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NeutroSemigroups Generated by Upside-Down Logic

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Abstract. Current models of NeutroAlgebras generally depend on static assignments to construct partial axioms. This paper introduces a deterministic method to generate NeutroAlgebraic structures over plithogenic spaces. We define a dual-threshold mode selector that combines the plithogenic intersection with the Three-Mode Upside-Down operator, strictly parameterized by the contradiction degree function. By introducing a novel binary operation $\star$, we prove that the classical associative law fails within specific topological sub-regions due to contradiction-triggered truth reversals. Since the associative property holds partially, fails partially, and remains indeterminate elsewhere, the space equipped with $\star$ forms a NeutroSemigroup. This approach provides a structural generation of NeutroAxioms rather than relying on arbitrary assignments. The theoretical results are verified through an illustration in dynamic threat evaluation. We conclude by posing open problems regarding the existence of NeutroIdentity and NeutroInverse elements under the defined operation.

Keywords: NeutroSemigroup; Upside-Down Logic; Plithogenic Set; Contradiction Degree; NeutroAxiom; NeutroAlgebra

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1. Introduction

Let X be a plithogenic space where elements possess multiple attribute values. The standard plithogenic intersection $\wedge_p$ aggregates these attributes using linear combinations of fuzzy t-norms and t-conorms. Smarandache defined this intersection to be strictly parameterized by a contradiction degree function $c(a,b)$, which measures the dissimilarity between attribute values [1,2]. This foundational framework effectively models multi-dimensional indeterminacy but assumes the universal validity of classical algebraic axioms.

To formalize the failure of classical axioms in indeterminate environments, Smarandache introduced the theory of NeutroAlgebras. He defined NeutroAlgebras as generalized algebraic structures where operations or axioms evaluate to partially true, partially indeterminate, and partially false [3, 4]. Current literature primarily establishes these NeutroAlgebraic spaces by utilizing static, finite Cayley table assignments to demonstrate the partiality of axioms [5].

Independent of algebraic spaces, logical evaluations exhibit context-dependent truth reversals. To model these contextual transitions, Fujita [6] defined the Three-Mode Upside-Down operator on plithogenic neutrosophic sets. This operator utilizes Smarandache's Upside-Down logic [7] to exchange the degrees of truth and falsity under high contradiction. However, Fujita restricted the application of this mechanism entirely to propositional truth valuations.

This paper introduces a deterministic method to analytically generate NeutroAlgebraic structures, advancing the previously mentioned frameworks. Instead of assigning NeutroAlgebraic partiality statically, and rather than restricting the Upside-Down operator to propositional logic, we embed this reversal mechanism directly into the structural axioms of an algebraic space. We define a deterministic dual-threshold mode selector $M(a, b)$ that integrates the plithogenic intersection with the Three-Mode Upside-Down operator. This piecewise function links the contradiction degree directly to the failure of the classical associative law, generating NeutroAxioms from topological boundaries. By introducing a novel binary operation $\star$ based on $M(a, b)$, we prove that contradiction-triggered truth reversals force the associative property to hold partially, fail partially, and remain indeterminate elsewhere. Consequently, the plithogenic space equipped with $\star$ forms a NeutroSemigroup. Finally, the theoretical failure of associativity is verified in a dynamic threat evaluation application.

2. Preliminaries

In this section, we review the foundational concepts of plithogenic sets, contradiction degree functions, and NeutroAlgebraic structures required for the subsequent theorems.

Definition 2.1 ([1]). Let U be a universe of discourse and $P \subseteq U$ be a non-empty set. A plithogenic set X is defined as a quintuple $X = (P, v, P_v, d, c)$, where v is a designated attribute, P_v is the domain of admissible values for the attribute v, $d : P \times P_v \to [0, 1]^s$ is the degree of appurtenance function (DAF), and $c : P_v \times P_v \to [0, 1]^t$ is the degree of contradiction function (DCF), with $s, t \in \mathbb{N}^+$. For any $x \in P$ and $a \in P_v$, $d(x, a)$ represents the appurtenance degree of x to the set P with respect to the attribute value a.

Definition 2.2 ([1]). The contradiction degree function $c : P_v \times P_v \to [0, 1]^t$ quantifies the dissimilarity between any two attribute values $a, b \in P_v$. It satisfies the following conditions:

(1) Reflexivity: $c(a, a) = 0$ for all $a \in P_v$.
(2) Symmetry: $c(a, b) = c(b, a)$ for all $a, b \in P_v$.

Definition 2.3 ([2]). Let $a, b \in P_v$ be two attribute values. Let $\wedge_F$ and $\vee_F$ denote a classical fuzzy t-norm and t-conorm, respectively. The plithogenic intersection $\wedge_p$ between the appurtenance degrees of a and b is defined as a linear combination modulated by the contradiction degree $c(a, b)$:

$$a \wedge_p b = (1 - c(a, b)) \cdot (a \wedge_F b) + c(a, b) \cdot (a \vee_F b). \tag{1}$$

To clarify the operational mechanics of the plithogenic intersection and its notation under a fixed contradiction degree, consider the following example:

Example 2.4. Let $a, b \in P_v$ be two attribute values with a contradiction degree $c(a, b) = 0.2$. Assume their truth components under a specific evaluation are $T_a = 0.8$ and $T_b = 0.5$. Utilizing the standard fuzzy operators $\wedge_F(u, v) = \min(u, v)$ and $\vee_F(u, v) = \max(u, v)$, the truth component of the plithogenic intersection computes as:

$$T_{a \wedge_p b} = (1 - 0.2) \cdot \min(0.8, 0.5) + 0.2 \cdot \max(0.8, 0.5) = 0.8(0.5) + 0.2(0.8) = 0.56.$$

This illustrates the algebraic interpolation between the t-norm and t-conorm bounds dictated by $c(a, b)$.

Definition 2.5 ([3]). Let X be a non-empty space equipped with a well-defined binary operation. An algebraic axiom defined on X is called a NeutroAxiom if it evaluates to true (T) for some elements in X, indeterminate (I) for other elements, and false (F) for the remaining elements, such that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

Definition 2.6 ([3]). Let X be a non-empty set endowed with a well-defined binary operation. If the associativity axiom on X operates as a NeutroAxiom, the algebraic structure is called a NeutroSemigroup.

Definition 2.7 ([8]). A fuzzy t-norm $t : [0, 1]^2 \to [0, 1]$ is a commutative, associative, and non-decreasing binary operation with the identity element 1. Dually, a fuzzy t-conorm $s : [0, 1]^2 \to [0, 1]$ satisfies the same algebraic properties with the identity element 0. These operators are used to model the generalized intersection and union, respectively.

3. The Operator and Deterministic Mode Selection

In this section, we formalize the modes of the Three-Mode Upside-Down operator and establish a deterministic selection mechanism. By coupling the contradiction degree directly to a dual-threshold function, we eliminate randomness in mode activation, ensuring that the resulting algebraic operations are strictly well-defined prior to evaluating the semigroup axioms.

Definition 3.1 ([6]). Let $x = (T, I, F) \in [0, 1]^3$ be a neutrosophic triplet representing an appurtenance degree. Let $s : [0, 1] \times [0, 1] \to [0, 1]$ be a standard fuzzy t-conorm. The three fundamental transformation modes operating on the plithogenic neutrosophic components are defined as mappings from $[0,1]^3$ to $[0,1]^3$:

$$Keep(T, I, F) = (T, I, F), \tag{2}$$

$$SwapTF(T, I, F) = (F, I, T), \tag{3}$$

$$EnrichI(T, I, F) = (0, s(I, s(T, F)), 0). \tag{4}$$

Definition 3.2. Let P_v be the domain of admissible attribute values and let $c : P_v \times P_v \to [0,1]$ be the contradiction degree function. We introduce two fixed activation thresholds $\tau_1, \tau_2 \in [0,1]$ such that $0 \leq \tau_1 < \tau_2 \leq 1$. The deterministic mode selector $M : P_v \times P_v \to \{Keep, EnrichI, SwapTF\}$ is defined as a piecewise function evaluated for any pair of attribute values $a, b \in P_v$:

$$M(a, b) = \begin{cases} Keep, & \text{if } 0 \leq c(a, b) < \tau_1, \\ EnrichI, & \text{if } \tau_1 \leq c(a, b) < \tau_2, \\ SwapTF, & \text{if } \tau_2 \leq c(a, b) \leq 1. \end{cases} \tag{5}$$

The piecewise construction of the mode selector M partitions the plithogenic space into three mutually exclusive regions based on the severity of the contradiction. Rather than leaving the mode selection to an external or random variable, this function strictly forces the system to either preserve the components, absorb the polar supports into indeterminacy, or completely invert the truth values, bounded entirely by the thresholds τ_1 and τ_2.

Remark 3.3. The activation thresholds τ_1 and τ_2 utilized in the deterministic mode selector are exogenous parameters rather than algebraic constants. Their numerical values are dictated by the contextual requirements of the given application. Also, the breakdown of the associative law is invariant with respect to these values. Provided the condition $0 \leq \tau_1 < \tau_2 \leq 1$ holds, the generation of the NeutroAxiom and the resulting NeutroSemigroup structure remain independent of the threshold selection.

Definition 3.4. Let X be a plithogenic space. For any two attribute values $a, b \in P_v$ with their respective neutrosophic appurtenance degrees $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$, we define the new binary operation $\star$ as the composition of the deterministic mode selector M and the standard plithogenic intersection $\wedge_p$:

$$x \star y = M(a, b)(x \wedge_p y). \tag{6}$$

Consequently, the operation $\star$ computes the plithogenic intersection of x and y, and subsequently maps the resulting neutrosophic components according to the exact mode dictated by the contradiction degree $c(a, b)$.

Example 3.5. To illustrate the mechanism of the mode selector M, assume the activation thresholds are fixed at $\tau_1 = 0.5$ and $\tau_2 = 0.8$. Let the standard plithogenic intersection of two elements x and y yield a resulting neutrosophic triplet $x \wedge_p y = (0.7, 0.2, 0.1)$. Assume the standard bounded sum $s(u, v) = \min\{1, u + v\}$ is utilized as the t-conorm. The binary operation $x \star y$ maps this triplet strictly based on the regional partition of the contradiction degree $c(a, b)$:

(1) Assume $c(a, b) = 0.3$. The condition $c(a, b) < \tau_1$ holds. The selector assigns $Keep$, yielding $x \star y = (0.7, 0.2, 0.1)$.
(2) Assume $c(a, b) = 0.6$. The condition $\tau_1 \leq c(a, b) < \tau_2$ holds. The selector assigns $EnrichI$. The polar components (truth and falsity) are mapped to 0, and the indeterminacy is enriched via the t-conorm s: $s(0.2, s(0.7, 0.1)) = \min\{1, 0.2 + 0.8\} = 1.0$. Thus, $x \star y = (0, 1.0, 0)$.
(3) Assume $c(a, b) = 0.9$. The condition $c(a, b) \geq \tau_2$ holds. The selector assigns $SwapTF$, interchanging the truth and falsity components. This yields $x \star y = (0.1, 0.2, 0.7)$.

This demonstrates how the continuous spectrum of the contradiction degree discretizes the algebraic output into distinct topological regions, ensuring the operation $\star$ remains strictly deterministic.

4. Main Results: The NeutroSemigroup Structure

In this section, we investigate the algebraic properties of the binary operation $\star$ defined in Section 3. First, we demonstrate the closure and well-definedness of the operation under the proposed dual-threshold mechanism. Then, we evaluate the associativity axiom, proving that the structural inversion triggered by the mode selector forces the associative law to break down in specific sub-regions. Finally, we establish that the plithogenic space equipped with the operation $\star$ constitutes a NeutroSemigroup.

Lemma 4.1. *The binary operation $\star$ is closed and well-defined on the plithogenic space X.*

Proof. Let $(x_1, y_1), (x_2, y_2) \in X \times X$ such that $x_1 = x_2$ and $y_1 = y_2$. Let a_1, b_1 be the attributes corresponding to x_1, y_1, and a_2, b_2 be the attributes corresponding to x_2, y_2. The equality of elements strictly implies $a_1 = a_2$ and $b_1 = b_2$.

Since the contradiction degree function $c : P_v \times P_v \to [0, 1]$ is a well-defined mapping, we have $c(a_1, b_1) = c(a_2, b_2)$. The thresholds τ_1 and τ_2 partition the codomain of c into disjoint intervals. Consequently, $c(a_1, b_1)$ and $c(a_2, b_2)$ map to the identical interval, yielding $M(a_1, b_1) = M(a_2, b_2) = M_0$, where $M_0 \in \{Keep, EnrichI, SwapTF\}$.

The standard plithogenic intersection $\wedge_p$ is well-defined on $[0,1]^3$, implying $x_1 \wedge_p y_1 = x_2 \wedge_p y_2$. Applying the mode operator M_0, we obtain:

$$x_1 \star y_1 = M_0(x_1 \wedge_p y_1) = M_0(x_2 \wedge_p y_2) = x_2 \star y_2.$$

Furthermore, $\wedge_p$ is closed on $[0,1]^3$, and the transformation modes defined in M map $[0,1]^3$ into $[0,1]^3$. Thus, $x_1 \star y_1 \in X$. This produces a unique and well-defined neutrosophic element in X. □

Theorem 4.2. *The associativity of the binary operation $\star$ operates as a NeutroAxiom in the plithogenic space X.*

Proof. Let $x, y, z \in X$ corresponding to the attributes $a, b, d \in P_v$. We evaluate $(x \star y) \star z = x \star (y \star z)$.

Case 1 (Degree of Truth T): Assume $c(a,b) < \tau_1$, $c(b,d) < \tau_1$, and $c(a,d) < \tau_1$. The mode selector yields *Keep* for all operations. The equation reduces to $(x \wedge_p y) \wedge_p z = x \wedge_p (y \wedge_p z)$. The standard plithogenic intersection preserves associativity on $[0,1]^3$. The equality holds.

Case 2 (Degree of Falsehood F): Assume $c(a,b) \geq \tau_2$ and $c(b,d) < \tau_1$. Let $c(a \wedge_p b, d) < \tau_1$ and $c(a, b \wedge_p d) \geq \tau_2$. The left-hand side evaluates as: $(x \star y) \star z = SwapTF(x \wedge_p y) \star z = (F_{xy}, I_{xy}, T_{xy}) \wedge_p (T_z, I_z, F_z)$. Let T_L denote the truth component of this expression. Letting $c_L = c(a \wedge_p b, d)$, the standard aggregation yields:

$$T_L = (1 - c_L) \cdot \min(F_{xy}, T_z) + c_L \cdot \max(F_{xy}, T_z). \tag{7}$$

The right-hand side evaluates as: $x \star (y \star z) = x \star (y \wedge_p z) = SwapTF(x \wedge_p (y \wedge_p z))$. Let T_R denote the truth component of the right-hand side. By definition, T_R equals the falsity component of the total intersection, denoted F_{xyz}.

Assume $(x \star y) \star z = x \star (y \star z)$ holds universally in this sub-space. This requires $T_L = T_R$ for all elements. The term $T_R = F_{xyz}$ is invariant with respect to the variable T_z. Conversely, T_L depends on T_z. Evaluating the condition $T_L = T_R$ at the boundary $T_z = 0$ yields $T_L = c_L \cdot F_{xy}$. Evaluating at $T_z = 1$ yields $T_L = (1 - c_L) \cdot F_{xy} + c_L$. Since T_R is invariant, we equate both evaluations:

$$c_L \cdot F_{xy} = (1 - c_L) \cdot F_{xy} + c_L. \tag{8}$$

For arbitrary $F_{xy} \in (0,1)$, this algebraic equality holds if and only if $c_L = 1$. This strictly contradicts the initial premise $c_L < \tau_1 \leq 1$. The assumption fails. Thus, $T_L \neq T_R$.

The associativity equation evaluates to true in Case 1 and false in Case 2. It constitutes a NeutroAxiom. □

Lemma 4.3. *The plithogenic space $(X, \star)$ lacks a universal identity element.*

Proof. Assume there exists a universal identity $e = (T_e, I_e, F_e) \in X$ such that $x \star e = x$ for all $x \in X$. This requires the mode selector to yield $Keep$, implying $c(a_x, a_e) < \tau_1$ for all attributes. The equation reduces to $x \wedge_p e = x$. Evaluating the truth component yields:

$$(1 - c) \cdot \min(T_x, T_e) + c \cdot \max(T_x, T_e) = T_x, \tag{9}$$

where $c = c(a_x, a_e)$. For this equality to hold for arbitrary $T_x \in [0, 1]$ under a fixed $c \in (0, 1)$, T_e must simultaneously satisfy $\min(T_x, T_e) = T_x \implies T_e = 1$ and $\max(T_x, T_e) = T_x \implies T_e = 0$. This is a contradiction. Thus, e does not exist. □

Remark 4.4. By Lemma 4.3, the absence of a universal identity element prevents the extension of the algebraic structure into a NeutroGroup. Consequently, the structural generation strictly terminalizes at a NeutroSemigroup. Formulating a NeutroGroup under $\star$ requires restricting the space to localized topological sub-regions where partial identities hold.

Corollary 4.5. *The plithogenic space X equipped with the binary operation $\star$ constitutes a NeutroSemigroup.*

Proof. By Lemma 4.1, the space X is endowed with a well-defined binary operation $\star$. By Theorem 4.2, the associativity of the operation $\star$ acts as a NeutroAxiom. These two conditions satisfy the requirements of a NeutroSemigroup as outlined in Definition 2.6. □

5. Illustrative Applications

To demonstrate the operational mechanics of the deterministic mode selector $M(a, b)$ across different parameter thresholds, we present two distinct applied scenarios. The first application examines the failure of associativity triggered by a strict truth reversal in dynamic threat evaluation. The second application illustrates a structural absorption into indeterminacy and its subsequent algebraic collapse in financial fraud detection.

5.1. *Dynamic Threat Evaluation*

To demonstrate the breakdown of associativity under the operation $\star$ in an applied context, we adapt the security patch deployment and threat evaluation model introduced by Fujita [6]. In this framework, the evaluation of a software security patch relies on multi-attribute threat intelligence reports.

Let X be the plithogenic space of threat evaluations. We define the classical fuzzy t-norm as $\wedge_F(u, v) = \min(u, v)$ and the t-conorm as $\vee_F(u, v) = \max(u, v)$. In accordance with Remark 3.3, the exogenous mode selector thresholds are fixed at $\tau_1 = 0.4$ and $\tau_2 = 0.7$ to reflect the true nature of specific security protocols. Consider three threat intelligence sources corresponding to the attribute values a (Heuristic Analysis), b (Signature Matching), and d (Network Behavior). The neutrosophic values assigned to the patch by these sources are given as:

$$\begin{aligned} x &= (0.7, 0.2, 0.1) \quad \text{for attribute } a, \\ y &= (0.6, 0.3, 0.2) \quad \text{for attribute } b, \\ z &= (0.8, 0.1, 0.2) \quad \text{for attribute } d. \end{aligned}$$

Assume the contradiction degrees between the intelligence sources are empirically established as follows: $c(a, b) = 0.8$, $c(b, d) = 0.2$, $c(a \wedge_p b, d) = 0.2$, and $c(a, b \wedge_p d) = 0.9$.

We evaluate the left-hand side of the associativity equation: $(x \star y) \star z$. For the inner operation $x \star y$, the contradiction degree is $c(a, b) = 0.8 \geq \tau_2$. The mode selector yields $M(a, b) = SwapTF$. The standard plithogenic intersection $x \wedge_p y$ is computed as:

$$\begin{aligned} x \wedge_F y &= (0.6, 0.2, 0.1) \\ x \vee_F y &= (0.7, 0.3, 0.2) \\ x \wedge_p y &= (1 - 0.8)(0.6, 0.2, 0.1) + 0.8(0.7, 0.3, 0.2) \\ &= (0.12, 0.04, 0.02) + (0.56, 0.24, 0.16) \\ &= (0.68, 0.28, 0.18). \end{aligned} \tag{10}$$

Applying the selected mode:

$$x \star y = SwapTF(0.68, 0.28, 0.18) = (0.18, 0.28, 0.68). \tag{11}$$

Let $w = x \star y$. The subsequent operation is $w \star z$. The contradiction degree is $c(a \wedge_p b, d) = 0.2 < \tau_1$. The mode selector yields $M(a \wedge_p b, d) = Keep$.

$$\begin{aligned} w \wedge_F z &= (\min(0.18, 0.8), \min(0.28, 0.1), \min(0.68, 0.2)) = (0.18, 0.10, 0.20) \\ w \vee_F z &= (\max(0.18, 0.8), \max(0.28, 0.1), \max(0.68, 0.2)) = (0.80, 0.28, 0.68) \\ w \wedge_p z &= (1 - 0.2)(0.18, 0.10, 0.20) + 0.2(0.80, 0.28, 0.68) \\ &= (0.144, 0.080, 0.160) + (0.160, 0.056, 0.136) \\ &= (0.304, 0.136, 0.296). \end{aligned} \tag{12}$$

Since the mode is $Keep$, the left-hand side evaluates to $(0.304, 0.136, 0.296)$.

We now evaluate the right-hand side of the equation: $x \star (y \star z)$. For the inner operation $y \star z$, the contradiction degree is $c(b,d) = 0.2 < \tau_1$. The mode selector yields $M(b,d) = Keep$.

$$\begin{aligned} y \wedge_F z &= (0.6, 0.1, 0.2) \\ y \vee_F z &= (0.8, 0.3, 0.2) \\ y \wedge_p z &= 0.8(0.6, 0.1, 0.2) + 0.2(0.8, 0.3, 0.2) \\ &= (0.48, 0.08, 0.16) + (0.16, 0.06, 0.04) \\ &= (0.64, 0.14, 0.20). \end{aligned} \tag{13}$$

Applying the selected mode yields $y \star z = Keep(0.64, 0.14, 0.20) = (0.64, 0.14, 0.20)$. Let $u = y \star z$. The subsequent operation is $x \star u$. The contradiction degree is $c(a, b \wedge_p d) = 0.9 \geq \tau_2$. The mode selector yields $M(a, b \wedge_p d) = SwapTF$.

$$\begin{aligned} x \wedge_F u &= (\min(0.7, 0.64), \min(0.2, 0.14), \min(0.1, 0.20)) = (0.64, 0.14, 0.10) \\ x \vee_F u &= (\max(0.7, 0.64), \max(0.2, 0.14), \max(0.1, 0.20)) = (0.70, 0.20, 0.20) \\ x \wedge_p u &= (1 - 0.9)(0.64, 0.14, 0.10) + 0.9(0.70, 0.20, 0.20) \\ &= (0.064, 0.014, 0.010) + (0.630, 0.180, 0.180) \\ &= (0.694, 0.194, 0.190). \end{aligned} \tag{14}$$

Applying the selected mode:

$$x \star u = SwapTF(0.694, 0.194, 0.190) = (0.190, 0.194, 0.694). \tag{15}$$

Since $(0.304, 0.136, 0.296) \neq (0.190, 0.194, 0.694)$, the associative property fails under the defined threshold parameters, verifying the theoretical results of Theorem 4.2.

5.2. *Algebraic Collapse in Transaction Legitimacy*

The deterministic generation of NeutroAlgebraic partiality extends into spaces governed by algorithmic legitimacy. Consider a transaction evaluation space where a request is verified against multiple attributes. Let a denote cryptographic authentication and b denote geospatial consistency. Assume an evaluation yields $x_a = (0.9, 0.1, 0.1)$, indicating a correct password, and $x_b = (0.2, 0.1, 0.8)$, indicating an anomalous location.

The contradiction degree between these evaluations evaluates to $c(a,b) = 0.5$. Under the thresholds $\tau_1 = 0.4$ and $\tau_2 = 0.7$, the condition $\tau_1 \leq c(a,b) < \tau_2$ holds. The deterministic selector $M(a,b)$ triggers the *EnrichI* mode. Algebraically, the polar components (Truth and Falsity) are mapped strictly to 0. The structural weight of the operation is absorbed into the indeterminacy component by the t-conorm. The transaction is neither approved nor rejected; it is structurally locked in an indeterminate state, demanding external manual resolution.

Suppose a third attribute d, representing physical impossibility (e.g., an ATM withdrawal on a different continent), enters the evaluation with $x_d = (0.1, 0.1, 0.9)$. The contradiction degree escalates to $c(a \wedge_p b, d) = 0.8 \geq \tau_2$. The threshold is breached. The selector M shifts to the $SwapTF$ mode. The operation $\star$ applies the truth reversal, forcing any residual truth into absolute falsity.

The associative equation $(x_a \star x_b) \star x_d = x_a \star (x_b \star x_d)$ fails because the intermediate $EnrichI$ collapse is irreversible. The topological transition between the sub-regions $[0, \tau_1)$, $[\tau_1, \tau_2)$, and $[\tau_2, 1]$ acts as a one-way trapdoor. The failure of associativity is not an operational defect; it guarantees that chronological re-evaluations cannot bypass the threshold-triggered lockdowns.

6. Conclusions and Open Problems

In this paper, we constructed an analytical generator for NeutroAlgebraic structures by synthesizing the plithogenic intersection and the Three-Mode Upside-Down operator. We defined a dual-threshold mode selector $M(a, b)$ modulated by the contradiction degree function. Based on this selector, we introduced the binary operation $\star$ and proved that the plithogenic space equipped with $\star$ forms a NeutroSemigroup. We demonstrated that the classical associative law fails to hold universally, as the contradiction degree dictates the sub-regions where associativity evaluates to true or false.

The transition from classical partial algebras to NeutroAlgebraic structures under threshold-based mode selectors presents the following open algebraic problems:

- By Lemma 4.3, the space lacks a universal identity element. A possible approach to construct partial identities (NeutroIdentities) is to restrict the space to localized topological sub-regions where the contradiction degree satisfies $c(a, b) < \tau_1$. Within these boundaries, the mode selector continuously yields $Keep$, allowing localized identities to emerge analytically. Establishing the formal conditions for these NeutroIdentities will determine whether the current NeutroSemigroup can be extended into a NeutroGroup.
- The algebraic classification of the operation $\star$ under asymmetric t-norms and t-conorms.

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