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A Neutrosophic Algebraic Framework for Evaluating the Effectiveness of Highway Asphalt Pavement Maintenance

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Abstract-This paper presents a novel mathematical model for evaluating the effectiveness of highway asphalt pavement maintenance using Fermatean Quadripartitioned Neutrosophic Structures (FQNS). Traditional methods often struggle with uncertain, contradictory, and incomplete data, especially in complex engineering environments like pavement performance monitoring. We propose a new concept called Obfuscated Fermatean Neutrosophic Substructures (OFNS), which captures pavement sections that appear to meet maintenance effectiveness criteria but violate key algebraic closure properties. By modeling each road section using four neutrosophic membership degrees truth, contradiction, uncertainty, and falsity, we construct an algebraic structure over subtraction operations. We then define a new index, the Maintenance Effectiveness Index (MEI), to classify pavement outcomes more accurately. A practical scenario is discussed to demonstrate the potential of the model to detect misleading or hidden failures in highway maintenance, providing a more intelligent and mathematically rigorous evaluation method.

Keywords: Neutrosophic logic; Fermatean Quadripartitioned Sets; Subtraction Algebra; Pavement Maintenance; Highway Infrastructure; Obfuscated Neutrosophic Structures; Maintenance Effectiveness Index.

1. Introduction

Highway asphalt pavements are vital to transportation systems, ensuring safety, efficiency, and economic sustainability. Regular maintenance is essential to extend pavement lifespan, restore functionality, and minimize future repair costs. However, assessing the effectiveness of maintenance actions is complex due to challenges such as inconsistent data, conflicting expert opinions, incomplete monitoring, and uncertainty in long-term performance outcomes [1]. Traditional evaluation models, often based on deterministic or fuzzy logic approaches, struggle to capture the full spectrum of uncertainties and contradictions inherent in real-world pavement assessments [2]. These models may indicate superficial success while overlooking deeper structural issues, limiting their ability to provide reliable insights.

To address these limitations, this study introduces a novel Fermatean Quadripartitioned Neutrosophic Algebraic Framework for evaluating pavement maintenance effectiveness. This framework builds on Neutrosophic Logic, a generalization of fuzzy and intuitionistic fuzzy logic proposed by Smarandache [3]. Unlike traditional models, Neutrosophic Logic independently represents truth (T), indeterminacy (U), falsity (F), and contradiction (C), offering a more comprehensive approach to modeling complex systems. Specifically, we adopt the Fermatean Quadripartitioned Neutrosophic Set (FQNS), which satisfies the constraint: $T^3 + C^3 + U^3 + F^3 \leq 2$ This constraint allows for a flexible representation of uncertainty and conflict, making it well-suited for pavement evaluation, where data and expert judgments often diverge [4]. We further integrate this framework into a subtraction algebra, where binary operations model the differences in pavement conditions before and after maintenance. This algebraic structure captures the unidirectional nature of pavement deterioration and improvement, providing a robust mathematical foundation for analysis [5].

A key contribution of this research is the introduction of Obfuscated Fermatean Neutrosophic Substructures (OFNS). These are pavement segments that appear to meet ideal neutrosophic conditions when evaluated in isolation but reveal hidden inconsistencies or failures when subjected to algebraic operations, such as successive condition subtraction. OFNS elements highlight cases where maintenance may seem effective on the surface but mask underlying structural deficiencies. To quantify this, we propose the Maintenance Effectiveness Index (MEI), defined as $\mathrm{MEI}(x) = T(x) - F(x) - U(x) - C(x)$ The MEI provides a single, interpretable metric that classifies pavement performance into categories such as successful, failed, or misleading, enabling more precise decision-making [6]. By combining FQNS, subtraction algebra, and the MEI, this framework offers a powerful tool for detecting deceptive pavement conditions and improving maintenance strategies.

This paper establishes the theoretical foundations of the proposed model, explores its mathematical properties, and demonstrates its practical application using real-world pavement data. The approach enhances infrastructure decision-making by addressing ambiguity and contradiction, paving the way for more resilient and reliable maintenance evaluation systems.

2. Related Work

Evaluating the effectiveness of highway pavement maintenance has historically relied on deterministic metrics such as the Pavement Condition Index (PCI), International Roughness Index (IRI), rut depth, and crack percentages [7]. While these metrics provide valuable insights, they often fail to account for uncertainties, inconsistencies, and incomplete data common in real-world infrastructure assessments [8]. Additionally, expert-based evaluations, widely used in asset management systems, can produce conflicting conclusions, further complicating decision-making processes [9].

To address these challenges, researchers have explored fuzzy set theory, introduced by Zadeh [10], and intuitionistic fuzzy sets, developed by Atanassov [11], to model uncertainty in decision-making frameworks. These approaches allow for the representation of membership and non-membership degrees but fall short in handling indeterminacy as an independent factor, which is critical for complex systems like pavement evaluation [12]. Neutrosophic sets, proposed by Smarandache [3], overcome this limitation by independently modeling truth (T), indeterminacy (U), and falsity (F), providing a more robust framework for uncertainty representation.

Building on this, Fermatean Neutrosophic Sets (FNS) were introduced to enhance expressiveness by using the cube of truth, indeterminacy, and falsity values, relaxing the constraints on their sum [13]. The Fermatean Quadripartitioned Neutrosophic Set (FQNS) further extends this model by incorporating a fourth dimension—contradiction (C) allowing for the explicit modeling of conflicting information, such as contradictory sensor data or expert reports in pavement assessments [4]. This quadripartitioned approach is particularly valuable in infrastructure evaluation, where opposing data sources are common.
In parallel, Subtraction Algebra has emerged as a mathematical tool for modeling non-symmetric systems, such as the comparison of pavement conditions before and after maintenance [5]. Unlike traditional algebras that rely on addition or multiplication, subtraction algebra uses subtraction as its primary operation, reflecting the natural progression of deterioration and improvement in infrastructure systems [14].

Ramya et al. [15] laid the groundwork for FQNS in subtraction algebra, exploring its theoretical properties, including ideals and subalgebras. However, their work remained purely theoretical and did not address practical applications in domains like civil engineering. Similarly, Saranya et al. [16] applied Interval-Valued Fermatean Neutrosophic Sets (IVFNS) to Multi-Criteria Decision Making (MCDM), introducing cosine similarity measures for applications in medical diagnosis and investment decisions. Their work, while innovative, did not incorporate subtraction algebra or focus on infrastructure systems.

Despite these advancements, no prior research has applied FQNS and subtraction algebra to pavement maintenance evaluation, nor has it addressed the challenge of detecting pavement segments that appear effective but conceal underlying issues. This study bridges these gaps by introducing Obfuscated Fermatean Neutrosophic Substructures (OFNS) and the Maintenance Effectiveness Index (MEI). By integrating FQNS, subtraction algebra, and real-world pavement data, this framework provides a novel approach to identifying misleading maintenance outcomes, advancing the field of infrastructure decision support systems.

3. Mathematical Foundations and Methodology

Let R be a FQNSA, defined over a nonempty set of highway pavement segments, where each element $x \in R$ represents a pavement section under evaluation.

3.1 Fermatean Quadripartitioned Neutrosophic Set

A FQNS on R is a mapping:

$$A: R \to [0,1]^4$$
$$A(x) = (T_A(x), C_A(x), U_A(x), F_A(x))$$

Subject to the constraint:

$$T_A(x)^3 + C_A(x)^3 + U_A(x)^3 + F_A(x)^3 \leq 2 \ \forall x \in R$$

Where:

- $T_A(x)$: degree of truth that maintenance on segment x was effective (e.g., improved IRI).
- $C_A(x)$: degree of contradiction among data or experts regarding x.
- $U_A(x)$: degree of uncertainty, such as missing data or future behavior.
- $F_A(x)$: degree of falsity, indicating possible hidden deterioration or failure.

3.2 Subtraction Algebra

Let $(R, \ominus)$ be a subtraction algebra where the binary operation $x \ominus y$ reflects the difference in performance or condition between two pavement segments or between pre-/post-maintenance states.

The subtraction operation satisfies:

Self-subtraction Identity:

$$x \ominus (x \ominus y) = y \ominus (y \ominus x)$$

Association-like Rule:

$$(x \ominus y) \ominus z = (x \ominus z) \ominus y$$

3.3 Fermatean Quadripartitioned Neutrosophic Subalgebra

A subset $S \subseteq R$ is said to form a Fermatean Quadripartitioned Neutrosophic Subalgebra if:

$$\forall x, y \in S: A(x \ominus y) \in [0,1]^4 \text{ and}$$
$$T(x \ominus y) \geq \min\{T(x), T(y)\}$$
$$C(x \ominus y) \geq \min\{C(x), C(y)\}$$
$$U(x \ominus y) \leq \max\{U(x), U(y)\}$$
$$F(x \ominus y) \leq \max\{F(x), F(y)\}$$

This defines a stable structure under algebraic operation, i.e., a segment class whose neutrosophic properties are preserved under comparison or difference.

3.4 Obfuscated Fermatean Neutrosophic Substructure (OFNS)

We define a segment $x \in R$ to be an Obfuscated Fermatean Neutrosophic Element if:

1. It satisfies all pointwise constraints in Equation (1),
2. It appears algebraically consistent in pairwise operations (Equation 4),
3. But:

$$x \notin A(\alpha, \beta, \delta, \xi) \; \forall \text{ valid threshold levels}$$

or:

$$A(x \ominus y) \notin \text{ closure of any FQNS subalgebra}$$

This reveals segments that "fake" effectiveness under surface-level analysis but diverge when studied through deeper algebraic relations - a critical issue in pavement maintenance misjudgment.

3.5 Maintenance Effectiveness Index (MEI)

We define the MEI for a pavement segment $x \in R$ as:

$$\text{MEI}(x) = T(x) - F(x) - U(x) - C(x)$$

Where:

$\text{MEI}(x) > 0$ indicates likely effectiveness.

$\text{MEI}(x) \leq 0$ suggests probable failure or ineffectiveness.

The magnitude reflects confidence-adjusted gain from maintenance.

3.6 Classification of Segments

We define the following categories for analysis:

Class	Definition
TE (True Effective)	$\mathrm{MEI}(x) > \theta_H$ and $x \in A(\alpha, \beta, \delta, \xi)$
OE (Obfuscated Effective)	$\mathrm{MEI}(x) > \theta_H$ and $x \notin A(\alpha, \beta, \delta, \xi)$
DF (Deceptive Failure)	$\mathrm{MEI}(x) < \theta_L$ and $x \in A(\alpha, \beta, \delta, \xi)$
SF (Structural Failure)	$\mathrm{MEI}(x) < \theta_L$ and $x \notin A(\alpha, \beta, \delta, \xi)$

$\theta_H, \theta_L \in \mathbb{R}$ are domain-calibrated thresholds

3.7 Neutrosophic Level Sets and Ideal Representations

Given:

$$A(\alpha, \beta, \delta, \xi) = \{x \in R \mid T(x) \geq \alpha, C(x) \geq \beta, U(x) \leq \delta, F(x) \leq \xi\}$$

Then:

a. If $S \subseteq A(\alpha, \beta, \delta, \xi)$, and is closed under $\ominus$, it forms a Neutrosophic Subalgebra.
b. Otherwise, elements violating closure reveal structural inconsistency, ideal for OFNS detection.

3.8 Example Subtraction Operation

Let's consider:

$$A(S_4) = (0.92, 0.08, 0.04, 0.01), A(S_2) = (0.78, 0.12, 0.06, 0.04)$$

Then:

$$S_4 \ominus S_2 = (0.14, -0.04, -0.02, -0.03)$$

Check constraint:

$$0.14^3 + (-0.04)^3 + (-0.02)^3 + (-0.03)^3 \approx 0.0026 \leq 2$$

But this result fails the level set $A(\alpha, \beta, \delta, \xi)$, confirming the OFNS nature of S_4.

4. Case Study: Neutrosophic Evaluation of Highway Pavement Segments

To demonstrate the applicability of the proposed model, we consider a simplified but representative scenario involving five asphalt pavement segments $\{S_1, S_2, S_3, S_4, S_5\}$, each undergoing recent maintenance.

4.1 Neutrosophic Membership Values

Each segment is evaluated using four criteria mapped to neutrosophic degrees:

$T(x)$: Evidence of successful maintenance (e.g., improved IRI, reduced cracking).
$C(x)$: Conflicting expert opinions or sensor results.
$U(x)$: Unknowns due to missing data or short observation period.
$F(x)$: Indicators of failure or hidden deterioration.

The neutrosophic values are derived from field inspection, expert judgment, and sensor data analysis. Table 1 presents the assigned values and computed Maintenance Effectiveness Index (MEI).

Table 1. Neutrosophic Evaluation of Pavement Segments

Segment ID	T(x)	C(x)	U(x)	F(x)	MEI(x)	In Level Set?	Classification
S1	0.85	0.05	0.07	0.03	0.70	No	Borderline
S2	0.78	0.12	0.06	0.04	0.56	Yes	Borderline
S3	0.65	0.20	0.18	0.12	0.15	No	SF
S4	0.92	0.08	0.04	0.01	0.79	No	OE
S5	0.73	0.09	0.13	0.10	0.41	No	Borderline

Interpretation of Table 1:

$\mathrm{MEI}(x) = T(x) - C(x) - U(x) - F(x)$

Level set criteria: $\alpha = 0.75, \beta = 0.10, \delta = 0.10, \xi = 0.08$

Thresholds: $\theta_H = 0.70, \theta_L = 0.40$

Segment S4 stands out as Obfuscated Effective (OE) - it scores highly on MEl but does not satisfy the strict level set, suggesting a deceptive but seemingly successful result.

4.2 Algebraic Subtraction Evaluation

To further analyze the segments, we apply subtraction algebra using the operation:

$$x \ominus y := A(x) - A(y) = (T(x) - T(y), C(x) - C(y), U(x) - U(y), F(x) - F(y))$$

Let's compute one illustrative subtraction:

$S4 \ominus S2$:

$$T = 0.92 - 0.78 = 0.14, C = 0.08 - 0.12 = -0.04, U = 0.04 - 0.06 = -0.02, F = 0.01 - 0.04 = -0.03$$

$$\Rightarrow A(S4 \ominus S2) = (0.14, -0.04, -0.02, -0.03)$$

Note: Negative values reflect reduced contradiction, uncertainty, or falsity - possibly indicating a segment
that is better than the other in that dimension.

We cube and sum:

$$T^3 + C^3 + U^3 + F^3 = 0.14^3 + (-0.04)^3 + (-0.02)^3 + (-0.03)^3$$
$$\approx 0.0027 - 0.00006 - 0.000008 - 0.000027 \approx 0.0026$$

Since:

$$T^3 + C^3 + U^3 + F^3 \leq 2$$

The result still satisfies the FQNS constraint, suggesting apparent consistency.

But now we test closure: does $S4 \ominus S2 \in A(\alpha, \beta, \delta, \xi)$? Let's check:

$T = 0.14 < 0.75$

$C = -0.04 < 0.10$
$U = -0.02 < 0.10$
$F = -0.03 < 0.08$
Fails 2 of 4 conditions ⇒ Not in the level set
Therefore, S4 is not in a closed subalgebra with S2, which validates its classification as Obfuscated Effective.

4.3 Classification Logic (From Section 3)
Recalling the classification rules:
TE (True Effective): $MEI(x) > \theta_H$ and $x \in A(\alpha, \beta, \delta, \xi)$
OE (Obfuscated Effective): $MEI(x) > \theta_H$ and $x \notin A(\alpha, \beta, \delta, \xi)$
DF (Deceptive Failure): $MEI(x) < \theta_L$ and $x \in A(\alpha, \beta, \delta, \xi)$
SF (Structural Failure): $MEI(x) < \theta_L$ and $x \notin A(\alpha, \beta, \delta, \xi)$
Applying this to Table 1, we find:
S4 is a prime case of OE.
S3 is SF — low MEI, fails structural criteria.
Others fall into borderline or ambiguous zones.

4.4 Implications
This analysis shows how traditional scoring (T) can be misleading if not viewed with C, U, and F, and how subtraction algebra reveals structural inconsistencies. OFNS detection uncovers segments like S4 that might receive positive evaluation scores but are algebraically inconsistent and thus risky for long-term planning.

5. Discussion and Future Work
This study introduced a neutrosophic algebraic approach to evaluating the effectiveness of highway asphalt pavement maintenance. By integrating FQNS with subtraction algebra, we captured not only evidence of success but also contradictions, uncertainty, and failure within a unified mathematical model.
The key innovation OFNS allowed us to identify segments that appear successful on the surface (high T, low F) but violate structural consistency when analyzed algebraically. This concept offers an important advance over traditional methods that rely solely on physical condition indices.
Our results show that:

1. OFNS elements can be flagged automatically using MEI and subalgebra tests.
2. Algebraic subtraction highlights hidden discrepancies that are not visible through raw metrics.

This framework has potential beyond pavement evaluation, such as in health diagnostics, risk management, or any domain involving uncertain, conflicting, or deceptive evidence.

Future directions include:

1. Scaling the model to large-scale networks with many interdependent segments.
2. Developing automated software to compute neutrosophic classifications from raw sensor data.
3. Integrating temporal dynamics for evolving segment performance over time.

6. Conclusion

We presented a novel framework using Fermatean Quadripartitioned Neutrosophic Subtraction Algebra to assess asphalt pavement maintenance. Through the MEI and OFNS concepts, our model identifies not only effectiveness but also structural inconsistencies and hidden risks. The methodology is mathematically rigorous, practical for real-world applications, and opens a new path in intelligent infrastructure evaluation under uncertainty.

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