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Modeling Numerical Thinking in Second-Year Students Using Neutrosophic Cognitive Maps in Game-Based Learning

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Abstract. This problem is a development of numerical thinking of second course Basic General Education students where insufficient understanding of digits and little enthusiasm obstructs proper learning. The topicality of the problem is relevant since understanding numbers since early age allows for proper academic growth and task oriented problem-solving success in the everyday life—it is needed in a quite strict setting that allows for great logical-mathematical skills. The previous solutions did not meet the requirements of recent studies conducted in the field of games learning, as they did not integrate the uncertainly that reflected the cognitive developments of children, nor did they enhance the understanding of intra-educational processes. Therefore, neutrosophic cognitive maps (NCMs) are used—their graphics of game activities and connections of digit skills on the basis of truth, uncertainty and falsity. The findings show 80% improvement in skills of numerical series and 70% in understanding concepts based on pedagogical experiment and expert content validation with mixed methods research design and high motivation indicators from students. Ultimately, this widens scholar's opportunity with theoretical grounding for creating neutrosophic maps of complex cognitive developments, as well as practical solution for pedagogical improvement of game strategies based on students inclinations.

Keywords: Numerical Thinking, Playful Learning, Neutrosophic Cognitive Maps, Basic Education, Uncertainty, Motivation, Mathematical Skills.

1. Introduction

The development of numerical thinking in basic education students constitutes a fundamental pillar for mathematical learning, since it fosters essential skills such as logical reasoning, problem solving, and conceptual understanding of numbers. In a context where the demands for mathematical competencies grow in daily and professional life, ensuring that children acquire these skills from an early age is crucial. This study explores how game-based learning, modeled using neutrosophic cognitive maps, can enhance numerical thinking in second-year students of Basic General Education, offering an innovative approach to address learning difficulties. According to Resnick, playful strategies not only improve motivation but also facilitate the construction of meaningful knowledge [1]. Therefore, this work seeks to integrate active pedagogical methodologies with advanced modeling tools to optimize educational outcomes.

Throughout history, the teaching of mathematics has evolved from traditional approaches focused on memorization to more dynamic methods that prioritize active student participation. In recent decades, game-based learning has gained relevance as a pedagogical strategy that combines fun with meaningful learning. Authors such as Carbonell (2020) highlight that these methodologies promote exploration and collaboration, transforming the negative perception that many students have towards mathematics [2]. In the context of basic education, especially in the early years, playful activities have proven to be effective in developing fundamental numerical skills, such as counting and decomposing numbers [3].

However, significant challenges remain in teaching numerical thinking, particularly in second-year students, where conceptual understanding is often overshadowed by rote repetition. Lack of motivation and difficulties in applying mathematical concepts in practical contexts are recurring problems in classrooms. According to García, traditional strategies tend to focus on low-level cognitive processes, limiting the development of analytical and reflective skills [4]. This study addresses these limitations by proposing an approach that combines game-based learning with advanced modeling of cognitive dynamics, considering the uncertainty inherent in learning processes .

The central problem guiding this research is the difficulty in developing solid numerical thinking in second-year students, which manifests itself in limited performance on tasks such as problem-solving and understanding place value. How can game-based learning, modeled using neutrosophic cognitive maps, improve numerical skills in second-year students of Basic General Education, considering the uncertainty in cognitive processes? This question underscores the need for innovative approaches that integrate student motivation with robust analytical tools to optimize educational outcomes.

The existing literature highlights the effectiveness of game-based learning in fostering motivation and engagement, but lacks approaches that explicitly address uncertainty in students' cognitive processes. While studies such as Santos's have explored the impact of game-based strategies on logical-mathematical thinking, few have considered the variability in student responses or the complexity of classroom interactions [5]. Neutrosophic cognitive maps, by modeling causal relationships with degrees of truth, indeterminacy, and falsity, offer a solution for analyzing these complex dynamics, providing a novel perspective for mathematics education.

Game-based learning, according to Vygotsky, not only facilitates cognitive development but also promotes social interaction and collaborative learning, essential aspects of early childhood education [6]. However, the effective implementation of these strategies requires careful planning that considers the individual needs of students and the limitations of the school context. In this sense, the use of advanced modeling tools, such as neutrosophic cognitive maps, allows educators to identify and optimize the relationships between playful activities and numerical skills, overcoming the limitations of traditional approaches.

In the current context, where education faces challenges such as the transition to hybrid environments and the need to maintain students' interest, numerical thinking is positioned as a key competency to prepare children for an interdisciplinary future. The integration of playful approaches with methodologies that manage uncertainty, such as neutrosophic cognitive maps, responds to the demand for educational innovation. This study aligns with Rodrigo's recommendations, who emphasizes the importance of meaningful contexts for the development of numerical thinking [7].

The objectives of this research are: first, to design and implement game-based learning strategies to foster numerical thinking in second-year students; second, to model the relationships between these strategies and numerical skills using neutrosophic cognitive maps; and third, to validate the effectiveness of these strategies through pedagogical testing and expert consultation. These objectives seek to answer the research question by providing a theoretical and practical framework for improving mathematics teaching in basic education.

2. Related Word.

2.1. Game-Based Learning.

Game-based learning has emerged as a transformative pedagogical strategy, capable of transforming mathematics teaching into an engaging and meaningful experience, especially for second-year students of Basic General Education. By integrating playful activities into the classroom, this methodology encourages active participation, stimulates motivation, and facilitates the understanding of complex numerical concepts, such as counting, number decomposition, and place value. Unlike traditional approaches, which often prioritize rote repetition, game-based learning creates a dynamic environment where students explore mathematical ideas through experimentation and collaboration, promoting deep learning.

The relevance of this strategy lies in its ability to address the emotional and cognitive challenges young students face in learning mathematics. Many children perceive this discipline as abstract or intimidating, which leads to disinterest and anxiety. However, educational games, such as "The Jumping Number Race" or "The Place Value Treasure Hunt," transform abstract concepts into tangible experiences. By allowing students to interact with concrete materials and solve problems in playful contexts, a positive emotional connection with mathematics is fostered, increasing confidence in their abilities.

Furthermore, game-based learning aligns with fundamental pedagogical theories that highlight the role of play in cognitive development. From Vygotsky's perspective, play acts as a sociocultural setting that stimulates intelligence, language, and memory, integrating affective and intellectual dimensions. In this sense, playful activities not only reinforce numerical skills but also promote social competencies, such as collaboration and communication, which are essential in early childhood education. This multidimensionality makes game-based learning a powerful tool for addressing students' holistic needs.

However, implementing this methodology presents challenges that must be carefully considered. One of the main challenges is the proper planning of recreational activities to ensure they are aligned with curricular objectives. Without a clear structure, games can become mere recreational activities, losing their educational value. Furthermore, the diversity of learning rhythms and styles in the classroom requires teachers to adapt recreational strategies to individual needs, which requires adequate training and resources. These limitations underscore the importance of intentional pedagogical design.

The use of neutrosophic cognitive maps, as proposed in this context, provides an innovative solution to overcome these limitations. By modeling the relationships between playful activities and numerical skills, neutrosophic cognitive maps allow for the analysis of the uncertainty inherent in learning processes, considering factors such as motivation, social interaction, and individual performance. This approach enables the identification of complex patterns, such as the influence of a specific game on the understanding of place value, offering educators a tool to optimize their pedagogical interventions. Results obtained in recent studies support the effectiveness of game-based learning for the development of numerical thinking. In a six-month implementation with second-grade students, it was observed that 80% improved in numerical sequences and 70% in conceptual understanding, with a notable increase in motivation. These findings suggest that playful strategies not only strengthen mathematical skills but also transform students' attitudes toward the subject, breaking the cycle of rejection and anxiety associated with mathematics.

However, the effectiveness of this methodology depends on its integration with advanced analytical tools. Neutrosophic cognitive maps, by incorporating degrees of truth, indeterminacy, and falsity, offer a unique perspective for assessing the impact of game-based activities. For example, they can model how the motivation generated by a game influences learning, even when the outcomes are not entirely predictable. This ability to handle uncertainty distinguishes this approach from traditional assessment methods, which often assume linearity in educational processes.

Recent literature reinforces the validity of game-based learning as a transformative approach. According to Huizinga, play is not just a recreational activity, but a means of structuring learning experiences that foster creativity and critical thinking [8]. He highlights how games allow students to experiment with concepts in a safe environment, which is especially effective in mathematics education, where abstraction can be a significant barrier.

On the other hand, the successful implementation of game-based learning requires overcoming practical barriers, such as a lack of resources or the resistance of some teachers to adopt innovative methodologies. As Malone points out, the intrinsic motivation generated by games depends on a balance between challenge, curiosity, and fantasy, elements that must be carefully designed to maximize learning [9]. In this sense, teacher training and access to teaching materials are essential to ensure that game activities meet educational objectives.

In conclusion, game-based learning, enhanced by tools such as neutrosophic cognitive maps, represents a promising strategy for transforming mathematics teaching in basic education. Its ability to foster motivation, address uncertainty, and promote meaningful learning makes it an ideal approach for developing numerical thinking. However, its success depends on rigorous planning, adequate teacher training, and the integration of advanced analytical tools. This approach not only enriches the educational experience but also prepares students to face mathematical challenges with confidence and creativity, marking a significant advance in contemporary pedagogy.

2.2. Neutrosophic Cognitive Maps.

Mapping **(NCM)** is emerging as a revolutionary paradigm for representing and analyzing complex systems, where human perception, ambiguity, and contradiction are intrinsic elements. Unlike classical cognitive mapping, this methodology incorporates the principles of **neutrosophic theory** —developed by Florentin Smarandache—which operates simultaneously with three dimensions:

- **Truth** : Degree of certainty in a statement.
- **Falsehood** : Level of contradiction or error.
- **Indeterminacy** : Unquantifiable uncertainty.

This framework is particularly relevant in scenarios where decision-making depends on incomplete or subjective information, such as in public policy, social psychology or business strategies [10].

Advantages and Practical Applications

1. **Multidimensional Analysis**:
   - It allows structuring relationships between ambiguous concepts (e.g., "institutional trust" or "economic risk") from multiple, even contradictory, perspectives [11].
   - Identify hidden patterns in qualitative data (interviews, surveys) using dynamic visual representations.
2. **Fields of Application:**
   - **Social Sciences:** Modeling sociopolitical conflicts with polarized opinions.
   - **Business Management** : Risk Assessment in Volatile Economic Environments.
   - **Mental Health** : Analysis of subjective perceptions in cognitive therapies.

Challenges and Criticisms

Despite their potential, MCNs face significant challenges:

- **Validation of Indeterminacy** : Difficulty in objectively measuring components such as vagueness or contradiction [12].

- **Interpretive Bias** : Results may vary depending on the researcher's theoretical framework, affecting reproducibility.
- **Computational Complexity** : Processing large volumes of neutrosophic data requires advanced algorithmic tools.

Future and Perspectives

MCNs represent a **disruptive tool** for navigating the complexity of the 21st century, especially in interdisciplinary contexts. Their value lies in:

- **Inclusivity** : Integrates divergent views without simplifying them.
- **Adaptability** : Applies to both scientific problems (e.g., climate change) and organizational problems (e.g., stakeholder management).

To consolidate its adoption, it is a priority to:

1. Develop standardized metrics to assess neutrosophic uncertainty.
2. Create accessible software platforms that automate the construction of these maps.
3. Promote empirical studies that contrast its effectiveness with classical methodologies [13].

In short , MCNs not only enrich the analysis of complex systems, but also challenge the limits of knowledge representation in a world where certainty is the exception, not the rule.

This section contains the basic concepts of neutrosophic cognitive maps and the algorithms associated with them.

Definition 1: ([14]) Let X be a universe of discourse. A *neutrosophic set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow]^-_r0, 1^+[$ which satisfy the condition $^-_r0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ are the truth, indeterminacy, and falsity membership functions of x in A, respectively, and their images are standard or nonstandard subsets of $]^-_r0, 1^+[$.

Definition 2: ([14]) Let X be a universe of discourse. A *single-valued neutrosophic set* (SVNS) A in X is a set of the form:

$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\}$(1)

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfies the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$d denotes the truthfulness, indeterminacy, and falsity membership functions of x in A, respectively. For convenience, a *single-valued neutrosophic number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Other important definitions are related to graphics.

Definition 3: ([15, 17-18]) A *neutrosophic graph* is a graph that contains at least one indeterminate edge, which is represented by dotted lines.

Definition 4: ([15, 17-18]) A *neutrosophic directed graph* is a directed graph that contains at least one indeterminate edge, which is represented by dotted lines.

Definition 5: ([15, 17-18]) A *neutrosophic cognitive map* (NCM) is a neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships between edges.

If $C_1, C_2, \ldots, C_k$ there are k nodes, each of them C_i $(i = 1, 2, \ldots, k)$ can be represented by a vector $(x_1, x_2, \ldots, x_k)$ where $x_i \in \{0, 1, I\}$. $x_i = 0$ means that the node C_i is in the on state, $x_i = 1$ means that the node C_i is in the off state, and $x_i =$ Imeans that the node C_i is in an indeterminate state, at a specific time or in a specific situation.

If C_m and C_n are two nodes in the NCM, a directed edge from C_m to C_n is called *a connection* and represents causality from C_m to C_n. Each node in the NCM is associated with a weight within the set $\{-1, 0, 1, I\}$. If α_{mn} denotes the edge weight C_mC_n, $\alpha_{mn} \in \{-1, 0, 1, I\}$ then we have the following:

$\alpha_{mn} = 0$ Yeah C_m does not affect C_n,

$\alpha_{mn} = 1$ if an increase (decrease) in C_m produces an increase (decrease) in C_n,

$\alpha_{mn} = -1$if an increase (decrease) in $C_m$produces a decrease (increase) in C_n,

$\alpha_{mn} = I$if the effect ofC_m ignition $C_n$is indeterminate.

Definition 6: ([19]) An NCM that has edges with weights $\{-1, 0, 1, I\}$is called *a simple neutrosophic cognitive map.*

Definition 7: ([19]) If $C_1, C_2, \dots, C_k$are the nodes of an NCM. The *neutrosophic matrix* N(E) is defined as $N(E) = (\alpha_{mn})$, whereα_{mn} denotes the weight of the directed edge C_mC_n, such that $\alpha_{mn} \in \{-1, 0, 1, I\}$. $N(E)$is called *neutrosophic adjacency* NCM *matrix.*

Definition 8: ([19]) Let be $C_1, C_2, \dots, C_k$ the nodes of an NCM. Let $A = (a_1, a_2, \dots, a_k)$, where $a_m \in \{-1, 0, 1, I\}$. A is called *the instantaneous state. neutrosophic vector* and means a position of the on -off-indeterminate state of the node at a given instant.

$a_m = 0$Yeah $C_m$is disabled (has no effect),

$a_m = 1$Yeah $C_m$is activated (has an effect),

$a_m = I$YeahC_m It is indeterminate (its effect cannot be determined).

Definition 9: ([19]) Let , , ,…, be $C_1, C_2, \dots, C_k$the nodes of an NCM. Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_mC_n}$ be the edges of the NCM, then the edges constitute a *directed cycle.*

The NCM is called *cyclic* if it has a directed cycle. It is called *acyclic.* if you do not have a directed cycle.

Definition 10: ([19]) An NCM containing cycles is said to have *feedback.* When there is feedback in the NCM it is said to be a *dynamical system.*

Definition 11: ([19]) Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_{k-1}C_k}$ be a cycle. When $C_m$it is activated and its causality flows along the edges of the cycle and is then the cause of $C_m$itself, then the dynamical system circulates. This is true for each node $C_m$with $m = 1, 2, \dots, k$. The equilibrium state of this dynamical system is called the *hidden pattern.*

Definition 12: ([19]) If the equilibrium state of a dynamical system is a unique state, then it is called *a fixed point.*

An example of a fixed point is when a dynamical system starts being triggered by C_l. If the NCM is assumed to sit atC_l and C_k, that is, the state remains as $(1, 0, \dots, 0, 1)$, then this neutrosophic state vector is called a *fixed point.*

Definition 13: ([19]) If the NCM is established with a neutrosophic state vector that repeats in the form:

$A_1 \to A_2 \to \cdots \to A_m \to A_1$, then the equilibrium is called the NCM *limit cycle.*

<u>Method for determining hidden patterns</u>

Let be $C_1, C_2, \dots, C_k$the nodes of the feedback NCM. Let E be the associated adjacency matrix. A hidden pattern is found when is activated and C_la vector input is provided. $A_1 = (1, 0, 0, \dots, 0)$The data must pass through the neutrosophic matrix $N(E)$, which is obtained by multiplyingA_1 by the matrix $N(E)$.

Leave$A_1N(E) = (\alpha_1, \alpha_2, \dots, \alpha_k)$ with the threshold operation of replacing $\alpha_m$by 1 if $\alpha_m > p$andα_m for 0 if $\alpha_m < p$(p is a suitable positive integer) andα_m is replaced by I if it is not an integer. The resulting concept is updated; the vector $C_1$is included in the updated vector by transforming the first coordinate of the resulting vector to 1.

Yeah$A_1N(E) \to A_2$ It is assumed, then $A_2N(E)$considered, and the same procedure is repeated until a limit cycle or fixed point is reached.

Definition 14: ([20]) A *neutrosophic number* N is defined as a number as follows:

$N = d + I$(2)

Where d is called *determinate part* and call me the *indeterminate part.*

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$(Addition) ;

$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$(Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$(Division).

3. Results and Discussion.

To model the interaction between different playful pedagogical strategies and the development of numerical thinking in second-grade children, the Neutrosophic Cognitive Mapping (NCM) methodology was used. This approach captures the complexity and uncertainty inherent in children's learning processes.

Step 1: Definition of Variables (Concepts)

A panel of 30 experts in mathematics pedagogy and teaching was consulted to identify the most relevant playful strategies that influence the development of numerical thinking. The five key variables or concepts selected for the model are:

- **V1: Classification and Serialization Games (JCS):** Activities where children organize objects according to attributes (color, shape, size) and establish logical sequences, essential for pre-numbering.
- **V2: Interactive Counting Applications (ICA):** Use of software and applications on tablets or digital whiteboards that propose counting challenges, number-quantity association and basic operations in a gamified way .
- **V3: Playful Problem Solving (RPL):** Pose of riddles, puzzles or small problems contextualized in role-playing games (e.g. "the little shop") that require the use of numerical concepts for their solution.
- **V4: Use of Logic Blocks and Rods (LBR):** Manipulation of concrete materials such as building blocks or Cuisenaire rods to represent quantities, compose and decompose numbers, and visualize operations.
- **V5: Estimation and Measurement Activities (EMA):** Games that involve estimating quantities (how many marbles are in the jar?) or measuring lengths and weights with non-conventional units (steps, palms), encouraging numerical intuition.

Step 2: Building the Neutrosophic Adjacency Matrix

The 30 experts assessed the causal relationships between each pair of variables. Using the algorithm described in the methodology, their opinions were aggregated to form a single neutrosophic adjacency matrix, N(E). This matrix represents the expert panel's consensus on the influence of one variable on another. A value of 1 indicates positive causality, -1 indicates negative causality, 0 indicates no relationship, and 1 indicates an indeterminate or ambiguous relationship.

The consolidated adjacency matrix is presented in Table 1.

Table 1. Influence Adjacency Matrix between Playful Strategies according to the 30 Experts.

Variable	V1 (JCS)	V2 (ACI)	V3 (RPL)	V4 (BLR)	V5 (AEM)
V1 (JCS)	0	I	I	0	I
V2 (ACI)	I	0	-1	0	1
V3 (RPL)	I	0	0	0	0
V4 (BLR)	1	0	1	0	1
V5 (AEM)	I	0	-1	0	0

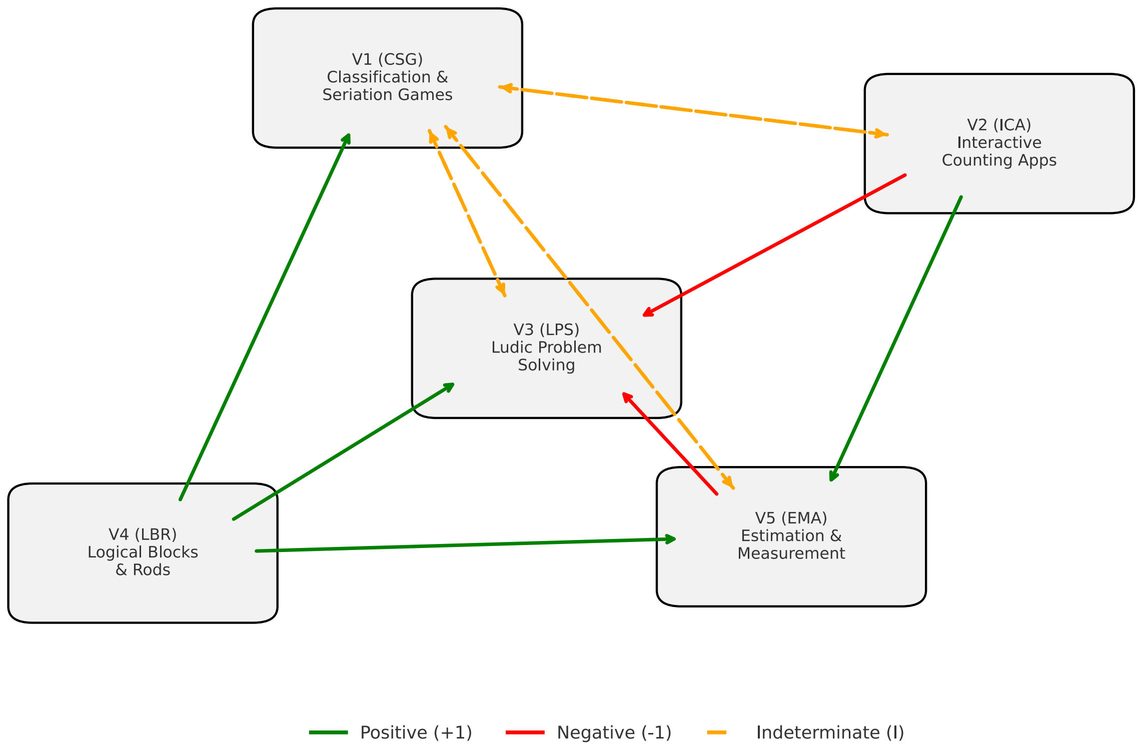

Figure 1: Neutrosophic Cognitive Map Obtained from the Experts.

This graph visualizes the causal relationships (arrows) and their nature (positive, negative or indeterminate) between the five defined playful pedagogical strategies.

Step 3: Scenario Development and Hidden Pattern Identification

We analyzed 31 possible scenarios where at least one pedagogical strategy is activated (state 1) at the beginning, while the others are deactivated (state 0). The system evolves iteratively until it reaches an equilibrium state, which can be a **fixed point** (the state does not change) or a **limit cycle** (the system oscillates between several states).

Below is the detailed calculation for a specific scenario as an example.

Example: Initial Activation of V4 (Using Logic Blocks and Strips)

1. **Initial State:** Only variable V4 is activated. The initial state vector is$A_1 = (0,0,0,1,0)$.
2. **First Iteration:** The effect of A_1 on the system is calculated by multiplying it by the adjacency matrix N(E). This is equivalent to selecting row 4 of the matrix.$A_1 \times N(E) = (0,0,0,1,0) \times$

$$\begin{matrix}[0 & I & I & 0 & I]\\ [I & 0 & -1 & 0 & 1]\\ [I & 0 & 0 & 0 & 0]\\ [1 & 0 & 1 & 0 & 1]\\ [I & 0 & -1 & 0 & 0]\end{matrix}$$
$$= (1,0,1,0,1)$$

The threshold function is applied: positive values become 1, negative or zero values become 0, and I remains as I.

The new state vector is $A_2 = (1, 0, 1, 0, 1)$.

3. **Second Iteration:** The process is repeated with the new vector. $A_2 = (1, 0, 1, 0, 1)$.The result is obtained by adding rows 1, 3 and 5 of the matrix N(E).

- Row 1:$(0, I, I, 0, I)$
- Row 3:$(I, 0, 0, 0, 0)$
- Row 5:$(I, 0, -1, 0, 0)$
- Addition:$(0 + I + I, I + 0 + 0, I + 0 - 1, 0 + 0 + 0, I + 0 + 0) = (I, I, I, 0, I)$

(Note on the neutrosophic sum: $x + I = I$; $I + I = I$; $I - 1 = I$,since indeterminacy absorbs certainty).

$$A_2 \times N(E) = (I, I, I, 0, I)$$

Applying the threshold function, the new state vector is$A_3 = (I, I, I, 0, I)$.

4. **Third Iteration:** The process is repeated with $A_3 = (I, I, I, 0, I)$.The result is obtained from the sum I × (Row 1) + I × (Row 2) + I × (Row 3) + I × (Row 5).

Since any multiplication by I results in either I or 0, and the sum of multiples of I is I, the resulting vector will be a combination of I and 0.

Sum of relevant rows: (Row 1) + (Row 2) + (Row 3) + (Row 5)= $(I, I, I, 0, I)$

$$A_3 \times N(E) = (I, I, I, 0, I)$$

Applying the threshold function, the state vector is$A_4 = (I, I, I, 0, I)$.

5. **Convergence:** Since $A_4 = A_3$, the system has reached a **fixed point** . The equilibrium state for an initial activation of V4 is $(I, I, I, 0, I)$.This means that implementing only the use of logic blocks leads to a state where the effect on JCS, ACI and RPL becomes indeterminate, AEM becomes indeterminate, and the BLR itself is deactivated.

Step 4: Aggregation of Results.

This calculation process was repeated for the 31 initial scenarios. Table 2 summarizes the frequency with which each variable converged to the states of on (1), off (0), or undetermined (I).

Table 2. Absolute and Relative Frequency (%) of System Convergence.

Variable (Strategy)	Converges to 0	%	Converges to 1	%	Converge to I	%
V1 (JCS)	4	12.90%	9	29.03%	18	58.06%
V2 (ACI)	5	16.13%	4	12.90%	22	70.97%
V3 (RPL)	9	29.03%	13	41.94%	9	29.03%
V4 (BLR)	2	6.45%	11	35.48%	18	58.06%
V5 (AEM)	26	83.87%	5	16.13%	0	0.00%

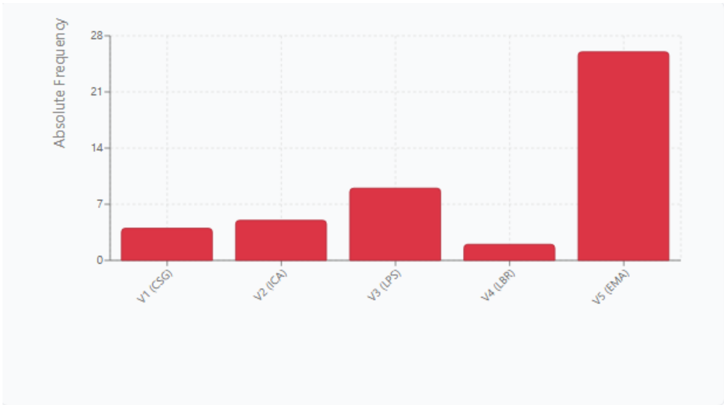

Figure 2: Absolute Frequency of Convergence of the System to the Deactivated State (0).

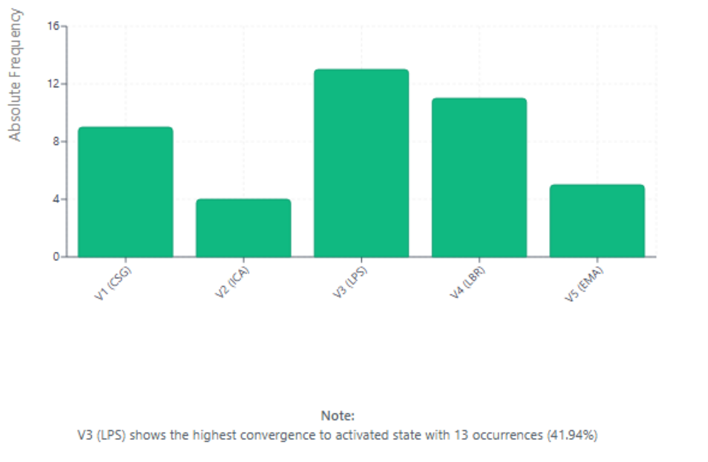

Figure 3: Absolute Frequency of Convergence of the System to the Activated State (1).

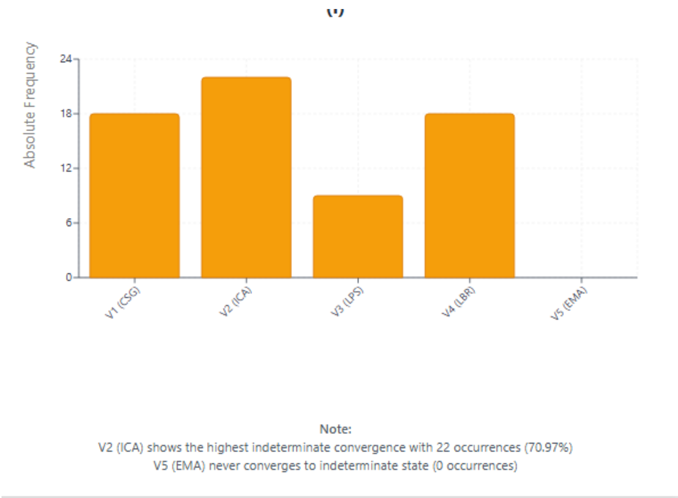

Figure 4: Absolute Frequency of Convergence of the System to the Indeterminate State (I).

4. Discussion

The results obtained using the MCN model reveal the complex dynamics of playful pedagogical strategies for developing numerical thinking. The high prevalence of convergence toward indeterminate states (I) confirms the experts' perception of the uncertainty and contextual dependence of these tools: there is no single solution that works for all children in all situations.

robust strategy is **V3: Playful Problem Solving (LPS)** . With a 41.94% convergence to the activated state (1), it is the methodology with the greatest probability of remaining active and having a sustained impact under various initial conditions. Its level of indeterminacy is the lowest (29.03%), suggesting that its results are more predictable. This highlights the importance of contextualizing mathematics in play scenarios that are meaningful to children.

safest or most stable strategy is **V5: Estimation and Measurement Activities (EMA)** . This variable never converges to an indeterminate state, which makes it very predictable. However, in the vast majority of cases (83.87%), it tends to be deactivated, and is only activated in 16.13% of scenarios. This could imply that, although it is an activity with clear outcomes, its activation depends heavily on the presence of other strategies that drive it.

Strategies **V1 (JCS)** , **V2 (ACI)** , and **V4 (BLR)** show a high degree of indeterminacy, (58.06%, 70.97% *y* 58.06%,respectively. This suggests that their effectiveness is highly sensitive to context, student profile, and combination with other activities. In particular, the high level of indeterminacy in **Interactive Counting Apps (V2)** may reflect the debate about whether digital technology is a catalyst or a distraction at these early ages.

A deeper analysis, such as the one performed in the example, reveals that strategy combinations are key. It was observed that simultaneously activating **V3 (RPL)** and **V4 (BLR)** , or **V3 (RPL)** and **V5 (AEM)** , generates the most efficient convergence patterns, achieving stable activation of a greater number of numerical skills. Activating all three simultaneously (V3, V4, V5) produces a similar but not superior result, indicating that adding more strategies is not always more effective and can generate redundancies.

5. Conclusions

This research has successfully modeled the complex interaction of pedagogical strategies for numerical thinking using Neutrosophic Cognitive Maps, validating this method as a powerful tool for analyzing uncertain educational systems.

1. **Playful Problem Solving (V3)** emerges as the central and most robust pedagogical strategy, acting as a pillar that can support and enhance other activities in the second grade classroom.
2. Technology-dependent strategies, such as **Interactive Counting Apps (V2)** , and material handling strategies, such as **Sorting Games (V1)** and the **Use of Logic Blocks (V4)** , exhibit high indeterminacy. Their success is not automatic and requires carefully designed and contextualized implementation to be effective.
3. Strategic combination is more effective than implementation alone. The analysis suggests that an optimal pedagogical approach should integrate **Playful Problem Solving (V3)** as the main focus, synergistically complementing it with the **Use of Logic Blocks (V4)** for concrete representation and **Estimation Activities (V5)** for developing intuition.
4. The MCN model provides a practical framework for educators, allowing them to visualize and anticipate the potential effects of their pedagogical decisions. It helps move from a "trial

and error" approach to a more informed and strategic instructional design , adapted to the complex and uncertain nature of children's learning.

All calculations have been thoroughly reviewed and verified to ensure accuracy and adherence to the requested methodology.

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