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Γ Neutrosophic Set

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Abstract. Γ Neutrosophic Set is introduced in this article , which is the extension of neutrosophic set. It is required, to study the real world problems in neutrosophic environment with two parameters. In this article the following are defined, Γ Neutrosophic Set, operations on Γ Neutrosophic Set, Relations and properties of Γ Neutrosophic Set. Also illustrated some example.

1. **Introduction**

Fuzzy set was introduced by L.A. Zadeh [1] in the year 1965 to deal with uncertainty. It represents the degrees of truth function. To introduce the concept, truth and falsity function, K. Atanassov [2] introduced Intuitionistic fuzzy set in 1986, Which is the extension of fuzzy set theory. Florentin Smarandache [4] added the indeterminacy concept with truth and falsity and named it as Neutrosophic set in 1998, which is a Generalization of the Intuitionistic Fuzzy Set. Haibin Wang et al. [9] defined the set theoretic operators on an instance of neutrosophic set called single Neutrosophic set in 2010. Juan-juan Peng and Jian-qiang Wang [8] proposed Multivalued neutrosophic set in 2015. To over come the problem of extending the multi valued in Single valued Nuetrosophic set, they defined Multi-valued Neutrosophic set. Soft set theory was introduced by Molodtsov [6] in 1999. T.Srinivasa Rao, B.Srinivasa Kumar and S. Hanumanth Rao [5] introduced Γ neutrosophic soft set in 2018.

Since Fuzzy set represents the degrees of truth, Intuitionistic fuzzy set represents degrees of truth and falsity, Neutrosophic set represents degrees of truth, false and indeterminacy(unclear). Many real time situations will occur with 2 parameters in Neutrosophic environment, instead of single parameter. To solve these problems Γ Neutrosophic Set is proposed in this article

2. Basic Definitions

2.1. *Fuzzy Set [1]*

Any set A in X is defined by a characteristic function $\mu_A(x)$ in which every point belongs to X is related with a real number in the interval [0,1] is called fuzzy set and grade of membership of $x \in A$ is $\mu_A(x)$ at x.

2.2. *Intuitionistic Fuzzy Set [2]*

An Intuitionistic Fuzzy Set A in X is defined by A $= \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ where $\{< x, \mu_A(x) >\}$ - truth membership function and $\{< x, \nu_A(x) >\}$ - falsity membership function.

2.3. *Neutrosophic Set [4]*

Let X be a universal set and Let P be a set in X, A neutrosophic set P is characterized by $x(T, I, F)$ where triplets t, i, f are defined as $t\%$ true in the set, $i\%$ indeterminate in the set, and $f\%$ false, where t varies in T, i varies in I, f varies in F

2.4. *Single valued Neutrosophic Set [9]*

Let X be a space of points, with generic element in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A, indeterminancy-membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$

When X is continuous, a SVNS A can be written as A= $\int_X < T(x), I(x), F(x) > /x, x \in X$.

When X is discrete, a SVNS A can be written as A= $\sum_{i=1}^{n} < T(x_i), I(x_i), F(x_i) > /x_i, x_i \in X$.

2.5. *Multi-Valued Neutrosophic Set [8]*

Let X be a space of points, with a generic element in X denoted by x. A MVNS A in X is characterized by three functions $T_A(x), I_A(x)$ and $F_A(x)$ in the form of subset of [0, 1], which can be denoted as follows:

$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ where $T_A(x), I_A(x)$ and $F_A(x)$ are three sets of discrete value number in [0,1], denoting the truth-membership, indeterminacy-membership and falsity-membership respectively, with the conditions: $0 \leq t, i, f \leq 1, 0 \leq t^+ + i^+ + f^+ \leq 3$, where $t \in T_A(x), i \in I_A(x), f \in F_A(x)$ and $t^+ =$ sup $T_A(x), i^+ =$ sup $I_A(x)$, and $f^+ =$ Sup $F_A(x)$.

3. Definition: Γ Neutrosophic Set

Let X be a universal set and Let P, Γ be sets in X, A Γ neutrosophic set $N(P \text{ X } \Gamma\ )$ is characterized by $x(T_{p\times\gamma}, I_{p\times\gamma}, F_{p\times\gamma})$ where $t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma}$ are defined as $t_{p\times\gamma}$% true $i_{p\times\gamma}$% indeterminate , and $f_{p\times\gamma}$% false in the set , where $t_{p\times\gamma}$ varies in $T_{p\times\gamma}$, $i_{p\times\gamma}$ varies in $I_{p\times\gamma}$, $f_{p\times\gamma}$ varies in $F_{p\times\gamma}$ and is defined as

$x < t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma>} =< max(t_p, t_\gamma), (i_p + i_r)/2, min(f_p, f_\gamma) >$

Example 3.1:

In a mobile company to increase the sales, the management conducted a survey with the salesmen to get the input that affordability of the customer for the present price(P) and quality satisfaction (Γ).

$x < 0.2, 0.8, 0.5 >$ belongs to P , $x < 0.7, 0.4, 0.8 >$ belongs to Γ then $x < 0.7, 0.6, 0.5 >$ belongs to $N(P \text{ X } \Gamma)$

4. Operations on Γ Neutrosophic Set

4.1. *Complement of a Γ Neutrosophic Set N(P X Γ)*

Let $N(P \text{ X } \Gamma)$ be a Γ Neutrosophic Set over the universal set X, Then the Complement of $N(P \text{ X } \Gamma)(t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma})$ is denoted by $N^{\neg}$ (P X Γ)$(t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma})$ and defined as

$$\begin{aligned} N^{\neg}(\text{ P X }\Gamma)(t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma}) &= < f_{p\times\gamma}, i_{p\times\gamma}, t_{p\times\gamma} > \\ &= < 1 - t_{p\times\gamma}, 1 - i_{p\times\gamma}, 1 - f_{p\times\gamma} > \\ &= < f_{p\times\gamma}, 1 - i_{p\times\gamma}, t_{p\times\gamma} > \\ &= < 1 - t_{p\times\gamma}, i_{p\times\gamma}, 1 - f_{p\times\gamma} > \end{aligned}$$

Example 4.1.1:

$N(P \text{ X } \Gamma) = < 0.7, 0.4, 0.5 >$ then

$$\begin{aligned} N^{\neg}\ (\text{ P X }\Gamma)(t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma}) &=< f_{p\times\gamma}, i_{p\times\gamma}, t_{p\times\gamma} >= < 0.5, 0.4, 0.7 > \\ &=< 1 - t_{p\times\gamma}, 1 - i_{p\times\gamma}, 1 - f_{p\times\gamma} >= < 0.3, 0.6, 0.5 > \\ &=< f_{p\times\gamma}, 1 - i_{p\times\gamma}, t_{p\times\gamma} >= < 0.5, 0.6, 0.7 > \\ &=< 1 - t_{p\times\gamma}, i_{p\times\gamma}, 1 - f_{p\times\gamma} >= < 0.3, 0.4, 0.5 > \end{aligned}$$

4.2. *Union of two Γ Neutrosophic Sets*

Let $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ be two Γ Neutrosophic sets over the universal set X, Then union between them is defined as

$$N(P \text{ X } \Gamma) \bigcup N(Q \text{ X } \Gamma)) = < t_{p\times\gamma} \vee t_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \vee f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \vee f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \wedge f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \wedge f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, 1 - \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \wedge f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, \left| i_{p\times\gamma} - i_{q\times\gamma} \right|, f_{p\times\gamma} \wedge f_{q\times\gamma} >$$

Example 4.2.1:
$N(P \text{ X } \Gamma) = < 0.6, 0.8, 0.1 >$ & $N(Q \text{ X } \Gamma) = < 0.3, 0.4, 0.2 >$ then

$$N(P \text{ X } \Gamma) \bigcup N(Q \text{ X } \Gamma)) = < t_{p\times\gamma} \vee t_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \vee f_{q\times\gamma} > = < 0.6, 0.8, 0.2 >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \wedge f_{q\times\gamma} > = < 0.6, 0.4, 0.1 >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \vee f_{q\times\gamma} > = < 0.6, 0.4, 0.2 >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \wedge f_{q\times\gamma} > = < 0.6, 0.6, 0.1 >$$

$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, 1 - \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \wedge f_{q\times\gamma} > = < 0.6, 0.4, 0.1 >$$
$$= < t_{p\times\gamma} \vee t_{q\times\gamma}, \left| i_{p\times\gamma} - i_{q\times\gamma} \right|, f_{p\times\gamma} \wedge f_{q\times\gamma} > = < 0.6, 0.4, 0.1 >$$

4.3. *Intersection of two Γ Neutrosophic Sets*

Let $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ be two Γ Neutrosophic Sets over the universal set X, Then intersection between them is defined as

$$N(P \text{ X } \Gamma) \bigcap N(Q \text{ X } \Gamma)) = < t_{p\times\gamma} \wedge t_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \wedge f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \wedge f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \vee f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \vee f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, 1 - \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \vee f_{q\times\gamma} >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, \left| i_{p\times\gamma} - i_{q\times\gamma} \right|, f_{p\times\gamma} \vee f_{q\times\gamma} >$$

Example 4.3.1:
$N(P \text{ X } \Gamma) = < 0.6, 0.8, 0.1 >$ & $N(Q \text{ X } \Gamma) = < 0.3, 0.4, 0.2 >$ then

$$N(P \text{ X } \Gamma) \bigcap N(Q \text{ X } \Gamma)) = < t_{p\times\gamma} \wedge t_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \wedge f_{q\times\gamma} > = < 0.3, 0.4, 0.1 >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \wedge f_{q\times\gamma} > = < 0.3, 0.8, 0.1 >$$
$$= < t_{p\times\gamma} \wedge t_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \vee f_{q\times\gamma} > = < 0.3, 0.8, 0.2 >$$

$$=< t_{p\times\gamma} \wedge t_{q\times\gamma}, \frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \vee f_{q\times\gamma} >=< 0.3, 0.6, 0.2 >$$
$$=< t_{p\times\gamma} \wedge t_{q\times\gamma}, 1-\frac{i_{p\times\gamma}+i_{q\times\gamma}}{2}, f_{p\times\gamma} \vee f_{q\times\gamma} >=< 0.3, 0.4, 0.2 >$$
$$=< t_{p\times\gamma} \wedge t_{q\times\gamma}, \left|i_{p\times\gamma} - i_{q\times\gamma}\right|, f_{p\times\gamma} \vee f_{q\times\gamma} >=< 0.3, 0.4, 0.2 >$$

4.4. *Difference of two* Γ *Neutrosophic Set*

Let $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ be two Γ Neutrosophic Sets over the universal set X, Then Difference between them is defined as

$$N(P \text{ X } \Gamma) \text{ - } N(Q \text{ X } \Gamma) = N((P \text{ X } \Gamma) \text{ - } (Q \text{ X } \Gamma))$$
$$= P \wedge \neg Q = P \wedge N^{\neg}(QX\Gamma) =$$
$$=< t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma} > \wedge < 1-t_{q\times\gamma}, 1-i_{q\times\gamma}, 1-f_{q\times\gamma} >$$
$$=< t_{p\times\gamma} \wedge (1-t_{q\times\gamma}), i_{p\times\gamma} \wedge (1-i_{q\times\gamma}), f_{p\times\gamma} \wedge (1-f_{q\times\gamma}) >$$
$$=< t_{p\times\gamma} \wedge (1-t_{q\times\gamma}), i_{p\times\gamma} \vee (1-i_{q\times\gamma}), f_{p\times\gamma} \wedge (1-f_{q\times\gamma}) >$$

$$P \wedge N^{\neg}(QX\Gamma) =< t_{p\times\gamma}, i_{p\times\gamma}, f_{p\times\gamma} > \wedge < f_{p\times\gamma}, i_{p\times\gamma}, t_{p\times\gamma} >$$
$$=< t_{p\times\gamma} \wedge f_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \wedge t_{q\times\gamma} >$$
$$=< t_{p\times\gamma} \wedge f_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \wedge t_{q\times\gamma} >$$

Similarly we can find all combination of intersection and negation

Example 4.4.1:

$N(P \text{ X } \Gamma) = < 0.6, 0.8, 0.1 >$ & $N(Q \text{ X } \Gamma) =< 0.3, 0.4, 0.2 >$ then

$N(P \text{ X } \Gamma)$ - $N(Q \text{ X } \Gamma)$=

$< t_{p\times\gamma} \wedge (1-t_{q\times\gamma}), i_{p\times\gamma} \wedge (1-i_{q\times\gamma}), f_{p\times\gamma} \wedge (1-f_{q\times\gamma}) > = < 0.6, 0.6, 0.1 >$

$< t_{p\times\gamma} \wedge (1-t_{q\times\gamma}), i_{p\times\gamma} \vee (1-i_{q\times\gamma}), f_{p\times\gamma} \wedge (1-f_{q\times\gamma}) > = < 0.6, 0.8, 0.1 >$

$< t_{p\times\gamma} \wedge (1-t_{q\times\gamma}), i_{p\times\gamma} \vee (1-i_{q\times\gamma}), f_{p\times\gamma} \vee (1-f_{q\times\gamma}) > = < 0.6, 0.8, 0.8 >$

$< t_{p\times\gamma} \wedge f_{q\times\gamma}, i_{p\times\gamma} \wedge i_{q\times\gamma}, f_{p\times\gamma} \wedge t_{q\times\gamma} > = < 0.2, 0.4, 0.1 >$

$< t_{p\times\gamma} \wedge f_{q\times\gamma}, i_{p\times\gamma} \vee i_{q\times\gamma}, f_{p\times\gamma} \wedge t_{q\times\gamma} > = < 0.2, 0.8, 0.1 >$

4.5. *Cartesian Product of two* Γ *Neutrosophic Sets*

Let $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ be two Γ Neutrosophic Sets over the universal set X, Then cartesian Product between them is defined as

If $x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in N(P \text{ X } \Gamma)$ & , $y(T_{(q\times\gamma)}, I_{(q\times\gamma)}, F_{(q\times\gamma)}) \in N(Q \text{ X } \Gamma)$ then
$x(T_{1(p\times\gamma)}, I_{1(p\times\gamma)}, F_{1(p\times\gamma)}), y(T_{2(p\times\gamma)}, I_{2(p\times\gamma)}, F_{2(p\times\gamma)}) \in N(P \text{ X } \Gamma) \text{ X } N(Q \text{ X } \Gamma)$

Example 4.5.1:

To increase the sales of a particular brand laptop, a survey was conducted by the quality team to the salesmen of 2 shops A & B,

Shop A salesmen gave the inputs $x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in N(P \text{ X } \Gamma)$ &

Shop B salesmen gave the inputs $y(T_{(q\times\gamma)}, I_{(q\times\gamma)}, F_{(q\times\gamma)}) \in N(Q \text{ X } \Gamma)$ where

$x_1 =< 0.5, 0.8, 0.2 >, x_2 =< 0.1, 0.25, 0.3 >, x_3 =< 0.41, 0.21, 0.15 >$

$y_1 =< 0.23, 0.17, 0.9 >, y_2 =< 0.3, 0.5, 0.23 >, y_3 =< 0.57, 0.14, 0.34 >$

$x(T_{1(p\times\gamma)}, I_{1(p\times\gamma)}, F_{1(p\times\gamma)}), y(T_{2(p\times\gamma)}, I_{2(p\times\gamma)}, F_{2(p\times\gamma)}) = (x_1 =< 0.5, 0.8, 0.2 >, y_1 =< 0.23, 0.17, 0.9 >), (x_1 =< 0.5, 0.8, 0.2 >, y_2 =< 0.3, 0.5, 0.23 >), (x_1 =< 0.5, 0.8, 0.2 >, y_3 =< 0.57, 0.14, 0.34 >), (x_2 =< 0.1, 0.25, 0.3 >, y_1 =< 0.23, 0.17, 0.9 >), (x_2 =< 0.1, 0.25, 0.3 >, y_2 =< 0.3, 0.5, 0.23 >), (x_2 =< 0.1, 0.25, 0.3 >, y_3 =< 0.57, 0.14, 0.34 >), (x_3 =< 0.41, 0.21, 0.15 >, y_1 =< 0.23, 0.17, 0.9 >), (x_3 =< 0.41, 0.21, 0.15 >, y_2 =< 0.3, 0.5, 0.23 >), (x_3 =< 0.41, 0.21, 0.15 >, y_3 =< 0.57, 0.14, 0.34 >)$

4.6. Γ *neutrosophic subset*

If $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ are two Γ neutrosophic set then $N(P \text{ X } \Gamma)$ is a subset of $N(Q \text{ X } \Gamma)$ if

$x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in N(P \text{ X } \Gamma)$ & , $y(T_{(q\times\gamma)}, I_{(q\times\gamma)}, F_{(q\times\gamma)}) \in N(Q \text{ X } \Gamma)$ then where $\inf T_{(p\times\gamma)} \leq \inf T_{(q\times\gamma)}$, $\sup T_{(p\times\gamma)} \leq \sup T_{(q\times\gamma)}$, $\inf F_{(p\times\gamma)} \geq \inf F_{(q\times\gamma)}$, $\sup F_{(p\times\gamma)} \geq \sup F_{(q\times\gamma)}$.

4.7. *Equality of two* Γ *neutrosophic sets*

If $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ are two Γ neutrosophic set then they are said to be equal if $N(P \text{ X } \Gamma)$ is a Subset of $N(Q \text{ X } \Gamma)$ and $N(Q \text{ X } \Gamma)$ is subset of $N(P \text{ X } \Gamma)$.

5. **Relations on Γ Neutrosophic Set**

5.1. *Relation from* $N(P \text{ X } \Gamma)$ *to* $N(Q \text{ X } \Gamma)$

Let $N(P \text{ X } \Gamma)$ & $N(Q \text{ X } \Gamma)$ are two Γ neutrosophic set. A relation R from $N(P \text{ X } \Gamma)$ to $N(Q \text{ X } \Gamma)$ is a subset of $N(P \text{ X } \Gamma)$ X $N(Q \text{ X } \Gamma)$. ie. $R \subseteq N(P \text{ X } \Gamma) \text{ X } N(Q \text{ X } \Gamma)$

If $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in R$, Then x is related to y by R.

5.2. *Reflexive*

If $x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in N(P \text{ X } \Gamma)$, R be the relation, then R is reflexive, if $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$

5.3. *Symmetric*

If $x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in N(P \text{ X } \Gamma)$ and R be the relation, then R is symmetric, if $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$ then $(y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$

5.4. *Transitive*

If $x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), z(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) \in N(P \text{ X } \Gamma)$ and R be the relation, then R is transitive, if $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$ and $(y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), z(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$ then $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), z(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$

5.5. *Antisymmetric*

If $x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), \in N(P \text{ X } \Gamma)$ and R be the relation, then R is an antisymmetric, if $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$ and $(y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}), x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)})) \in R$ then $(x(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}) = y(T_{(p\times\gamma)}, I_{(p\times\gamma)}, F_{(p\times\gamma)}))$

6. **Properties of Γ neutrosophic set**

6.1. *Commutative Property*

The union and intersection of Γ neutrosophic sets satisfies the commutative property

- $N(P \text{ X } \Gamma) \cap N(Q \text{ X } \Gamma) = N(Q \text{ X } \Gamma) \cap N(P \text{ X } \Gamma)$
- $N(P \text{ X } \Gamma) \cup N(Q \text{ X } \Gamma) = N(Q \text{ X } \Gamma) \cup N(P \text{ X } \Gamma)$

6.2. *Associative Property*

The union and intersection of Γ neutrosophic sets satisfies the associative property

- $(N(P \text{ X } \Gamma) \cap N(Q \text{ X } \Gamma)) \cap N(R \text{ X } \Gamma) = N(P \text{ X } \Gamma) \cap (N(Q \text{ X } \Gamma) \cap N(R \text{ X } \Gamma))$
- $(N(P \text{ X } \Gamma) \cup N(Q \text{ X } \Gamma)) \cup N(R \text{ X } \Gamma) = N(P \text{ X } \Gamma) \cup (N(Q \text{ X } \Gamma) \cup N(R \text{ X } \Gamma))$

6.3. *Distributive Property*

The union and intersection of Γ neutrosophic sets satisfies the distributive property

- $N(P \text{ X } \Gamma) \cup (N(Q \text{ X } \Gamma) \cap N(R \text{ X } \Gamma)) = (N(P \text{ X } \Gamma) \cup N(Q \text{ X } \Gamma)) \cap (N(P \text{ X } \Gamma)) \cup (N(R \text{ X } \Gamma))$
- $N(P \text{ X } \Gamma) \cap (N(Q \text{ X } \Gamma) \cup N(R \text{ X } \Gamma)) = (N(P \text{ X } \Gamma) \cap N(Q \text{ X } \Gamma)) \cup (N(P \text{ X } \Gamma)) \cap (N(R \text{ X } \Gamma))$

6.4. *Identity Property*

The union and intersection of Γ neutrosophic sets satisfies the identity property

- $N(P \text{ X } \Gamma) \cap \text{X} = N(P \text{ X } \Gamma)$
- $N(P \text{ X } \Gamma) \cup \emptyset = N(P \text{ X } \Gamma)$

6.5. *Idempotent Property*

The union and intersection of Γ neutrosophic sets satisfies the idempotent property

- $N(P \text{ X } \Gamma) \cap N(P \text{ X } \Gamma) = N(P \text{ X } \Gamma)$
- $N(P \text{ X } \Gamma) \cup N(P \text{ X } \Gamma) = N(P \text{ X } \Gamma)$

7. **Conclusion**

In this paper the Γ neutrosophic set was introduced and basic definitions and properties were discussed. Some examples were given. Furture one can discuss the application of Γ neutrosophic set in MCDM Problems.

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