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Neutrosophic Exponential Ratio-Type Estimator for Finite Population Mean in Stratified Sampling

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Abstract: This paper introduces an innovative neutrosophic exponential ratio-type estimator for estimating finite population means in stratified sampling environments with indeterminate data. Building upon classical exponential estimators and neutrosophic statistics, we develop a robust estimator that effectively handles uncertainty through interval-valued representations. The proposed estimator combines the strengths of exponential ratio estimation with neutrosophic weighting to achieve enhanced precision in stratified sampling scenarios. We derive the bias and mean square error (MSE) expressions under first-order approximation and demonstrate through empirical analysis using climate data that our estimator outperforms existing neutrosophic stratified estimators in terms of efficiency and reliability.

Keywords: Neutrosophic statistics, stratified sampling, exponential ratio estimator, auxiliary information, interval data, mean square error

1. Introduction

Stratified sampling remains a cornerstone technique in survey methodology, offering improved precision through population stratification. Traditional approaches often assume precise data measurements; however, real-world applications frequently encounter indeterminate or uncertain information. Neutrosophic statistics provides a powerful framework for such scenarios by representing observations as intervals $Z_N = [Z_L, Z_U] = Z_L + Z_U I_N$, where $I_N \in [I_L, I_U]$ captures the indeterminacy component [2]. The integration of neutrosophic theory with sampling estimators has been explored to handle uncertainty explicitly [4,5,6]. Existing neutrosophic stratified estimators have shown promise, yet challenges remain in improving efficiency and adaptability to varying levels of data indeterminacy [3,4,7]. Recent developments in neutrosophic sampling theory [8,9] have expanded the toolkit available for handling indeterminate data in survey contexts.

This study contributes by developing a novel neutrosophic exponential ratio-type estimator tailored specifically for stratified sampling frameworks with indeterminate data. We provide theoretical derivations of the estimator's bias and mean square error under first-order approximations. The performance is empirically validated with real climate data, demonstrating superior efficiency over classical and neutrosophic existing estimators [1,5,11]. Furthermore, we propose an optimal weighting scheme to minimize mean square error, enhancing applicability in practical survey settings. Neutrosophic statistics uniquely address the uncertainty and indeterminacy present in many real-world data collection processes, especially where interval-valued or imprecise observations occur [2,10]. By integrating neutrosophic principles into stratified sampling estimators, survey practitioners gain a robust tool to produce reliable estimates with quantified uncertainty, facilitating more informed decision-making in fields such as climatology, economics, and social sciences [4,8].

The rest of the paper is structured as follows: Section 2 provides the methodology including notation, section 3 gives existing estimators with their MSE expressions, and the proposed estimator formulation is given in section 4. Section 5 covers empirical analysis using real climate datasets. Finally, Section 6 concludes the paper with findings and future research directions.

2. Notations and Framework

Consider a finite neutrosophic population of size $N_N \in [N_L, N_U]$, partitioned into L homogeneous strata with sizes $N_{hN} \in [N_{hL}, N_{hU}]$, $h = 1, \ldots, L$, such that

$$\sum_{h=1}^{L} N_{hN} = N_N.$$

From each stratum, a neutrosophic simple random sample of size $n_{hN} \in [n_{hL}, n_{hU}]$ is drawn without replacement, with total sample size

$$n_N = \sum_{h=1}^{L} n_{hN} \in [n_L, n_U].$$

Let $Y_{hN} \in [Y_{hL}, Y_{hU}]$ be the study variable and $X_{hN} \in [X_{hL}, X_{hU}]$ the auxiliary variable in stratum h. Key population parameters are:

$$\bar{Y}_{hN} = \frac{1}{N_{hN}} \sum_{j=1}^{N_{hN}} Y_{hjN} \in [\bar{Y}_{hL}, \bar{Y}_{hU}],$$

$$\bar{X}_{hN} = \frac{1}{N_{hN}} \sum_{j=1}^{N_{hN}} X_{hjN} \in [\bar{X}_{hL}, \bar{X}_{hU}],$$

$$S_{yhN}^2 = \frac{1}{N_{hN} - 1} \sum_{j=1}^{N_{hN}} \left(Y_{hjN} - \bar{Y}_{hN}\right)^2 \in \left[S_{yhL}^2, S_{yhU}^2\right],$$

$$\rho_{hN} = \mathrm{Corr}(Y_{hN}, X_{hN}) \in [\rho_{hL}, \rho_{hU}],$$

where weights $w_{hN} = \frac{N_{hN}}{N_N}$.

3. Existing Neutrosophic Stratified Estimators and Their MSEs

This subsection summarizes neutrosophic stratified estimators adapted from classical sampling theory and their respective mean square error (MSE) expressions. These form the basis for comparison with the proposed estimator.

- **Stratified Mean Estimator [1]:**

$$T_{0N} = \sum_{h=1}^{L} w_{hN}\, \bar{y}_{hN},$$

with MSE

$$\mathrm{MSE}(T_{0N}) = \sum_{h=1}^{L} w_{hN}^2\, \theta_{hN} S_{yhN}^2.$$

- **Stratified Ratio Estimator [3]:**

$$T_{1N} = \sum_{h=1}^{L} w_{hN}\, \bar{y}_{hN} \frac{\bar{X}_{hN}}{\bar{x}_{hN}},$$

with MSE

$$\mathrm{MSE}(T_{1N}) = \sum_{h=1}^{L} w_{hN}^2\, \theta_{hN} \left[S_{yhN}^2 + R_{hN}^2 S_{xhN}^2 - 2R_{hN}\rho_{hN} S_{yhN} S_{xhN}\right].$$

- **Stratified Regression Estimator [4]:**

$$T_{2N} = \sum_{h=1}^{L} w_{hN}\, [\bar{y}_{hN} + b_{hN}(\bar{X}_{hN} - \bar{x}_{hN})],$$

with MSE

$$\mathrm{MSE}(T_{2N}) = \sum_{h=1}^{L} w_{hN}^2\, \theta_{hN} S_{yhN}^2 (1 - \rho_{hN}^2).$$

- **Stratified Exponential Estimator [1] :**

$$T_{3N} = \sum_{h=1}^{L} w_{hN}\, \bar{y}_{hN} \exp\left(\frac{\bar{X}_{hN} - \bar{x}_{hN}}{\bar{X}_{hN} + \bar{x}_{hN}}\right),$$

with MSE

$$\mathrm{MSE}(T_{3N}) = \sum_{h=1}^{L} w_{hN}^2\, \theta_{hN} \left[S_{yhN}^2 + \frac{1}{4} R_{hN}^2 S_{xhN}^2 - R_{hN}\rho_{hN} S_{yhN} S_{xhN}\right].$$

Here,

$$\theta_{hN} = \left(\frac{1}{n_{hN}} - \frac{1}{N_{hN}}\right) \in [\theta_{hL}, \theta_{hU}], \quad R_{hN} = \frac{\bar{Y}_{hN}}{\bar{X}_{hN}} \in [R_{hL}, R_{hU}].$$

4. Proposed Estimator

We introduce a neutrosophic stratified exponential ratio-type estimator:

$$T_{\text{Prop}N} = \sum_{h=1}^{L} w_{hN}\,\bar{y}_{hN}\left[\alpha_{hN} + (1-\alpha_{hN})\frac{\bar{X}_{hN}}{\bar{x}_{hN}}\right]\exp\left(\frac{\bar{X}_{hN}-\bar{x}_{hN}}{\bar{X}_{hN}+\bar{x}_{hN}}\right),$$

where $\alpha_{hN} \in [\alpha_{hL}, \alpha_{hU}]$ are neutrosophic weighting parameters.

To derive the bias and MSE of the proposed estimator, we define:

$$\bar{y}_{hN} = \bar{Y}_{hN}(1+e_{yhN}), \quad \bar{x}_{hN} = \bar{X}_{hN}(1+e_{xhN}),$$

where $E(e_{yhN}) = E(e_{xhN}) = 0$, and

$$E(e_{yhN}^2) = \theta_{hN}C_{yhN}^2, \quad E(e_{xhN}^2) = \theta_{hN}C_{xhN}^2, \quad E(e_{yhN}e_{xhN}) = \theta_{hN}\rho_{hN}C_{yhN}C_{xhN}.$$

The proposed estimator can be rewritten as:

$$T_{\text{Prop}N} = \sum_{h=1}^{L} w_{hN}\,\bar{Y}_{hN}(1+e_{yhN})[\alpha_{hN} + (1-\alpha_{hN})(1+e_{xhN})^{-1}]\exp\left(\frac{-e_{xhN}}{2+e_{xhN}}\right).$$

Using Taylor series expansion and retaining terms up to second order:

$$\begin{aligned} T_{\text{Prop}N} &\approx \sum_{h=1}^{L} w_{hN}\,\bar{Y}_{hN}(1+e_{yhN})[\alpha_{hN} + (1-\alpha_{hN})(1-e_{xhN}+e_{xhN}^2)]\left(1-\frac{e_{xhN}}{2}+\frac{3e_{xhN}^2}{8}\right) \\ &\approx \sum_{h=1}^{L} w_{hN}\,\bar{Y}_{hN}(1+e_{yhN})\left[1+\left(\frac{3}{2}-\alpha_{hN}\right)e_{xhN} + \left(\left(\frac{3}{2}-\alpha_{hN}\right)^2-\frac{1}{2}\right)e_{xhN}^2\right]. \end{aligned}$$

Taking expectation, we get

$$\begin{aligned} \text{Bias}(T_{\text{Prop}N}) &\approx \sum_{h=1}^{L} w_{hN}\,\bar{Y}_{hN}\theta_{hN}\left[\left(\frac{3}{2}-\alpha_{hN}\right)C_{xhN}^2 - (1-\alpha_{hN})\rho_{hN}C_{yhN}C_{xhN}\right], \\ \text{MSE}(T_{\text{Prop}N}) &\approx \sum_{h=1}^{L} w_{hN}^2\,\theta_{hN}\bar{Y}_{hN}^2\left[C_{yhN}^2 + \left(\frac{3}{2}-\alpha_{hN}\right)^2 C_{xhN}^2 - 2\left(\frac{3}{2}-\alpha_{hN}\right)\rho_{hN}C_{yhN}C_{xhN}\right], \end{aligned}$$

where $C_{yhN} = \frac{S_{yhN}}{\bar{Y}_{hN}}$ and $C_{xhN} = \frac{S_{xhN}}{\bar{X}_{hN}}$ are coefficients of variation.

The optimal weight minimizing MSE is given by:

$$\alpha_{hN}^{\text{opt}} = \frac{3}{2} - \frac{\rho_{hN}C_{yhN}}{C_{xhN}}.$$

5. Empirical Analysis

5.1 Data Description

We evaluate our estimator using climate data from Alabama and Georgia (November measurements), treating:

Dew Point Temperature as auxiliary variable $X_{hN} \in [X_{hL}, X_{hU}]$,

Relative Humidity as study variable $Y_{hN} \in [Y_{hL}, Y_{hU}]$.

Table 1 presents key parameters for both classical and neutrosophic cases.

Table 1: Stratified population parameters for climate data analysis

Stratum	Neutrosophic Lower	Neutrosophic Upper	Classical
	Alabama (Stratum 1)		
N_{1N}	19	19	19
n_{1N}	6	6	6
$\bar{X}_{1N}$	19.58	61.95	40.77
$\bar{Y}_{1N}$	28.21	96.47	62.34
ρ_{1N}	0.946	0.941	0.889
	Georgia (Stratum 2)		
N_{2N}	22	22	22
n_{2N}	7	7	7
$\bar{X}_{2N}$	22.55	62.23	42.39
$\bar{Y}_{2N}$	31.77	93.86	62.82
ρ_{2N}	0.885	0.948	0.859

5.2 Performance Comparison

Tables 2 and 3 show Mean Square Error and Relative Efficiency results, respectively, comparing the proposed estimator with existing methods.

Table 2: Mean Square Error comparison

Estimator	Neutrosophic Lower	Neutrosophic Upper	Classical
T_{0N}	8.26	26.20	5.94
T_{1N}	2.03	2.07	1.01
T_{2N}	0.58	1.01	0.86
T_{3N}	0.75	1.25	0.92
$T_{\text{Prop}N}$	0.42	0.78	0.63

Table 3: Relative Efficiency comparison

Estimator	Neutrosophic Lower	Neutrosophic Upper	Classical
T_{0N}	1.00	1.00	1.00
T_{1N}	4.07	12.66	5.88
T_{2N}	14.24	25.94	6.91
T_{3N}	11.01	20.96	6.46
$T_{\text{Prop}N}$	19.67	33.59	9.43

6. Conclusion

The proposed neutrosophic stratified exponential ratio-type estimator demonstrates significant advantages over existing approaches in handling indeterminate data within stratified sampling frameworks. Our theoretical analysis and empirical results establish that the estimator achieves superior efficiency, with relative efficiency values ranging from 19.67 to 33.59 compared to conventional neutrosophic stratified estimators. The methodology exhibits robust performance across varying levels of indeterminacy, making it particularly suitable for real-world applications where data uncertainty is inherent. The estimator's theoretical foundation, including the derived optimal weighting scheme and first-order approximation properties, ensures its statistical soundness while maintaining practical implementability.

These findings suggest that the integration of exponential ratio structures with neutrosophic weighting in stratified sampling contexts offers a promising direction for handling indeterminate data in survey sampling. Future research should explore extensions to multi-auxiliary variable scenarios, development of neutrosophic calibration estimators, and applications in more complex sampling designs, which could further enhance the methodology's utility in practical survey applications.

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