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Mappings Of Plithogenic Cubic Sets

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Abstract:

Objectives:

To introduce novel mappings for plithogenic fuzzy cubic structures and derive core properties of stable and almost stable plithogenic neutrosophic cubic mappings.

Methods:

This study employs a theoretical approach to develop mappings for plithogenic cubic sets. We conducted a comprehensive analysis of stable and almost stable mappings, utilizing comparative techniques to assess their properties. Unique modifications include the introduction of specific stability criteria and a detailed examination of their implications in decision-making contexts.

Findings:

Our findings reveal that the newly introduced mappings exhibit distinct stability characteristics, enhancing the theoretical framework of Plithogenic cubic mapping. These mappings not only align with previous reports but also provide novel insights into their applications in optimization and data analysis. The advancements made in this study contribute valuable knowledge to the field, addressing gaps in literature and offering new avenues for future research.

Novelty:

This study presents innovative mappings and stability properties, advancing the theoretical understanding of Plithogenic cubic sets.

Keywords: Plithogenic cubic sets, Mappings, Stable mappings, Almost stable mappings, Decision-making.

1. Introduction

The Plithogenic Set (PS), a generalized extension of neutrosophic sets, was first proposed by Smarandache [10] in 2017. One or more characteristics each with a wide range of potential values, determine the components of a PS. There are appurtenance degrees linked to a certain element (like the PS) for every attribute value. Smarandache additionally improved the accuracy of plithogenic aggregation techniques by introducing for the first time the idea of a variation degree between each attribute value and the most dominating attribute value. Later, cubic sets and stable cubic sets—which blend fuzzy sets with interval-valued fuzzy sets—were created by Y.B. Jun et al. [2,4]. Additionally, they investigated and examined some of the features of internal as well as external cubic sets.

In addition to the notions of internal as well as external truth, falsehood, and indeterminacy values, Y.B. Jun, Smarandache, and Kim [3] developed neutrosophic cubic sets. Additionally, for both internal and external neutrosophic cubic sets, they suggested a variety of P and R order operations.

Recently, Priyadharshini et al. proposed the concept of plithogenic cubic sets [5, 6]. Examples are given to discuss the related internal and external cubic sets after a summary of the concepts of the Plithogenic fuzzy cubic set, Plithogenic intuitionistic fuzzy cubic set, and Plithogenic neutrosophic cubic set. The main features of the Plithogenic neutrosophic cubic sets were also studied, along with their uses.

The concepts of P and R order of Plithogenic neutrosophic cubic sets were studied by Priyadharshini et al. [7, 8]. The aforementioned principle plays significant importance in the analysis of problems involving multi-attribute decision making as Plithogenic neutrosophic cubic sets are characterized by a multitude of attribute values and very accurate data consistency.

Stable plithogenic cubic sets, which define stable, unstable, stable cut, and unstable cut, were also presented by Priyadharshini et al. [9]. Stable degrees were specified for plithogenic cubic sets. The fundamental characteristics of almost stable plithogenic cubic sets under P and R orders were also studied.

Cubic mappings, which are a type of mapping between cubic sets, have been deliberated by many researchers. However, the study of cubic mappings is still in its early stages and many questions remain unanswered.

The literature review highlights the evolution of uncertainty modeling from fuzzy and intuitionistic fuzzy sets to more advanced frameworks like cubic sets and plithogenic sets. Fuzzy sets introduced the concept of partial truth, while intuitionistic fuzzy sets added non-membership degrees. Cubic sets combined fuzzy and interval-valued fuzzy sets, providing a more comprehensive approach to ambiguity. Neutrosophic cubic sets further incorporated truth, falsity, and indeterminacy components, expanding analytical capabilities. Plithogenic cubic sets enhanced these frameworks by introducing variation degrees, enabling precise multi-attribute decision-making. Despite these advancements, the study of cubic mappings and their integration with plithogenic principles remains underdeveloped. Practical applications, such as supply chain risk management, showcase the potential of plithogenic cubic sets in addressing complex, real-world problems through accurate modeling and analysis of multi-dimensional uncertainties.

Motivation and Novelties

The study of cubic mappings is motivated by the need to develop more advanced mathematical tools for dealing with multipart and ambiguous systems. Traditional fuzzy sets and intuitionistic fuzzy sets have limitations in handling complex systems, and cubic mappings offer a more general and flexible framework for modeling and analyzing such systems.

Despite the recent developments in cubic mappings, there are still several research gaps that need to be addressed. One of the main gaps is the lack of a comprehensive study on the properties and behavior of cubic mappings. Another research gap is the need for more advanced mathematical tools for analyzing and processing cubic mappings.

The objective of this work is to fill in these research gaps by introducing new concepts and developments in cubic mappings. Specifically, we will study the properties and behavior of cubic mappings, and develop new mathematical tools for analyzing and processing these mappings.

Research Gaps

The paper aims to address the following gaps:

i. A lack of in-depth studies focusing on the properties, structures, and behaviors of cubic mappings.

ii. The absence of advanced mathematical tools to analyze and process cubic mappings, especially in complex or uncertain systems.

iii. Limited research on the interrelationships between cubic mappings and other mapping types, such as fuzzy mappings and intuitionistic fuzzy mappings.

These gaps highlight the need for further exploration to enhance both the theoretical understanding and practical applications of cubic mappings.

Novelties

The key novelties of the paper include:

i. The introduction of new concepts related to plithogenic cubic mappings.

ii. An analytical study of the unique properties and behaviors of plithogenic cubic mappings.

iii. The development of mathematical methods specifically designed for the analysis and processing of plithogenic cubic mappings.

These contributions aim to provide a foundation for advancing the theory and applications of Plithogenic cubic mappings in addressing complex mathematical and real-world problems.

The comparison study highlights that the incorporation of variation degrees in plithogenic sets and mappings significantly enhances their flexibility and accuracy, making them more effective for multi-attribute decision-making. Cubic mappings, as an extension of cubic sets, provide new avenues for exploring relationships and transformations between sets, with plithogenic cubic mappings offering greater precision and adaptability due to their unique structural characteristics. The proposed analytical tools address existing limitations by enabling more accurate analysis of variation-based mappings, particularly in complex and uncertain systems. These advancements demonstrate the applicability of plithogenic cubic mappings across diverse domains in multi-criteria decision-making, underscoring their potential for solving real-world problems involving complex attribute relationships.

2. Preliminaries

Definition 2.1 **[9]** Let $\Gamma =< X,\mu >$ be a Plithogenic fuzzy cubic sets in A, then the evaluation set of $\Gamma =< X,\mu >$ is specified to be a structure $\Xi_\Gamma = \{(a, E_\Gamma(a))|a \in A\}$ where $\Xi_\Gamma(a) = \langle L(E_\Gamma(a)), R(E_\Gamma(a))\rangle$ with $L(E_\Gamma(a)) = \mu(a) - X(a)^-$ and $R(E_\Gamma(a)) = X(a)^+ - \mu(a)$ which are called the left evaluation point and the right evaluation point respectively of $\Gamma =< X,\mu >$ at $a \in A$. We say that $\Xi_\Gamma(a)$ is the evaluation point of $\Gamma =< X,\mu >$ at $a \in A$.

Definition 2.2 [9] Let $\Gamma =< X,\mu >$ be a Plithogenic fuzzy cubic sets in A with the evaluation set $\Xi_\Gamma = \{(a, E_\Gamma(a))|a \in A\}$. An element $a \in A$ is called a stable element of $\Gamma =< X,\mu >$ in A if it satisfies $L(E_\Gamma(a)) = \mu(a) - X(a)^- \geq 0$ and $R(E_\Gamma(a)) = X(a)^+ - \mu(a) \geq 0$. Otherwise we say that 'a' is an unstable element of $\Gamma =< X,\mu >$ in A.

The set of all stable elements of $\Gamma =< X,\mu >$in A is called the stable cut of $\Gamma =< X,\mu >$in A and is denoted by SC_{Γ}.The set of all unstable elements of $\Gamma =< X,\mu >$in A is called the unstable cut of $\Gamma =< X,\mu >$in A and is denoted by USC_{Γ}. We say that $\Gamma =< X,\mu >$ is a stable Plithogenic fuzzy cubic set if $SC_{\Gamma} = A$. Otherwise $\Gamma =< X,\mu >$ is called an unstable Plithogenic fuzzy cubic set.

Definition 2.3 [9] Let $\Psi =< Y,\delta >$be a Plithogenic intuitionistic fuzzy cubic sets in A, then the evaluation set of $\Psi =< Y,\delta >$ is specified to be a structure $\Xi_{\Psi} = \{(a, E_{\Psi}(a))|a \in A\}$ where $\Xi_{\Psi}(a) = \langle L(E_{\Psi}(a)), R(E_{\Psi}(a))\rangle$ with $L(E_{\Psi}(a)) = \delta(a) - Y(a)^{-}$ and $R(E_{\Psi}(a)) = Y(a)^{+} - \delta(a)$ which are called the left evaluation point and the right evaluation point respectively of $\Psi =< Y,\delta >$at $a \in A$. We say that $\Xi_{\Psi}(a)$ is the evaluation point of $\Psi =< Y,\delta >$at $a \in A$.

Definition 2.4 [9] Let $\Psi =< Y,\delta >$be a Plithogenic intuitionistic fuzzy cubic sets in A with the evaluation set $\Xi_{\Psi} = \{(a, E_{\Psi}(a))|a \in A\}$. An element $a \in A$ is called a stable element of $\Psi =< Y,\delta >$ in A if it satisfies $L(E_{c}(a)) = \delta(a) - Y(a)^{-} \geq 0$ and $R(E_{\Psi}(a)) = Y(a)^{+} - \delta(a) \geq 0$. Otherwise we say that it is an unstable element of $\Psi =< Y,\delta >$in A.

The set of all stable elements of $\Psi =< Y,\delta >$in A is called the stable cut of $\Psi =< Y,\delta >$in A and is denoted by SC_{Ψ}.The set of all unstable elements of $\Psi =< Y,\delta >$in A is called the unstable cut of $\Psi =< Y,\delta >$in A and is denoted by USC_{Ψ}. We say that $\Psi =< Y,\delta >$ is a stable plithogenic intuitionistic fuzzy cubic set if $SC_{\Psi} = A$. Otherwise $\Psi =< Y,\delta >$ is called an unstable plithogenic intuitionistic fuzzy cubic set.

Definition 2.5 [9] Let $\Lambda =< Z, \lambda >$be a Plithogenic neutrosophic cubic sets in A, then the evaluation set of $\Lambda =< Z, \lambda >$ is specified to be a structure $\Xi_\Lambda = \{(a, E_\Lambda(a)) | a \in A\}$ where $E_\Lambda(a) = \langle L(E_\Lambda(a)), R(E_\Lambda(a))\rangle$ with $L(E_\Lambda(a)) = \lambda(a) - Z(a)^-$ and $R(E_\Lambda(a)) = Z(a)^+ - \lambda(a)$ which are called the left evaluation point and the right evaluation point respectively of $\Lambda =< Z, \lambda >$at $a \in A$.

We say that $\Xi_\Lambda(a)$ is the evaluation point of $\Lambda =< Z, \lambda >$at $a \in A$.

Definition 2.6 [9] Let $\Lambda =< Z, \lambda >$be a Plithogenic neutrosophic cubic sets in A with the evaluation set $\Xi_\Lambda = \{(a, E_\Lambda(a)) | a \in A\}$. An element $a \in A$ is called a stable element of $\Lambda =< Z, \lambda >$ in A if it satisfies $L(E_\Lambda(a)) = \lambda(a) - Z(a)^- \geq 0$ and $R(E_\Lambda(a)) = Z(a)^+ - \lambda(a) \geq 0$. Otherwise, we refer to unstable element of $\Lambda =< Z, \lambda >$in A.

The collection of all stable elements of $\Lambda =< Z, \lambda >$in A is named the stable cut of $\Lambda =< Z, \lambda >$in A and is denoted by SC_Λ.The set of all unstable elements of $\Lambda =< Z, \lambda >$in A is termed the unstable cut of $\Lambda =< Z, \lambda >$in A and is represented by USC_Λ. We say that $\Lambda =< Z, \lambda >$ is a stable Plithogenic neutrosophic cubic set if $SC_\Lambda = A$. Else $\Lambda =< Z, \lambda >$ is called an unstable Plithogenic neutrosophic cubic set.

3. Mappings of Plithogenic cubic set

Definition 3.1 Let $p : H \to N$ be a mapping and let $C =< K, \Phi >$ be a Plithogenic fuzzy Cubic set in H. then the image of $C =< K, \Phi >$under p is represented by $p(C) = \langle p(K), p(\Phi)\rangle$and is specified by $(\forall h \in H)(p(C)(h) = \langle p(K)(h), p(\Phi)(h)\rangle)$.

Definition 3.2 Let $p : H \to N$ be a mapping and let $D =< M, \Omega >$ be a Plithogenic fuzzy Cubic set in H. Then the image of inverse $D =< M, \Omega >$ under p is denoted by

$p^{-1}(D) = \langle p^{-1}(M), p(\Omega) \rangle$ and is specified by $(\forall h \in H)(p^{-1}(C)(h) = \langle p^{-1}(M)(h), p(\Omega)(h) \rangle)$

Example 3.3

Let $C =< K, \Phi >$ be a Plithogenic fuzzy Cubic set in N. For a set

Measures of contradiction	0	0.3	0.7	1
Value of traits	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure K(h)	[0.5,0.6]	[0.8,0.9]	[0.2,0.8]	[0.1,0.3]
$\Phi(h)$	0.2	0.5	0.1	0.6

Let g be a function given as $p: H \to N,\ h \mapsto \begin{cases} c & if \quad h = a \\ d & if \quad h = b \\ a & if \quad h = c \\ b & if \quad h = d \end{cases}$

Measures of contradiction	0	0.3	0.7	1
Value of traits	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure $p^{-1}\big(K(h)\big)$	[0.2,0.8]	[0.1,0.3]	[0.5,0.6]	[0.8,0.9]
$p^{-1}\big(\Phi(h)\big)$	0.1	0.6	0.2	0.5

Definition 3.4 Let $p : H \to N$ be a mapping and let $C =< K, \Phi >$ be a Plithogenic Intuitionistic fuzzy Cubic set in H. then the image of $C =< K, \Phi >$ under g is denoted by $p(C) = \langle p(K), p(\Phi) \rangle$ and is specified by $(\forall h \in H)(p(C)(h) = \langle p(K)(h), p(\Phi)(h) \rangle)$.

Definition 3.5 Let $g : H \to N$ be a mapping and let $C =< K, \Phi >$ be a Plithogenic Intuitionistic fuzzy Cubic set in H. then the image of $C =< K, \Phi >$ under g is denoted by $g(C) = \langle g(K), g(\Phi) \rangle$and is specified by $(\forall h \in H)(g(C)(h) = \langle g(K)(h), g(\Phi)(h) \rangle)$.

Example 3.6

Let $C =< K, \Phi >$ be a Plithogenic Intuitionistic fuzzy Cubic set in N. For a set

Measures of contradiction	0	0.3	0.7	1
Value of traits	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure K(h)	([0.3, 0.5], [0.7, 0.8])	([0.2, 0.9], [0.1, 0.2])	([0.5, 0.7], [0.6, 0.8])	([0.1, 0.5], [0.2, 0.8])
$\Phi(h)$	([0.1,0.4])	([0.4,0.1])	([0.6, 0.1])	([0.2, 0.7])

Let g be a function given as $g: H \to N,\ h \mapsto \begin{cases} b & if \quad h = a \\ c & if \quad h = b \\ d & if \quad h = c \\ a & if \quad h = d \end{cases}$

Measures of contradiction	0	0.3	0.7	1
Value of traits	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure $g^{-1}(K(h))$	([0.2, 0.9], [0.1, 0.2])	([0.5, 0.7], [0.6, 0.8])	([0.1, 0.5], [0.2, 0.8])	([0.3, 0.5], [0.7, 0.8])
$g^{-1}(\Phi(h))$	([0.4,0.1])	([0.6, 0.1])	([0.2, 0.7])	([0.1,0.4])

Definition 3.7 Let $g: H \to N$ be a mapping and let $C = < K, \Phi >$ be a Plithogenic Neutrosophic Cubic set in H. then the image of $C = < K, \Phi >$under g is denoted by $g(C) = \langle g(K), g(\Phi)\rangle$and is specified by $(\forall h \in H)(g(C)(h) = \langle g(K)(h), g(\Phi)(h)\rangle)$.

Definition 3.8 Let $g: H \to N$ be a mapping and let $D = < M, \Omega >$ be a Plithogenic Neutrosophic Cubic set in H. Then the image of inverse $D = < M, \Omega >$ under $g$is denoted by

$g^{-1}(D) = \langle g^{-1}(M), g(\Omega)\rangle$ and is specified by $(\forall h \in H)(g^{-1}(C)(h) = \langle g^{-1}(M)(h), g(\Omega)(h)\rangle)$

Example 3.9

Let $C = < K, \Phi >$ be a Plithogenic Neutrosophic Cubic set in N.For a set

Measures of contradiction	0	0.3	0.7	1
Value of traits	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure K(h)	([0.5, 0.1], [0.3, 0.6], [0.4, 0.8])	([0.3, 0.5], [0.2, 0.8], [0.1, 0.4])	([0.1, 0.4], [0.2, 0.3], [0.6, 0.8])	([0.3, 0.5], [0.1,0.5], [0.6, 0.8])
$\Phi(h)$	([0.1,0.2, 0.4])	([0.4,0.6, 0.7])	([0.5,0.6,0.1])	([0.2,0.5,0.3])

Let g be a function given as $g: H \to N,\ h \mapsto \begin{cases} d & if\ \ h = a \\ a & if\ \ h = b \\ b & if\ \ h = c \\ c & if\ \ h = d \end{cases}$

Measures of contradiction	0	0.3	0.7	1
Value of traits	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure	([0.3, 0.5], [0.1,0.5],	([0.5, 0.1], [0.3, 0.6],	([0.3, 0.5], [0.2, 0.8],	([0.1, 0.4], [0.2, 0.3],

$g^{-1}(K(h))$	[0.6, 0.8])	[0.4, 0.8])	[0.1, 0.4])	[0.6, 0.8])
$g^{-1}(\Phi(h))$	([0.2,0.5,0.3])	([0.1,0.2, 0.4])	([0.4,0.6, 0.7])	([0.5,0.6,0.1])

Theorem 3.10 Let $g : H \rightarrow N$ be a mapping. If $C =< K, \Phi >$ is a stable Plithogenic Neutrosophic cubic set in H, then $g(C) = \langle g(K), g(\Phi)\rangle$ is a stable Plithogenic Neutrosophic cubic set in N.

Proof: Let $C =< K, \Phi >$ be a Plithogenic neutrosophic cubic sets in H. Then $H = S_c = \{h \in H | L(\Xi_c(H)) \geq 0,\ R(\Xi_c(H)) \geq 0\}$.Hence $\Phi(h) - K(h)^- \geq 0$ and $K(h)^+ - \Phi(h) \geq 0$ for all $h \in H$.

For each $n \in N,\ if \quad g^{-1}(n) \neq \phi$, then

$$g(\Phi)(n) = \underset{n=g(h)}{Sup} \Phi(h) \geq \underset{n=g(h)}{Sup} (\Phi(h))^- = g(K)(n)^-$$

And

$$g(K)(n) = \underset{n=g(h)}{Sup} (K(h))^+ \geq \underset{n=g(h)}{Sup} \Phi(h) = g(\Phi)(n).$$

Obviously $g(\Phi)(n) \geq g(K)(n)^-$ and $g(K)(n)^+ \geq g(\Phi)(n)$ if $g^{-1}(n) \neq \phi$.This shows that $S_{g(c)} = N,$ and therefore $g(C) = \langle g(K), g(\Phi)\rangle$ is a stable Plithogenic Neutrosophic cubic set in N.

Theorem 3.11 Let $g : H \rightarrow N$ be a mapping. If $C =< K, \Phi >$ is a stable Plithogenic Neutrosophic cubic set in H, then the negation of $g(C) = \langle g(K), g(\Phi)\rangle$ is a stable Plithogenic Neutrosophic cubic set in N.

Theorem 3.12 Let $g : H \to N$ be a mapping. If $C =< K, \Phi >$and $D =< M, \Omega >$ are stable Plithogenic Neutrosophic cubic set in H, then the P-order of $g(C) = \langle g(K), g(\Phi)\rangle$and $g(D) = \langle g(M), g(\Omega)\rangle$ is a stable Plithogenic Neutrosophic cubic set in N.

Theorem 3.13 Let $g : H \to N$ be a mapping. If $C =< K, \Phi >$and $D =< M, \Omega >$ are stable Plithogenic Neutrosophic cubic set in H, then $g(C \cup D)$ and $g(C \cap D)$ are stable Plithogenic Neutrosophic cubic set in N.

Theorem 3.14 Let $g : H \to N$ be an 1-1 mapping. If $C =< K, \Phi >$is an almost stable Plithogenic Neutrosophic cubic set in H, then $g(C) = \langle g(K), g(\Phi)\rangle$is an almost stable Plithogenic Neutrosophic cubic set in N.

Proof Let $C =< K, \Phi >$be an almost stable Plithogenic Neutrosophic cubic set in H, then there exists a stable degree SDG_C such that

$$\sum_{h \in H} L(\Xi_C)(h) = \sum_{h \in H} (\Phi(h) - K(h)^-) \geq 0,$$

and

$$\sum_{h \in H} R(\Xi_C)(h) = \sum_{h \in H} \left(K(h)^+ - \Phi(h)\right) \geq 0.$$

We have to prove that $\sum_{h \in H} L(\Xi_C)(h) \geq 0$ and $\sum_{h \in H} R(\Xi_C)(h) \geq 0$ in the stable degree $SDG_{g(C)}$

For every $n \in N$, Let $N^* = \{n \in N | g^{-1}(n) \neq \varphi\}$ and $N^{**} = N \setminus N^*$. Then N is the disjoint union of N and N^* and N^{**}.If $n \in N^{**}$, then $g(K)(h) = 0$ and $g(\Phi)(h) = 0$ which imply that

$$\sum_{n \in N^{**}} \left(g(K)(n)^+ - g(\Phi)(n)\right) = 0 \text{ and } \sum_{n \in N^{**}} (g(\Phi)(n) - g(K)(n)^-) = 0.$$

Since g is 1-1, $g^{-1}(n)$ is a singleton set for all $n \in N^*$ and $\{g^{-1}(n) | n \in N^*\} = H$.

Therefore $\sum_{h \in H} L(\Xi_C)(h) = \sum_{n \in N}(g(\Phi)(n) - g(K)(n)^-)$

$$= \sum_{n \in N^*} \left(g(\Phi)(n) - g(K)(n)^-\right) + \sum_{n \in N^{**}} \left(g(\Phi)(n) - g(K)(n)^-\right)$$

$$= \sum_{n \in N^*} \left(g(\Phi)(n) - g(K)(n)^-\right)$$

$$= \sum_{h \in H} \left(\Phi(h) - K(h)^-\right) \geq 0$$

And $\sum_{h \in H} R\left(\Xi_{g(C)}\right)(h) = \sum_{n \in N}\left(g(K)(n)^+ - g(\Phi)(n)\right)$

$$= \sum_{n \in N^*} \left(g(K)(n)^+ - g(\Phi)(n)\right) + \sum_{n \in N^{**}} \left(g(K)(n)^+ - g(\Phi)(n)\right)$$

$$= \sum_{n \in N^*} \left(g(K)(n)^+ - g(\Phi)(n)\right)$$

$$= \sum_{h \in H} \left(K(h)^+ - \Phi(h)\right) \geq 0$$

Hence $g(C) = \langle g(K), g(\Phi)\rangle$is an almost stable Plithogenic Neutrosophic cubic set in N.

Theorem 3.15 Let $g : H \to N$ be an 1-1 mapping. If $C =< K, \Phi >$is an almost stable Plithogenic Neutrosophic cubic set in H, then the negation of $g(C) = \langle g(K), g(\Phi)\rangle$is an almost stable Plithogenic Neutrosophic cubic set in N.

Theorem 3.16 Let $g : H \to N$ be a mapping. If $D =< M, \Omega >$is a stable Plithogenic Neutrosophic cubic set in N, then $g^{-1}(D) = \langle g^{-1}(M), g^{-1}(\Omega)\rangle$ is a stable Plithogenic Neutrosophic cubic set in H.

Proof: Let $D =< M, \Omega >$be a stable Plithogenic Neutrosophic cubic set in N. Then

$S_D = \left\{n \in N | L\left(\Xi_D(n)\right) \geq 0,\ R\left(\Xi_D(n)\right) \geq 0\right\} = N.$

Therefore $\Omega(n) - M(n)^- \geq 0$ and $M(n)^+ - \Omega(n) \geq 0$ for all $n \in N.$

It follows that

$$L\left(\Xi_{g^{-1}(D)}(h)\right) = g^{-1}(\Omega)(h) - g^{-1}(M)(h)^{-} = \Omega\big(g(h)\big) - M\big(g(h)\big)^{-} \geq 0$$

And

$$R\left(\Xi_{g^{-1}(D)}(h)\right) = g^{-1}(M)(h)^{+} - g^{-1}(\Omega)(h) = M\big(g(h)\big)^{+} - M\big(\Omega(h)\big) \geq 0 \ for\ all\ h \in H.$$

Thus

$$S_{g^{-1}(D)} = \left\{n \in N | L\left(\Xi_{g^{-1}(D)}(n)\right) \geq 0,\ R\left(\Xi_{g^{-1}(D)}(n)\right) \geq 0\right\} = N.$$

Which implies $g^{-1}(D) = \langle g^{-1}(M), g^{-1}(\Omega)\rangle$ is a stable Plithogenic Neutrosophic cubic set in H.

Corollary 3.17 Let $g : H \to N$ be a mapping. If $C =< K, \Phi >$is a stable Plithogenic Neutrosophic cubic set in N, then $g^{-1}(D)^{c}$ is a stable Plithogenic Neutrosophic cubic set in H.

Corollary 3.18 Let $g : H \to N$ be a mapping. If $C =< K, \Phi >$and $D =< M, \Omega >$ are stable Plithogenic Neutrosophic cubic set in N, then the P-union and the P-intersection of $g^{-1}(C)$and $g^{-1}(D)$ are stable Plithogenic Neutrosophic cubic set in H

Corollary 3.19 Let $g : H \to N$ be a mapping. If $C =< K, \Phi >$and $D =< M, \Omega >$ are stable Plithogenic Neutrosophic cubic set in N, then $g^{-1}(C \cup D)$ and $g^{-1}(C \cap D)$ are stable Plithogenic Neutrosophic cubic set in H.

Corollary 3.20 Let $g : H \to N$ be a mapping. If $C =< K, \Phi >$is a stable Plithogenic Neutrosophic cubic set in H, then $g^{-1}\big(g(C)\big)$ is a stable Plithogenic Neutrosophic cubic set in H.

Corollary 3.21 Let $g : H \to N$ be a mapping. If $D =< M, \Omega >$is a stable Plithogenic Neutrosophic cubic set in H, then $g^{-1}\big(g(D)\big)$ is a stable Plithogenic Neutrosophic cubic set in H.

4. Application

Let T represents a set of environmental factors related to climate change, such as temperature (t_1), precipitation (t_2), humidity (t_3), cloudiness(t_4) and carbon emission(t_5).

Define a mapping $g : H \to N$.If $C = < K, \Phi >$is a stable Plithogenic Neutrosophic cubic set in H. Here K(h) denotes the interval valued neutrosophic set and $\Phi(h)$ denotes the neutrosophic set.

Measures of contradiction	0	0.3	0.5	0.7	1
Value of traits	temperature	precipitation	humidity	cloudiness	carbon emission
Appurtenance Measure K(h)	([0.6, 0.7], [0.3, 0.7], [0.1, 0.5])	([0.4, 0.6], [0.1, 0.4], [0.2, 0.5])	([0.3, 0.5], [0.2, 0.3], [0.7, 0.8])	([0.4, 0.7], [0.1,0.3], [0.6, 0.9])	([0.1, 0.5], [0.4,0.6], [0.7, 0.9])
$\Phi(h)$	([0.1,0.3, 0.4])	([0.3,0.2, 0.3])	([0.4,0.2,0.2])	([0.2,0.1,0.3])	([0.2,0.5,0.8])

The image of inverse of plithogenic cubic mapping is crucial for reversing cause-effect relationships, setting thresholds validating models which is calculated below. In this application, the analytical tool transforms complex relationships into actionable in sights, driving more effective interventions and strategies

Measures of contradiction	0	0.3	0.5	0.7	1
Value of traits	temperature	precipitation	humidity	cloudiness	carbon emission
Appurtenance Measure $g^{-1}(K(h))$	([0.3, 0.5], [0.2, 0.3], [0.7, 0.8])	([0.4, 0.7], [0.1,0.3], [0.6, 0.9])	([0.1, 0.5], [0.4,0.6], [0.7, 0.9])	([0.6, 0.7], [0.3, 0.7], [0.1, 0.5])	([0.4, 0.6], [0.1, 0.4], [0.2, 0.5])
$g^{-1}(\Phi(h))$	([0.4,0.2,0.2])	([0.2,0.1,0.3])	([0.2,0.5,0.8])	([0.1,0.3, 0.4])	([0.3,0.2, 0.3])

Inferences:

(i) Attributes like temperature and cloudiness have higher truth memberships in their original mapping, indicating stronger, more direct effects on the environmental system.

(ii) High Measures of contradictions in humidity and carbon emissions suggest significant uncertainty or variability in their impact.

(iii) The inverse mapping shows decreased truth memberships for most attributes, indicating weaker backward contributions.

(iv) The increase in Measures of contradictions in attributes like temperature and humidity reflects more uncertainty when tracing causes from observed effects.

5. Conclusion

In this article, we have examined the process of determining the inverse image for plithogenic cubic sets. We have also presented the essential features of stable and almost stable plithogenic cubic maps. Our study has shown that plithogenic cubic sets and their mappings can be used to model and analyze complex systems in a more general and flexible way.

6.Future Research

By exploring these areas of future research, we hope to further advance the state of the art in plithogenic cubic sets and their mappings and contribute to the growth of novel mathematical tools for dealing with complex and uncertain classifications.

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