NSS

Neutrosophic Sets and Systems, Vol. 87, 2025

University of New Mexico

Fuzzy parameterized extensions of hypersoft set embedded with possibility-degree settings: an overview

Atiqe Ur Rahman[1], Florentin Smarandache[2,*], Muhammad Saeed[1], Khuram Ali Khan[3]

[1] Department of Mathematics, University of Management and Technology, Lahore, 54000, Pakistan; aurkhb@gmail.com; muhammad.saeed@umt.edu.pk

[2] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA; smarand@unm.edu

[3] Department of Mathematics, University of Sargodha, Sargodha, 40100, Pakistan; khuram.ali@uos.edu.pk

*Correspondence: smarand@unm.edu

Abstract. The Hypersoft set marks a significant advancement in modeling vagueness and uncertainty. Concurrently, fuzzy parameterization and possibility-degree setting have been pivotal. The Hypersoft set addresses the complexity of a multi-argument domain through approximate mapping; fuzzy parameterization tackles the uncertain characteristics of parameters and their associated sub-parametric tuples; while the possibility-degree setting offers a dependable framework for evaluating the acceptance level of decision-makers' views. This research aims to introduce several hybrid set structures by integrating the above-mentioned three ideas with fuzzy set extensions. The novelty of each structure is self-explanatory and justified. To grasp each structure's true meaning, numerical examples have been used to explain the concept.

Keywords: Possibility degree, Fuzzy parameterization, multi-argument domain, Hypersoft Set, Attribute-valued Sets.

1. Introduction

The possibility theory was first introduced by Dubois and Prade [1] for assessing acceptance and rejection level in the decision analysis. It uses possibility and necessity measures to capture upper and lower bounds of uncertainty, making it particularly useful in situations with incomplete or imprecise data. Later on, Dubois [2] studied possibility theory and statistical reasoning collectively. Dubois and Prade [3] studied fuzzy information with possibility theory. Zadeh [4,5] proved the fuzzy set [6] as a basis for possibility theory. Fuzzy possibility theory is an extension of possibility theory that integrates the principles of fuzzy set theory to better handle vagueness and imprecision in uncertain information. In this framework, possibility distributions are modeled using fuzzy sets, allowing for a more flexible representation of uncertainty than traditional crisp methods. Fuzzy possibility theory is particularly useful in complex decision-making and inference systems where both uncertainty and fuzziness coexist. Fuzzy extensions [7–12] have been introduced by modifying the membership function of fuzzy sets. Molodtsov [13] introduced soft set (S-Set) as a parameterized family of objects under observation. The researchers [14–17] integrated the fuzzy extensions with possibility theory to assess the acceptance level of decision-makers' opinions regarding the approximations of alternatives based on evaluating parameters. In such concepts, a possibility grade is assigned to each approximation. In various decision-making situations, decision-makers are uncertain regarding the selection of parameters for evaluation process. Such uncertainty is managed by the concept of fuzzy set-like parameterization. The researchers [18–30] discussed various decision-making problems using the parameterization of various fuzzy set-like structures. In fuzzy parameterization, a fuzzy-valued grade is attached to each parameter for assessing its uncertainty. To equip S-Set with multi-argument domain, hypersoft set (HySS) [31] is introduced by Smarandache. Saeed et al. [32] introduced the notions of several fundamentals of HySSs and presented their numerical examples. Between 2018-2024, Smarandache [33–35] introduced six new types of S-Sets, such as: the HyperSoft Set (2018), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), SuperHyperSoft Set, TreeSoft Set (2022) and ForestSoft Set (2024). Smarandache, & Gifu [36] introduced several new extensions of S-Set and HySS that are meant to manage parametric indeterminacies. Smarandache [37] and Fujita & Smarandache [38] introduced advanced version of HySS called superhypersoft and its extensions. Rahman et al. [39–44] discussed fuzzy set-like parameterization in HySS environment and applied the idea to decision-making situations such heart disease diagnosis, evaluation of hand sanitizers, evaluation of mobile tablets and mobiles, and liver disorder diagnosis. Rahman et al. [45], Zhang et al. [46], Alballa et al. [47] and Rahman et al. [48] discussed the concepts of fuzzy parameterization and possibility grade collectively in HySS environment. Rahman et al. [49–52], Zhao et al. [53], Khan et al. [54], Saeed et al. [55], Romdhini et al. [56], Khan et al. [57], Al-Hagery and Abdalla Musa [58], Hussein et al. [59, 60], and Abdalla Musa and Al-Hagery [61] applied possibility grade settings with HySSs and discussed their applications in agri-automobile evaluation, human resource management pattern recognition, heart disease diagnosis, investment selection, educational institutions evaluation, passport quality assessment, international football rankings, network security enhancement, plant disease detection, and others.

Rahman et al. [62] introduced 36 novel mathematical hybrid set structures by combining fuzzy parameterization and HySS settings, illustrating them with numerical examples. Inspired by that study, this paper further presents 36 new mathematical structures (frameworks) by integrating HySSs, fuzzy parameterization, and possibility grade settings. Some of these have already been introduced and published, while others are being presented for the first time in this paper.

2. Preliminaries

This part presents some essential terms for proper understanding of the main results. The notation $2^{\tilde{S}}$ represents the power set of $\tilde{S}$ (initial space of objects) and $\mathcal{I}$ stands for [0,1].

Definition 2.1. A S-Set A is a set consisting of objects $(\Psi_A, \hat{E})$ defined by mapping $\Psi_A : \hat{E} \rightarrow 2^{\tilde{S}}$ such that

$$A = \left\{(\Psi_A(e), e) : e \in \hat{E} \wedge \Psi_A(e) \subseteq 2^{\tilde{S}}\right\}$$

where $\hat{E}$ is a collection of different parameters and $\Psi_A(e)$ is e-approximate element of the S-Set A with respect to parameter e.

Definition 2.2. A HySS H is a set consisting of objects $(\Psi_H, \tilde{Q})$ defined by mapping $\Psi_H : \tilde{Q} \rightarrow 2^{\tilde{S}}$ such that

$$H = \left\{(\Psi_H(\ddot{q}), \ddot{q}) : \ddot{q} \in \tilde{Q} \wedge \Psi_H(\ddot{q}) \subseteq 2^{\tilde{S}}\right\}$$

where $\Psi_H(\ddot{q})$ is $\ddot{q}-$approximate element of H with respect to multi-argument tuple $\ddot{q}$ and $\tilde{Q} = \tilde{Q}_1 \times \tilde{Q}_2 \times ... \times \tilde{Q}_n$. The sets $\tilde{Q}_1, \tilde{Q}_2, ..., \tilde{Q}_n$ are attribute-valued non-overlapping sets.

3. Hybrid-Set Structures of Fuzzy Parameterized Hypersoft Sets with Possibility Settings

In this section, some new hybrid-set structures of fuzzy parameterized hypersoft set with possibility settings are discussed with illustrative examples. Full names of all Abbreviations are presented in Table 1 and complete structure mechanism is presented by Figure 1.

TABLE 1. Abbreviations

Abbreviations	Full name	Abbreviations	Full names
FppFHSS	Fuzzy parameterized fuzzy hypersoft set	FpPiFHSS	Fuzzy parameterized picture fuzzy hypersoft set
FpIFHS	Fuzzy parameterized intuitionistic fuzzy hypersoft set	FpSVNHSS	Fuzzy parameterized single-valued neutrosophic hypersoft set
FpPFHSS	Fuzzy parameterized Pythagorean fuzzy hypersoft set	FpIVFHSS	Fuzzy parameterized interval-valued fuzzy hypersoft set

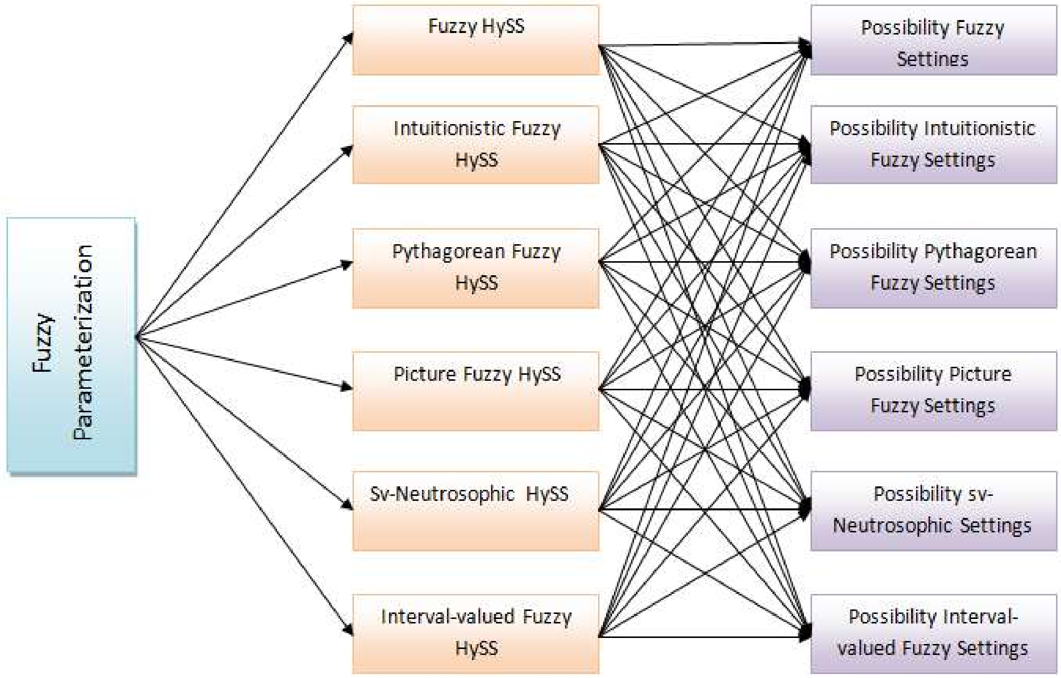

FIGURE 1. Structure Mechanism

3.1. *Hybrid-Set Structures of Fuzzy Parameterized Fuzzy Hypersoft Sets with Possibility Settings*

In this section, possibility degree is considered in terms of fuzzy membership grade, and other fuzzy set-like membership grades, e.g., intuitionistic fuzzy membership grades, neutrosophic membership grades, etc. The set $\tilde{\mathbb{Q}} = \tilde{\mathbb{Q}}_1 \times \tilde{\mathbb{Q}}_2 \times ... \times \tilde{\mathbb{Q}}_n$ is the product of attribute-valued disjoint sets $\tilde{\mathbb{Q}}_i$, $i = 1, 2, 3, ..., n$ with respect to n different attributes e_i. The set $\tilde{\mathbb{Q}}_F$ is the fuzzy set over the multi-argument tuples.

Definition 3.1. FpFHSS with fuzzy valued possibility settings
A FpFHSS of kind-I A_1 is stated as

$$A_1 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_F$ and $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 1. Mrs. Robin is intended to purchase an electric oven from the market for her domestic use. She is very much concerned regarding the diversity of sub standard ovens. Therefore, she is accompanied by two of her relatives (cousins) who have good experience of purchasing electronics appliances. Four models of reputed brands are available that constitute the set $\tilde{\mathbb{S}} = \{\ddot{s}_1, \ddot{s}_2, \ddot{s}_3, \ddot{s}_4\}$ and the set $\tilde{\mathbb{Q}}_F = \left\{ \frac{\ddot{q}_1}{.275}, \frac{\ddot{q}_2}{.321}, \frac{\ddot{q}_3}{.453}, \frac{\ddot{q}_4}{.572} \right\}$ contains fuzzy parameterized multi-argument tuples obtained by the Cartesian product $\tilde{\mathbb{Q}} = \tilde{\mathbb{Q}}_1 \times \tilde{\mathbb{Q}}_2 \times \tilde{\mathbb{Q}}_3 \times \tilde{\mathbb{Q}}_4$ of attribute-valued sets $\tilde{\mathbb{Q}}_1 = \{1.5, 2.5\}$, $\tilde{\mathbb{Q}}_2 = \{45, 95\}$, $\tilde{\mathbb{Q}}_3 = \{\textit{mineral wool}\}$, and $\tilde{\mathbb{Q}}_4 = \{220\}$

with respect to evaluating attributes e_1= load in kilowatts, e_2= volume in litres, e_3= insulation, and e_4= power supply in volts respectively then FpFHSS kind-I A_1 can be constructed by considering the approximations of experts

$$A_1 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{\left\langle \frac{\ddot{s}_1}{.211}, .312\right\rangle, \left\langle \frac{\ddot{s}_2}{.311}, .412\right\rangle, \left\langle \frac{\ddot{s}_3}{.411}, .512\right\rangle, \left\langle \frac{\ddot{s}_4}{.511}, .612\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{\left\langle \frac{\ddot{s}_1}{.221}, .322\right\rangle, \left\langle \frac{\ddot{s}_2}{.321}, .422\right\rangle, \left\langle \frac{\ddot{s}_3}{.421}, .522\right\rangle, \left\langle \frac{\ddot{s}_4}{.521}, .622\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{\left\langle \frac{\ddot{s}_1}{.231}, .332\right\rangle, \left\langle \frac{\ddot{s}_2}{.331}, .432\right\rangle, \left\langle \frac{\ddot{s}_3}{.431}, .532\right\rangle, \left\langle \frac{\ddot{s}_4}{.531}, .632\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{\left\langle \frac{\ddot{s}_1}{.241}, .342\right\rangle, \left\langle \frac{\ddot{s}_2}{.341}, .442\right\rangle, \left\langle \frac{\ddot{s}_3}{.441}, .542\right\rangle, \left\langle \frac{\ddot{s}_4}{.541}, .642\right\rangle\right\}\right), \end{array} \right\}.$$

Definition 3.2. FpFHSS with intuitionistic fuzzy valued possibility settings
A FpFHSS of kind-II A_2 is stated as

$$A_2 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \subseteq \mathfrak{A}_F, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IF}$ and $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $0 \leq \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 2. Assuming the data of Example 1, the FpFHSS of kind-II A_2 can be constructed as $A_2 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{\left\langle \frac{\ddot{s}_1}{.211}, \langle .311, .211\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.311}, \langle .411, .311\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.411}, \langle .511, .412\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.511}, \langle .611, .212\rangle\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{\left\langle \frac{\ddot{s}_1}{.221}, \langle .321, .221\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.321}, \langle .421, .323\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.421}, \langle .523, .421\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.521}, \langle .621, .224\rangle\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{\left\langle \frac{\ddot{s}_1}{.231}, \langle .331, .233\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.331}, \langle .435, .336\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.431}, \langle .534, .437\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.531}, \langle .634, .233\rangle\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{\left\langle \frac{\ddot{s}_1}{.241}, \langle .341, .243\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.341}, \langle .446, .342\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.441}, \langle .542, .444\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.541}, \langle .641, .242\rangle\right\rangle\right\}\right), \end{array} \right\}.$$

Definition 3.3. FpFHSS with Pythagorean fuzzy valued possibility settings
A FpFHSS of kind-III A_3 is stated as

$$A_3 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \subseteq \mathcal{A}^F, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PyF}$ and $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $0 \leq \tilde{\mathbb{T}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 3. Assuming the data of Example 1, the FpFHSS of kind-III A_3 can be constructed as $A_3 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{\left\langle \frac{\ddot{s}_1}{.211}, \langle .514, .614\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.311}, \langle .712, .512\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.411}, \langle .512, .812\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.511}, \langle .612, .513\rangle\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{\left\langle \frac{\ddot{s}_1}{.221}, \langle .913, .413\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.321}, \langle .412, .811\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.421}, \langle .411, .722\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.521}, \langle .522, .833\rangle\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{\left\langle \frac{\ddot{s}_1}{.231}, \langle .722, .511\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.331}, \langle .811, .533\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.431}, \langle .633, .522\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.531}, \langle .514, .614\rangle\right\rangle\right\}\right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{\left\langle \frac{\ddot{s}_1}{.241}, \langle .433, .922\rangle\right\rangle, \left\langle \frac{\ddot{s}_2}{.341}, \langle .811, .422\rangle\right\rangle, \left\langle \frac{\ddot{s}_3}{.441}, \langle .722, .433\rangle\right\rangle, \left\langle \frac{\ddot{s}_4}{.541}, \langle .833, .512\rangle\right\rangle\right\}\right), \end{array} \right\}.$$

Definition 3.4. FpFHSS with picture fuzzy valued possibility settings
A FpFHSS of kind-IV A_4 is stated as

$$A_4 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \subseteq \mathfrak{A}_F, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PF}$ and $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $0 \leq \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 4. Assuming the data of Example 1, the FpFHSS of kind-IV A_4 can be constructed as

$$A_4 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{.212}, \langle .511, .122, .233 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{.313}, \langle .211, .522, .233 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{.414}, \langle .422, .233, .344 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{.515}, \langle .622, .133, .144 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{.313}, \langle .111, .444, .444 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{.414}, \langle .333, .433, .133 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{.515}, \langle .222, .333, .333 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{.616}, \langle .211, .411, .211 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{.414}, \langle .344, .544, .144 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{.515}, \langle .244, .544, .244 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{.616}, \langle .622, .122, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{.717}, \langle .511, .222, .211 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{.515}, \langle .411, .111, .111 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{.616}, \langle .211, .211, .211 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{.717}, \langle .722, .122, .122 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{.818}, \langle .444, .233, .133 \rangle \right\rangle \end{array} \right\} \right), \end{array} \right\}.$$

Definition 3.5. FpFHSS with single-valued neutrosophic possibility settings
A FpFHSS of kind-V A_5 is stated as

$$A_5 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \subseteq \mathfrak{A}_F, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{svN}$ and $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in [0, 3]$.

Example 5. Assuming the data of Example 1, the FpFHSS of kind-V A_5 can be constructed as

$$A_5=\left\{\begin{array}{l}\left(\frac{\ddot{q}_1}{.275},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{.212},\langle .511,.611,.611\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{.313},\langle .611,.711,.711\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{.414},\langle .711,.811,.811\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{.515},\langle .811,.911,.911\rangle\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_2}{.321},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{.313},\langle .622,.522,.522\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{.414},\langle .722,.622,.622\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{.515},\langle .822,.722,.722\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{.616},\langle .933,.833,.833\rangle\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_3}{.453},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{.414},\langle .733,.633,.733\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{.515},\langle .833,.733,.833\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{.616},\langle .944,.844,.944\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{.717},\langle .644,.544,.944\rangle\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_4}{.572},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{.515},\langle .855,.755,.644\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{.616},\langle .911,.622,.511\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{.717},\langle .611,.722,.522\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{.818},\langle .611,.822,.833\rangle\right\rangle\end{array}\right\}\right),\end{array}\right\}.$$

Definition 3.6. FpFHSS with interval valued fuzzy possibility settings
A FpFHSS of kind-VI A_6 is stated as

$$A_6=\left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right):\ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\subseteq\mathfrak{A}_F,\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\subseteq\mathfrak{A}_{IVF}$ and $\varrho^F(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in\mathcal{I},\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})=\left[L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right]$ with $L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})})\in\mathcal{I}$.

Example 6. Assuming the data of Example 1, the FpFHSS of kind-VI A_6 can be constructed as

$$A_6=\left\{\begin{array}{l}\left(\frac{\ddot{q}_1}{.275},\left\{\left\langle\frac{\ddot{s}_1}{.211},[.211,.311]\right\rangle,\left\langle\frac{\ddot{s}_2}{.311},[.311,.411]\right\rangle,\left\langle\frac{\ddot{s}_3}{.411},[.411,.511]\right\rangle,\left\langle\frac{\ddot{s}_4}{.511},[.211,.611]\right\rangle\right\}\right),\\ \left(\frac{\ddot{q}_2}{.321},\left\{\left\langle\frac{\ddot{s}_1}{.221},[.221,.321]\right\rangle,\left\langle\frac{\ddot{s}_2}{.321},[.321,.421]\right\rangle,\left\langle\frac{\ddot{s}_3}{.421},[.421,.521]\right\rangle,\left\langle\frac{\ddot{s}_4}{.521},[.221,.621]\right\rangle\right\}\right),\\ \left(\frac{\ddot{q}_3}{.453},\left\{\left\langle\frac{\ddot{s}_1}{.231},[.231,.331]\right\rangle,\left\langle\frac{\ddot{s}_2}{.331},[.331,.431]\right\rangle,\left\langle\frac{\ddot{s}_3}{.431},[.431,.531]\right\rangle,\left\langle\frac{\ddot{s}_4}{.531},[.232,.632]\right\rangle\right\}\right),\\ \left(\frac{\ddot{q}_4}{.572},\left\{\left\langle\frac{\ddot{s}_1}{.241},[.243,.342]\right\rangle,\left\langle\frac{\ddot{s}_2}{.341},[.341,.442]\right\rangle,\left\langle\frac{\ddot{s}_3}{.441},[.443,.543]\right\rangle,\left\langle\frac{\ddot{s}_4}{.541},[.245,.645]\right\rangle\right\}\right),\end{array}\right\}.$$

3.2. *Hybrid-Set Structures of Fuzzy Parameterized Intuitionistic Fuzzy Hypersoft Sets with Possibility Settings*

Definition 3.7. FpIFHSS with fuzzy valued possibility settings
A FpIFHSS of kind-I B_1 is stated as

$$B_1=\left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right):\ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})=\left\langle\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right\rangle\subseteq\mathfrak{A}_{IF},\ \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})\subseteq\mathfrak{A}_F$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\ \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})\in\mathcal{I}$ such that $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in\mathcal{I}$.

Example 7. Assuming the data of Example 1, the FpIFHSS of kind-I B_1 can be constructed as

$$B_1 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .211,.511 \rangle}, .311 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .311,.611 \rangle}, .411 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .422,.533 \rangle}, .511 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .533,.244 \rangle}, .611 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .222,.333 \rangle}, .321 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .312,.413 \rangle}, .421 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .431,.221 \rangle}, .521 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .512,.121 \rangle}, .621 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .213,.515 \rangle}, .331 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .313,.313 \rangle}, .431 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .412,.113 \rangle}, .531 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .512,.231 \rangle}, .631 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .212,.231 \rangle}, .341 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .312,.513 \rangle}, .441 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .441,.512 \rangle}, .541 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .514,.313 \rangle}, .641 \right\rangle \right\} \right), \end{array} \right\}.$$

Definition 3.8. FpIFHSS with intuitionistic fuzzy valued possibility settings

A FpIFHSS of kind-II B_2 is stated as

$$B_2 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{IF}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IF}$ and $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle, \tilde{\mathbb{T}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 8. Assuming the data of Example 1, the FpIFHSS of kind-II B_2 can be constructed as

$$B_2 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .212,.414 \rangle}, \langle .311, .211 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .312,.413 \rangle}, \langle .411, .311 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .421,.521 \rangle}, \langle .512, .413 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.231 \rangle}, \langle .612, .213 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .232,.334 \rangle}, \langle .321, .224 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .332,.211 \rangle}, \langle .412, .312 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .421,.441 \rangle}, \langle .523, .412 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.121 \rangle}, \langle .612, .225 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .121,.512 \rangle}, \langle .331, .233 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .112,.421 \rangle}, \langle .413, .331 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .512,.121 \rangle}, \langle .531, .413 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .421,.223 \rangle}, \langle .632, .231 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .323,.312 \rangle}, \langle .341, .242 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .321,.512 \rangle}, \langle .442, .314 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .441,.511 \rangle}, \langle .541, .414 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .151,.333 \rangle}, \langle .614, .214 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.9. FpIFHSS with Pythagorean fuzzy valued possibility settings

A FpIFHSS of kind-III B_3 is stated as

$$B_3 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{IF}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PyF}$ and $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ and $\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}^2_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}^2_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 9. Assuming the data of Example 1, the FpIFHSS of kind-3 λ_3 can be constructed as $B_3 =$

$$\left\{\begin{array}{l}
\left(\frac{\ddot{q}_1}{.275},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .211,.121\rangle},\langle .521,.612\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .212,.321\rangle},\langle .723,.521\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .214,.421\rangle},\langle .534,.851\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .215,.521\rangle},\langle .635,.511\rangle\right\rangle\end{array}\right\}\right),\\
\left(\frac{\ddot{q}_2}{.321},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .311,.121\rangle},\langle .711,.411\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .315,.221\rangle},\langle .422,.833\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .315,.421\rangle},\langle .455,.722\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .315,.521\rangle},\langle .533,.811\rangle\right\rangle\end{array}\right\}\right),\\
\left(\frac{\ddot{q}_3}{.453},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .411,.122\rangle},\langle .711,.555\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .415,.221\rangle},\langle .822,.544\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .415,.321\rangle},\langle .622,.511\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .415,.521\rangle},\langle .566,.622\rangle\right\rangle\end{array}\right\}\right),\\
\left(\frac{\ddot{q}_4}{.572},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .515,.121\rangle},\langle .433,.611\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .415,.261\rangle},\langle .822,.433\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .415,.361\rangle},\langle .766,.444\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .415,.461\rangle},\langle .811,.511\rangle\right\rangle\end{array}\right\}\right)
\end{array}\right\}.$$

Definition 3.10. FpIFHSS with picture fuzzy valued possibility settings

A FpIFHSS of kind-IV $B_4$is stated as

$$B_4=\left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right):\ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right\rangle \subseteq \mathfrak{A}_{IF}$, and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PF}$, $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),\tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),\tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),\tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),\tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 10. Assuming the data of Example 1, the FpIFHSS of kind-IV B_4 can be constructed as

$$B_4=\left\{\begin{array}{l}
\left(\frac{\ddot{q}_1}{.275},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .211,.121\rangle},\langle .515,.121,.211\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .212,.321\rangle},\langle .215,.521,.211\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .214,.421\rangle},\langle .415,.221,.321\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .215,.521\rangle},\langle .612,.122,.211\rangle\right\rangle\end{array}\right\}\right),\\
\left(\frac{\ddot{q}_2}{.321},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .311,.121\rangle},\langle .111,.415,.421\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .315,.221\rangle},\langle .315,.421,.122\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .315,.421\rangle},\langle .212,.321,.311\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .315,.521\rangle},\langle .214,.421,.211\rangle\right\rangle\end{array}\right\}\right),\\
\left(\frac{\ddot{q}_3}{.453},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .411,.122\rangle},\langle .315,.521,.121\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .415,.221\rangle},\langle .215,.521,.211\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .415,.321\rangle},\langle .611,.211,.121\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .415,.521\rangle},\langle .415,.261,.211\rangle\right\rangle\end{array}\right\}\right),\\
\left(\frac{\ddot{q}_4}{.572},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .515,.121\rangle},\langle .411,.122,.111\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .415,.261\rangle},\langle .212,.213,.214\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .415,.361\rangle},\langle .711,.121,.122\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .415,.461\rangle},\langle .415,.221,.121\rangle\right\rangle\end{array}\right\}\right)
\end{array}\right\}.$$

Definition 3.11. FpIFHSS with single-valued neutrosophic possibility settings

A FpIFHSS of kind-V B_5 is stated as

$$B_5=\left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right):\ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{IF}$, and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{svN}$, $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in [0,3]$.

Example 11. Assuming the data of Example 1, the FpIFHSS of kind-V B_5 can be constructed as $B_5 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .211,.121\rangle}, \langle .511,.622,.633\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .212,.321\rangle}, \langle .622,.733,.744\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .214,.421\rangle}, \langle .722,.833,.844\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .215,.521\rangle}, \langle .811,.922,.911\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .311,.121\rangle}, \langle .611,.415,.451\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.221\rangle}, \langle .744,.633,.622\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .315,.421\rangle}, \langle .866,.722,.711\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.521\rangle}, \langle .911,.866,.822\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .411,.122\rangle}, \langle .755,.644,.711\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.221\rangle}, \langle .877,.722,.811\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.321\rangle}, \langle .911,.811,.922\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.521\rangle}, \langle .666,.566,.933\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .515,.121\rangle}, \langle .844,.744,.633\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.261\rangle}, \langle .922,.611,.511\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.361\rangle}, \langle .677,.788,.577\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.461\rangle}, \langle .655,.855,.844\rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.12. FpIFHSS with interval valued fuzzy possibility settings

A FpIFHSS of kind-VI B_6 is stated as

$$B_6 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{IF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ such that $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IVF}$ with $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left[L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right]$ and $L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 12. Assuming the data of Example 1, the FpIFHSS of kind-VI B_6 can be constructed as $B_6 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .211,.121\rangle}, [.211,.311] \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .212,.321\rangle}, [.311,.412] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .214,.421\rangle}, [.412,.512] \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .215,.521\rangle}, [.213,.613] \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .311,.121\rangle}, [.224,.324] \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.221\rangle}, [.321,.424] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .315,.421\rangle}, [.425,.525] \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.521\rangle}, [.225,.652] \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .411,.122\rangle}, [.263,.336] \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.221\rangle}, [.331,.436] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.321\rangle}, [.473,.537] \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.521\rangle}, [.237,.673] \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .515,.121\rangle}, [.284,.348] \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.261\rangle}, [.341,.448] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.361\rangle}, [.494,.549] \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.451\rangle}, [.284,.649] \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

3.3. *Hybrid-Set Structures of Fuzzy Parameterized Pythagorean Fuzzy Hypersoft Sets with Possibility Settings*

Definition 3.13. FpPFHSS with fuzzy valued possibility settings

A FpPFHSS of kind-I C_1 is stated as

$$C_1 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PyF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$such that $\tilde{\mathbb{T}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_F$ with $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 13. Assuming the data of Example 1, the FpPFHSS of kind-I C_1 can be constructed as $C_1 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .511,.622 \rangle}, .311 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .522,.733 \rangle}, .411 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .522,.833 \rangle}, .511 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.813 \rangle}, .611 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .611,.522 \rangle}, .321 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .622,.633 \rangle}, .421 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .622,.733 \rangle}, .521 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .611,.733 \rangle}, .621 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .711,.522 \rangle}, .331 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .722,.633 \rangle}, .431 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .721,.733 \rangle}, .531 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .741,.423 \rangle}, .631 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \left\langle \frac{\ddot{s}_1}{\langle .811,.522 \rangle}, .341 \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .822,.533 \rangle}, .441 \right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .821,.413 \rangle}, .541 \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .841,.423 \rangle}, .641 \right\rangle \right\} \right), \end{array} \right\}.$$

Definition 3.14. FpPFHSS with intuitionistic fuzzy valued possibility settings

A FpPFHSS of kind-II C_2 is stated as

$$C_2 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PyF}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle, \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 14. Assuming the data of Example 1, the FpPFHSS of kind-II C_2 can be constructed as $C_2 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .511,.622 \rangle}, \langle .311, .212 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .522,.733 \rangle}, \langle .412, .321 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .522,.833 \rangle}, \langle .521, .412 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.813 \rangle}, \langle .621, .212 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.522 \rangle}, \langle .321, .222 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .622,.633 \rangle}, \langle .422, .312 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .622,.733 \rangle}, \langle .522, .412 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .611,.733 \rangle}, \langle .612, .242 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .711,.522 \rangle}, \langle .331, .234 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .722,.633 \rangle}, \langle .453, .353 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .721,.733 \rangle}, \langle .563, .473 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .741,.423 \rangle}, \langle .613, .273 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .811,.522 \rangle}, \langle .341, .247 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .822,.533 \rangle}, \langle .447, .374 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .821,.413 \rangle}, \langle .514, .474 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .841,.423 \rangle}, \langle .614, .244 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.15. FpPFHSS with Pythagorean fuzzy valued possibility settings
A FpPFHSS of kind-III C_3 is stated as

$$C_3 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PyF}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PyF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ and $\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 15. Assuming the data of Example 1, the FpPFHSS of kind-III C_3 can be constructed as $C_3 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .511,.622\rangle}, \langle .511, .623\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .522,.733\rangle}, \langle .723, .512\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .522,.833\rangle}, \langle .512, .832\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.813\rangle}, \langle .632, .545\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.522\rangle}, \langle .654, .465\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .622,.633\rangle}, \langle .465, .865\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .622,.733\rangle}, \langle .476, .745\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .611,.733\rangle}, \langle .521, .876\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .711,.422\rangle}, \langle .723, .555\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .722,.533\rangle}, \langle .811, .534\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .721,.613\rangle}, \langle .634, .565\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .741,.733\rangle}, \langle .544, .677\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .811,.422\rangle}, \langle .466, .633\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .822,.533\rangle}, \langle .822, .499\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .821,.513\rangle}, \langle .766, .422\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .841,.423\rangle}, \langle .822, .511\rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.16. FpPFHSS with picture fuzzy valued possibility settings
A FpPFHSS of kind-IV $C_4$is stated as

$$C_4 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PyF}$, and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PF}$, $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 16. Assuming the data of Example 1, the FpPFHSS of kind-IV C_4can be constructed as $C_4 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .511,.622\rangle}, \langle .515,.121,.244\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .522,.733\rangle}, \langle .215,.521,.233\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .522,.833\rangle}, \langle .415,.221,.321\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.813\rangle}, \langle .612,.144,.255\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.522\rangle}, \langle .122,.415,.421\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .622,.633\rangle}, \langle .315,.421,.122\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .622,.733\rangle}, \langle .212,.321,.352\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .611,.733\rangle}, \langle .214,.421,.282\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .711,.422\rangle}, \langle .315,.521,.121\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .722,.533\rangle}, \langle .215,.521,.274\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .721,.613\rangle}, \langle .676,.211,.121\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .741,.733\rangle}, \langle .415,.261,.232\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .811,.422\rangle}, \langle .411,.122,.143\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .822,.533\rangle}, \langle .342,.451,.111\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .821,.513\rangle}, \langle .721,.167,.292\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .841,.423\rangle}, \langle .415,.221,.121\rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.17. FpPFHSS with single-valued neutrosophic possibility settings

A FpPFHSS of kind-V C_5 is stated as

$$C_5 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PyF}$, and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}^2_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{svN}$, $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in [0,3]$.

Example 17. Assuming the data of Example 1, the FpPFHSS of kind-V C_5 can be constructed as $C_5 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .511,.622\rangle}, \langle .522,.633,.644\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .522,.733\rangle}, \langle .655,.744,.733\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .522,.833\rangle}, \langle .744,.855,.822\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.813\rangle}, \langle .833,.911,.933\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.522\rangle}, \langle .644,.415,.451\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .622,.633\rangle}, \langle .744,.612,.634\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .622,.733\rangle}, \langle .822,.711,.764\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .611,.733\rangle}, \langle .911,.856,.823\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .711,.422\rangle}, \langle .733,.612,.732\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .722,.533\rangle}, \langle .867,.722,.834\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .721,.613\rangle}, \langle .945,.834,.922\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .741,.733\rangle}, \langle .688,.574,.953\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .811,.422\rangle}, \langle .844,.733,.622\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .822,.433\rangle}, \langle .911,.622,.533\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .821,.513\rangle}, \langle .623,.734,.544\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .841,.533\rangle}, \langle .633,.866,.877\rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.18. FpPFHSS with interval valued fuzzy possibility settings

A FpPFHSS of kind-VI $C_6$is stated as

$$C_6 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PyF}$ and $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ such that $\tilde{\mathbb{T}}^2_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}^2_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IVF}$ with $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left[L_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right]$ and $L_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_{\zeta}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 18. Assuming the data of Example 1, the FpPFHSS of kind-VI C_6 can be constructed as $C_6 =$

$$\left\{\begin{array}{l}
\left(\frac{\ddot{q}_1}{.275}, \left\{\begin{array}{l}\left\langle \frac{\ddot{s}_1}{\langle .511,.622\rangle}, [.213,.318]\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .522,.733\rangle}, [.311,.413]\right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .522,.833\rangle}, [.413,.516]\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .521,.813\rangle}, [.213,.617]\right\rangle \end{array}\right\}\right), \\
\left(\frac{\ddot{q}_2}{.321}, \left\{\begin{array}{l}\left\langle \frac{\ddot{s}_1}{\langle .611,.522\rangle}, [.224,.327]\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .622,.633\rangle}, [.321,.425]\right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .622,.733\rangle}, [.421,.525]\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .611,.733\rangle}, [.221,.623]\right\rangle \end{array}\right\}\right), \\
\left(\frac{\ddot{q}_3}{.453}, \left\{\begin{array}{l}\left\langle \frac{\ddot{s}_1}{\langle .711,.422\rangle}, [.231,.323]\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .722,.533\rangle}, [.331,.413]\right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .721,.613\rangle}, [.433,.513]\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .741,.733\rangle}, [.233,.613]\right\rangle \end{array}\right\}\right), \\
\left(\frac{\ddot{q}_4}{.572}, \left\{\begin{array}{l}\left\langle \frac{\ddot{s}_1}{\langle .811,.422\rangle}, [.243,.342]\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .822,.533\rangle}, [.341,.443]\right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .821,.413\rangle}, [.441,.544]\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .841,.533\rangle}, [.246,.647]\right\rangle \end{array}\right\}\right)
\end{array}\right\}.$$

3.4. *Hybrid-Set Structures of Fuzzy Parameterized Picture Fuzzy Hypersoft Sets with Possibility Settings*

Definition 3.19. FpPiFHSS with fuzzy valued possibility settings
A FpPiFHSS of kind-I D_1 is stated as

$$D_1 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PF}$ and $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$such that $\tilde{\mathbb{T}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_{\varrho}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_F$ with $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 19. Assuming the data of Example 1, the FpPiFHSS of kind-I D_1 can be constructed as $D_1 =$

$$\left\{\begin{array}{l}
\left(\frac{\ddot{q}_1}{.275}, \left\{\left\langle \frac{\ddot{s}_1}{\langle .215,.521,.121\rangle}, .323\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.521,.121\rangle}, .434\right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .415,.421,.122\rangle}, .534\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.261,.121\rangle}, .623\right\rangle\right\}\right), \\
\left(\frac{\ddot{q}_2}{.321}, \left\{\left\langle \frac{\ddot{s}_1}{\langle .212,.321,.222\rangle}, .265\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.421,.234\rangle}, .322\right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .415,.221,.234\rangle}, .456\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .515,.121,.223\rangle}, .533\right\rangle\right\}\right), \\
\left(\frac{\ddot{q}_3}{.453}, \left\{\left\langle \frac{\ddot{s}_1}{\langle .215,.521,.383\rangle}, .134\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.321,.387\rangle}, .291\right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .411,.122,.382\rangle}, .345\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.261,.281\rangle}, .483\right\rangle\right\}\right), \\
\left(\frac{\ddot{q}_4}{.572}, \left\{\left\langle \frac{\ddot{s}_1}{\langle .277,.214,.421\rangle}, .599\right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.521,.121\rangle}, .671\right\rangle, \left\langle \frac{\ddot{s}_3}{\langle .415,.421,.122\rangle}, .772\right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.361,.121\rangle}, .882\right\rangle\right\}\right),
\end{array}\right\}.$$

Definition 3.20. FpPiFHSS with intuitionistic fuzzy valued possibility settings
A FpPiFHSS of kind-II D_2 is stated as

$$D_2 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$, with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IF}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 20. Assuming the data of Example 1, the FpPiFHSS of kind-II D_2 can be constructed as $D_2 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .214,.421,.122 \rangle}, \langle .315, .221 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.421,.122 \rangle}, \langle .415, .321 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.421,.122 \rangle}, \langle .415, .461 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.261,.121 \rangle}, \langle .613, .223 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .212,.321,.271 \rangle}, \langle .311, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.221,.232 \rangle}, \langle .415, .221 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.421,.122 \rangle}, \langle .415, .361 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .515,.121,.281 \rangle}, \langle .631, .251 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .133,.415,.361 \rangle}, \langle .315, .321 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .126,.415,.321 \rangle}, \langle .315, .421 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .515,.121,.381 \rangle}, \langle .315, .521 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.221,.321 \rangle}, \langle .315, .621 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .315,.321,.371 \rangle}, \langle .315, .421 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.521,.261 \rangle}, \langle .315, .521 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.425,.221 \rangle}, \langle .315, .621 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.325,.221 \rangle}, \langle .315, .321 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.21. FpPiFHSS with Pythagorean fuzzy valued possibility settings
A FpPiFHSS of kind-III D_3 is stated as

$$D_3 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$.
Similarly
$\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PyF}$ and $\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 21. Assuming the data of Example 1, the FpPiFHSS of kind-III D_3 can be constructed as $D_3 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .211,.121,.145\rangle}, \langle .521, .687\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .212,.321,.121\rangle}, \langle .781, .533\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .214,.421,.122\rangle}, \langle .532, .871\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .215,.521,.121\rangle}, \langle .671, .561\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .311,.121,.233\rangle}, \langle .788, .433\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.221,.412\rangle}, \langle .481, .861\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .315,.421,.244\rangle}, \langle .466, .777\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.521,.121\rangle}, \langle .512, .815\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .411,.122,.334\rangle}, \langle .781, .516\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.221,.321\rangle}, \langle .871, .571\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.321,.121\rangle}, \langle .666, .521\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.421,.122\rangle}, \langle .571, .611\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .515,.121,.251\rangle}, \langle .461, .781\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.261,.121\rangle}, \langle .811, .451\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.361,.121\rangle}, \langle .721, .461\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.261,.121\rangle}, \langle .718, .455\rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.22. FpPiFHSS with picture fuzzy valued possibility settings
A FpPiFHSS of kind-IV $D_4$is stated as

$$D_4 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PF}$, and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PF}$, $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 22. Assuming the data of Example 1, the FpPiFHSS of kind-IV D_4 can be constructed as $D_4 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .211,.121,.133\rangle}, \langle .515, .121, .255\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .212,.321,.121\rangle}, \langle .215, .521, .241\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .214,.421,.122\rangle}, \langle .415, .221, .321\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .215,.521,.121\rangle}, \langle .631, .144, .211\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .311,.121,.233\rangle}, \langle .166, .415, .421\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.221,.277\rangle}, \langle .315, .421, .122\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .315,.421,.233\rangle}, \langle .212, .321, .388\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.521,.121\rangle}, \langle .214, .421, .233\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .411,.122,.243\rangle}, \langle .315, .521, .121\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.221,.121\rangle}, \langle .215, .521, .273\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.321,.121\rangle}, \langle .652, .211, .121\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.421,.122\rangle}, \langle .415, .261, .232\rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .515,.121,.276\rangle}, \langle .411, .122, .175\rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.261,.121\rangle}, \langle .254, .298, .253\rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.361,.121\rangle}, \langle .435, .133, .199\rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .214,.421,.122\rangle}, \langle .415, .221, .121\rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.23. FpPiFHSS with single-valued neutrosophic possibility settings
A FpPiFHSS of kind-V D_5 is stated as

$$D_5 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PF}$, and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{svN}$, $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in [0,3]$.

Example 23. Assuming the data of Example 1, the FpPiFHSS of kind-V D_5 can be constructed as $D_5 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .211,.121,.123 \rangle}, \langle .523, .645, .634 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .212,.321,.121 \rangle}, \langle .622, .755, .721 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .214,.421,.122 \rangle}, \langle .722, .811, .855 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .215,.521,.121 \rangle}, \langle .851, .931, .926 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .311,.121,.223 \rangle}, \langle .623, .415, .451 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .315,.221,.211 \rangle}, \langle .723, .625, .671 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .315,.421,.223 \rangle}, \langle .811, .771, .731 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.521,.121 \rangle}, \langle .911, .822, .855 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .411,.122,.326 \rangle}, \langle .722, .631, .726 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.221,.321 \rangle}, \langle .881, .781, .823 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.325,.221 \rangle}, \langle .911, .812, .922 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .415,.421,.122 \rangle}, \langle .626, .531, .911 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .515,.121,.257 \rangle}, \langle .811, .723, .677 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .415,.261,.121 \rangle}, \langle .922, .611, .534 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .415,.361,.121 \rangle}, \langle .622, .755, .511 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .315,.421,.122 \rangle}, \langle .682, .882, .834 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.24. FpPiFHSS with interval valued fuzzy possibility settings
A FpPiFHSS of kind-VI $D_6$is stated as

$$D_6 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{PF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ such that $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IVF}$ with $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left[L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right]$ and $L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 24. Assuming the data of Example 1, the FpPiFHSS of kind-VI D_6 can be constructed as $D_6 =$

$$\left\{\begin{array}{l}\left(\frac{\ddot{q}_1}{.275},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .211,.121,.121\rangle},[.212,.312]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .212,.321,.121\rangle},[.314,.414]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .214,.421,.122\rangle},[.434,.543]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .215,.521,.121\rangle},[.525,.634]\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_2}{.321},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .311,.121,.242\rangle},[.611,.712]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .315,.221,.281\rangle},[.711,.888]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .315,.421,.221\rangle},[.811,.981]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .315,.521,.121\rangle},[.213,.413]\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_3}{.453},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .411,.122,.429\rangle},[.314,.515]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .415,.221,.299\rangle},[.434,.634]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .415,.321,.121\rangle},[.554,.756]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .415,.421,.122\rangle},[.611,.899]\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_4}{.572},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .515,.121,.211\rangle},[.781,.923]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .415,.261,.121\rangle},[.213,.513]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .415,.361,.121\rangle},[.315,.616]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .214,.421,.122\rangle},[.434,.723]\right\rangle\end{array}\right\}\right)\end{array}\right\}.$$

3.5. *Hybrid-Set Structures of Fuzzy Parameterized Single-valued Neutrosophic Hypersoft Sets with Possibility Settings*

Definition 3.25. FpSVNHSS with fuzzy valued possibility settings

A FpSVNHSS of kind-I E_1 is stated as

$$E_1=\left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right):\ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right\rangle\subseteq\mathfrak{A}_{svN}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in\mathcal{I}$such that $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in[0,3]$. Similarly $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})\subseteq\mathfrak{A}_F$ with $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})})\in\mathcal{I}$.

Example 25. Assuming the data of Example 1, the FpSVNHSS of kind-I D_1 can be constructed as $D_1 =$

$$\left\{\begin{array}{l}\left(\frac{\ddot{q}_1}{.275},\left\{\left\langle\frac{\ddot{s}_1}{\langle .812,.834,.921\rangle},.312\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .812,.821,.854\rangle},.456\right\rangle,\left\langle\frac{\ddot{s}_3}{\langle .871,.856,.743\rangle},.547\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .834,.856,.643\rangle},.651\right\rangle\right\}\right),\\ \left(\frac{\ddot{q}_2}{.321},\left\{\left\langle\frac{\ddot{s}_1}{\langle .712,.734,.956\rangle},.226\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .776,.756,.821\rangle},.312\right\rangle,\left\langle\frac{\ddot{s}_3}{\langle .755,.733,.722\rangle},.456\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .765,.755,.611\rangle},.547\right\rangle\right\}\right),\\ \left(\frac{\ddot{q}_3}{.453},\left\{\left\langle\frac{\ddot{s}_1}{\langle .611,.622,.933\rangle},.191\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .633,.644,.855\rangle},.226\right\rangle,\left\langle\frac{\ddot{s}_3}{\langle .633,.644,.711\rangle},.312\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .633,.988,.622\rangle},.456\right\rangle\right\}\right),\\ \left(\frac{\ddot{q}_4}{.572},\left\{\left\langle\frac{\ddot{s}_1}{\langle .955,.944,.833\rangle},.547\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .966,.922,.745\rangle},.651\right\rangle,\left\langle\frac{\ddot{s}_3}{\langle .988,.977,.623\rangle},.723\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .967,.945,.534\rangle},.811\right\rangle\right\}\right),\end{array}\right\}.$$

Definition 3.26. FpSVNHSS with intuitionistic fuzzy valued possibility settings

A FpSVNHSS of kind-II E_2 is stated as

$$E_2=\left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right):\ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right\rangle \subseteq \mathfrak{A}_{svN}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in\mathcal{I}$, with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in$

$[0,3]$. Similarly $\zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IF}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 26. Assuming the data of Example 1, the FpSVNHSS of kind-II E_2 can be constructed as $E_2 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .812,.834,.921 \rangle}, \langle .315, .221 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .812,.821,.854 \rangle}, \langle .415, .321 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .871,.856,.743 \rangle}, \langle .415, .461 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .834,.856,.643 \rangle}, \langle .623, .226 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .712,.734,.956 \rangle}, \langle .311, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .776,.756,.821 \rangle}, \langle .415, .221 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .755,.733,.722 \rangle}, \langle .415, .361 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .765,.755,.611 \rangle}, \langle .634, .191 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.622,.933 \rangle}, \langle .315, .321 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .633,.644,.855 \rangle}, \langle .315, .421 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .633,.644,.711 \rangle}, \langle .315, .521 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .633,.988,.622 \rangle}, \langle .315, .621 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .955,.944,.833 \rangle}, \langle .315, .421 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .966,.922,.745 \rangle}, \langle .315, .521 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .988,.977,.623 \rangle}, \langle .315, .621 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .967,.945,.534 \rangle}, \langle .315, .321 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.27. FpSVNHSS with Pythagorean fuzzy valued possibility settings
A FpSVNHSS of kind-III E_3 is stated as

$$E_3 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{svN}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in [0,3]$. Similarly $\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PyF}$ and $\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 27. Assuming the data of Example 1, the FpSVNHSS of kind-III E_3 can be constructed as $E_3 =$

$$\left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .812,.834,.921 \rangle}, \langle .512, .651 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .812,.821,.854 \rangle}, \langle .781, .547 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .871,.856,.743 \rangle}, \langle .512, .811 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .834,.856,.643 \rangle}, \langle .671, .547 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .712,.734,.956 \rangle}, \langle .911, .456 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .776,.756,.821 \rangle}, \langle .432, .811 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .755,.733,.722 \rangle}, \langle .432, .723 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .765,.755,.611 \rangle}, \langle .512, .811 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.622,.933 \rangle}, \langle .781, .547 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .633,.644,.855 \rangle}, \langle .851, .547 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .633,.644,.711 \rangle}, \langle .671, .547 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .633,.988,.622 \rangle}, \langle .512, .651 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .955,.944,.833 \rangle}, \langle .432, .926 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .966,.922,.745 \rangle}, \langle .851, .456 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .988,.977,.623 \rangle}, \langle .781, .456 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .967,.945,.534 \rangle}, \langle .851, .547 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}$$

Definition 3.28. FpSVNHSS with picture fuzzy valued possibility settings

A FpSVNHSS of kind-IV $E_4$is stated as

$$E_4 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{svN}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in [0,3]$. Similarly $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PF}$, $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 28. Assuming the data of Example 1, the FpSVNHSS of kind-IV E_4can be constructed as $E_4 =$

$$\left\{ \begin{array}{l}
\left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .812,.834,.921 \rangle}, \langle .515, .121, .226 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .812,.821,.854 \rangle}, \langle .215, .521, .226 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .871,.856,.743 \rangle}, \langle .415, .221, .321 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .834,.856,.643 \rangle}, \langle .671, .123, .226 \rangle \right\rangle \end{array} \right\} \right), \\
\left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .712,.734,.956 \rangle}, \langle .126, .415, .421 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .776,.756,.821 \rangle}, \langle .315, .421, .122 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .755,.733,.722 \rangle}, \langle .212, .321, .312 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .765,.755,.611 \rangle}, \langle .214, .421, .226 \rangle \right\rangle \end{array} \right\} \right), \\
\left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .611,.622,.933 \rangle}, \langle .315, .521, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .633,.644,.855 \rangle}, \langle .215, .521, .226 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .633,.644,.711 \rangle}, \langle .671, .211, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .633,.988,.622 \rangle}, \langle .415, .261, .226 \rangle \right\rangle \end{array} \right\} \right), \\
\left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{\langle .955,.944,.833 \rangle}, \langle .411, .122, .191 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{\langle .966,.922,.745 \rangle}, \langle .234, .282, .226 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{\langle .988,.977,.623 \rangle}, \langle .781, .174, .191 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{\langle .967,.945,.534 \rangle}, \langle .415, .221, .121 \rangle \right\rangle \end{array} \right\} \right)
\end{array} \right\}.$$

Definition 3.29. FpSVNHSS with single-valued neutrosophic possibility settings

A FpSVNHSS of kind-V E_5 is stated as

$$E_5 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle \tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right\rangle \subseteq \mathfrak{A}_{svN}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$ with $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) + \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in [0,3]$. Similarly $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{svN}$, $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in [0,3]$.

Example 29. Assuming the data of Example 1, the FpSVNHSS of kind-V E_5 can be constructed as $E_5 =$

$$\left\{\begin{array}{l}\left(\frac{\ddot{q}_1}{.275},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .812,.834,.921\rangle},\langle .512,.634,.651\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .812,.821,.854\rangle},\langle .671,.721,.723\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .871,.856,.743\rangle},\langle .781,.823,.811\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .834,.856,.643\rangle},\langle .851,.911,.926\rangle\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_2}{.321},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .712,.734,.956\rangle},\langle .671,.415,.451\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .776,.756,.821\rangle},\langle .781,.623,.651\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .755,.733,.722\rangle},\langle .851,.735,.723\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .765,.755,.611\rangle},\langle .911,.811,.811\rangle\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_3}{.453},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .611,.622,.933\rangle},\langle .781,.681,.723\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .633,.644,.855\rangle},\langle .851,.721,.811\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .633,.644,.711\rangle},\langle .911,.823,.926\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .633,.988,.622\rangle},\langle .671,.581,.926\rangle\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_4}{.572},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .955,.944,.833\rangle},\langle .851,.722,.651\rangle\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .966,.922,.745\rangle},\langle .911,.633,.547\rangle\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .988,.977,.623\rangle},\langle .671,.744,.547\rangle\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .967,.945,.534\rangle},\langle .671,.855,.811\rangle\right\rangle\end{array}\right\}\right)\end{array}\right\}.$$

Definition 3.30. FpSVNHSS with interval valued fuzzy possibility settings
A FpSVNHSS of kind-VI $E_6$is stated as

$$D_6 = \left\{\left(\frac{\ddot{q}}{\nu_F(\ddot{q})},\left\langle\frac{\ddot{s}}{\varrho^{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})},\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right\rangle\right) : \ddot{s}\in\tilde{\mathbb{S}}\wedge\frac{\ddot{q}}{\nu_F(\ddot{q})}\in\tilde{\mathbb{Q}}_F,\nu_F(\ddot{q})\in\mathcal{I}\right\}$$

where $\varrho^{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left\langle\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right\rangle\subseteq\mathfrak{A}_{svN}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}),\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in\mathcal{I}$ such that $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{I}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})+\tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\in[0,3]$. Similarly $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})\subseteq\mathfrak{A}_{IVF}$ with $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left[L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right]$ and $L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}),U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})})\in\mathcal{I}$.

Example 30. Assuming the data of Example 1, the FpSVNHSS of kind-VI E_6 can be constructed as

$$E_6 = \left\{\begin{array}{l}\left(\frac{\ddot{q}_1}{.275},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .812,.834,.921\rangle},[.212,.312]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .812,.821,.854\rangle},[.314,.414]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .871,.856,.743\rangle},[.434,.543]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .834,.856,.643\rangle},[.525,.634]\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_2}{.321},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .712,.734,.956\rangle},[.634,.723]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .776,.756,.821\rangle},[.734,.811]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .755,.733,.722\rangle},[.811,.926]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .765,.755,.611\rangle},[.213,.413]\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_3}{.453},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .611,.622,.933\rangle},[.314,.515]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .633,.644,.855\rangle},[.434,.634]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .633,.644,.711\rangle},[.554,.756]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .633,.988,.622\rangle},[.622,.811]\right\rangle\end{array}\right\}\right),\\ \left(\frac{\ddot{q}_4}{.572},\left\{\begin{array}{l}\left\langle\frac{\ddot{s}_1}{\langle .955,.944,.833\rangle},[.766,.926]\right\rangle,\left\langle\frac{\ddot{s}_2}{\langle .966,.922,.745\rangle},[.213,.513]\right\rangle,\\ \left\langle\frac{\ddot{s}_3}{\langle .988,.977,.623\rangle},[.315,.616]\right\rangle,\left\langle\frac{\ddot{s}_4}{\langle .967,.945,.534\rangle},[.434,.723]\right\rangle\end{array}\right\}\right)\end{array}\right\}.$$

3.6. *Hybrid-Set Structures of Fuzzy Parameterized Interval-valued Fuzzy Hypersoft Sets with Possibility Settings*

Definition 3.31. FpIVFHSS with fuzzy valued possibility settings

A FpIVFHSS of kind-I $\tilde{\mathbb{F}}_1$ is stated as

$$\tilde{\mathbb{F}}_1 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left[L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right] \subseteq \mathfrak{A}_{IVF}$ and $L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_F$ with $\zeta_F(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 31. Assuming the data of Example 3.1.1, the FpIVFHSS of kind-I $\tilde{\mathbb{F}}_1$ can be constructed as

$$\tilde{\mathbb{F}}_1 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \left\langle \frac{\ddot{s}_1}{[.212,.312]}, .311 \right\rangle, \left\langle \frac{\ddot{s}_2}{[.213,.413]}, .411 \right\rangle, \left\langle \frac{\ddot{s}_3}{[.213,.513]}, .511 \right\rangle, \left\langle \frac{\ddot{s}_4}{[.214,.614]}, .611 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \left\langle \frac{\ddot{s}_1}{[.314,.414]}, .321 \right\rangle, \left\langle \frac{\ddot{s}_2}{[.314,.515]}, .421 \right\rangle, \left\langle \frac{\ddot{s}_3}{[.315,.616]}, .521 \right\rangle, \left\langle \frac{\ddot{s}_4}{[.316,.717]}, .621 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \left\langle \frac{\ddot{s}_1}{[.434,.543]}, .331 \right\rangle, \left\langle \frac{\ddot{s}_2}{[.434,.634]}, .431 \right\rangle, \left\langle \frac{\ddot{s}_3}{[.434,.723]}, .531 \right\rangle, \left\langle \frac{\ddot{s}_4}{[.421,.812]}, .631 \right\rangle \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \left\langle \frac{\ddot{s}_1}{[.525,.634]}, .341 \right\rangle, \left\langle \frac{\ddot{s}_2}{[.554,.756]}, .441 \right\rangle, \left\langle \frac{\ddot{s}_3}{[.589,.888]}, .541 \right\rangle, \left\langle \frac{\ddot{s}_4}{[.554,.987]}, .641 \right\rangle \right\} \right), \end{array} \right\}.$$

Definition 3.32. FpIVFHSS with intuitionistic fuzzy valued possibility settings

A FpIVFHSS of kind-II $\tilde{\mathbb{F}}_2$ is stated as

$$\tilde{\mathbb{F}}_2 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left[L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \right] \subseteq \mathfrak{A}_{IVF}$ and $L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IF}$ with $\zeta_{IF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle, \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ such that $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 32. Assuming the data of Example 3.1.1, the FpIVFHSS of kind-II $\tilde{\mathbb{F}}_2$ can be constructed as

$$\tilde{\mathbb{F}}_2 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.212,.312]}, \langle .311, .211 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.213,.413]}, \langle .411, .313 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.213,.513]}, \langle .513, .411 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.214,.614]}, \langle .612, .213 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.314,.414]}, \langle .321, .232 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.314,.515]}, \langle .423, .322 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.315,.616]}, \langle .521, .412 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.316,.717]}, \langle .612, .212 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.434,.543]}, \langle .331, .213 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.434,.634]}, \langle .413, .331 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.434,.723]}, \langle .531, .413 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.421,.812]}, \langle .632, .223 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.525,.634]}, \langle .341, .242 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.554,.756]}, \langle .442, .324 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.589,.888]}, \langle .524, .434 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.554,.987]}, \langle .624, .242 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.33. FpIVFHSS with Pythagorean fuzzy valued possibility settings
A FpIVFHSS of kind-III $\tilde{\mathbb{F}}_3$ is stated as

$$\tilde{\mathbb{F}}_3 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left[L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right] \subseteq \mathfrak{A}_{IVF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PyF}$ with$\zeta_{PyF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$, $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}^2_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 33. Assuming the data of Example 3.1.1, the FpIVFHSS of kind-III $\tilde{\mathbb{F}}_3$ can be constructed as

$$\tilde{\mathbb{F}}_3 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.212,.312]}, \langle .511, .622 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.213,.413]}, \langle .711, .522 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.213,.513]}, \langle .511, .822 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.214,.614]}, \langle .611, .533 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.314,.414]}, \langle .811, .422 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.314,.515]}, \langle .422, .811 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.315,.616]}, \langle .411, .722 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.316,.717]}, \langle .511, .833 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.434,.543]}, \langle .722, .533 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.434,.634]}, \langle .833, .544 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.434,.723]}, \langle .633, .544 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.421,.812]}, \langle .533, .644 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.525,.634]}, \langle .433, .711 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.554,.756]}, \langle .833, .422 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.589,.888]}, \langle .711, .444 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.554,.987]}, \langle .822, .566 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.34. FpIVFHSS with picture fuzzy valued possibility settings
A FpIVFHSS of kind-IV $\tilde{\mathbb{F}}_4$is stated as

$$\tilde{\mathbb{F}}_4 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left[L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right] \subseteq \mathfrak{A}_{IVF}$, and $L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{PF}$, $\zeta_{PF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 34. Assuming the data of Example 3.1.1, the FpIVFHSS of kind-IV $\tilde{\mathbb{F}}_4$can be constructed as

$$\tilde{\mathbb{F}}_4 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.212,.312]}, \langle .515, .121, .211 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.213,.413]}, \langle .215, .521, .211 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.213,.513]}, \langle .415, .221, .321 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.214,.614]}, \langle .622, .111, .211 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.314,.414]}, \langle .122, .315, .421 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.314,.515]}, \langle .315, .421, .122 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.315,.616]}, \langle .212, .321, .322 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.316,.717]}, \langle .214, .421, .232 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.434,.543]}, \langle .315, .521, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.434,.634]}, \langle .215, .521, .211 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.434,.723]}, \langle .611, .211, .121 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.421,.812]}, \langle .415, .261, .222 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.525,.634]}, \langle .411, .122, .221 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.554,.756]}, \langle .222, .255, .332 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.589,.888]}, \langle .117, .441, .133 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.554,.987]}, \langle .415, .221, .121 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.35. FpIVFHSS with single-valued neutrosophic possibility settings

A FpIVFHSS of kind-V $\tilde{\mathbb{F}}_5$ is stated as

$$\tilde{\mathbb{F}}_5 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left[L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right] \subseteq \mathfrak{A}_{IVF}$, and $L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{svN}$, $\zeta_{svN}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left\langle \tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle$ with $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$ and $\tilde{\mathbb{T}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{I}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) + \tilde{\mathbb{F}}_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in [0,3]$.

Example 35. Assuming the data of Example 3.1.1, the FpIVFHSS of kind-V $\tilde{\mathbb{F}}_5$ can be constructed as

$$\tilde{\mathbb{F}}_5 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.212,.312]}, \langle .511, .622, .633 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.213,.413]}, \langle .622, .733, .711 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.213,.513]}, \langle .722, .811, .833 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.214,.614]}, \langle .833, .911, .912 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.314,.414]}, \langle .623, .415, .451 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.314,.515]}, \langle .722, .612, .632 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.315,.616]}, \langle .823, .732, .711 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.316,.717]}, \langle .922, .811, .855 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.434,.543]}, \langle .744, .633, .755 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.434,.634]}, \langle .855, .766, .822 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.434,.723]}, \langle .922, .855, .933 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.421,.812]}, \langle .633, .544, .955 \rangle \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.525,.634]}, \langle .855, .744, .633 \rangle \right\rangle, \left\langle \frac{\ddot{s}_2}{[.554,.756]}, \langle .933, .644, .511 \rangle \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.589,.888]}, \langle .633, .744, .511 \rangle \right\rangle, \left\langle \frac{\ddot{s}_4}{[.554,.987]}, \langle .622, .811, .844 \rangle \right\rangle \end{array} \right\} \right) \end{array} \right\}.$$

Definition 3.36. FpIVFHSS with interval valued fuzzy possibility settings

A FpIVFHSS of kind-VI $\tilde{\mathbb{F}}_6$is stated as

$$\tilde{\mathbb{F}}_6 = \left\{ \left(\frac{\ddot{q}}{\nu_F(\ddot{q})}, \left\langle \frac{\ddot{s}}{\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})}, \zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \right\rangle \right) : \ddot{s} \in \tilde{\mathbb{S}} \wedge \frac{\ddot{q}}{\nu_F(\ddot{q})} \in \tilde{\mathbb{Q}}_F, \nu_F(\ddot{q}) \in \mathcal{I} \right\}$$

where $\varrho^{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) = \left[L_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), U_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s})\right] \subseteq \mathfrak{A}_{IVF}$ and $\tilde{\mathbb{T}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}), \tilde{\mathbb{F}}_\varrho(\frac{\ddot{q}}{\nu_F(\ddot{q})})(\ddot{s}) \in \mathcal{I}$. Similarly $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \subseteq \mathfrak{A}_{IVF}$ with $\zeta_{IVF}(\frac{\ddot{q}}{\nu_F(\ddot{q})}) = \left[L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})})\right]$ and $L_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}), U_\zeta(\frac{\ddot{q}}{\nu_F(\ddot{q})}) \in \mathcal{I}$.

Example 36. Assuming the data of Example 3.1.1, the FpIVFHSS of kind-VI $\tilde{\mathbb{F}}_6$ can be constructed as

$$\tilde{\mathbb{F}}_6 = \left\{ \begin{array}{l} \left(\frac{\ddot{q}_1}{.275}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.212,.312]}, [.212,.312] \right\rangle, \left\langle \frac{\ddot{s}_2}{[.213,.413]}, [.313,.413] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.213,.513]}, [.414,.514] \right\rangle, \left\langle \frac{\ddot{s}_4}{[.214,.614]}, [.214,.614] \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_2}{.321}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.314,.414]}, [.221,.321] \right\rangle, \left\langle \frac{\ddot{s}_2}{[.314,.515]}, [.321,.421] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.315,.616]}, [.426,.526] \right\rangle, \left\langle \frac{\ddot{s}_4}{[.316,.717]}, [.224,.623] \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_3}{.453}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.434,.543]}, [.233,.336] \right\rangle, \left\langle \frac{\ddot{s}_2}{[.434,.634]}, [.332,.432] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.434,.723]}, [.432,.531] \right\rangle, \left\langle \frac{\ddot{s}_4}{[.421,.812]}, [.231,.631] \right\rangle \end{array} \right\} \right), \\ \left(\frac{\ddot{q}_4}{.572}, \left\{ \begin{array}{l} \left\langle \frac{\ddot{s}_1}{[.525,.634]}, [.241,.341] \right\rangle, \left\langle \frac{\ddot{s}_2}{[.554,.756]}, [.344,.443] \right\rangle, \\ \left\langle \frac{\ddot{s}_3}{[.589,.888]}, [.443,.543] \right\rangle, \left\langle \frac{\ddot{s}_4}{[.554,.987]}, [.248,.648] \right\rangle \end{array} \right\} \right), \end{array} \right\}.$$

4. Conclusions

Based on hypersoft settings, fuzzy parameterized settings and possibility grade settings, this research proposes a suite of hybrid set structures that merge the strengths of these concepts with various fuzzy set extensions such as intuitionistic, interval-valued, and neutrosophic fuzzy sets, thereby enhancing representational flexibility and analytical depth. The novelty of each hybrid model is substantiated both conceptually and practically, with carefully designed numerical examples illuminating their distinct functionalities and real-world applicability. The hypersoft set represents a pivotal evolution in managing vagueness and layered uncertainty within complex decision-making environments. By incorporating multi-argument domains and enabling approximate mappings, it provides a refined means to capture intricate relational data. When combined with fuzzy parameterization, which accommodates the imprecise nature of parameters and their nested sub-tuples, and possibility-degree settings, which quantify the plausibility of expert judgments, a more expressive and realistic modeling framework emerges. These hybrid set structures can be applied to any decision-making situation for modeling uncertainties and vagueness.

Funding: "This research received no external funding".

Data Availability Statement: "No copyright data associated with this research".

Conflicts of Interest: "The authors declare no conflict of interest".

References

1. Dubois, D., & Prade, H. (1998). Possibility theory: qualitative and quantitative aspects. Quantified representation of uncertainty and imprecision, 169-226.

2. Dubois, D. (2006). Possibility theory and statistical reasoning. Computational statistics & data analysis, 51(1), 47-69.
3. Dubois, D., & Prade, H. (1992). Combination of fuzzy information in the framework of possibility theory. Data fusion in robotics and machine intelligence, 12, 481-505.
4. Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 1(1), 3-28.
5. Zadeh, L. A. (1999). Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 100, 9-34.
6. Zadeh, L. (1965). Fuzzy sets. Information Control, 8, 338-353.
7. Gorzałczany, M. B. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets and Systems, 21(1), 1-17.
8. Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87-96.
9. Yager, R. R. (2013). Pythagorean membership grades in multicriteria decision making. IEEE Transactions on Fuzzy Systems, 22(4), 958-965.
10. Yager, R. R., & Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. International Journal of Intelligent Systems, 28(5), 436-452.
11. Cuong, B. C. (2014). Picture fuzzy sets. Journal of Computer Science and Cybernetics, 30(4), 409-409.
12. Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. International Journal of Pure and Applied Mathematics, 24(3), 287.
13. Molodtsov, D. (1999). Soft set theory - first results. Computers & Mathematics with Applications, 37(4-5), 19-31.
14. Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Possibility fuzzy soft set. Advances in Decision Sciences, 2011, 1-18.
15. Bashir, M., Salleh, A. R., & Alkhazaleh, S. (2012). Possibility intuitionistic fuzzy soft set. Advances in Decision Sciences, 2012, 1-24.
16. Jia-hua, D., Zhang, H., & He, Y. (2019). Possibility Pythagorean fuzzy soft set and its application. Journal of Intelligent & Fuzzy Systems, 36(1), 413-421.
17. Karaaslan, F. (2017). Possibility neutrosophic soft sets and PNS-decision making method. Applied Soft Computing, 54, 403-414.
18. Çağman, N., Çıtak, F., & Enginoğlu, S. (2010). Fuzzy parameterized fuzzy soft set theory and its applications. Turkish Journal of Fuzzy Systems, 1(1), 21-35.
19. Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Fuzzy parameterized interval-valued fuzzy soft set. Applied Mathematical Sciences, 5(67), 3335-3346.
20. Deli, I., & Çağman, N. (2015). Intuitionistic fuzzy parameterized soft set theory and its decision making. Applied Soft Computing, 28, 109-113.
21. Joshi, B. P., Kumar, A., Singh, A., Bhatt, P. K., & Bharti, B. K. (2018). Intuitionistic fuzzy parameterized fuzzy soft set theory and its application. Journal of Intelligent & Fuzzy Systems, 35(5), 5217-5223.
22. Kamacı, H. (2019). Interval-valued fuzzy parameterized intuitionistic fuzzy soft sets and their applications. Cumhuriyet Science Journal, 40(2), 317-331.
23. Karaaslan, F. (2016). Intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets with applications in decision making. Annals of Fuzzy Mathematics and Informatics, 11(4), 607-619.
24. Aydın, T., & Enginoğlu, S. (2021). Interval-valued intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy soft sets and their application in decision-making. Journal of Ambient Intelligence and Humanized Computing, 12(1), 1541-1558.
25. Sulukan, E., Çağman, N., & Aydın, T. (2019). Fuzzy parameterized intuitionistic fuzzy soft sets and their application to a performance-based value assignment problem. Journal of New Theory, (29), 79-88.
26. Ihsan, M., Saeed, M., & Rahman, A. U. (2023). Optimizing hard disk selection via a fuzzy parameterized single-valued neutrosophic soft set approach. Journal of operational and strategic analytics, 1(2), 62-69.

27. Memiş, S. (2023). Picture fuzzy parameterized picture fuzzy soft sets and their application in a performance-based value assignment problem to salt-and-pepper noise removal filters. International Journal of Fuzzy Systems, 25(7), 2860-2875.
28. Deli, I., & Karataş, S. (2016). Interval valued intuitionistic fuzzy parameterized soft set theory and its decision making. Journal of Intelligent & Fuzzy Systems, 30(4), 2073-2082.
29. Rahman, A. U., El-Morsy, S., Rahman, H. U., & Malik, M. Y. H. (2024). A Synergistic Multi-Attribute Decision-Making Strategy for Selecting Sustainable Educational Institution Site Using Parameterized Single-Valued Neutrosophic Soft Sets. Journal of Uncertain Systems, 18(4), 2450024.
30. Rahman, A. U., Alballa, T., Alqahtani, H., & Khalifa, H. A. E. W. (2023). A Fuzzy Parameterized Multiattribute Decision-Making Framework for Supplier Chain Management Based on Picture Fuzzy Soft Information. Symmetry, 15(10), 1872.
31. Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets and Systems, 22, 168-170.
32. Saeed, M., Rahman, A. U., Ahsan, M., & Smarandache, F. (2022). Theory of Hypersoft Sets: Axiomatic Properties, Aggregation Operations, Relations, Functions and Matrices. Neutrosophic Sets and Systems, 51, 744-765.
33. Smarandache, F. (2022). Introduction to the IndetermSoft Set and IndetermHyperSoft Set. Neutrosophic Sets and Systems, 50, 629-650.
34. Smarandache, F. (2023). New Types of Soft Sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set. International Journal of Neutrosophic Science (IJNS), 20(4), 58-64.
35. Smarandache, F. (2022). Practical Applications of IndetermSoft Set and IndetermHyperSoft Set and Introduction to TreeSoft Set as an extension of the MultiSoft Set. Neutrosophic Sets and Systems, 51, 939-947.
36. Smarandachea, F., & Gifu, D. (2024). Soft Sets Extensions used in Bioinformatics. Procedia Computer Science, 246, 2185-2193.
37. Smarandache, F. (2023). Foundation of the superhypersoft set and the fuzzy extension superhypersoft set: A new vision. Neutrosophic Systems with Applications, 11, 48-51.
38. Fujita, T., & Smarandache, F. (2025). An introduction to advanced soft set variants: Superhypersoft sets, indetermsuperhypersoft sets, indetermtreesoft sets, bihypersoft sets, graphicsoft sets, and beyond. Neutrosophic Sets and Systems, 82, 817-843.
39. Rahman, A. U., Saeed, M., Mohammed, M. A., Jaber, M. M., & Garcia-Zapirain, B. (2022). A novel fuzzy parameterized fuzzy hypersoft set and riesz summability approach based decision support system for diagnosis of heart diseases. Diagnostics, 12(7), 1546.
40. Rahman, A. U., Saeed, M., Alodhaibi, S. S., & El-Wahed Khaifa, H. A. (2021). Decision making algorithmic approaches based on parameterization of neutrosophic set under hypersoft set environment with fuzzy, intuitionistic fuzzy and neutrosophic settings. Computer Modeling in Engineering & Sciences, 128(2), 743-777.
41. Rahman, A. U., Saeed, M., & Khalifa, H. A. W. (2022). Decision making application based on parameterization of fuzzy hypersoft set with fuzzy setting. Italian Journal of Pure and Applied Mathematics, 48, 1033-1048.
42. Rahman, A. U., Saeed, M., Alburaikan, A., & Khalifa, H. A. E. W. (2022). An Intelligent Multiattribute Decision-Support Framework Based on Parameterization of Neutrosophic Hypersoft Set. Computational Intelligence and Neuroscience, 2022(1), 6229947.
43. Rahman, A. U., Saeed, M., & Dhital, A. (2021). Decision Making Application Based on Neutrosophic Parameterized Hypersoft Set Theory. Neutrosophic Sets and Systems, 41, 1-15.
44. Rahman, A. U., Saeed, M., Mohammed, M. A., Abdulkareem, K. H., Nedoma, J., & Martinek, R. (2023). An innovative mathematical approach to the evaluation of susceptibility in liver disorder based on fuzzy parameterized complex fuzzy hypersoft set. Biomedical signal processing and control, 86, 105204.

45. Rahman, A. U., Saeed, M., Mohammed, M. A., Al-Waisy, A. S., Kadry, S., & Kim, J. (2023). An innovative fuzzy parameterized MADM approach to site selection for dam construction based on sv-complex neutrosophic hypersoft set. Aims Mathematics, 8, 4907-4929.
46. Zhang, H., Zhang, Y., Rahman, A. U., & Saeed, M. (2022). An intelligent sv-neutrosophic parameterized MCDM approach to risk evaluation based on complex fuzzy hypersoft set for real estate investments. Management Decision, 61(2), 486-505.
47. Alballa, T., Ihsan, M., Rahman, A. U., Alsorayea, N. A., & Khalifa, H. A. E. W. (2024). Solid Waste Management: A MADM Approach Using Fuzzy Parameterized Possibility Single-Valued Neutrosophic Hypersoft Expert Settings. CMES-Computer Modeling in Engineering and Sciences, 142(1), 531-553.
48. Rahman, A. U., Saeed, M., Mohammed, M. A., Abdulkareem, K. H., Nedoma, J., & Martinek, R. (2023). Fppsv-NHSS: Fuzzy parameterized possibility single valued neutrosophic hypersoft set to site selection for solid waste management. Applied soft computing, 140, 110273.
49. Rahman, A. U., Saeed, M., & Garg, H. (2022). An innovative decisive framework for optimized agri-automobile evaluation and HRM pattern recognition via possibility fuzzy hypersoft setting. Advances in Mechanical Engineering, 14(10), 16878132221132146.
50. Rahman, A. U., Saeed, M., Khalifa, H. A. E. W., & Afifi, W. A. (2022). Decision making algorithmic techniques based on aggregation operations and similarity measures of possibility intuitionistic fuzzy hypersoft sets. AIMS Mathematics, 7(3), 3866-3895.
51. Rahman, A. U., Saeed, M., Mohammed, M. A., Krishnamoorthy, S., Kadry, S., & Eid, F. (2022). An integrated algorithmic MADM approach for heart diseases' diagnosis based on neutrosophic hypersoft set with possibility degree-based setting. Life, 12(5), 729.
52. Rahman, A. U., Saeed, M., & Abd El-Wahed Khalifa, H. (2024). Multi-attribute decision-making based on aggregations and similarity measures of neutrosophic hypersoft sets with possibility setting. Journal of Experimental & Theoretical Artificial Intelligence, 36(2), 161-186.
53. Zhao, J., Li, B., Rahman, A. U., & Saeed, M. (2023). An intelligent multiple-criteria decision-making approach based on sv-neutrosophic hypersoft set with possibility degree setting for investment selection. Management Decision, 61(2), 472-485.
54. Khan, K. A., Ishfaq, S. M., Rahman, A. U., & El-Morsy, S. (2024). A Synergistic Multi-Attribute Decision-Making Method for Educational Institutions Evaluation Using Similarity Measures of Possibility Pythagorean Fuzzy Hypersoft Sets. Computer Modeling in Engineering & Sciences, 142(1), 501-530.
55. Saeed, M., Wahab, A., Ali, M., Ali, J., & Bonyah, E. (2023). An innovative approach to passport quality assessment based on the possibility q-rung ortho-pair fuzzy hypersoft set. Heliyon, 9(9), e19379.
56. Romdhini, M. U., Al-Sharqi, F., Al-Obaidi, R. H., & Rodzi, Z. M. (2025). Modeling uncertainties associated with decision-making algorithms based on similarity measures of possibility belief interval-valued fuzzy hypersoft setting. Neutrosophic Sets and Systems, 77, 282-309.
57. Khan, S., Asjad, M. I., Riaz, M. B., Wahab, A., Ashaq, H., & Muhammad, T. (2024). Innovative player evaluation: Dual-possibility Pythagorean fuzzy hypersoft sets for accurate international football rankings. Heliyon, 10(17), e36993.
58. Al-Hagery, M. A., & Abdalla Musa, A. I. (2025). Enhancing Network Security using Possibility Neutrosophic Hypersoft Set for Cyberattack Detection. International Journal of Neutrosophic Science, 25(1), 38-50.
59. Hussein, E., Al-Qudah, Y., Jaradat, H. M., Al-Sharqi, F., Elnajar, S., & Jaradat, A. (2025). Similarity measure and sine exponential measure of possibility interval-valued neutrosophic hypersoft sets and their applications. Neutrosophic Sets and Systems, 81, 41-61.
60. Hussein, E., Al-Qudah, Y., Hamadameen, A. O., Al-Obaidi, R. H., Al-Jawarneh, A. S., Al-Sharqi, F., & Owledat, A. (2025). Matrices and Correlation Coefficient for possibility interval-valued neutrosophic hypersoft sets and their applications in real-life. International Journal of Neutrosophic Science, 26(1), 254-265.

61. Abdalla Musa, A. I., & Al-Hagery, M. A. (2025). An Efficient Plant Disease Detection: Possibility Neutrosophic Hypersoft Set Approach with Whale Optimization Algorithm. International Journal of Neutrosophic Science, 25(2), 11-21.
62. Rahman, A. U., Smarandache, F., Saeed, M., & Khan, K. A. (2025). Development of Some New Hybrid Structures of Hypersoft Set With Possibility-Degree Settings. In F. Smarandache, M. Jdid, & M. Leyva-Vazquez (Eds.), Neutrosophic and Plithogenic Inventory Models for Applied Mathematics (pp. 337-386). IGI Global Scientific Publishing. https://doi.org/10.4018/979-8-3693-3204-7.ch014

Received: Dec. 15, 2024. Accepted: June 21, 2025