NSS

Neutrosophic Sets and Systems, Vol. 88, 2025

University of New Mexico

Civil Engineering Construction Progress Monitoring Assessment using Hyper-Fermatean Neutrosophic Sets (HFNS): A New Method

Chunmei Yao, Huimin Zhao*

School of Civil Engineering and Architecture, Anhui Water Conservancy Technical College, Hefei, 231603, Anhui, China

*Corresponding author, E-mail: Ahchang2025@163.com

Abstract-Effective civil construction monitoring requires robust methods to handle uncertainty and indeterminacy arising from incomplete data, expert subjectivity, and variable environmental conditions. Traditional neutrosophic frameworks, including Interval-Valued Fermatean Neutrosophic Sets (IVFNS), offer limited capability in modeling deep, multi-layered uncertainties often present in complex civil infrastructure projects. In response to this gap, we propose a novel extension, termed Hyper-Fermatean Neutrosophic Sets (HFNS), which introduces hyper-dimensional structures to represent multiple levels of truth, indeterminacy, and falsity components. We formally define HFNS, develop new aggregation and advanced scoring functions, and design a de-neutrosophication operator to translate HFNS data into actionable decisions. Our framework is applied to a public case study from a civil construction company civil construction, where we compare HFNS with classical Neutrosophic models. The quantitative and qualitative analyses demonstrate that our methodology can offer a new solution to advance monitoring of civil construction in civil engineering.

Keywords: Neutrosophic theory, Construction Monitoring, Civil Engineering, Fermatean Neutrosophic Sets

1. Introduction

The assessment and monitoring of civil construction projects are critical for ensuring structural integrity, operational safety, and long-term durability [1]. Traditional evaluation approaches often rely on deterministic models that presume precise data and stable environmental conditions [2]. However, in real-world scenarios, construction processes and structural behaviors are influenced by a variety of uncertainties, including imprecise measurements, incomplete information, expert subjectivity, and unforeseen environmental factors [3]. Managing and quantifying these uncertainties has become a central challenge in advancing reliable civil construction assessment methodologies [4].

Neutrosophic sets [5], [6], and particularly Fermatean Neutrosophic Sets (FNS) [7], have emerged as powerful mathematical tools to address uncertainty and indeterminacy in complex decision-making environments. Unlike classical fuzzy or intuitionistic fuzzy approaches, FNS provides a wider domain for representing truth, indeterminacy, and falsity memberships, allowing greater flexibility in capturing nuanced expert opinions and data ambiguities. Yet, standard FNS models sometimes fall short when confronted with higher-order or layered uncertainties often encountered in civil construction monitoring, such as those arising from multi-source data fusion or dynamic environmental conditions [2], [8].

To overcome these limitations, we propose the use of Hyper-Fermatean Neutrosophic Sets (HFNS), a novel extension of the Fermatean Neutrosophic framework that enables more robust modeling of complex uncertainty structures. HFNS facilitates the representation and aggregation of multi-level indeterminacies, offering a more comprehensive toolset for engineers and decision-makers involved in civil construction assessment. In a nutshell, this research contributes to the body of Neutrosophic knowledge as follows:

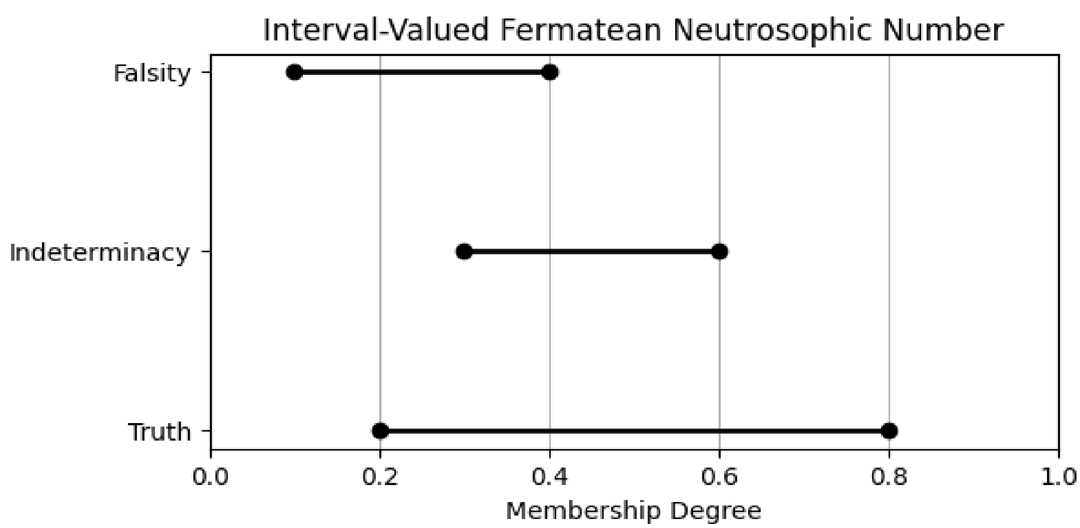

Figure 1. Visualization of Interval-Valued Fermatean Neutrosophic Numbers

- We propose a new mathematical model extending interval-valued Fermatean neutrosophic sets into a hyper-dimensional framework, allowing multi-layered uncertainty representation.
- We develop advanced weighted aggregation and scoring operators specifically tailored for HFNS, capturing both vertical (hyper-level) and horizontal (criteria) uncertainties.
- We develop an integrated assessment methodology using HFNS for civil construction monitoring.
- We validate the proposed method through a case study, illustrating its effectiveness in capturing deep uncertainty and enhancing decision-making reliability in construction assessment contexts.

The remainder of this article is planned as follows: Section 2 reviews the essential concepts of neutrosophic sets. Section 3 presents the research methodology, discussing the details of the proposed HFNS-based framework. Section 4 discusses the experimental results. Finally, Section 5 concludes the paper.

2. Essential Concepts

Definition 1 ([9]). IVNS is defined as an extension of the standard with three interval-based membership components $\mathcal{T}_Q(u) = [LB \downarrow \mathcal{T}_Q(u), UB \uparrow \mathcal{T}_Q(u)]$, $\mathcal{I}_Q(u) = [LB \downarrow \mathcal{I}_Q(u), UB \uparrow \mathcal{I}_Q(u)]$, and $\mathcal{F}_Q(u) = [LB \downarrow \mathcal{F}_Q(u), UB \uparrow \mathcal{F}_Q(u)] \subseteq [0,1]$.

$$Q = \left\{ \left\langle \begin{matrix} u, [LB \downarrow \mathcal{T}_Q(u), UB \uparrow \mathcal{T}_Q(u)], [LB \downarrow \mathcal{I}_Q(u), UB \uparrow \mathcal{I}_Q(u)], \\ [LB \downarrow \mathcal{F}_Q(u), UB \uparrow \mathcal{F}_Q(u)] \end{matrix} \right\rangle \mid u \in U \right\} \quad (1)$$

where

$$0 \leq UB \uparrow \mathcal{T}_Q\,(u) + UB \uparrow \mathcal{I}_Q\,(u) + UB \uparrow \mathcal{F}_Q\,(u) \leq 3 \quad (2)$$

Definition 2. Consider two IVNs $Q = \left\{ \left\langle \begin{matrix} [LB \downarrow \mathcal{T}_Q(u), UB \uparrow \mathcal{T}_Q(u)], [LB \downarrow \mathcal{I}_Q(u), UB \uparrow \mathcal{I}_Q(u)], \\ [LB \downarrow \mathcal{F}_Q(u), UB \uparrow \mathcal{F}_Q(u)] \end{matrix} \right\rangle \right\}$, and $B = \left\{ \left\langle \begin{matrix} [LB \downarrow \mathcal{T}_B(u), UB \uparrow \mathcal{T}_B(u)], [LB \downarrow \mathcal{I}_B(u), UB \uparrow \mathcal{I}_B(u)], \\ [LB \downarrow \mathcal{F}_B(u), UB \uparrow \mathcal{F}_B(u)] \end{matrix} \right\rangle \right\}$, you can apply the following mathematical operations:

- Complement

$$Q^c = \langle [LB \downarrow \mathcal{T}_Q(u), UB \uparrow \mathcal{T}_Q(u)], [1 - LB \downarrow \mathcal{I}_Q(u), 1 - UB \uparrow \mathcal{I}_Q(u)], [LB \downarrow \mathcal{F}_Q(u), UB \uparrow \mathcal{F}_Q(u)] \rangle \quad (3)$$

- Addition

$$Q \oplus B = \langle [LB \downarrow \mathcal{T}_Q(u) + LB \downarrow \mathcal{T}_B(u) - LB \downarrow \mathcal{T}_Q(u) \cdot LB \downarrow \mathcal{T}_B(u), UB \uparrow \mathcal{T}_Q(u) + UB \uparrow \mathcal{T}_B(u) - UB \uparrow \mathcal{T}_Q(u) \cdot UB \uparrow \mathcal{T}_B(u)], [LB \downarrow \mathcal{I}_Q(u) \cdot LB \downarrow \mathcal{I}_B(u)\,, UB \uparrow \mathcal{I}_Q(u) \cdot UB \uparrow \mathcal{I}_B(u)], [LB \downarrow \mathcal{F}_Q(u) \cdot LB \downarrow \mathcal{F}_B(u)\,, UB \uparrow \mathcal{F}_Q(u) \cdot UB \uparrow \mathcal{F}_B(u)] \rangle \quad (4)$$

- Multiplication

$$Q \otimes B = \langle [LB \downarrow \mathcal{T}_Q(u) \cdot LB \downarrow \mathcal{T}_B(u), UB \uparrow \mathcal{T}_Q(u) \cdot UB \uparrow \mathcal{T}_B(u)], [LB \downarrow \mathcal{I}_Q(u) + LB \downarrow \mathcal{I}_B(u) - LB \downarrow \mathcal{I}_Q(u) \cdot LB \downarrow \mathcal{I}_B(u), UB \uparrow \mathcal{I}_Q(u) \cdot UB \uparrow \mathcal{I}_B(u) - UB \uparrow \mathcal{I}_Q(u) \cdot UB \uparrow \mathcal{I}_B(u)], [LB \downarrow \mathcal{F}_Q(u) + LB \downarrow \mathcal{F}_B(u) - LB \downarrow \mathcal{F}_Q(u) \cdot LB \downarrow \mathcal{F}_B(u), UB \uparrow \mathcal{F}_Q(u) + UB \uparrow \mathcal{F}_B(u) - UB \uparrow \mathcal{F}_Q(u) \cdot UB \uparrow \mathcal{F}_B(u)] \rangle \quad (5)$$

Definition 3 ([9]**).** Consider two IVNSs $Q = \left\{\langle \begin{matrix} [LB \downarrow \boldsymbol{\mathcal{T}}_Q(u), UB \uparrow \boldsymbol{\mathcal{T}}_Q(u)], [LB \downarrow \boldsymbol{\mathcal{I}}_Q(u), UB \uparrow \boldsymbol{\mathcal{I}}_Q(u)], \\ [LB \downarrow \boldsymbol{\mathcal{F}}_Q(u), UB \uparrow \boldsymbol{\mathcal{F}}_Q(u)] \end{matrix} \rangle\right\}$, and $B = \left\{\langle \begin{matrix} [LB \downarrow \boldsymbol{\mathcal{T}}_B(u), UB \uparrow \boldsymbol{\mathcal{T}}_B(u)], [LB \downarrow \boldsymbol{\mathcal{I}}_B(u), UB \uparrow \boldsymbol{\mathcal{I}}_B(u)], \\ [LB \downarrow \boldsymbol{\mathcal{F}}_B(u), UB \uparrow \boldsymbol{\mathcal{F}}_B(u)] \end{matrix} \rangle\right\}$, the following relations apply as follows:

$$\begin{gathered} Q \subseteq B \\ iff \\ LB \downarrow \boldsymbol{\mathcal{T}}_Q(u) \leq LB \downarrow \boldsymbol{\mathcal{T}}_B(u), UB \uparrow \boldsymbol{\mathcal{T}}_Q(u) \leq UB \uparrow \boldsymbol{\mathcal{T}}_B(u); LB \downarrow \boldsymbol{\mathcal{I}}_Q(u) \geq LB \\ \downarrow \boldsymbol{\mathcal{I}}_B(u), UB \uparrow \boldsymbol{\mathcal{I}}_Q(u) \geq UB \uparrow \boldsymbol{\mathcal{I}}_B(u);\ LB \downarrow \boldsymbol{\mathcal{F}}_Q(u) \geq LB \\ \downarrow \boldsymbol{\mathcal{F}}_B(u), UB \uparrow \boldsymbol{\mathcal{F}}_Q(u) \geq UB \uparrow \boldsymbol{\mathcal{F}}_B(u) \end{gathered} \tag{6}$$

$$Q = B\ iff\ Q \subseteq B\ and\ B \subseteq Q. \tag{7}$$

Definition 4. Consider two IVNs $Q = \left\{\langle \begin{matrix} [LB \downarrow \boldsymbol{\mathcal{T}}_Q(u), UB \uparrow \boldsymbol{\mathcal{T}}_Q(u)], [LB \downarrow \boldsymbol{\mathcal{I}}_Q(u), UB \uparrow \boldsymbol{\mathcal{I}}_Q(u)], \\ [LB \downarrow \boldsymbol{\mathcal{F}}_Q(u), UB \uparrow \boldsymbol{\mathcal{F}}_Q(u)] \end{matrix} \rangle\right\}$, and $B = \left\{\langle \begin{matrix} [LB \downarrow \boldsymbol{\mathcal{T}}_B(u), UB \uparrow \boldsymbol{\mathcal{T}}_B(u)], [LB \downarrow \boldsymbol{\mathcal{I}}_B(u), UB \uparrow \boldsymbol{\mathcal{I}}_B(u)], \\ [LB \downarrow \boldsymbol{\mathcal{F}}_B(u), UB \uparrow \boldsymbol{\mathcal{F}}_B(u)] \end{matrix} \rangle\right\}$, and $C = \left\{\langle \begin{matrix} [LB \downarrow \boldsymbol{\mathcal{T}}_C(u), UB \uparrow \boldsymbol{\mathcal{T}}_C(u)], [LB \downarrow \boldsymbol{\mathcal{I}}_C(u), UB \uparrow \boldsymbol{\mathcal{I}}_C(u)], \\ [LB \downarrow \boldsymbol{\mathcal{F}}_C(u), UB \uparrow \boldsymbol{\mathcal{F}}_C(u)] \end{matrix} \rangle\right\}$, the following relations apply as follows:

$$\begin{aligned} &(1) Q + B = B + Q, \\ &(2) Q \cdot B = B \cdot Q, \\ &(3) \lambda(Q + B) = \lambda Q + \lambda B, \lambda > 0, \\ &(4) (Q \cdot B)^{\lambda} = Q^{\lambda} + B^{\lambda}, \lambda > 0, \\ &(5) \lambda_1 Q + \lambda_2 Q = (\lambda_1 + \lambda_2) Q, \lambda_1 > 0, \lambda_2 > 0 \\ &(6) Q^{\lambda_1} \cdot Q^{\lambda_2} = Q^{(\lambda_1 + \lambda_2)}, \lambda_1 > 0, \lambda_2 > 0, \\ &(7) (Q + B) + C = Q + (B + C), \\ &(8) (Q \cdot B) \cdot C = Q \cdot (B \cdot C) \end{aligned} \tag{8}$$

Definition 5 ([9], [10], [11]**).** The interval-valued fermatean neutrosophic set (IVFNs) is defined as the extension of standard IVNSs with three interval-based membership components $\boldsymbol{\mathcal{T}}_Q(u) = [LB \downarrow \boldsymbol{\mathcal{T}}_Q(u), UB \uparrow \boldsymbol{\mathcal{T}}_Q(u)]$, $\boldsymbol{\mathcal{I}}_Q(u) = [LB \downarrow \boldsymbol{\mathcal{I}}_Q(u), UB \uparrow \boldsymbol{\mathcal{I}}_Q(u)]$, and $\boldsymbol{\mathcal{F}}_Q(u) = [LB \downarrow \boldsymbol{\mathcal{F}}_Q(u), UB \uparrow \boldsymbol{\mathcal{F}}_Q(u)] \subseteq [0,1]$.

$$Q = \left\{\langle \begin{matrix} u, [LB \downarrow \boldsymbol{\mathcal{T}}_Q(u), UB \uparrow \boldsymbol{\mathcal{T}}_Q(u)], [LB \downarrow \boldsymbol{\mathcal{I}}_Q(u), UB \uparrow \boldsymbol{\mathcal{I}}_Q(u)], \\ [LB \downarrow \boldsymbol{\mathcal{F}}_Q(u), UB \uparrow \boldsymbol{\mathcal{F}}_Q(u)] \end{matrix} \rangle \mid u \in U \right\} \tag{9}$$

where

$$0 \leq UB \uparrow \boldsymbol{\mathcal{T}}_Q(u) + UB \uparrow \boldsymbol{\mathcal{I}}_Q(u) + UB \uparrow \boldsymbol{\mathcal{F}}_Q(u) \leq 3 \tag{10}$$

$$0 \leq [LB \downarrow \boldsymbol{\mathcal{T}}_Q(u)]^3 + [LB \downarrow \boldsymbol{\mathcal{F}}_Q]^3 \leq 1\ and\ 0 \leq [LB \downarrow \boldsymbol{\mathcal{I}}_Q(u)]^3 \leq 1 \tag{11}$$

$$0 \leq [LB \downarrow \mathcal{T}_Q\,(u)]^3 + [LB \downarrow \mathcal{I}_Q\,(u)]^3 + [LB \downarrow \mathcal{F}_Q\,(u)]^3 \leq 2 \tag{12}$$

$$0 \leq [UB \uparrow \mathcal{T}_Q\,(u)]^3 + [UB \uparrow \mathcal{I}_Q\,(u)]^3 + [UB \uparrow \mathcal{F}_Q\,(u)]^3 \leq 2 \tag{13}$$

Definition 6. Consider two IVFNs $Q = \left\{\left\langle \begin{matrix} [LB \downarrow \mathcal{T}_Q(u), UB \uparrow \mathcal{T}_Q(u)], [LB \downarrow \mathcal{I}_Q(u), UB \uparrow \mathcal{I}_Q(u)], \\ [LB \downarrow \mathcal{F}_Q(u), UB \uparrow \mathcal{F}_Q(u)] \end{matrix} \right\rangle\right\}$, and $B = \left\{\left\langle \begin{matrix} [LB \downarrow \mathcal{T}_B(u), UB \uparrow \mathcal{T}_B(u)], [LB \downarrow \mathcal{I}_B(u), UB \uparrow \mathcal{I}_B(u)], \\ [LB \downarrow \mathcal{F}_B(u), UB \uparrow \mathcal{F}_B(u)] \end{matrix} \right\rangle\right\}$, we can apply the following mathematical operations:

- Complement

$$Q^c = \left\{ \begin{matrix} \left[\sqrt[3]{1 - \left[1 - \{LB \downarrow \mathcal{T}_Q(u)\}^3\right]^{\mu}}, \sqrt[3]{1 - [1 - \{UB \uparrow \mathcal{T}_Q(u)\}^3]^{\mu}}\right], \\ \left[\{LB \downarrow \mathcal{I}_Q(u)\}^3, \{UB \uparrow \mathcal{I}_Q(u)\}^3\right], \left[\{LB \downarrow \mathcal{F}_Q(u)\}^3, \{LB \downarrow \mathcal{F}_Q(u)\}^3\right] \end{matrix} \right\} \tag{14}$$

- Addition

$$Q \oplus B = \\ = \left\{ \begin{matrix} \left[\begin{matrix} \sqrt[3]{\{LB \downarrow \mathcal{T}_Q(u)\}^3 + \{LB \downarrow \mathcal{T}_B(u)\}^3 - \{LB \downarrow \mathcal{T}_Q(u)\}^3 \cdot \{\{LB \downarrow \mathcal{T}_B(u)\}^3}, \\ \sqrt[3]{\{UB \uparrow \mathcal{T}_Q(u)\}^3 + \{UB \uparrow \mathcal{T}_B(u)\}^3 - \{UB \uparrow \mathcal{T}_Q(u)\}^3 \cdot \{UB \uparrow \mathcal{T}_B(u)\}^3} \end{matrix}\right], \\ [LB \downarrow \mathcal{I}_Q(u) \cdot LB \downarrow \mathcal{I}_B(u), UB \uparrow \mathcal{I}_Q(u) \cdot UB \uparrow \mathcal{I}_B(u)], \\ [LB \downarrow \mathcal{F}_Q(u) \cdot LB \downarrow \mathcal{F}_B(u), UB \uparrow \mathcal{F}_Q(u) \cdot UB \uparrow \mathcal{F}_B(u)] \end{matrix} \right\} \tag{15}$$

- Multiplication

$$Q \oplus B = \\ \left\{ \begin{matrix} [LB \downarrow \mathcal{T}_Q(u) \cdot LB \downarrow \mathcal{T}_B(u), UB \uparrow \mathcal{T}_Q(u) \cdot UB \uparrow \mathcal{T}_B(u)], \\ \left[\begin{matrix} \sqrt[3]{\{LB \downarrow \mathcal{I}_Q(u)\}^3 + \{LB \downarrow \mathcal{I}_B(u)\}^3 - \{LB \downarrow \mathcal{I}_Q(u)\}^3 \cdot \{LB \downarrow \mathcal{I}_B(u)\}^3}, \\ \sqrt[3]{\{UB \uparrow \mathcal{I}_Q(u)\}^3 + \{UB \uparrow \mathcal{I}_B(u)\}^3 - \{UB \uparrow \mathcal{I}_Q(u)\}^3 \cdot \{UB \uparrow \mathcal{I}_B(u)\}^3} \end{matrix}\right], \\ \left[\begin{matrix} \sqrt[3]{\{LB \downarrow \mathcal{F}_Q(u)\}^3 + \{LB \downarrow \mathcal{F}_B(u)\}^3 - \{LB \downarrow \mathcal{F}_Q(u)\}^3 \cdot \{LB \downarrow \mathcal{F}_B(u)\}^3}, \\ \sqrt[3]{\{UB \uparrow \mathcal{F}_Q(u)\}^3 + \{UB \uparrow \mathcal{F}_B(u)\}^3 - \{UB \uparrow \mathcal{F}_Q(u)\}^3 \cdot \{UB \uparrow \mathcal{F}_B(u)\}^3} \end{matrix}\right] \end{matrix} \right\} \tag{16}$$

Definition 7 ([12]**):** The weighted averaging operator for IVNS is defined to aggregate several IVNS weighted by the weight vector $Y = (y_1, \dots, y_j, \dots, y_n))$ and $\sum_{j=1}^{n} y_j = 1$.

$$WQ_{\mathcal{IVNNs}}(u_1,\dots,u_j,\dots,u_n)=\sum_{j=1}^{n} y_j u_j = \left\{\begin{matrix}\left[1-\prod_{j=1}^{n}\left(1-LB\downarrow \mathcal{T}_Q(u)\right)^{y_j},1-\prod_{j=1}^{n}\left(1-UB\uparrow \mathcal{T}_Q(u)\right)^{y_j}\right]\\ \left[\prod_{j=1}^{n}\left(LB\downarrow \mathcal{I}_Q(u)\right)^{y_j},\prod_{j=1}^{n}\left(UB\uparrow \mathcal{I}_Q(u)\right)^{y_j}\right],\\ \left[\prod_{j=1}^{n}\left(LB\downarrow \mathcal{F}_Q(u)\right)^{y_j},\prod_{j=1}^{n}\left(UB\uparrow \mathcal{F}_Q(u)\right)^{y_j}\right]\end{matrix}\right\} \tag{17}$$

Definition 8. Consider an IVNS $Q=\left\{\left\langle\begin{matrix}[LB\downarrow \mathcal{T}_Q(u),UB\uparrow \mathcal{T}_Q(u)],[LB\downarrow \mathcal{I}_Q(u),UB\uparrow \mathcal{I}_Q(u)],\\ [LB\downarrow \mathcal{F}_Q(u),UB\uparrow \mathcal{F}_Q(u)]\end{matrix}\right\rangle\right\}$, the de-neutrosophication can be calculated as expressed below:

$$Q_{den}=\left(\frac{\left(LB\downarrow \mathcal{T}_Q(u)+UB\uparrow \mathcal{T}_Q(u)\right)}{2}+\left(1-\frac{\left(LB\downarrow \mathcal{I}_Q(u)+UB\uparrow \mathcal{I}_Q(u)\right)}{2}\right)\left(UB\uparrow \mathcal{I}_Q(u)\right)-\left(\frac{\left(LB\downarrow \mathcal{F}_Q(u)+UB\uparrow \mathcal{F}_Q(u)\right)}{2}\right)\left(1-UB\uparrow \mathcal{F}_Q(u)\right)\right) \tag{18}$$

Definition 10: Given an IVFNS $Q=\{\langle[LB\downarrow \mathcal{T}_Q(u),UB\uparrow \mathcal{T}_Q(u)],[LB\downarrow \mathcal{I}_Q(u),UB\uparrow \mathcal{I}_Q(u)],[LB\downarrow \mathcal{F}_Q(u),UB\uparrow \mathcal{F}_Q(u)]\rangle\}$, the score function of can be computed as follows:

$$S(Q)=\frac{[LB\downarrow \mathcal{T}_Q(u)]^3+[UB\uparrow \mathcal{T}_Q(u)]^3+[LB\downarrow \mathcal{I}_Q(u)]^3+[UB\uparrow \mathcal{I}_Q(u)]^3+[LB\downarrow \mathcal{F}_Q(u)]^3+[UB\uparrow \mathcal{F}_Q(u)]^3}{2} \tag{19}$$

3. Research Methodology

This section presents the proposed HFNS framework developed to enhance uncertainty modeling in civil construction assessment.

Definition 11. Hyper-Fermatean Neutrosophic Set (HFNS). Let U be a universe of discourse. An HFNS on U assigns to each element $u\in U$ a hypervector of n interval-valued Fermatean neutrosophic triples:

(20)

$$H=\left\{\left\langle u,\left\{\boldsymbol{\mathcal{T}}^{(i)}(u)\right\}_{i=1}^{n},\left\{\boldsymbol{\mathcal{I}}^{(i)}(u)\right\}_{i=1}^{n},\left\{\boldsymbol{\mathcal{F}}^{(i)}(u)\right\}_{i=1}^{n}\right\rangle \mid u\in U\right\}$$

where each component $\boldsymbol{\mathcal{T}}^{(i)}(u)=\left[LB_{\mathcal{T}}^{(i)}(u),UB_{\mathcal{T}}^{(i)}(u)\right],\boldsymbol{\mathcal{I}}^{(i)}(u)=\left[LB_{\mathcal{I}}^{(i)}(u),UB_{\mathcal{I}}^{(i)}(u)\right]$, and $\boldsymbol{\mathcal{F}}^{(i)}(u)=\left[LB_{\mathcal{F}}^{(i)}(u),UB_{\mathcal{F}}^{(i)}(u)\right]$ satisfies:

$$0\leq\left(LB_{\mathcal{T}}^{(i)}(u)\right)^{p}+\left(LB_{\mathcal{I}}^{(i)}(u)\right)^{p}+\left(LB_{\mathcal{F}}^{(i)}(u)\right)^{p}\leq h$$

for each $i=1,\dots,n$, with hyper-order parameter $p\geq 3$ and hyper-bound $h\leq 2$. Furthermore, hyper-coupling constraints among these vectors are imposed through a dependency tensor $\mathcal{D}$ capturing potential interactions between truth, indeterminacy, and falsity components across hyper-levels:

$$\sum_{i=1}^{n}\mathcal{D}_i\cdot\left(\left(LB_{\mathcal{T}}^{(i)}(u)\right)^{p}+\left(LB_{\mathcal{I}}^{(i)}(u)\right)^{p}+\left(LB_{\mathcal{F}}^{(i)}(u)\right)^{p}\right)\leq H_{\max} \tag{21}$$

where $\mathcal{D}_i$ reflects the weight or relevance of the i^{th} component (based on, e.g., sensor trust, expert reliability), and $H_{\max}$ is a threshold controlling global consistency. This definition enables multi-source or multi-phase uncertainty representation. Also, it offers a generalized Fermatean condition adjustable via p. It allows controlled aggregation via coupling tensor $\mathcal{D}$, providing both local and global constraint mechanisms.

Inspired by these advantages, we propose extending the classical weighted averaging operator to fit the above definition of HFNS.

Definition 12. Hyper-Fermatean Weighted Aggregation Operator (HF-WAO)**.** Given a set of n HFNS elements $\{u_1,u_2,\dots,u_n\}$, each associated with a weight vector $Y=(y_1,y_2,\dots,y_n)$ where $\sum_{j=1}^{n}y_j=1$, the HF-WAO aggregates the hyper-dimensional membership components as follows:

(22)

$$H\mathcal{F}_{WAO}(u_1,\ldots,u_n)=\left\{\begin{matrix}\left[\begin{matrix}1-\prod_{j=1}^{n}\prod_{i=1}^{m}\left(1-LB_{\mathcal{T}}^{(i)}(u_j)\right)^{y_j\alpha_i},\\ 1-\prod_{j=1}^{n}\prod_{i=1}^{m}\left(1-UB_{\mathcal{T}}^{(i)}(u_j)\right)^{y_j\alpha_i}\end{matrix}\right]\\ \left[\prod_{j=1}^{n}\prod_{i=1}^{m}\left(LB_{\mathcal{I}}^{(i)}(u_j)\right)^{y_j\alpha_i},\prod_{j=1}^{n}\prod_{i=1}^{m}\left(UB_{\mathcal{I}}^{(i)}(u_j)\right)^{y_j\alpha_i}\right]\\ \left[\prod_{j=1}^{n}\prod_{i=1}^{m}\left(LB_{\mathcal{F}}^{(i)}(u_j)\right)^{y_j\alpha_i},\prod_{j=1}^{n}\prod_{i=1}^{m}\left(UB_{\mathcal{F}}^{(i)}(u_j)\right)^{y_j\alpha_i}\right]\end{matrix}\right\}$$

where m is the number of hyper-levels, and α_i is the hyper-level weight (with $\sum_{i=1}^{m}\alpha_i=1$) reflecting the relative importance of each hyper-dimension (e.g., expert confidence, sensor fidelity, or temporal relevance). This operator respects both the vertical aggregation across HFNS hyper-levels via α_i, in addition to the horizontal aggregation across HFNS elements via y_j. It generalizes the traditional weighted average by incorporating hyper-dimensional consensus, capturing deeper uncertainty structures across multi-source, multi-layered HFNS information.

Definition 13. Hyper-Fermatean De-Neutrosophication Function (HF-DeN). Given an HFNS element $H=\langle u,\{\mathcal{T}^{(i)}(u)\},\{\mathcal{I}^{(i)}(u)\},\{\mathcal{F}^{(i)}(u)\}\rangle$ with $i=1,\ldots,m$, we define its de-neutrosophication value as:

$$H_{den}=\frac{1}{m}\sum_{i=1}^{m}\left(\begin{matrix}\frac{LB_{\mathcal{T}}^{(i)}(u)+UB_{\mathcal{T}}^{(i)}(u)}{2}+\left[1-\frac{LB_{\mathcal{I}}^{(i)}(u)+UB_{\mathcal{I}}^{(i)}(u)}{2}\right]\cdot UB_{\mathcal{I}}^{(i)}(u)\\ -\frac{LB_{\mathcal{F}}^{(i)}(u)+UB_{\mathcal{F}}^{(i)}(u)}{2}\cdot\left(1-UB_{\mathcal{F}}^{(i)}(u)\right)\end{matrix}\right) \tag{23}$$

Where the result averages across m hyper-level components, providing a composite measure integrating multi-dimensional uncertainty.

Definition 14. Advanced Hyper-Fermatean Score Function (HF-Score). Given a HFNS element as before, we define:

$$S(H)=\frac{1}{2m}\sum_{i=1}^{m}\left(\begin{matrix}\left[LB_{\mathcal{T}}^{(i)}(u)\right]^{\frac{1}{p}}+\left[UB_{\mathcal{T}}^{(i)}(u)\right]^{\frac{1}{p}}+\left[1-\left(LB_{\mathcal{I}}^{(i)}(u)\right)^{\frac{1}{p}}\right]+\\ \left[1-\left(UB_{\mathcal{I}}^{(i)}(u)\right)^{\frac{1}{p}}\right]+\left[1-\left(LB_{\mathcal{F}}^{(i)}(u)\right)^{\frac{1}{p}}\right]+\left[1-\left(UB_{\mathcal{F}}^{(i)}(u)\right)^{\frac{1}{p}}\right]\end{matrix}\right) \tag{24}$$

where $p \geq 3$ controls the curve's steepness but inversely via the root $\frac{1}{p}$, preventing premature suppression. Subtracting indeterminacy and falsity components from 1 reflects their negative contribution toward the score, in line with neutrosophic interpretation.

Definition 15.Operations on Hyper-Fermatean Neutrosophic Sets (HFNS). Let $H_Q = \left\langle u, \left\{\boldsymbol{\mathcal{T}}_Q^{(i)}(u)\right\}, \left\{\boldsymbol{\mathcal{I}}_Q^{(i)}(u)\right\}, \left\{\boldsymbol{\mathcal{F}}_Q^{(i)}(u)\right\}\right\rangle$ and $H_B = \left\langle u, \left\{\boldsymbol{\mathcal{T}}_B^{(i)}(u)\right\}, \left\{\boldsymbol{\mathcal{I}}_B^{(i)}(u)\right\}, \left\{\boldsymbol{\mathcal{F}}_B^{(i)}(u)\right\}\right\rangle$ be two HFNS elements with $i = 1, \dots, m$. Then, the following operations are defined

Complement. For each hyper-level i :

$$\left(\boldsymbol{\mathcal{T}}_Q^{(i)}\right)^c = \left[\left(1-\left(1-\left(LB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p\right)^\mu\right)^{1/p}, \left(1-\left(1-\left(UB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p\right)^\mu\right)^{1/p}\right]$$
$$\left(\boldsymbol{\mathcal{I}}_Q^{(i)}\right)^c = \left[\left(LB_{\boldsymbol{\mathcal{I}}}^{(i)}(u)\right)^p, \left(UB_{\boldsymbol{\mathcal{I}}}^{(i)}(u)\right)^p\right] \quad (25)$$
$$\left(\boldsymbol{\mathcal{F}}_Q^{(i)}\right)^c = \left[\left(LB_{\boldsymbol{\mathcal{F}}}^{(i)}(u)\right)^p, \left(UB_{\boldsymbol{\mathcal{F}}}^{(i)}(u)\right)^p\right]$$

where $p \geq 3$ is the hyper-order parameter, and $\mu \in (0,1]$ is an uncertainty adjustment factor.

Addition. For each hyper-level i :

$$\boldsymbol{\mathcal{T}}_{Q\oplus B}^{(i)} = \left[\left(\left(LB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p + \left(LB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p - \left(LB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p \cdot \left(LB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p\right)^{1/p}\right.$$
$$\left.\left(\left(UB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p + \left(UB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p - \left(UB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p \cdot \left(UB_{\boldsymbol{\mathcal{T}}}^{(i)}(u)\right)^p\right)^{1/p}\right] \quad (26)$$
$$\boldsymbol{\mathcal{I}}_{Q\oplus B}^{(i)} = \left[LB_{\boldsymbol{\mathcal{I}}}^{(i)}(u) \cdot LB_{\boldsymbol{\mathcal{I}}}^{(i)}(u), UB_{\boldsymbol{\mathcal{I}}}^{(i)}(u) \cdot UB_{\boldsymbol{\mathcal{I}}}^{(i)}(u)\right]$$
$$\boldsymbol{\mathcal{F}}_{Q\oplus B}^{(i)} = \left[LB_{\boldsymbol{\mathcal{F}}}^{(i)}(u) \cdot LB_{\boldsymbol{\mathcal{F}}}^{(i)}(u), UB_{\boldsymbol{\mathcal{F}}}^{(i)}(u) \cdot UB_{\boldsymbol{\mathcal{F}}}^{(i)}(u)\right]$$

Multiplication. For each hyper-level i :

(27)

$$\mathcal{T}_{Q\otimes B}^{(i)} = \left[LB_{\mathcal{T}}^{(i)}(u) \cdot LB_{\mathcal{T}}^{(i)}(u), UB_{\mathcal{T}}^{(i)}(u) \cdot UB_{\mathcal{T}}^{(i)}(u)\right]$$

$$\mathcal{I}_{Q\otimes B}^{(i)} = \left[\left(\left(\left(LB_{\mathcal{I}}^{(i)}(u)\right)^p + \left(LB_{\mathcal{I}}^{(i)}(u)\right)^p - \left(LB_{\mathcal{I}}^{(i)}(u)\right)^p \cdot \left(LB_{\mathcal{I}}^{(i)}(u)\right)^p\right)^{1/p}\right.$$
$$\left.\left(\left(\left(UB_{\mathcal{I}}^{(i)}(u)\right)^p + \left(UB_{\mathcal{I}}^{(i)}(u)\right)^p - \left(UB_{\mathcal{I}}^{(i)}(u)\right)^p \cdot \left(UB_{\mathcal{I}}^{(i)}(u)\right)^p\right)^{1/p}\right]$$

$$\mathcal{F}_{Q\otimes B}^{(i)} = \left[\left(\left(\left(LB_{\mathcal{F}}^{(i)}(u)\right)^p + \left(LB_{\mathcal{F}}^{(i)}(u)\right)^p - \left(LB_{\mathcal{F}}^{(i)}(u)\right)^p \cdot \left(LB_{\mathcal{F}}^{(i)}(u)\right)^p\right)^{1/p}\right.$$
$$\left.\left(\left(\left(UB_{\mathcal{F}}^{(i)}(u)\right)^p + \left(UB_{\mathcal{F}}^{(i)}(u)\right)^p - \left(UB_{\mathcal{F}}^{(i)}(u)\right)^p \cdot \left(UB_{\mathcal{F}}^{(i)}(u)\right)^p\right)^{1/p}\right]$$

4. Results and Discussions

This section presents a quantitative case study grounded in a real-world scenario from a large-scale civil construction firm evaluating potential investments in infrastructure projects aimed at enhancing urban resilience. The firm is considering five alternative projects: Project A focuses on constructing earthquake-resistant mid-rise residential complexes in densely populated urban centers. Project B involves developing flood-resilient transport infrastructure in a coastal metropolitan area [13], [14]. Project C centers on retrofitting aging bridges with advanced structural health monitoring systems in an industrial corridor. Project D targets the development of eco-friendly low-income housing clusters in peri-urban regions. Project E emphasizes building smart modular healthcare facilities in rapidly expanding urban districts. These projects differ in scope, scale, and operational challenges, which translate into varying degrees of risk and cost implications. The evaluation criteria considered include Construction Complexity (C1), Environmental Impact (C2), Community Disruption (C3), Resilience Enhancement Potential (C4), and Long-Term Maintenance Requirements (C5).

The HFNS membership valuations for the afore-mentioned alternatives are gathered from three independent domain experts specializing in civil infrastructure risk assessment, environmental engineering, and construction management. Each expert provided their evaluations regarding the criteria across the five alternative projects. The corresponding aggregated HFNS membership values reflecting their assessments are presented in Table 1.

Table 1. Aggregated HFNS Membership Values From Different Experts Across Five Alternative Projects

	C_1	C_2	C_3	C_4	C_5
Alt 1	T[0.19, 0.96], I[0.37, 0.75], F[0.08, 0.22]; T[0.03, 0.87],	T[0.11, 0.18], I[0.14, 0.28], F[0.46, 0.9]; T[0.32, 0.91],	T[0.16, 0.79], I[0.32, 0.9], F[0.33, 0.71]; T[0.05, 0.4],	T[0.48, 0.86], I[0.28, 0.72], F[0.21, 0.41]; T[0.18, 0.8],	T[0.31, 0.38], I[0.04, 0.71], F[0.04, 0.83]; T[0.35, 0.4],

	I[0.3, 0.8], F[0.01, 0.97]; T[0.42, 0.54], I[0.09, 0.26], F[0.15, 0.6]	I[0.4, 0.51], F[0.45, 0.75]; T[0.4, 0.94], I[0.16, 0.25], F[0.11, 0.49]	I[0.13, 0.34], F[0.49, 0.69]; T[0.45, 0.8], I[0.4, 0.7], F[0.29, 0.64]	I[0.01, 0.12], F[0.02, 0.06]; T[0.43, 0.83], I[0.24, 0.31], F[0.25, 0.61]	I[0.04, 0.99], F[0.19, 0.49]; T[0.41, 0.97], I[0.49, 0.87], F[0.19, 0.26]
Alt 2	T[0.22, 0.45], I[0.31, 0.41], F[0.15, 0.46]; T[0.23, 0.83], I[0.1, 0.56], F[0.3, 0.33]; T[0.3, 0.42], I[0.03, 0.95], F[0.48, 0.9]	T[0.41, 0.92], I[0.0, 0.51], F[0.21, 0.39]; T[0.06, 0.38], I[0.47, 0.64], F[0.26, 0.78]; T[0.18, 0.98], I[0.48, 0.61], F[0.25, 0.48]	T[0.1, 0.75], I[0.14, 0.16], F[0.32, 0.44]; T[0.47, 0.98], I[0.46, 0.66], F[0.01, 0.93]; T[0.21, 0.97], I[0.48, 0.92], F[0.15, 0.48]	T[0.09, 0.48], I[0.2, 0.69], F[0.32, 0.35]; T[0.19, 0.7], I[0.25, 0.89], F[0.33, 0.44]; T[0.04, 0.66], I[0.01, 0.59], F[0.47, 0.78]	T[0.39, 0.73], I[0.21, 0.93], F[0.06, 0.52]; T[0.01, 0.47], I[0.03, 0.15], F[0.06, 0.67]; T[0.37, 0.74], I[0.48, 0.67], F[0.14, 0.89]
Alt 3	T[0.15, 0.23], I[0.34, 0.63], F[0.06, 0.53]; T[0.02, 0.91], I[0.13, 0.71], F[0.16, 0.6]; T[0.27, 0.4], I[0.48, 0.88], F[0.47, 0.94]	T[0.14, 0.17], I[0.3, 0.65], F[0.03, 0.3]; T[0.45, 0.58], I[0.07, 0.53], F[0.49, 0.61]; T[0.34, 0.84], I[0.12, 0.76], F[0.18, 0.7]	T[0.43, 0.61], I[0.08, 0.59], F[0.47, 0.84]; T[0.29, 0.36], I[0.31, 0.99], F[0.07, 0.55]; T[0.44, 0.85], I[0.35, 0.81], F[0.18, 0.42]	T[0.19, 0.71], I[0.23, 0.65], F[0.47, 0.67]; T[0.48, 0.95], I[0.1, 0.16], F[0.05, 0.07]; T[0.05, 0.7], I[0.04, 0.35], F[0.42, 0.43]	T[0.11, 0.97], I[0.01, 0.97], F[0.02, 0.89]; T[0.26, 0.99], I[0.04, 0.57], F[0.48, 0.75]; T[0.31, 0.79], I[0.23, 0.71], F[0.29, 0.93]
Alt 4	T[0.3, 0.95], I[0.04, 0.23], F[0.02, 0.34]; T[0.19, 0.41], I[0.41, 0.62], F[0.14, 0.61]; T[0.07, 0.82], I[0.04, 0.99], F[0.39, 0.51]	T[0.32, 0.68], I[0.05, 0.84], F[0.16, 0.32]; T[0.02, 0.6], I[0.34, 0.35], F[0.26, 0.43]; T[0.32, 0.44], I[0.35, 0.6], F[0.47, 0.54]	T[0.4, 0.89], I[0.43, 0.95], F[0.26, 0.63]; T[0.4, 0.79], I[0.35, 0.87], F[0.45, 0.64]; T[0.19, 0.27], I[0.29, 0.32], F[0.23, 0.65]	T[0.41, 0.58], I[0.06, 0.71], F[0.31, 0.92]; T[0.37, 0.88], I[0.14, 0.29], F[0.38, 0.88]; T[0.5, 0.71], I[0.19, 0.82], F[0.17, 0.94]	T[0.02, 0.3], I[0.48, 0.94], F[0.23, 0.71]; T[0.14, 0.3], I[0.23, 0.5], F[0.29, 0.35]; T[0.49, 0.99], I[0.35, 0.7], F[0.15, 0.84]
Alt 5	T[0.0, 0.82], I[0.35, 0.82], F[0.39, 0.44]; T[0.18, 0.28], I[0.43, 0.79], F[0.17, 0.22]; T[0.16, 0.43], I[0.36, 0.77], F[0.44, 0.7]	T[0.17, 0.26], I[0.46, 0.93], F[0.13, 0.7]; T[0.41, 0.74], I[0.26, 0.44], F[0.05, 0.9]; T[0.45, 0.8], I[0.17, 0.46], F[0.36, 0.93]	T[0.14, 0.65], I[0.02, 0.06], F[0.41, 0.62]; T[0.06, 0.55], I[0.38, 0.51], F[0.31, 0.37]; T[0.03, 0.55], I[0.27, 0.74], F[0.36, 0.98]	T[0.43, 0.67], I[0.38, 0.85], F[0.05, 0.91]; T[0.25, 0.87], I[0.16, 0.91], F[0.19, 0.2]; T[0.45, 0.5], I[0.16, 0.96], F[0.48, 0.78]	T[0.34, 0.45], I[0.46, 0.9], F[0.47, 0.85]; T[0.31, 0.6], I[0.47, 0.93], F[0.02, 0.05]; T[0.19, 0.85], I[0.49, 0.57], F[0.3, 0.57]
Alt 6	T[0.06, 0.73], I[0.38, 0.73],	T[0.44, 0.88], I[0.32, 0.38],	T[0.26, 0.5], I[0.4, 0.56],	T[0.32, 0.62], I[0.15, 0.43],	T[0.48, 0.92], I[0.42, 0.69],

	F[0.39, 0.69];	F[0.08, 0.91];	F[0.22, 0.28];	F[0.34, 0.84];	F[0.21, 0.43];
	T[0.26, 0.58],	T[0.3, 0.31],	T[0.01, 0.96],	T[0.4, 0.87],	T[0.03, 0.87],
	I[0.01, 0.12],	I[0.05, 0.68],	I[0.42, 0.82],	I[0.05, 0.52],	I[0.41, 1.0],
	F[0.02, 0.64];	F[0.0, 0.16];	F[0.2, 0.34];	F[0.03, 0.56];	F[0.5, 0.78];
	T[0.16, 0.59],	T[0.27, 0.78],	T[0.08, 0.31],	T[0.22, 0.91],	T[0.38, 0.97],
	I[0.45, 0.59],	I[0.33, 0.48],	I[0.27, 0.79],	I[0.18, 0.28],	I[0.42, 0.56],
	F[0.21, 0.81]	F[0.36, 0.51]	F[0.33, 0.52]	F[0.07, 0.78]	F[0.23, 0.33]

Figure 2 presents the variation in lower bounds of the truth membership component across six alternatives and five criteria in the constructed HFNS decision matrix. Each subplot corresponds to a criterion, showing trends across hyper-levels, thereby illustrating how uncertainty structures differ among criteria and levels within the HFNS framework.

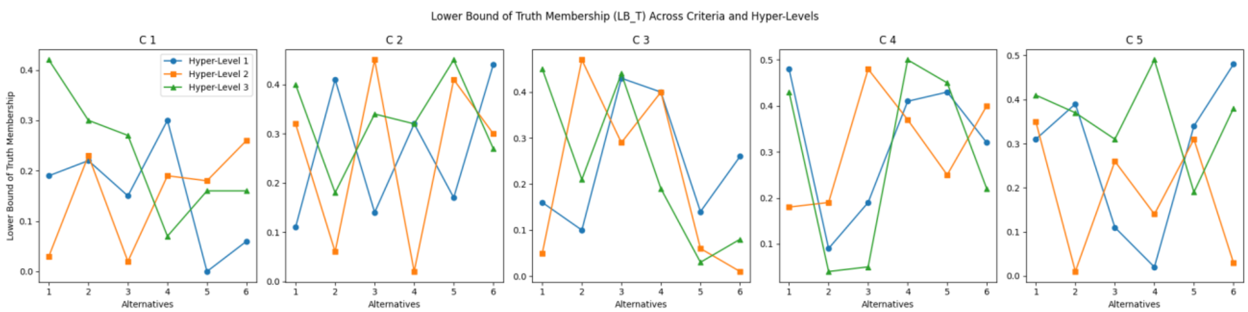

Figure 2. Lower Bound of Truth Membership values across six alternatives for each criterion and hyper-level in the HFNS decision matrix.

Then, we can translate the HFNS matrix into a crisp matrix utilizing our de-neutrosophication operator.

Table 2. The HF-DeN Matrix according to Definition 13.

Criteria / Alternative	C1	C2	C3	C4	C5
A_1	0.7093	0.5856	0.5833	0.6996	0.7328
A_2	0.6341	0.64	0.6776	0.5626	0.62
A_3	0.5583	0.6453	0.7169	0.6129	0.9119
A_4	0.6214	0.5361	0.6038	0.879	0.5397
A_5	0.4543	0.6942	0.4246	0.8067	0.6285
A_6	0.509	0.7098	0.4906	0.7483	0.734

Then, we can translate the HFNS matrix into a crisp matrix by means of our scoring operator.

Table 3. HF-Score matrix according to Definition 14, averaging across hyper-level components.

Criteria / Alternative	C1	C2	C3	C4	C5
A_1	1.4108	1.3086	1.1683	1.6476	1.4002
A_2	1.3211	1.3372	1.3862	1.209	1.3721
A_3	1.1214	1.3472	1.3063	1.5033	1.3768
A_4	1.4272	1.2747	1.2059	1.3496	1.1344
A_5	1.0345	1.2743	1.2157	1.3102	1.2558
A_6	1.3222	1.5081	1.1482	1.5024	1.2366

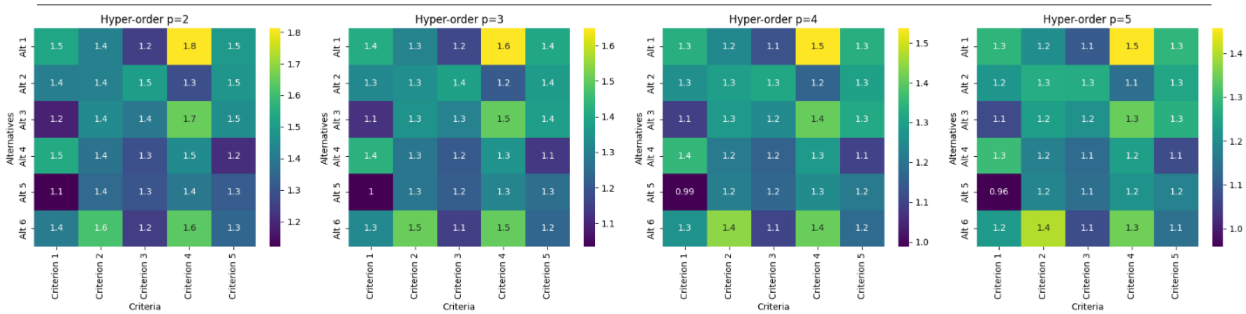

Figure 3. sensitivity analysis results in terms of HF-Score matrices across different sets of hyper-order parameter p.

In this study, a comprehensive sensitivity analysis was conducted to evaluate the robustness of the proposed Hyper-Fermatean Neutrosophic framework under varying hyperparameters. Specifically, we examined how changes in the hyper-order parameter p and the hyper-level weights. α_i Influence the computed HF-Score values and resulting alternative rankings. In Figure 3, the analysis revealed that while absolute scores shifted slightly with different p values, the relative rankings of alternatives remained largely stable, demonstrating the framework's resilience to changes in parameter settings. Similarly, in Figure 4, adjusting the hyper-level weights α_i which reflects the relative importance of hyper-level components, allowed us to test the model's capacity to adapt to differing expert confidence or sensor fidelity scenarios. The observed consistency in rankings and patterns across multiple heatmaps underscores the robustness and practical reliability of our HFNS-based decision-making approach in handling layered uncertainty structures.

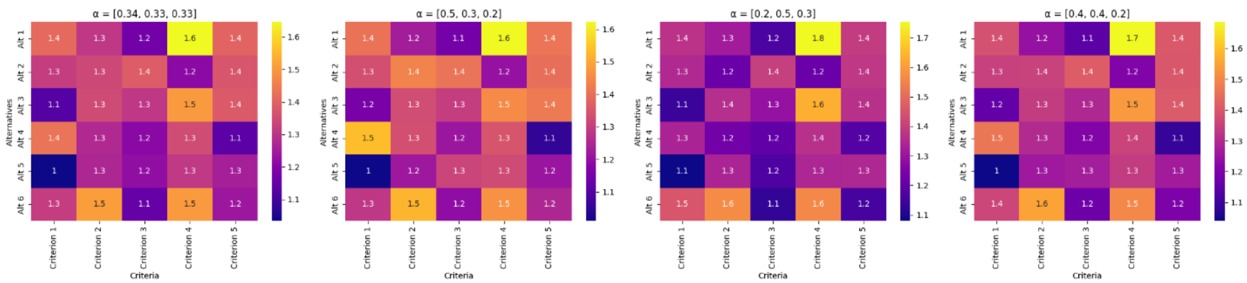

Figure 4. sensitivity analysis results in terms of HF-Score matrices across different sets of hyper-level weights α_i.

Additionally, we conducted a comparative analysis, in which we evaluated and ranked the alternatives using our proposed HFNS framework alongside classical IVFNS and FNS methods. In Figure 5, the results demonstrate noticeable changes in rankings across frameworks, which reflect the HFNS model's enhanced capacity to discriminate among alternatives under multi-level

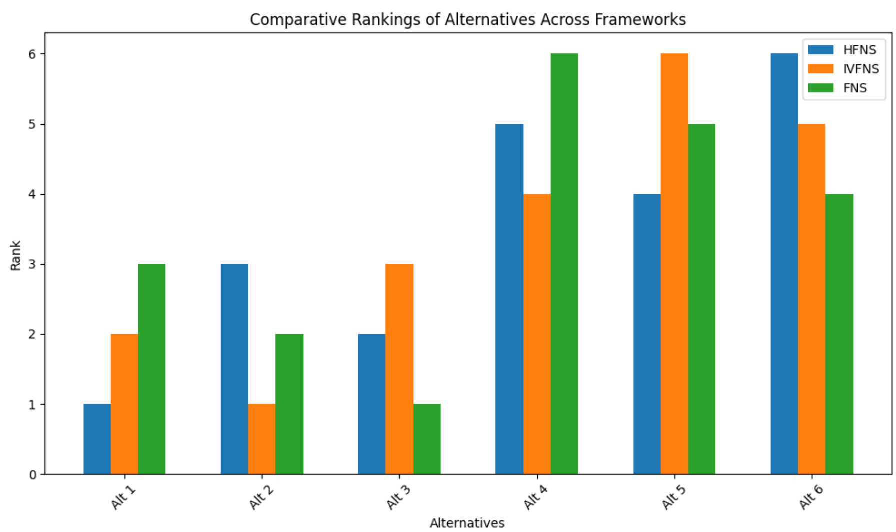

Figure 5. Comparative bar chart showing alternative rankings under HFNS, IVFNS, and FNS frameworks

uncertainty. In Figure 6, correlation analysis of the rankings further reveals moderate alignment

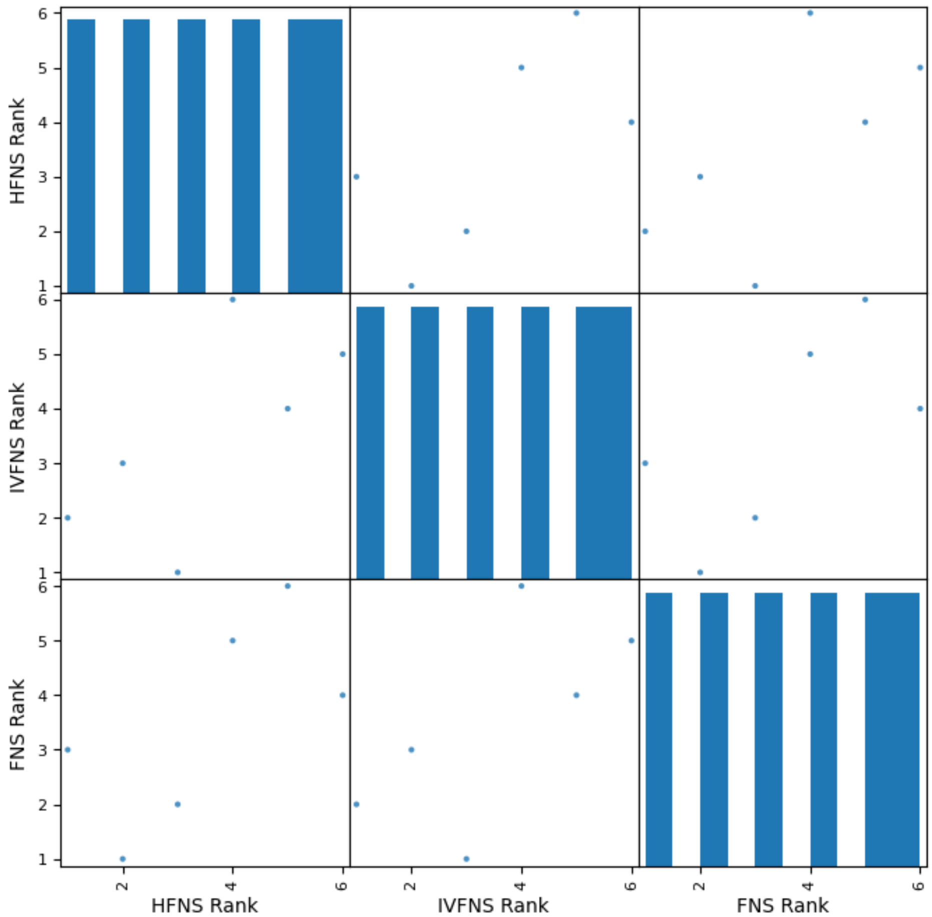

Figure 6. Scatter plot matrix illustrating relationships and correlations between rankings across the three frameworks.

with IVFNS and FNS, yet distinct prioritization patterns emerge in HFNS. This suggests that HFNS provides more nuanced and sensitive assessment outcomes, underscoring its practical value for complex decision environments in civil construction monitoring where layered uncertainties prevail.

In Figure 7, radar plots reveal clear differences in performance patterns between frameworks. HFNS generally shows more nuanced discrimination across criteria, with broader variation and sharper peaks reflecting its advanced uncertainty modeling capability. IVFNS results exhibit moderate distinction, while FNS curves are comparatively flatter, underscoring limited sensitivity to multidimensional uncertainty. This visual evidence demonstrates HFNS's potential to provide richer, more differentiated assessments in complex decision scenarios.

5. Conclusions

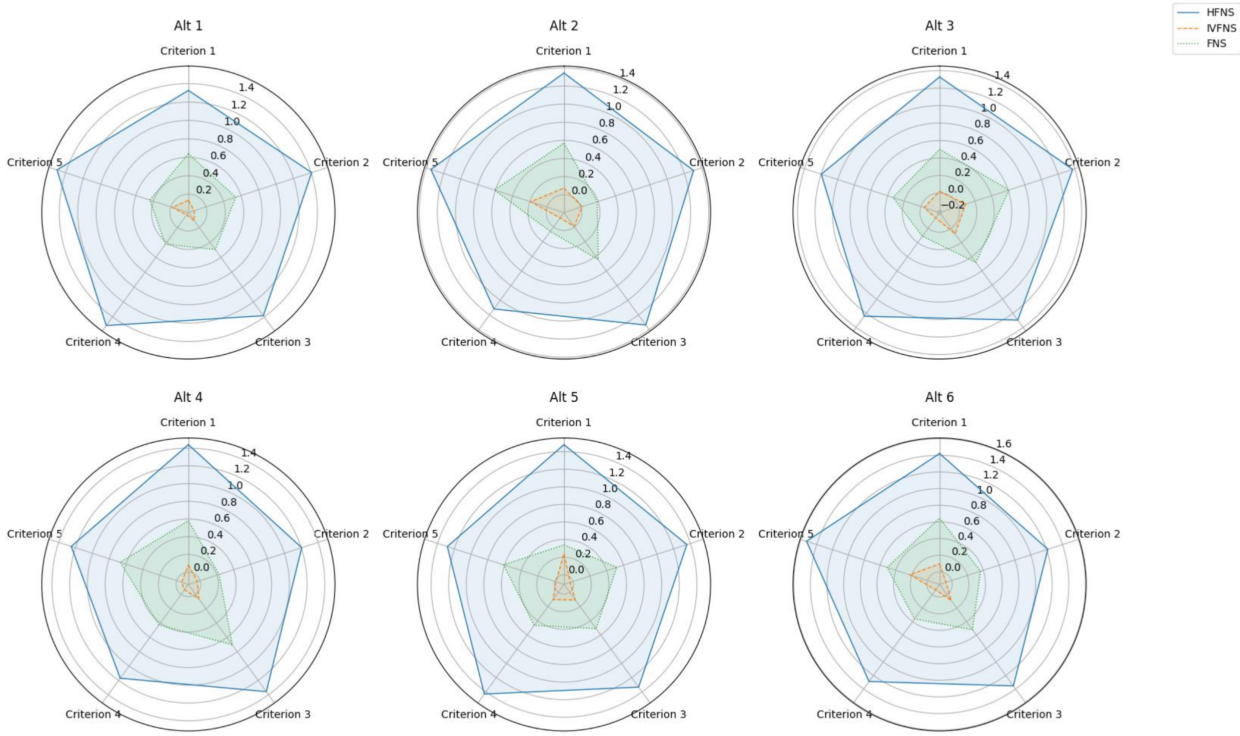

Figure 7. Radar charts comparing alternative performances across five criteria using HFNS, IVFNS, and FNS frameworks. Solid lines represent HFNS, dashed lines IVFNS, and dotted lines FNS results.

In this study, we proposed a novel framework for civil construction assessment based on Hyper-Fermatean Neutrosophic Sets (HFNS). By extending the conventional interval-valued Fermatean neutrosophic model into a hyper-dimensional structure, our approach enables a more nuanced representation of multi-source and multi-layered uncertainty inherent in civil construction monitoring. We defined advanced aggregation and scoring functions tailored to the HFNS context and demonstrated their practical applicability through a detailed case study. Comparative analyses with classical IVFNS and FNS frameworks revealed the superior discriminative power and flexibility of the proposed HFNS-based methodology. Future research will focus on developing adaptive parameter selection techniques and applying HFNS models to large-scale, real-time monitoring systems in civil engineering projects.

Acknowledgment

This work was supported by Anhui Province Quality Engineering，research on Municipal Engineering Technology Characteristic High-Level Specialty.(Project No. 2022tsgsp029) and China Higher Education Institutions Industry-University-Research Innovation Fund，Research on Smart Construction Site Safety Helmet Based on 5G and Internet of Things (Project No. 2023IT095).

References

[1] R. Sacks, I. Brilakis, E. Pikas, H. S. Xie, and M. Girolami, "Construction with digital twin information systems," *Data-Centric Eng.*, 2020, doi: 10.1017/dce.2020.16.

[2] W. Der Yu, S. T. Cheng, W. C. Ho, and Y. H. Chang, "Measuring the sustainability of construction projects throughout their lifecycle: A Taiwan Lesson," *Sustain.*, 2018, doi: 10.3390/su10051523.

[3] J. Yang, M. W. Park, P. A. Vela, and M. Golparvar-Fard, "Construction performance monitoring via still images, time-lapse photos, and video streams: Now, tomorrow, and the future," *Adv. Eng. Informatics*, 2015, doi: 10.1016/j.aei.2015.01.011.

[4] S. M. Harle, "Advancements and challenges in the application of artificial intelligence in civil engineering: a comprehensive review," 2024. doi: 10.1007/s42107-023-00760-9.

[5] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability*. 1999.

[6] F. Smarandache, *Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis*. 1998.

[7] A. Iampan, P. Jayaraman, S. D. Sudha, and N. Rajesh, "Interval-Valued Neutrosophic Ideals of Hilbert Algebras," *Int. J. Neutrosophic Sci.*, 2022, doi: 10.54216/IJNS.180420.

[8] Y. Shen, M. Xu, Y. Lin, C. Cui, X. Shi, and Y. Liu, "Safety Risk Management of Prefabricated Building Construction Based on Ontology Technology in the BIM Environment," *Buildings*, 2022, doi: 10.3390/buildings12060765.

[9] S. Broumi, R. Sundareswaran, M. Shanmugapriya, P. K. Singh, M. Voskoglou, and M. Talea, "Faculty Performance Evaluation through Multi-Criteria Decision Analysis Using Interval-Valued Fermatean Neutrosophic Sets," *Mathematics*, 2023, doi: 10.3390/math11183817.

[10] S. Broumi *et al.*, "INTERVAL- VALUED FERMATEAN NEUTROSOPHIC GRAPHS," *Decis. Mak. Appl. Manag. Eng.*, 2022, doi: 10.31181/dmame0311072022b.

[11] M. Palanikumar, A. Iampan, and S. Broumi, "MCGDM based on VIKOR and TOPSIS proposes neutrsophic Fermatean fuzzy soft with aggregation operators," *Int. J. Neutrosophic Sci.*, 2022, doi: 10.54216/IJNS.190308.

[12] P. Liu, "The Aggregation Operators Based on Archimedean t-Conorm and t-Norm for Single-Valued Neutrosophic Numbers and their Application to Decision Making," *Int. J. Fuzzy Syst.*, 2016, doi: 10.1007/s40815-016-0195-8.

[13] N. Nabeeh, "Assessment and Contrast the Sustainable Growth of Various Road Transport Systems using Intelligent Neutrosophic Multi-Criteria Decision-Making Model," *Sustain. Mach. Intell. J.*, vol. 2, 2023, doi: 10.61185/smij.2023.22102.

[14] M. Abouhawwash and M. Jameel, "Evaluation Factors of Solar Power Plants to Reduce Cost Under Neutrosophic Multi-Criteria Decision Making Model," *Sustain. Mach. Intell. J.*, 2023, doi: 10.61185/smij.2023.22101.

Received: Dec. 8, 2024. Accepted: July 2, 2025