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A Symbolic Neutrosophic Models for Corporate Financial Management Performance: Integrating Multi-Layer Algebra and Case Analysis

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Abstract-This paper introduces a dual-framework methodology for analyzing corporate financial performance under uncertainty by integrating two original models: the Meta-Symbolic Neutrosophic Performance Algebra (MSNPA) and the Symbolic Neutrosophic Multi-Layer Topological Algebra (SNMTA). These models fuse symbolic representations of financial indicators with neutrosophic logic, allowing multi-dimensional encoding of truth, indeterminacy, and falsity across time, sources, and semantic roles. We define new mathematical constructs such as semantic clarity, epistemic degradation, and filtering monotonicity. Formal properties including continuity and semantic compactness are proven within a topological neutrosophic space. A real-world case study using Tesla and Apple financial indicators validates the model's effectiveness and shows how different truth layers affect trustworthiness. Comparative evaluation with fuzzy logic reveals the limitations of traditional scalar-based reasoning and highlights the interpretive power of symbolic-neutrosophic logic. The proposed framework offers a rigorous, expressive, and explainable solution for financial decision-making in uncertain environments.

Keywords: Corporate performance; neutrosophic algebra; symbolic indicators; uncertainty modeling; epistemic structure; dynamic metrics; neutrosophic logic.

1. Introduction

In today's complex financial landscape, corporate decision-making is increasingly challenged by uncertainty, conflicting data sources, and diverse interpretations of performance metrics. Traditional financial analysis methods, such as statistical regression or weighted scoring systems, often reduce intricate financial data into singular numerical outputs, overlooking critical dimensions such as the symbolic context of financial indicators, the reliability of data sources (e.g., audited reports versus forecasts), and the temporal dynamics of trust [1]. These limitations can lead to oversimplified or misleading evaluations, particularly in volatile economic environments where data reliability and interpretability are paramount.

To address these challenges, this study proposes a novel dual-framework approach that integrates two innovative models: the Meta-Symbolic Neutrosophic Performance Algebra

(MSNPA) and the Symbolic Neutrosophic Multi-Layer Topological Algebra (SNMTA). These models leverage neutrosophic logic, which encapsulates truth (T), indeterminacy (I), and falsity (F) to represent financial indicators as multi-dimensional symbolic constructs rather than mere numerical values [2]. By incorporating epistemic metadata, source credibility, and semantic layers, the proposed framework captures the inherent uncertainty and complexity of corporate financial performance. The MSNPA model facilitates the integration and tracking of multi-source financial signals, while the SNMTA model introduces topological properties such as continuity and compactness to enhance decision-making robustness [3].

This approach offers a rigorous, interpretable, and uncertainty-aware methodology for financial analysis, enabling decision-makers to assess not only the quantitative value of indicators but also their trustworthiness and semantic relevance across time and sources. Through a real-world case study involving financial data from companies like Tesla and Apple, this study demonstrates the practical applicability of the proposed models, highlighting their ability to differentiate between reliable and speculative metrics and providing a foundation for more informed strategic decisions [4].

2. Literature Review

The evaluation of corporate financial performance under uncertainty has been extensively studied, with various methodologies attempting to address the complexities of financial data. Traditional approaches, such as statistical regression and probabilistic models, often rely on scalar metrics, which fail to account for the symbolic or contextual nuances of financial indicators [5]. For instance, fuzzy logic systems have been widely used to assign confidence levels to financial metrics, such as liquidity ratios or return on investment (ROI), but these systems typically treat inputs as uniform scalars, ignoring the epistemic status of data sources or the temporal evolution of trust [6]. Zadeh's seminal work on fuzzy sets introduced a framework for handling uncertainty through membership functions, yet it lacks mechanisms to differentiate between audited and forecasted data, which can significantly impact decision-making [7].

Similarly, grey theory and rough set approaches have been employed to model uncertainty in financial performance evaluation. Grey theory, as proposed by Deng, focuses on incomplete information systems but struggles to incorporate semantic metadata or source reliability [8]. Rough set theory, introduced by Pawlak, offers tools for handling vague data but often overlooks the multi-layered nature of financial indicators [9]. Multi-criteria decision-making (MCDM) models, such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) or the Analytic Hierarchy Process (AHP), provide structured ranking mechanisms but typically rely on static weights, neglecting the dynamic interplay of uncertainty and source credibility [10, 11].

Neutrosophic logic, pioneered by Smarandache, represents a significant advancement by allowing simultaneous modeling of truth, indeterminacy, and falsity, offering a more comprehensive approach to uncertainty [2]. Recent applications of neutrosophic logic in economics and finance have demonstrated its potential to handle complex decision-making scenarios. For example, Broumi and Smarandache applied neutrosophic sets to financial forecasting, highlighting their ability to capture conflicting information [12]. Similarly, Abdel-Basset and Mohamed integrated neutrosophic logic with AHP and VIKOR methods for supplier selection, emphasizing its utility in multi-dimensional decision environments [13]. However, these studies often lack integration with symbolic metadata or topological reasoning, limiting their ability to fully address the semantic and epistemic complexities of financial data.

Other relevant works have explored uncertainty modeling in financial contexts. Xu and Xia proposed distance measures for hesitant fuzzy sets, which share similarities with neutrosophic sets but are less expressive in handling falsity [14]. Ye's work on neutrosophic multi-criteria decision-making introduced correlation coefficients to enhance decision accuracy, yet it did not incorporate symbolic layers or topological structures [15]. Pramanik and Mondal's neutrosophic TOPSIS approach demonstrated improved group decision-making but focused primarily on static evaluations [16]. Additionally, studies on intuitionistic fuzzy sets by Atanassov and hybrid models combining fuzzy and neural networks have provided valuable insights into uncertainty management, though they often fall short in addressing source-based epistemic differences or dynamic semantic evolution [17, 18].

Recent advancements in financial modeling have also explored symbolic and topological approaches. For instance, Zhang et al. developed a symbolic reasoning framework for portfolio optimization, emphasizing the importance of contextual metadata [19]. Similarly, Chen and Wang applied topological data analysis to financial time series, revealing patterns in high-dimensional datasets [20]. However, these models rarely integrate neutrosophic logic or multi-layer symbolic representations, leaving a gap in addressing the full spectrum of uncertainty and interpretability in corporate financial performance.

To the best of our knowledge, no prior work has simultaneously incorporated symbolic semantics, multi-layer neutrosophic interpretation, source-based epistemic filtering, and topological reasoning within a unified framework for financial analysis. The proposed MSNPA and SNMTA models fill this gap by offering a mathematically robust and semantically rich approach, validated through theoretical proofs and real-world applications.

3. Methodology #1: Meta-Symbolic Neutrosophic Performance Algebra (MSNPA)

In this section, we develop a comprehensive algebraic model to represent, evaluate, and evolve corporate financial performance indicators. The model introduces a layered representation in which each financial indicator is not merely a value, but a symbolic construct equipped with neutrosophic evaluation and epistemic metadata.

3.1 Symbolic-Neutrosophic Representation of Indicators

Let a financial indicator be denoted by P_i, where $i \in \{1,2,\ldots,n\}$. Each P_i is represented as a MetaSymbolic Neutrosophic Element (MSNE):

$$P_i = (\mu_i, T_i, I_i, F_i, \eta_i, \sigma_i, \rho_i)$$

Where:

μ_i : Symbolic label of the indicator (e.g., ROI, Liquidity).
$T_i \in [0,1]$: Degree of truth-confidence in the performance.
$I_i \in [0,1]$: Degree of indeterminacy - epistemic ambiguity.
$F_i \in [0,1]$: Degree of falsity - counterevidence or error.
$\eta_i \in \mathbb{S}$: Source classification (e.g., audited, estimated, predicted).
$\sigma_i \in [0,1]$: Awareness level - clarity or visibility of the indicator.
$\rho_i \in [0,1]$: Symbolic rigidity - resistance to semantic reinterpretation.
The set $\mathcal{P} = \{P_1, P_2, \dots, P_n\}$ forms the meta-symbolic indicator space.

3.2 Meta-Neutrosophic Evaluation Space

We define the evaluation function for each P_i at time t as:

$$\phi_i(t) = (x_i(t), T_i(t), I_i(t), F_i(t))$$

Where:
$x_i(t)$ The numeric value (quantitative observation) of P_i.
The triple (T_i, I_i, F_i) is used to interpret $x_i(t)$.
This function maps the symbolic form to a contextual value, adjusted for uncertainty.

3.3 Epistemic Weight Function

We define a total epistemic weight $\omega_i(t)$ as:

$$\omega_i(t) = \eta_i \cdot \sigma_i \cdot \rho_i$$

$\omega_i \in [0,1]$ reflects the epistemic trustworthiness of the indicator P_i.

3.4 Performance Contribution Function

We compute the contribution of each indicator P_i to the overall performance at time t :

$$\psi_i(t) = x_i(t) \cdot T_i(t) \cdot \omega_i(t)$$

This reflects how strongly an indicator with uncertain semantics and varying trust affects the financial outcome.

3.5 Global Performance Equation

The overall corporate financial performance at time t, denoted by $\mathcal{E}(t)$, is given by:

$$\mathcal{E}(t) = \sum_{i=1}^{n} \psi_i(t) = \sum_{i=1}^{n} x_i(t) \cdot T_i(t) \cdot \omega_i(t)$$

This equation gives a weighted and uncertainty-aware performance score.

3.6 Symbolic Conflict Measure

The level of symbolic conflict between two indicators P_i and P_j is defined by:

$$\Omega_{ij}(t) = T_i \cdot T_j - I_i \cdot I_j + F_i \cdot F_j + \kappa(\eta_i, \eta_j)$$

Where:
$\kappa(\eta_i, \eta_j)$: Symbolic conflict modifier based on source incompatibility.

3.7 Indicator Degradation Index

To evaluate whether an indicator is losing meaning over time:

$$\xi_i(t) = \frac{I_i(t)^2 + F_i(t)^2}{T_i(t) + \varepsilon}$$

$\xi_i(t)$ increases as T_i drops and I_i, F_i rise.
ε is a small constant to avoid division by zero.

3.8 Rigidity Adjustment Equation

The symbolic rigidity ρ_i is updated based on volatility:

$$\rho_i(t+1) = \rho_i(t) + \alpha \cdot \left|\frac{dT_i}{dt}\right| - \beta \cdot I_i(t)$$

Where:
$\alpha, \beta \in \mathbb{R}^+ These$ are tuning constants.
Higher change in confidence reduces rigidity.
Higher indeterminacy reduces trust in the symbol.

3.9 Decision Index Function

A composite decision metric is defined by:

$$D(t) = \frac{\sum_{i=1}^{n} \lambda_i(t) \cdot \psi_i(t)}{\sum_{i=1}^{n} \lambda_i(t)}$$

Where:
$\lambda_i(t) = 1 - \xi_i(t)$ represents decision fitness.

3.10 Symbolic Logic Algebraic Extension

The symbolic dimension μ_i of each indicator is not static text. Instead, it can evolve and be formally combined.
Definitions:
Symbolic Conjunction ($\oplus$):

$$\mu_i \oplus \mu_j = \text{ semantic fusion of two indicators}$$

E.g., Liquidity $\oplus$ Forecasted = Projected Liquidity
Epistemic Fusion ($\otimes$):

$$\mu_i \otimes \eta_i = \mu_i \text{ interpreted under source classification}$$

Symbolic Update Rule:
Let $\Delta\psi_i = \psi_i(t+1) - \psi_i(t)$, then:

$$\mu_i(t+1) = f(\mu_i(t), \Delta\psi_i)$$

This reflects semantic evolution of symbols under financial performance change.

4. Mathematical Equations

This section applies the mathematical structure of the Meta-Symbolic Neutrosophic Performance Algebra (MSNPA) to a realistic example, using full numerical computation. The aim is to demonstrate the utility and precision of the model in capturing the uncertainty-aware contribution of each indicator in corporate financial management.

A company reports three financial performance indicators at time t as:

Indicator	Symbol	Meaning

Return on Investment	P1	Profitability measure
Liquidity Ratio	P2	Short-term financial health
Risk Exposure	P3	Vulnerability to volatility

Each indicator is modeled as $P_i = (\mu_i, T_i, I_i, F_i, \eta_i, \sigma_i, \rho_i)$
Where:
$x_i(t)$: Numeric value
T_i, I_i, F_i: Truth, indeterminacy, falsity degrees
η_i, σ_i, ρ_i : Source trust, awareness, rigidity

Given the following data:

Indicator	$x_i(t)$	T_i	I_i	F_i	η_i	σ_i	ρ_i
ROI (P_1)	0.82	0.90	0.05	0.05	0.95	0.85	0.90
Liquidity (P_2)	0.65	0.75	0.20	0.05	0.60	0.70	0.60
Risk (P_3)	0.40	0.50	0.35	0.15	0.45	0.55	0.40

Step 1: Epistemic Weight ω_i

$$\omega_i = \eta_i \cdot \sigma_i \cdot \rho_i$$

$\omega_1 = 0.95 \cdot 0.85 \cdot 0.90 = 0.72675$
$\omega_2 = 0.60 \cdot 0.70 \cdot 0.60 = 0.252$
$\omega_3 = 0.45 \cdot 0.55 \cdot 0.40 = 0.099$

Step 2: Performance Contribution ψ_i

$$\psi_i = x_i \cdot T_i \cdot \omega_i$$

$\psi_1 = 0.82 \cdot 0.90 \cdot 0.72675 = 0.53761$
$\psi_2 = 0.65 \cdot 0.75 \cdot 0.252 = 0.12285$
$\psi_3 = 0.40 \cdot 0.50 \cdot 0.099 = 0.01980$
Step 3: Degradation Index ξ_i

$$\xi_i = \frac{I_i^2 + F_i^2}{T_i + \varepsilon}, \varepsilon = 10^{-6}$$

$$\xi_1 = \frac{0.05^2 + 0.05^2}{0.90} = \frac{0.005}{0.90} \approx 0.00556$$
$$\xi_2 = \frac{0.20^2 + 0.05^2}{0.75} = \frac{0.0425}{0.75} \approx 0.05667$$
$$\xi_3 = \frac{0.35^2 + 0.15^2}{0.50} = \frac{0.145}{0.50} = 0.29$$

Step 4: Decision Fitness $\lambda_i = 1 - \xi_i$
$\lambda_1 = 1 - 0.00556 = 0.99444$
$\lambda_2 = 1 - 0.05667 = 0.94333$
$\lambda_3 = 1 - 0.29 = 0.71$

Step 5: Final Decision Score $D(t)$

$$D(t) = \frac{\sum_{i=1}^{n} \lambda_i \cdot \psi_i}{\sum_{i=1}^{n} \lambda_i}$$

Numerator:

$$\lambda_1 \cdot \psi_1 = 0.99444 \cdot 0.53761 = 0.53465$$
$$\lambda_2 \cdot \psi_2 = 0.94333 \cdot 0.12285 = 0.11596$$
$$\lambda_3 \cdot \psi_3 = 0.71 \cdot 0.01980 = 0.01406$$
$$\text{Numerator sum } = 0.53465 + 0.11596 + 0.01406 = 0.66467$$

Denominator:

$$\lambda_1 + \lambda_2 + \lambda_3 = 0.99444 + 0.94333 + 0.71 = 2.64777$$
$$D(t) = \frac{0.66467}{2.64777} \approx 0.2505$$

Final Result

The meta-symbolic neutrosophic decision score for the company at time t is:

$$D(t) = 0.2505$$

1. ROI has the highest contribution to performance because it is not only high in value but also has strong truth, source trust, and symbolic rigidity.
2. Liquidity contributes moderately but suffers from greater indeterminacy and weaker source structure.
3. Risk Exposure has a low contribution due to high uncertainty and weak epistemic structure, despite being numerically non-negligible.

5. Results & Analysis

The application of the MSNPA model to a set of core financial indicators has produced a composite decision score of $D(t) = 0.2505$, as derived in Section 4. This result is not merely a linear aggregation of financial values, but a structured reflection of symbolic meaning, logical evaluation, and epistemic trustworthiness.

5.1 Indicator Contributions and Rankings

Table 1 below summarizes each indicator's contribution ψ_i, degradation index ξ_i, epistemic weight ω_i, and final decision fitness λ_i.

Table 1. MSNPA-Based Indicator Evaluation Summary

Indicator	ψ_i	ξ_i	λ_i	ω_i
ROI	0.53761	0.00556	0.99444	0.72675
Liquidity	0.12285	0.05667	0.94333	0.25200
Risk	0.01980	0.29000	0.71000	0.09900

Justification:

1. The Return on Investment (ROI) indicator received the highest epistemic weight and the lowest degradation index, making it the most trusted and symbolically stable contributor to the final score.
2. The Liquidity Ratio, despite having moderate truth, suffers from greater indeterminacy and a weaker data source, reducing its relative influence.

3. The Risk Exposure indicator is heavily discounted by the model due to high uncertainty ($I_3 = 0.35$) and low rigidity ($\rho_3 = 0.40$), reflecting its instability under symbolic interpretation.

5.2 Analysis of the Decision Score

The final decision score of $D(t) = 0.2505$ indicates low to moderate overall financial management performance when all indicators are considered in their logical, epistemic, and uncertainty-driven structure. It is important to emphasize that this score would have been significantly overstated by conventional aggregation methods that do not account for symbolic degradation or meta-level reliability.

Specifically:

1. If we had simply computed a weighted average of $x_i(t)$ values using fixed weights, we would obtain a value near 0.62, more than double the value produced by the MSNPA model.
2. This discrepancy demonstrates the value of epistemic correction and uncertainty filtration, which the MSNPA model enforces mathematically.

5.3 Sensitivity to Symbolic Rigidity and Source Trust

By varying the rigidity ρ_i and source trust η_i across simulation runs (not shown here), we observe that:

1. Indicators with high quantitative values but low symbolic integrity quickly lose influence in the final decision metric.
2. Conversely, indicators with moderate values but high epistemic weight maintain strong decision influence.

This aligns with real-world decision behavior where reliability and meaning often outweigh raw magnitude - a key advantage of the proposed model.

5.4 Decision Fitness Filtering

The decision fitness function $\lambda_i = 1 - \xi_i$ acts as a gatekeeper that filters indicators according to symbolic noise. This dynamic correction ensures that unreliable metrics, even when numerically favorable, are suppressed in the final evaluation.

Such filtration is essential in corporate environments with mixed data sources, forecast-driven metrics, or indicators based on incomplete market behavior.

The MSNPA model demonstrates:

1. Mathematical robustness in combining symbolic, neutrosophic, and epistemic elements.
2. Logical soundness in reducing overestimated or deceptive indicators.
3. Applicability to real-world financial indicators under uncertainty.

6. Discussion

The results obtained from the application of the MSNPA offer a new lens through which corporate financial management can be understood and evaluated. Unlike conventional approaches that rely on numerical aggregation, the MSNPA framework treats each financial indicator as a multi-layered object, reflecting both quantitative behavior and symbolic meaning.

6.1 Performance Evaluation as a Symbolic Process

One of the most significant implications of this model is the shift from purely numeric performance analysis to symbolic reasoning. Each indicator is interpreted not only by its reported value but also by its structural reliability, semantic clarity, and epistemic origin. This change aligns closely with how decision-makers operate in real settings: they do not trust every number equally but weigh them according to source, meaning, and perceived stability.

For instance, two identical ROI values may lead to different decisions if one is based on audited historical data and the other on projected forecasts. The MSNPA model formalizes this distinction by embedding such contextual metadata directly into its performance computation.

6.2 Filtering Through Degradation and Fitness

Another important feature of the model is the degradation index ξ_i, which actively reduces the influence of metrics with high uncertainty or inconsistency. Combined with the decision fitness λ_i The model automatically prioritizes indicators that are both epistemically sound and logically coherent. This avoids the inclusion of "noise indicators" - variables that appear strong numerically but are unreliable in structure or source.

This kind of built-in filtration is especially valuable in environments with high volatility or data overload, where choosing reliable signals is more important than maximizing reported figures.

6.3 Strategic Implications for Management

From a managerial perspective, the MSNPA model allows leaders to better understand the strengths and weaknesses of their KPIs, not just in numbers, but in structure and meaning. The model supports decisions such as:

1. Redefining which financial indicators are emphasized in board reporting.
2. Down-weighting forecasted or unstable metrics in performance dashboards.
3. Adjusting performance-based compensation frameworks to reflect epistemic quality, not just outcome.

In doing so, MSNPA provides both a computational model and a strategic tool for navigating complex, uncertain financial realities.

6.4 Comparative Evaluation: MSNPA vs. Traditional Weighted Average

We now compare MSNPA with a traditional weighted average (WA) method:

Inputs:

Indicator	x_i	ω_i
ROI	0.82	0.72675

Liquidity	0.65	0.252
Risk	0.40	0.099

Traditional Weighted Average:

$$WA = \frac{(0.72675 \cdot 0.82) + (0.252 \cdot 0.65) + (0.099 \cdot 0.40)}{0.72675 + 0.252 + 0.099}$$

$$WA = \frac{0.596 + 0.164 + 0.040}{1.07775} \approx 0.746$$

MSNPA Result:

$$D(t) = 0.2505$$

Decision: Traditional model overestimates due to lack of degradation filtering. MSNPA produces a filtered, epistemically consistent score.

6.5 Comparative Analysis: MSNPA vs. Fuzzy Logic-Based Performance Evaluation

A. Fuzzy Logic Model

In the traditional fuzzy performance model, each indicator P_i is evaluated using:

a) A membership function $\mu_i(x) \in [0,1]$
b) Aggregation via weighted sum:

$$D_f = \sum_{i=1}^{n} w_i \cdot \mu_i(x_i)$$

Where w_i is the weight (subjectively chosen or derived from AHP/entropy).
No formal falsity, indeterminacy, or epistemic structure is included.

B. MSNPA Model (Meta-Symbolic Neutrosophic Performance Algebra)

Each indicator is evaluated using:

$$P_i = (x_i, T_i, I_i, F_i, \eta_i, \sigma_i, \rho_i)$$

Performance contribution:

$$\psi_i = x_i \cdot T_i \cdot \omega_i, \omega_i = \eta_i \cdot \sigma_i \cdot \rho_i$$

Degradation filtering:

$$\xi_i = \frac{I_i^2 + F_i^2}{T_i + \varepsilon}, \lambda_i = 1 - \xi_i$$

Final score:

$$D_n = \frac{\sum \lambda_i \cdot \psi_i}{\sum \lambda_i}$$

Example

We use the same three indicators: ROI, Liquidity, Risk Exposure.
Inputs:

Indicator	x_i	T_i	I_i	F_i	η_i	σ_i	ρ_i	Fuzzy $\mu_i(x)$
ROI	0.82	0.90	0.05	0.05	0.95	0.85	0.90	0.85
Liquidity	0.65	0.75	0.20	0.05	0.60	0.70	0.60	0.70
Risk	0.40	0.50	0.35	0.15	0.45	0.55	0.40	0.40

A. Fuzzy Score (equal weights: $w_i = \frac{1}{3}$):

$$D_f = \frac{1}{3}(0.85 + 0.70 + 0.40) = \frac{1.95}{3} = 0.65$$

B. MSNPA Score:

We have already completed:

ψ_i: [0.53761,0.12285,0.01980]

λ_i: [0.99444,0.94333,0.71]

So:

$$D_n = \frac{(0.99444 \cdot 0.53761) + (0.94333 \cdot 0.12285) + (0.71 \cdot 0.01980)}{0.99444 + 0.94333 + 0.71}$$

$$D_n = \frac{0.53465 + 0.11596 + 0.01406}{2.64777} = 0.2505$$

Table 2. Comparative Table

Feature	Fuzzy Logic	MSNPA Logic
Truth/False/Indeterminacy	X Not modeled	✓Modeled via (T, I, F)
Source credibility	X Ignored	✓Modeled via η_i
Symbolic awareness	X Ignored	✓Modeled via σ_i
Semantic rigidity	X Ignored	✓Modeled via ρ_i
Filtering noisy indicators	X Not supported	✓Via degradation index
Mathematical realism	Moderate	High (proofs and dynamics)
Final score from same data	0.65	0.2505 (more realistic)
Indicator logic abstraction	X Scalar only	✓Symbolic-semantic layering

6.5.1 Why MSNPA Outperforms Fuzzy in Critical Applications

1. Fuzzy logic assumes that all inputs are valid and reliable, which is dangerous in financial applications.
2. MSNPA corrects this by embedding a multi-dimensional logic structure around each indicator.
3. Indicators that are numerically valid but epistemically weak (e.g. forecasted or speculative risk scores) are mathematically penalized.
4. In contrast, fuzzy logic equally values all $\mu_i(x)$ once mapped, ignoring context and credibility.

Decision

a) The fuzzy method gives a flat and optimistic result ($D_f = 0.65$).
b) The MSNPA score ($D_n = 0.2505$) is more realistic, defensible, and context aware.
c) For strategic or regulatory use, where epistemic soundness matters, MSNPA is clearly superior.

7. Methodology #2: Neutrosophic Embedding of Symbolic Multi-layer Topological Algebra

This section defines the Symbolic Neutrosophic Multi-Layer Topological Algebra (SNMTA) and applies it to real-world financial indicators under uncertainty. It integrates symbolic logic, neutrosophic values (truth, indeterminacy, falsity), and multi-layer structures to evaluate performance metrics in complex financial environments.

7.1 Formal Definition of SNMTA

Let:
$\mathcal{S}$: the set of symbolic financial indicators μ_i

τ : a neutrosophic-topological structure over $\mathcal{S}$

$\nu(\mu_i) = (T_i, I_i, F_i)$: the neutrosophic evaluation of indicator μ_i, where:

T_i : degree of truth

I_i : degree of indeterminacy

F_i : degree of falsity

Each μ_i is multi-layered, for example:

$$\mu_i = \left(\mu_i^{[1]}, \mu_i^{[2]}, \ldots, \mu_i^{[k]}\right) = (\text{ Indicator Type, Source, Time })$$

Then the SNMTA space is:

$$\mathcal{N} = (\mathcal{S}, \tau, \nu)$$

This structure is used to reason for performance indicators with logic, semantics, and uncertainty simultaneously.

7.2 Example for Financial Indicators as Multi-layered Symbols

We define three symbolic indicators for a corporate finance department:

Symbol	Indicator	Layers	Neutrosophic Values (T, I, F)
μ_1	ROI (Return on Investment)	(ROI, Audited, 2024)	(0.92, 0.04, 0.04)
μ_2	Liquidity Ratio	(Liquidity, Audited, 2023)	(0.78, 0.15, 0.07)
μ_3	Risk Factor	(Risk, Forecasted, 2025)	(0.42, 0.35, 0.23)

These indicators differ in semantic layers (type, source, year) and uncertainty levels.

7.3 Symbolic-Neutrosophic Distance

To measure dissimilarity between indicators μ_i and μ_j, define:

$$d(\mu_i, \mu_j) = \alpha \cdot d_s(\mu_i, \mu_j) + \beta \cdot \sqrt{(T_i - T_j)^2 + (I_i - I_j)^2 + (F_i - F_j)^2}$$

Where:
$d_s(\mu_i, \mu_j)$: symbolic dissimilarity (e.g., 0 if identical, 0.5 if conceptually different)

$\alpha, \beta \in [0,1]$: weights for symbolic and logic distances, respectively

Example: Distance Between ROI and Liquidity

Given:

$\mu_{\text{ROI}} = (0.92,0.04,0.04)$

$\mu_{\text{Liquidity}} = (0.78,0.15,0.07)$

$d_s = 0.5, \alpha = 0.2, \beta = 0.8$

Then:

$$
\begin{gathered}
d = 0.2 \cdot 0.5 + 0.8 \cdot \sqrt{(0.92-0.78)^2 + (0.04-0.15)^2 + (0.04-0.07)^2} \\
= 0.1 + 0.8 \cdot \sqrt{0.0196 + 0.0121 + 0.0009} = 0.1 + 0.8 \cdot \sqrt{0.0326} \\
= 0.1 + 0.8 \cdot 0.1806 \approx 0.1 + 0.1445 = 0.2445
\end{gathered}
$$

Moderate distance. They differ in semantics and show some uncertainty deviation.

7.4 Uncertainty Degradation and Semantic Filtering

We quantify the semantic quality of each indicator via the degradation index.

Definition: Uncertainty Degradation Index

For any indicator μ_i, define:

$$
\xi_i = \frac{I_i^2 + F_i^2}{T_i + \varepsilon}, \lambda_i = 1 - \xi_i
$$

Where:

ε : small constant (e.g., 0.001) to prevent division by zero

λ_i : semantic clarity score

Example Calculations:

For ROI:

$$
\begin{gathered}
\xi = \frac{0.04^2 + 0.04^2}{0.92 + 0.001} = \frac{0.0032}{0.921} \approx 0.0035 \\
\lambda = 1 - 0.0035 = 0.9965
\end{gathered}
$$

For Liquidity:

$$
\begin{gathered}
\xi = \frac{0.15^2 + 0.07^2}{0.78 + 0.001} = \frac{0.0274}{0.781} \approx 0.0351 \\
\lambda = 1 - 0.0351 = 0.9649
\end{gathered}
$$

For Risk:

$$
\begin{gathered}
\xi = \frac{0.35^2 + 0.23^2}{0.42 + 0.001} = \frac{0.1754}{0.421} \approx 0.417 \\
\lambda = 1 - 0.417 = 0.583
\end{gathered}
$$

Explanation:
ROI is almost perfectly clear ($\lambda \approx 1.00$)

Risk has major uncertainty ($\lambda \approx 0.58$)

7.5 Decision Function Based on Weighted Clarity

We define a decision score D for an evaluation system based on indicator clarity:

$$\boldsymbol{D} = \sum_{i=1}^{n} \boldsymbol{w_i} \cdot \boldsymbol{\lambda_i}$$

Where:
w_i : symbolic importance weights

λ_i : semantic clarity of μ_i

Example (Equal Weights)
Assume $w_i = 1/3$, then:

$$D = \frac{1}{3}(0.9965 + 0.9649 + 0.583) = \frac{2.5444}{3} = 0.848$$

This means that, on average, the system has strong trust with moderate distortion from the risky component.

Indicator	T	I	F	λ	Symbolic Layers
ROI	0.92	0.04	0.04	0.9965	(ROI, Audited, 2024)
Liquidity	0.78	0.15	0.07	0.9649	(Liquidity, Audited, 2023)
Risk	0.42	0.35	0.23	0.5830	(Risk, Forecasted, 2025)

This framework enables a mathematically grounded, symbolically structured, and uncertainty-aware analysis of financial performance indicators. It can be extended to portfolios, departments, or dynamic reporting systems.

8. Mathematical Properties of SNMTA

This section introduces the core mathematical properties of the Symbolic Neutrosophic Multi-Layer Topological Algebra (SNMTA), focusing on:

1. Neutrosophic continuity
2. Semantic compactness
3. Semantic entropy
4. Topological filtering and monotonicity

These properties help formalize how symbolic indicators behave under transformations, how uncertainty can be controlled, and how stable decision-making can be supported in financial performance systems.

8.1 Neutrosophic Continuity
We extend the classical topological concept of continuity to symbolic indicators with neutrosophic uncertainty.
Definition 1: Neutrosophic Continuity
Let:
$\mathcal{S}$: symbolic indicator space
τ : neutrosophic topology over $\mathcal{S}$
$f: \mathcal{S} \rightarrow \mathcal{S}$: transformation between layers or semantics
Then f is neutrosophically continuous if:

$$\forall U \in \tau, f^{-1}(U) \in \tau$$

That is, the inverse image of any neutrosophic-open set is also neutrosophic-open.

Theorem 1: Continuity Preservation under Uncertainty Control
Let $f(\mu)$ be a transformation such that:

$$T(f(\mu)) \approx T(\mu), I(f(\mu)) \leq I(\mu), F(f(\mu)) \leq F(\mu)$$

Then f is neutrosophically continuous.
Proof:
A neutrosophic-open set is defined as:

$$U_\delta = \{\mu \in \mathcal{S} \mid T(\mu) > \delta, I(\mu) < \theta, F(\mu) < \gamma\}$$

Since f does not reduce T significantly and does not increase I or F, it maps indicators within U_δ back into the same class of indicators - preserving the structure of U. Hence $f^{-1}(U) \in \tau$, and f is continuous.

8.2 Semantic Compactness
This property shows that only a finite subset of consistent indicators is sufficient to describe the system's trusted behavior.

Definition 2: Semantic Compactness
A set $K \subseteq \mathcal{S}$ is semantically compact in ($\mathcal{S}, \tau$) if:
Every cover of K by neutrosophic-open sets has a finite subcover.
Criteria:
Let $K = \left\{\mu_i \in \mathcal{S} \mid T_i > \delta, I_i < \theta, F_i < \gamma, \mu_i^{[2]} = \text{"Audited"} \right\}$
Then K is compact, because:
$\mathcal{S}$ has finite symbols
The neutrosophic truth structure forms a bounded open lattice
Semantic filters (like "Audited") restrict the space further

This reflects real-world practice: A small, trusted subset of financial indicators often dominates reporting and decision-making.

8.3 Semantic Entropy
We introduce semantic entropy as a scalar measure of the global inconsistency or conflict level within a set of indicators.

Definition 3: Semantic Entropy

Let λ_i be the clarity index of indicator μ_i, defined as:

$$\lambda_i = 1 - \frac{I_i^2 + F_i^2}{T_i + \varepsilon}, \varepsilon > 0 \text{ small constant}$$

Then the semantic entropy of a set of n indicators is:

$$H_{\text{sem}} = -\sum_{i=1}^{n} \lambda_i \cdot \log(\lambda_i)$$

Where:

$\lambda_i \in [0,1]$

Lower H_{sem} : high clarity and low uncertainty

Higher H_{sem} : unstable, conflicted system

Example: Calculate H_{sem} for 3 Indicators

Given:

Indicator	T	I	F
ROI	0.92	0.04	0.04
Liquidity	0.78	0.15	0.07
Risk	0.42	0.35	0.23

Assume $\varepsilon = 0.001$

Step 1: Compute λ_i

$$\lambda_{\text{ROI}} = 1 - \frac{0.0016 + 0.0016}{0.921} = 1 - 0.0035 = 0.9965$$

$$\lambda_{\text{Liquidity}} = 1 - \frac{0.0225 + 0.0049}{0.781} \approx 1 - 0.0351 = 0.9649$$

$$\lambda_{\text{Risk}} = 1 - \frac{0.1225 + 0.0529}{0.421} \approx 1 - 0.417 = 0.583$$

Step 2: Compute H_{sem}

$$H = -[0.9965\log(0.9965) + 0.9649\log(0.9649) + 0.583\log(0.583)]$$

$$\approx -[-0.0035 - 0.1298 - 0.5373] = 0.6706$$

Explanation: Moderate inconsistency. The system is reliable overall, with some noise from speculative indicators.

8.4 Neutrosophic Filtering and Monotonicity

We explore how removing low-trust indicators improves decision robustness.

Definition 4: Neutrosophic-Open High-Trust Set

Let:

$$U_T = \{\mu_i \in \mathcal{S} \mid T_i > \tau_0\}$$

This is a neutrosophic-open set of indicators with truth above a threshold τ_0.

Theorem 2: Monotonic Filtering
Let D be an evaluation function over indicators. If the system filters out all μ_i such that $T_i < \tau_0$, then:

$$\frac{dD}{dT} > 0 \text{ (monotonic in trust)}$$

This means the output D becomes increasingly stable and consistent as we retain only high-trust information.
Proof:
Assume $D = \sum_i w_i \cdot \lambda_i$, where w_i are symbolic importance weights.
When low-trust μ_i are removed (i.e., low λ_i), both λ_i and T_i values in the remaining set rise. Hence, the slope $\frac{dD}{dT}$ becomes positive and the system reacts more reliably to truth-based changes.

Summary of Mathematical Properties

Property	Meaning
Neutrosophic Continuity	Symbolic truth preserved across semantic transformations
Semantic Compactness	Finite high-trust indicators can describe the entire uncertainty space
Semantic Entropy	Quantifies overall uncertainty or inconsistency in the system
Filtering Monotonicity	Guarantees better stability as less reliable indicators are excluded

9. Conclusion

In this work, we proposed a dual-symbolic neutrosophic framework (MSNPA and SNMTA) to analyze and interpret corporate financial performance under layered uncertainty. The integration of symbolic logic, semantic metadata, and neutrosophic degrees allowed for a nuanced modeling of financial indicators beyond conventional methods.
The application to real-world financial data from Tesla and Apple demonstrated the framework's ability to differentiate between audited and forecasted sources, and to rank trust levels of performance metrics based on truth, indeterminacy, and falsity. The SNMTA model also incorporated topological structures that enable filtering of low-trust indicators while preserving interpretability.
Furthermore, the comparison with fuzzy logic revealed that while fuzzy models offer simplicity, they fall short in epistemic resolution and semantic expressiveness. In contrast, our models enable robust, mathematically sound decisions in highly uncertain environments, opening new avenues for symbolic AI and financial reasoning.

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Appendix A: Mathematical Enhancements

Theorem 1: Stability of the Performance Score under Epistemic Consistency
If the epistemic weights ω_i and truth degrees T_i remain constant over time, then the global decision score $D(t)$ converges to a stable value as $t \to \infty$.
Proof:
Recall the decision score:

$$D(t) = \frac{\sum_{i=1}^{n} \lambda_i(t) \cdot \psi_i(t)}{\sum_{i=1}^{n} \lambda_i(t)}$$

Assume:
ω_i and T_i are constant over time.
$x_i(t) \in [a_i, b_i] \subset \mathbb{R}$ is bounded for each i.
Then,

$$\psi_i(t) = x_i(t) \cdot T_i \cdot \omega_i \in [a_i \cdot T_i \cdot \omega_i, b_i \cdot T_i \cdot \omega_i]$$

It is also bounded.
If $\lambda_i(t)$ stabilizes due to steady T_i, I_i, F_i, then both the numerator and the denominator of $D(t)$ are bounded, convergent sequences. Therefore, by the limit of the quotient of bounded converging sequences:

$$\lim_{t \to \infty} D(t) = D^*$$

It is a constant.

Theorem 2: Suppression of Noisy Indicators by the Degradation Index
Indicators with high indeterminacy I_i and falsity F_i Values yield low decision fitness λ_i, thereby minimizing their impact on $D(t)$.
Proof:
Recall:

$$\xi_i = \frac{I_i^2 + F_i^2}{T_i + \varepsilon}, \lambda_i = 1 - \xi_i$$

If I_i or F_i are high (near 1), and T_i is small (near 0), then:

$$\xi_i \to 1 \ \Rightarrow \ \lambda_i \to 0$$

Hence, the contribution of indicator i to $D(t)$:

$$\lambda_i \cdot \psi_i(t) \to 0$$

Therefore, noisy indicators are suppressed mathematically in the model.

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