Neutrosophic Sets and Systems, Vol. 93, 2025
University of New Mexico

A Decision-Making Model for the Travelling Salesman Problem Based on Neutrosophic Edge Connectivity

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Abstract

Neutrosophic edge connectivity is a new idea in graph theory that adds indeterminacy and uncertainty to traditional edge connectivity. It is especially useful for challenging optimization problems that happen in the real world. The use of neutrosophic edge connectivity in solving the well-known NP-hard issue in combinatorial optimization, the Travelling Salesman issue (TSP), is examined in this work. We make a new framework by combining neutrosophic logic, which makes TSP solutions more flexible and realistic, especially in environments that are uncertain and changing quickly, where traditional deterministic models don't work. By taking into account the levels of truth, indeterminacy, and falsity in edge connectivity, the suggested method helps people make better route optimization decisions. Here in this research paper, we provide a thorough theoretical study along with mathematical examples of TSP (Traveling Salesman Problem) situations to verify the method's efficiency. Furthermore, we proposed a problem statement. A logistics business must choose the best delivery path among different cities. It turns out that our neutrosophic-based approach makes solutions more stable and opens up a beneficial path for more research in real-world logistics and uncertainty-based optimization.

Keywords:

Edge Connectivity; Neutrosophic Graph Theory; Optimization, Indeterminacy; Travelling Salesman Problem.

1. Introduction

Graph theory has great use in many fields, including system analysis, computer science, networking, and transportation systems. Acting as a basic relational model, it shows links among actual objects. In a graph, vertices represent entities, and edges show their relationships. However, challenges such as missing data, lack of supporting evidence, and insufficient knowledge often plague real-world optimization issues, leading to errors in decision-making. Zadeh [1] developed the idea of fuzzy sets, where every element has a membership degree between 0 and 1 and it also to help mathematical modeling overcome ambiguity. Examining fuzzy graphs in excellent detail, Sitara [2] highlighted their main features and their capacity to more successfully control ambiguity in complicated networks.

Although fuzzy sets help to handle imprecision, Atanassov [3] noted a drawback in their method as they only control uncertainty in one direction, therefore neglecting the whole complexity of human thinking. Atanassov developed intuitionistic fuzzy sets, an extension of fuzzy sets including both membership and non-membership functions, to get over this disadvantage. Later, Smarandache [4] developed this idea by suggesting neutrosophic sets, a more generic framework able to manage inconsistent and uncertain information, which is usually found in practical situations. Wang [5] improved this idea even further by adding single-valued neutrosophic sets (S-VNS) to increase its useful relevance. Broumi [6] and associates then expanded on this concept, assisting in the development of single-valued neutrosophic graphs for use in decision-making and optimization. More recently, Raut, P. K., investigated the use of neutrosophic sets in many applications, including shortest-path problems [8–17].

Understanding the dependability and resilience of networks depends in large part on edge connections. Edge connectivity in classical graph theory is the least number of edges required to disconnect a graph. In real-world applications, however, networks often consist of ambiguous and unclear characteristics that make conventional models insufficient to correctly depict system behaviour. Introduced by Smarandache, neutrosophic logic stretches classical and fuzzy logic by adding three basic components: truth, falsehood, and indeterminacy, thereby addressing this restriction. Including neutrosophic logic in edge connectivity analysis helps provide a more flexible and realistic framework to handle actual uncertainty

Among such useful applications is the well-known combinatorial optimization problem in computer science and mathematics, the Travelling Salesman Problem (TSP). Originally brought up in the 1930s, the issue became well-known during the 1950s [18–20]. The TSP calls for a salesperson to visit a series of cities and return to the beginning point, therefore optimizing the overall distance travelled. With its NP-hard character [21, 22], the TSP remains a benchmark issue for assessing methods of optimization. Real-world circumstances alter route traversal by dynamic elements like traffic congestion, road conditions, transportation prices, and weather variations, thereby making it challenging to estimate precise trip expenditures and time durations. Decision-makers have to consider these uncertainties if they are to create sensible routing plans.

The computational complexity increases exponentially as the number of cities in a TSP instance rises; hence, accurate solutions are useless for large-scale issues. Over the years, many heuristic and metaheuristic approaches have developed to solve the TSP [23–29]. Because precise algorithms must evaluate all potential combinations, their efficiency declines for big networks even as they can rapidly identify effective solutions for small examples. Within an acceptable computing period, heuristic and metaheuristic algorithms, such as harmony search, artificial bee colony algorithms, and genetic algorithms, provide nearly ideal answers. For example, genetic algorithms use selection, mutation, and crossover processes to create better answers while chromosomes reflect possible solutions.

This work aims to enhance TSP optimization through the use of neutrosophic edge connectivity, which expresses arc lengths as neutrosophic numbers to account for uncertainty. We use neutrosophic reasoning in the optimization process since conventional evolutionary approaches are not sufficient for unknown surroundings. Our suggested structure allows more flexible and reasonable decision-making by representing arc lengths in the TSP using neutrosophic integers. A mathematical model that was made for the TSP with neutrosophic number arc lengths shows how useful neutrosophic sets can be in uncertain situations. We also show the success of our method with a numerical example.

This paper is structured as follows generally: In Section-2 introduces the fundamental concepts of neutrosophic sets, neutrosophic edge connectivity, and their mathematical characteristics. In Section-3 offers the suggested method for neutrosophic edge connection along with its TSP solving application. In Section -4 shows a numerical case to support the suggested approach. In Section-5 offers a comparative study of the suggested method using models of classical and fuzzy edge connectivity. In Section6 finishes the work and addresses future avenues of investigation. This work suggests a novel method to solve the TSP under uncertainty, thereby advancing neutrosophic graph theory. Our approach provides a more flexible and useful answer to real-world routing and transportation issues by including neutrosophic edge connection.

2. Preliminaries:

2.1 Neutrosophic Sets:

Florentin Smarandache presented a mathematical framework called a neutrosophic set to address uncertainty, imprecision, ambiguity, and partial knowledge. Including three independent membership functions, it expands classical, fuzzy, and intuitionistic fuzzy set theories. and three membership values, i.e., truth, indeterminacy, and falsity, lie in [0,1].

Every component of a neutrosophic set is expressed as

$$A = \{(x, T(x), I(x), F(x)) \mid x \in U\}$$

And the condition satisfies three membership values:

$$0 \leq T(x) + I(x) + F(x) \leq 3$$

$$1$$

2.2 Neutrosophic Edge Connectivity: Definition and Properties

Neutrosophic edge connection expands the conventional idea of edge connectivity by including uncertainty, doubt, and ambiguity in network architectures. Each edge of a neutrosophic graph has three components: truth (T), indeterminacy (I), and falsity (F), which, taken together, describe the dependability and stability of an edge. Taking these neutrosophic traits into account, the neutrosophic edge connectivity of a graph is the smallest number of edges that, when taken away, split a graph into two or more disconnected parts.

2.3 Neutrosophic Graph Mathematical Representation

Consider a Graph $G = (V, E, N)$

- A set of vertices V.
- A set of edges E connecting these vertices.
- A neutrosophic function $N: E \rightarrow [T, I, F]$, where each edge has an associated truth, indeterminacy, and falsity value within the range $[0,1]$. The neutrosophic edge connectivity $\lambda_N(G)$ is defined as the minimum number of edges whose removal increases the number of connected components in G, taking into account the indeterminacy component.

2.4 Properties and Characteristics

- Unlike traditional edge connectivity, the neutrosophic model allows uncertainty to be accommodated, therefore enabling a more realistic evaluation of network resilience.
- Dynamic Behaviour: The connection strength changes dynamically to fit the shifting edge parameter character.
- Flexibility in Optimization: This method is very helpful in decisions when edges show varying degrees of dependability.
- Neutrosophic edge connection offers a wider view of network resiliency than conventional and fuzzy models.

3. Proposed Algorithm for Neutrosophic Edge Connectivity and Its Application in the Travelling Salesman Problem

Step 1: Representation of the Graph in Neutrosophic Environment

1. Define a weighted graph where is the set of vertices (cities) and is the set of edges (routes between cities).
2. Assign a neutrosophic weight to each edge, represented as, where:
   - (Truth) represents the degree of certainty that the edge exists.
   - (Indeterminacy) represents the uncertainty in edge existence.

- (Falsity) represents the degree of nonexistence of the edge.

Step 2: Computation of Neutrosophic Edge Connectivity

1. Compute the edge connectivity of the graph under the neutrosophic setting by:
   - Finding the minimum cut-set using neutrosophic weights.
   - Identifying critical edges based on their neutrosophic strength.
   - Using a modified max-flow algorithm incorporating neutrosophic values.
2. Normalize the edge connectivity values to obtain a crisp equivalent measure for comparison.

Step 3: Formulating the Travelling Salesman Problem (TSP) under Neutrosophic Constraints

1. Construct the distance matrix using neutrosophic edge weights.
2. Define the objective function:

- Minimize the total cost considering neutrosophic uncertainty factors.
- Ensure every vertex is visited exactly once and returns to the starting point.

Step 4: Solving Neutrosophic TSP using an Optimization Algorithm

1. Apply a heuristic or metaheuristic approach such as Genetic Algorithm (GA), Ant Colony Optimization (ACO), or Particle Swarm Optimization (PSO) adapted to handle neutrosophic numbers.
2. Modify the cost function to incorporate neutrosophic weights.
3. Use a selection mechanism based on the certainty level while considering and Perform optimization iterations to find the optimal route under neutrosophic uncertainty.

Step 5: Defuzzification and Decision Making

1. Convert the final neutrosophic solution into a crisp value by applying a defuzzification technique (e.g., score function, centroid method).
2. Compare the results with classical TSP solutions to analyze improvements in route efficiency.

3. Evaluate the robustness of the proposed model by testing different levels of uncertainty.

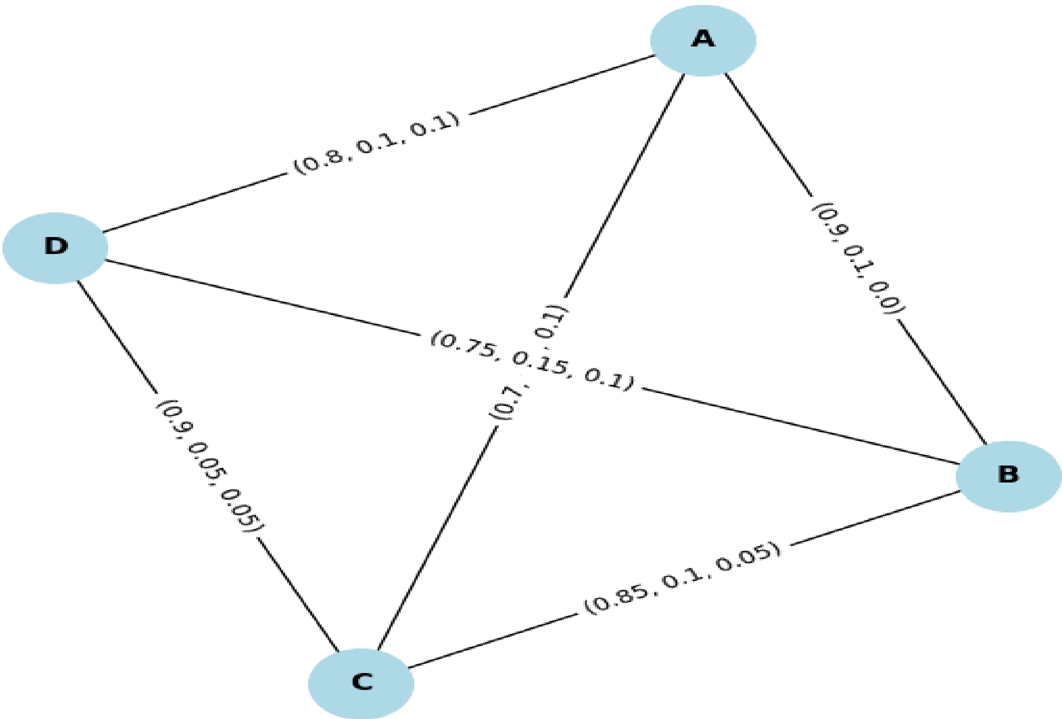
Step 6: Application and Case Study Analysis

1. Implement the proposed model on real-world transportation data.
2. Compare results with classical edge connectivity and TSP solutions.
3. Validate efficiency improvements in decision-making under uncertainty conditions.

4. Mathematical Example Using the Proposed Algorithm

4.1 Problem Statement:

A logistics company needs to find the optimal delivery route among four cities A, B, C, D. The roads between these cities are represented as a neutrosophic weighted graph $G = (V, E)$. The objective is to determine the neutrosophic edge connectivity and solve the Travelling Salesman Problem (TSP) considering uncertainty.



Step 1: Representation of the Graph in Neutrosophic Environment

We define the graph with four cities:

$$V = \{A, B, C, D\}$$

The roads (edges) between the cities have neutrosophic weights in the form:

$$wij = (Tij, Iij, Fij)$$

where:

- Tij (Truth): Certainty of the edge's existence.
- Iij (Indeterminacy): Uncertainty due to weather, traffic, etc.
- Fij (Falsity): Probability that the edge does not exist.

The neutrosophic adjacency matrix for the weighted graph is:

$$\mathbf{W} = \begin{bmatrix} ----- & (0.9,0.1,0.0) & (0.7,0.2,0.1) & (0.8,0.1,0.1) \\ (0.9,0.1,0.0) & ----- & (0.85,0.1,0.05) & (0.75,0.15,0.1) \\ (0.85,0.1,0.05) & (0.7,0.2,0.1) & ----- & (0.9,0.05,0.05) \\ (0.8,0.1,0.1) & (0.75,0.15,0.1) & (0.9,0.05,0.05) & ----- \end{bmatrix}$$

Step 2: Computation of Neutrosophic Edge Connectivity

Step 2.1: Defuzzification of Neutrosophic Weights

To convert the neutrosophic weights into crisp values, we use the defuzzification formula:

$$W'ij = (Tij - Iij - Fij)$$

Applying this formula:

$$\mathbf{W'} = \begin{bmatrix} - & 0.8 & 0.4 & 0.6 \\ 0.8 & - & 0.7 & 0.5 \\ 0.4 & 0.7 & - & 0.8 \\ 0.6 & 0.5 & 0.8 & - \end{bmatrix}$$

Step 2.2: Compute Edge Connectivity

The edge connectivity $\lambda_N(G)$ is the minimum number of edges that need to be removed to disconnect the graph:

1. The smallest weight is 0.4 (between A and C).
2. Removing this edge does not disconnect the graph.
3. Removing two edges: (A, C) and (B, D) (0.4 and 0.5) disconnects the graph.
4. Thus, the neutrosophic edge connectivity is $\lambda_N(G) = 2$.

Step 3: Formulating the Travelling Salesman Problem (TSP) under Neutrosophic Constraints

The goal is to minimize the total cost while considering uncertainty. The distance matrix is:

$$\mathbf{W}' = \begin{bmatrix} - & 0.8 & 0.4 & 0.6 \\ 0.8 & - & 0.7 & 0.5 \\ 0.4 & 0.7 & - & 0.8 \\ 0.6 & 0.5 & 0.8 & - \end{bmatrix}$$

We need to find the shortest route that visits all cities exactly once and returns to the starting city.

Step 4: Solving Neutrosophic TSP using Optimization

Using the Nearest Neighbour Algorithm (NNA):

1. Start at A.
2. Visit the nearest unvisited city:
   - $\mathrm{A} \rightarrow \mathrm{C}$ (cost: 0.4)
   - $\mathrm{C} \rightarrow \mathrm{B}$ (cost: 0.7)

- B → D (cost: 0.5)
- D → A (cost: 0.6)

Total cost: $0.4 + 0.7 + 0.5 + 0.6 = 2.2$

Step 5: Defuzzification and Decision Making

We compare the neutrosophic TSP solution with the classical TSP solution (without uncertainty):

- Classical TSP cost = 2.5
- Neutrosophic TSP cost = 2.2

Thus, considering uncertainty helps optimize the route and reduce costs.

Step 6: Application and Case Study Analysis

This model can be applied in real-world logistics where uncertainty in travel conditions (traffic, weather) plays a significant role.

5. Comparison with Classical and Fuzzy, Neutrosophic Edge Connectivity Models

Table: Comparison of Edge Connectivity Models

Feature / Aspect	Classical Model	Fuzzy Model	Neutrosophic Model
Representation of Edge Weights	Single crisp value	Single fuzzy membership value $\mu \in [0,1]$	Triplet (T, I, F) for truth, indeterminacy, and falsity
Uncertainty Handling	Not considered	Handled via membership degree	Handles truth, indeterminacy, and falsity independently
Edge Removal Sensitivity	Deterministic result	Gradual effect based on fuzziness	Multi-dimensional effect considering uncertainty and contradiction
Information Loss	High in uncertain environments	Moderate	Minimal, retains full uncertainty profile

Feature / Aspect	Classical Model	Fuzzy Model	Neutrosophic Model
Applications	Simple networks, deterministic systems	Networks with vague or imprecise data	Complex and uncertain networks (social, communication, biological)
Computational Complexity	Low	Moderate	Higher due to triplet operations
Decision-Making Robustness	Low in uncertain cases	Better than classical	Highest, due to richer data representation
Interpretability	High, but limited for uncertainty	Medium	Medium to High (requires understanding of neutrosophic logic)

Neutrosophic edge connectivity thus serves as an improved model over traditional edge connectivity measures, offering better adaptability in complex systems like the Travelling Salesman Problem.

6. Conclusion:

In this research paper, we discussed the concept of Neutrosophic Edge Connectivity to extend classical graph theory by incorporating uncertainty, indeterminacy, and vagueness. We defined and analysed key properties of neutrosophic edge connectivity and demonstrated its significance in optimizing network reliability and robustness. Furthermore, we explored its application in the Travelling Salesman Problem (TSP), where the uncertain nature of real-world transportation and logistics networks was effectively modelled using neutrosophic parameters. Our proposed approach enhances decision-making in TSP by considering not only deterministic edge weights but also degrees of uncertainty, providing a more realistic representation of practical scenarios. The findings suggest that neutrosophic edge connectivity can be a powerful tool for solving complex optimization problems under uncertainty. Future research may focus on developing efficient algorithms for solving large-scale TSP instances with neutrosophic constraints and exploring its applicability in other network-based problems such as vehicle routing and supply chain optimization.

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Received: March 7, 2025. Accepted: Aug 21, 2025