]	[0.401,0.979,0.166]	[0.362,0.977,0.152]	[0.523,0.989,0.195]
B_2	[0.398,0.985,0.182]	[0.396,0.978,0.172]	[0.603,0.994,0.192]	[0.702,0.997,0.174]	[0.473,0.985,0.193]	[0.739,0.998,0.162]	[0.535,0.991,0.194]
B_3	[0.324,0.971,0.138]	[0.315,0.982,0.159]	[0.754,0.999,0.159]	[0.377,0.983,0.173]	[0.639,0.995,0.189]	[0.585,0.995,0.205]	[0.519,0.989,0.191]
B_4	[0.527,0.989,0.191]	[0.535,0.991,0.194]	[0.597,0.993,0.195]	[0.259,0.97,0.119]	[0.449,0.989,0.192]	[0.449,0.989,0.192]	[0.444,0.983,0.178]
B_5	[0.25,0.962,0.105]	[0.324,0.971,0.148]	[0.304,0.976,0.135]	[0.187,0.97,0.073]	[0.456,0.988,0.197]	[0.555,0.991,0.194]	[0.656,0.996,0.186]
B_6	[0.331,0.978,0.162]	[0.504,0.992,0.203]	[0.535,0.989,0.196]	[0.577,0.992,0.198]	[0.359,0.98,0.175]	[0.492,0.992,0.201]	[0.535,0.991,0.194]
B_7	[0.527,0.989,0.191]	[0.331,0.978,0.162]	[0.267,0.978,0.127]	[0.611,0.994,0.19]	[0.378,0.982,0.181]	[0.284,0.966,0.118]	[0.527,0.989,0.191]
B_8	[0.57,0.992,0.193]	[0.409,0.981,0.178]	[0.571,0.992,0.196]	[0.359,0.98,0.175]	[0.255,0.971,0.108]	[0.661,0.996,0.183]	[0.379,0.983,0.168]
B_9	[0.488,0.987,0.193]	[0.412,0.985,0.178]	[0.214,0.955,0.091]	[0.555,0.991,0.194]	[0.331,0.984,0.173]	[0.528,0.989,0.197]	[0.642,0.995,0.187]
B_{10}	[0.661,0.996,0.183]	[0.444,0.983,0.178]	[0.523,0.989,0.195]	[0.115,0.969,0.037]	[0.555,0.991,0.194]	[0.481,0.985,0.19]	[0.519,0.989,0.194]
B_{11}	[0.149,0.964,0.055]	[0.481,0.985,0.183]	[0.386,0.983,0.17]	[0.365,0.976,0.156]	[0.702,0.997,0.174]	[0.53,0.989,0.189]	[0.396,0.978,0.172]
B_{12}	[0.597,0.993,0.195]	[0.341,0.98,0.166]	[0.308,0.976,0.145]	[0.331,0.978,0.162]	[0.527,0.989,0.191]	[0.437,0.987,0.193]	[0.752,0.999,0.16]

The aggregated neutrosophic decision matrix of SVNS, IVNS and PNS is tabulated in table 12.

Table 12: Aggregated neutrosophic decision matrix

	SVNS	IVNS	PNS
L_1	[1.694,4.264,0.7970]	[2.091,2.685,0.6650]	[1.671,4.264,0.63]

L_2	[1.605,4.257,0.8510]	[1.985,2.907,0.7330]	[1.585,4.264,0.675]
L_3	[1.788,4.271,0.7810]	[2.154,2.590,0.6600]	[1.769,4.271,0.636]
L_4	[1.581,4.259,0.7720]	[1.915,2.854,0.6040]	[1.57,4.263,0.587]
L_5	[1.799,4.275,0.8150]	[2.151,2.641,0.6990]	[1.763,4.278,0.681]
L_6	[1.996,4.282,0.7940]	[2.381,2.341,0.7110]	[1.948,4.287,0.695]
L_7	[2.118,4.292,0.7880]	[2.544,2.098,0.6910]	[2.051,4.294,0.705]

The Jaccard Similarity, Hamming Distance and Cosine Similarity of cooperative NSs is illustrated in table 13, 14 and 15, respectively.

Table 13: Jaccard Similarity measures

L_1	2.666667	2.666666667	3	3	3	3
L_2	3	3	3	3	3	3
L_3	3	3	2.666667	3	3	3
L_4	3	3	3	3	3	3
L_5	3	3	3	3	3	3
L_6	3	3	3	3	3	3
L_7	3	3	3	3	3	3

Table 14: Hamming Distance measures

L_1	2.666667	2.666666667	3	3	3	3
L_2	3	3	3	3	3	3
L_3	3	3	2.666667	3	3	3
L_4	3	3	3	3	3	3
L_5	3	3	3	3	3	3
L_6	3	3	3	3	3	3
L_7	3	3	3	3	3	3

Table 15: Cosine Similarity measures

L_1	0.0811	0.0706	0.0852	0.0713	0.082	0.1121
L_2	0.0716	0.0584	0.0777	0.0595	0.0738	0.1081
L_3	0.0859	0.0774	0.0886	0.0781	0.0857	0.1123
L_4	0.0697	0.0566	0.0761	0.0562	0.0728	0.1083

L_5	0.0829	0.0741	0.0857	0.0753	0.0826	0.1092
L_6	0.1142	0.1101	0.1133	0.1119	0.1108	0.1288
L_7	0.149	0.148	0.1455	0.15	0.1434	0.1557

Thus, the criteria weight is calculated using Shannon entropy from the Jaccard Similarity, Hamming Distance and Cosine Similarity of cooperative NSs, which is multiplied by the $\delta = [0.3192, 0.3617, 0.3192]$. Table 16 provides the criteria weight using Shannon entropy.

Table 16: Criteria weight using Shannon entropy.

	Criteria weight
L_1	0.863121
L_2	1.378783
L_3	0.591673
L_4	1.378783
L_5	1.148835
L_6	1.148835
L_7	1.378783

Now ranking the attributes based on the criteria weight using Shannon entropy is evaluated and presented in the table 17.

Table 17: Ranking

		Alternatives Weight	**Rank**
Ahmedabad	B_1	15.65062871	3
Bangalore	B_2	16.08829168	2
Bhopal	B_3	-32.22633914	12
Chennai	B_4	6.678372606	5
Delhi	B_5	40.12591441	1
Indore	B_6	-24.08748374	11
Jaipur	B_7	5.015008479	6
Mumbai	B_8	2.240523177	7

Mysore	B_9	-9.920934637	9
Pune	B_{10}	-9.26354624	8
Rajkot	B_{11}	-17.53773155	10
Surat	B_{12}	7.237296245	4

Delhi is ranked first with respect to alternative weight as it has higher performance (40.13). Bangalore and Ahmedabad follow behind with alternatives slightly better than the rest. Bhopal and Indore are the last-countries having large negative weights, indicating a poor evaluation of ranking. Weights essentially dictate the ranking of cities in this particular MCDM analysis.

7. Sensitive Analysis

The sensitivity analysis is very critical in testing and confirming the robustness and validity of any given MCDM model, especially when working within uncertain and complex environments. This research insists upon thoroughly testing the efficiency of the cooperative neutrosophic-set-based model through the evaluation, independent ranking and ranking of the performances of three different kinds of neutrosophic sets SVNS, IVNS and PNS. By isolating and considering the ranks generated by such neutrosophic environments, the study brings forth how variations in representing truth, indeterminacy and falsity could affect the decision-making outcomes. The alternative weights and ranks arrived at for each neutrosophic set are presented in table 18 for an easy and straightforward evaluation of the model's sensitivity to the different types of neutrosophic input. This sets out the knock for the proposed cooperative system by emphasizing consistency and flexibility for decision-making in sustainable transportation under many dimensions of uncertainty. Figure 1 illustrates the graphical image of sensitive analysis.

Table 18: Alternative weights and ranks for each neutrosophic sets

		SVNS		IVNS		PNS	
		Alternative Weights	Rank	Alternative Weights	Rank	Alternative Weights	Rank
Ahmedabad	B_1	27.96702703	3	21.34113514	3	27.34394595	3
Bangalore	B_2	28.19837838	2	22.78389189	2	27.40174324	2
Bhopal	B_3	-62.2	12	-47.5675	12	-60.81125	12
Chennai	B_4	9.582702703	6	8.796013514	5	9.170844595	6
Delhi	B_5	71.82540541	1	56.44502703	1	70.01318919	1
Indore	B_6	-43.16108108	11	-33.4394054	11	-42.13733784	11
Jaipur	B_7	9.821081081	5	6.411405405	6	9.723337838	5
Mumbai	B_8	1.48	7	4.2125	7	1.06875	7
Mysore	B_9	-18.25891892	9	-14.6175946	9	-17.74366216	9
Pune	B_{10}	-14.1527027	8	-12.3725135	8	-13.65659459	8
Rajkot	B_{11}	-28.33189189	10	-23.2839595	10	-27.46096622	10
Surat	B_{12}	17.23	4	11.291	4	17.088	4

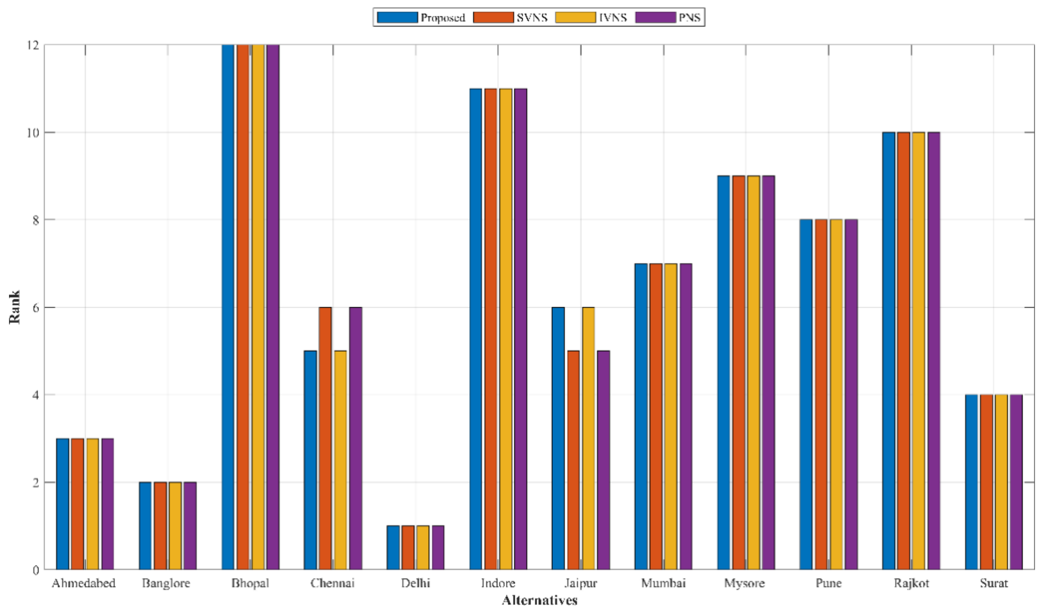

Figure 1: Sensitive Analysis graph

This analysis confirms the viability and hence the preference of the said cooperative neutrosophic-based decision-making model. The ranking of alternatives does not vary markedly across the SVNS, IVNS, and PNS: Delhi remains first, followed by Bangalore and Ahmedabad and Bhopal and Indore at the lowest levels. Slight differences among weights can arise with differing types of neutrosophic representations, but broadly speaking, the ranking patterns remain the same. This strongly endorses the model for being viable in scenarios that depict uncertainty in different forms and establishes that the decision-making conclusions reached are not primarily dependent upon the type of neutrosophic environment used.

7.1 Comparative analysis with Other Neutrosophic Based Methods

Literature highlights heightened interest in neutrosophic and rough-based perspectives within MCDM, Das et al. (2021) intertwined neutrosophic soft sets with RS theory, confirming the benefits of hybridization when managing uncertainty [19]. Martina and Deepa (2023) used multi-valued rough NSs and matrices for MCDM, noting their validity and flexibility for different data types. More recently [20]. Rogulj et al. (2021) proposed a hybrid approach merging VIKOR and cross-entropy and rough–neutrosophic sets, interpreting rough approximations to show advantages associated with rank-based approaches. Some concurrent research focused on similarity measures [21]. Bhatia et al. (2023) developed cosine similarity measures with Pythagorean fuzzy sets [23], while Arora and Naithani (2022) proposed logarithmic similarity measures; both recognized the utility of using similarity-based approaches within a decision context [24].

Even with the advancements, the existing approaches have drawbacks like:

- Focus on one similarity measure, introducing bias
- Only use a few weighting schemes, making the importance assignment partly subjective
- One neutrosophic environment (e.g, PNS) with no systematic comparison
- Limit potential with narrow issue domains, not large-scale sustainable use.

On the contrary, the proposed Cooperative NS framework:

- Can incorporate multiple similarity measures (Jaccard, Hamming and Cosine) for soundness
- Incorporates Shannon entropy and Dual RSs to achieve objective weighting
- Compares the three types of NSs systematically
- Validates results with data from twelve Indian cities, proving applicability to sustainable urban transportation.

8. Conclusion

This research utilized a strong framework that relied on Cooperative NSs, Dual RS Theory, RIORs, Shannon Entropy and Multi-Similarity Measures to manage the difficulties involved in planning sustainable transportation. With the help of SVNS, IVNS and PNS, the model was able to handle both uncertainty, imprecision and vagueness in the transportation sector. The system determined the relative importance of each attribute using Shannon entropy, evaluated the proximity of each alternative with Jaccard Similarity, Hamming Distance and Cosine Similarity and came to a recommendation. Delhi always received the top ranking in sustainable transportation options for all neutrosophic models, whereas Bangalore and Ahmedabad came second and third, respectively. Bhopal and Indore, on the other hand, performed the worst among these cities because they did not do well according to the considered sustainability factors. In addition, the sensitivity analysis confirmed that the outputs from the model could be trusted since the rankings did not change so much despite different weights for the alternatives. In comparison with traditional neutrosophic MCDM methods, the proposed framework demonstrated greater robustness through multi-similarity integration, improved weighting via entropy–RS, and a systematic comparison of neutrosophic variants. It proves that the suggested framework can stay steady and adjust to various neustrosophic conditions. All in all, this approach offered here for sustainable urban transportation planning manages uncertainty and worked well for complex domains due to its detailed and data-based nature.

8.1 Limitations

Despite the proposed framework providing strong results, there are limitations to recognize, the model is dependent on the quality and timeliness of real-world transportational data. Outdated or noisy inputs can affect how well data can be ranked. Shannon entropy is based on independence between criteria and does not provide a construct to consider projects with dependencies like "$population \times vehicle\ ownership$". The amount of computer processing power increases with multiple neutrosophic variants, double rough approximations and three similarity measurements. The application has been validated with data from cities in India. The application has not been evaluated using international datasets that have different cultural/transportation contexts; this would take further study.

8.2 Directions for Future Research:

Entropy and learning-based techniques could be further combined in an efficient hybrid weighting strategy to comprehend the relationships between criteria. Based on the vast volumes of continually available IoT and sensor data, the model may then be used to the decision-making process in dynamic contexts (smart cities). In urban planning situations, the various stakeholder preferences are taken into account by deconstructing the group decision-making element on a multi-level decision-making foundation. Lastly, to test the model's generalizability and robustness, it should be used in other sustainable imperative areas, such as water management and renewable energy.

ACKNOWLEDGEMENTS:

The authors sincerely thank the editor and anonymous reviewers for their valuable suggestions, which have significantly enhanced the quality of this paper.

DECLARATIONS:

FUNDING

On Behalf of all authors the corresponding author states that they did not receive any funds for this project.

CONFLICTS OF INTEREST

The authors declare that we have no conflict of interest.

COMPETING INTERESTS

The authors declare that we have no competing interest.

DATA AVAILABILITY STATEMENT

All the data is collected from the simulation reports of the software and tools (MATLAB) used by the authors. Authors are working on implementing the same using real world data with appropriate permissions.

References:

[1] Ünver, M., Türkarslan, E., elik, N., Olgun, M., & Ye, J. (2022). Intuitionistic fuzzy-valued neutrosophic multi-sets and numerical applications to classification. *Complex & Intelligent Systems*, *8*(2), 1703-1721.

[2] Smarandache, F. (2025). Introduction to Neutrosophy, Neutrosophic Set, Neutrosophic Probability, Neutrosophic Statistics and Their Applications to Decision Making. In *Neutrosophic Paradigms: Advancements in Decision Making and Statistical Analysis: Neutrosophic Principles for Handling Uncertainty* (pp. 3-21). Cham: Springer Nature Switzerland.

[3] Voskoglou, M. G. (2022). Managing the Uncertainty: From Probability to Fuzziness, Neutrosophy and Soft Sets. *TFSS*, *1*(2), 46-58.

[4] Hussain, W. (2024). Strategic Decision-Making in Uncertain Environments: A Management Science Perspective. *The Management Science Letter*, *1*(3), 64-77.

[5] Jelti, F., Allouhi, A., &TabetAoul, K. A. (2023). Transition paths towards a sustainable transportation system: a literature review. *Sustainability*, *15*(21), 15457.

[6] He, Y., Li, X., Huang, P., & Wang, J. (2022). Exploring the road toward environmental sustainability: natural resources, renewable energy consumption, economic growth, and greenhouse gas emissions. *Sustainability*, *14*(3), 1579.

[7] Olaleye, I. A., Mokogwu, C., Olufemi-Phillips, A. Q., & Adewale, T. T. (2024). Innovative frameworks for sustainable transportation coordination to reduce carbon footprints in logistics. *International Journal of Science and Technology Research Archive*, *7*(02), 68-75.

[8] Jiang, H., & Hu, B. Q. (2021). A decision-theoretic fuzzy rough set in hesitant fuzzy information systems and its application in multi-attribute decision-making. *Information Sciences*, *579*, 103-127.

[9] Yao, J., Medina, J., Zhang, Y., &Ślęzak, D. (2022). Formal concept analysis, rough sets, and three-way decisions. *International Journal of Approximate Reasoning*, *140*, 1-6.

[10] Naeem, M., Khan, M. A., Abdullah, S., Qiyas, M., & Khan, S. (2021). Extended TOPSIS method based on the entropy measure and probabilistic hesitant fuzzy information and their application in decision support system. *Journal of Intelligent & Fuzzy Systems*, *40*(6), 11479-11490.

[11] Xia, S., Bai, X., Wang, G., Cheng, Y., Meng, D., Gao, X., ... & Giem, E. (2022). An efficient and accurate rough set for feature selection, classification, and knowledge representation. *IEEE Transactions on Knowledge and Data Engineering*, *35*(8), 7724-7735.

[12] Singh, M., & Singla, S. K. (2025). EFI-SATL: An EfficientNet and Self-Attention Based Biometric Recognition for Finger-Vein Using Deep Transfer Learning. *Computer Modeling in Engineering & Sciences (CMES)*, *142*(3).

[13] Dhawan, S., Singh, K., & Yadav, M. (2024, August). Recommendation System Evaluation with Various Similarity Metrics. In *2024 1st International Conference on Advanced Computing and Emerging Technologies (ACET)* (pp. 1-6). IEEE.

[14] Jin, C., Mi, J., Li, F., & Liang, M. (2022). A novel probabilistic hesitant fuzzy rough set based multi-criteria decision-making method. *Information Sciences*, *608*, 489-516.

[15] Li, D., Zhang, H., Li, T., Bouras, A., Yu, X., & Wang, T. (2021). Hybrid missing value imputation algorithms using fuzzy c-means and vaguely quantified rough set. *IEEE Transactions on Fuzzy Systems*, *30*(5), 1396-1408.

[16] Naeem, K., &Divvaz, B. (2023). Information measures for MADM under m-polar neutrosophic environment. *Granular Computing*, *8*(3), 597-616.

[17] Uluçay, V., & Deli, İ. (2024). TOPSIS-Based Entropy Measure for N-Valued Neutrosophic Trapezoidal Numbers and Their Application to Multi-Criteria Decision-Making Problems. In *Analytical Decision Making and Data Envelopment Analysis: Advances and Challenges* (pp. 433-454). Singapore: Springer Nature Singapore.

[18] Yolcu, A., Benek, A., & Öztürk, T. Y. (2023). A new approach to neutrosophic soft rough sets. *Knowledge and information systems*, *65*(5), 2043-2060.

[19] Das, M., Mohanty, D., & Parida, K. C. (2021). On the neutrosophic soft set with rough set theory. *Soft Computing*, *25*(21), 13365-13376.

[20] Martina, D. J. S., & Deepa, G. (2023). Application of multi-valued rough neutrosophic set and matrix in multi-criteria decision-making: Multi-valued neutrosophic rough set and matrix. *Mathematics in Applied Sciences and Engineering*, *4*(3), 227-248.

[21] Rogulj, K., KilićPamuković, J., &Ivić, M. (2021). Hybrid MCDM based on VIKOR and cross entropy under rough neutrosophic set theory. *Mathematics*, *9*(12), 1334.

[22] Mohammad, M. M. S., Abdullah, S., & Al-Shomrani, M. M. (2022). Some linear Diophantine fuzzy similarity measures and their application in decision making problem. *IEEE Access*, *10*, 29859-29877.

[23] Bhatia, M., Arora, H. D., Nathani, A., & Kumar, V. (2023). Cosine similarity measures for Pythagorean fuzzy sets with applications in decision making. *Asia-Pacific Journal of Science and Technology*, *28*(05).

[24] Arora, H. D., & Naithani, A. (2022). Logarithmic similarity measures on Pythagorean fuzzy sets in admission process. *Operations Research and Decisions*, *32*(1), 5-24.

[25] DELİ, İ., & ULUÇAY, V. (2025). Some harmonic aggregation operators for N-valued neutrosophic trapezoidal numbers and their application to multi-criteria decision-making. *Neutrosophic Sets and Systems*, *79*(1), 20.

Received: May 17, 2025. Accepted: Oct 10, 2025