



Spherical Neutrosophic Projection Transform: Modeling Knowledge Evolution in Digital Transformation for Basic Education High-Quality Development

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Abstract: This paper presents a new mathematical model called the Spherical Neutrosophic Projection Transform (SNPT). The model is based on spherical neutrosophic numbers, which represent logical states using three values: truth (t), indeterminacy (i), and falsehood (f). These values follow the rule $t^2 + i^2 + f^2 \leq 3$. We define a special transform that maps each triplet to another point inside the same space. This allows us to study how knowledge or beliefs change over time in uncertain systems. We prove that the transform is mathematically safe, bounded, and stable. We also create a system of differential equations that describes how the three components interact dynamically. These tools allow us to model systems where understanding, confusion, and errors change step by step. To show a real application, we use this model to track the learning progress of students in basic education. Each student's knowledge is modeled as a changing neutrosophic triplet. The results show that our model can describe how digital transformation in education affects learning quality in a measurable and logical way. This work gives a new way to connect neutrosophic mathematics with educational improvement in the digital age.

Keywords: spherical neutrosophic numbers, projection transform, internal evolution, truth-indeterminacy-falsehood, neutrosophic dynamics, bounded logic, mathematical modeling

1. Introduction

Digital transformation in education transcends the mere integration of technology into classrooms; it fundamentally redefines the educational process, particularly at the basic education level, where foundational cognitive structures are established. This transformation leverages tools such as online learning platforms, smart classrooms, and real-time data analytics to enhance teaching and assessment [1]. However, these tools predominantly focus on measurable outcomes—test scores, attendance, or participation—often overlooking the qualitative, internal processes of learning. For instance, a student may provide a correct answer yet harbor uncertainty, or conversely, exhibit confidence in an incorrect belief. These nuances are critical to understanding authentic learning but are frequently neglected in conventional educational models [2].

Traditional assessment systems typically adopt a binary approach to knowledge evaluation, categorizing responses as either correct or incorrect. While some advanced models incorporate partial credit or fuzzy logic to account for degrees of correctness, they still fail to capture the complex mental states of learners, such as uncertainty, mixed understanding, or conceptual conflicts [3, 4]. These states are particularly prevalent in digitally transformed educational environments, where rapid technological changes introduce dynamic and multifaceted challenges [5]. The question arises: how can we effectively model the internal evolution of a learner's knowledge in such contexts?

To address this, this paper proposes a novel mathematical framework grounded in spherical neutrosophic logic, an extension of traditional logic that accounts for three independent dimensions: truth (t), representing the correctness of an idea; indeterminacy (i), capturing uncertainty or instability; and falsehood (f), indicating incorrectness [6]. Unlike classical or fuzzy logic, neutrosophic logic provides a more nuanced representation of a learner's cognitive state, accommodating the inherent uncertainties and contradictions in learning processes [7]. To operationalize this framework, we introduce a spherical constraint, defined as $(t^2 + i^2 + f^2 \leq 3)$, which ensures that all possible cognitive states are bounded within a geometrically structured, logically valid space [8].

Central to our approach is the Spherical Neutrosophic Projection Transform (SNPT), a mathematical function that maps one neutrosophic triplet $((t, i, f))$ to another within this constrained space. The SNPT models the dynamic evolution of a learner's cognitive state over time, influenced by internal cognitive processes or external educational inputs, such as interactive digital lessons or feedback [9]. This transform is supported by a system of differential equations that describe the interactions among truth, indeterminacy, and falsehood, capturing how digital learning interventions such as real-time feedback, gamified lessons, or exposure to conflicting information drive changes in a learner's knowledge state [10, 11].

The application of this model focuses on tracking knowledge evolution in a digitally transformed basic education environment. By simulating realistic classroom scenarios, we demonstrate how the SNPT can provide insights into the quality of learning beyond traditional metrics like test scores [12]. For example, the model can reveal how a student's uncertainty decreases as they engage with interactive digital content or how exposure to conflicting online resources increases indeterminacy [13]. This approach does not aim to replace existing educational tools but rather to enhance them by offering educators a deeper, more granular understanding of learning dynamics [14].

The theoretical foundation of this work builds on recent advances in neutrosophic set theory, which has been applied in diverse fields such as decision-making, medical diagnosis, and image processing [15, 16]. Specifically, spherical neutrosophic sets, with

their constrained geometric structure, have shown promise in modeling complex systems with uncertainty [17]. In education, neutrosophic logic has been used to evaluate student performance and decision-making processes, but its application to dynamic knowledge evolution remains underexplored [18]. Our model extends these efforts by integrating spherical neutrosophic logic with differential equations, providing a robust framework for analyzing learning in digital contexts [19].

This study bridges advanced mathematical modeling with practical educational applications, offering a new lens for understanding and guiding learning in the digital age. By combining the SNPT with real-world data from digitally transformed classrooms, we aim to empower educators to make informed decisions that enhance learning outcomes [20]. The proposed framework not only contributes to the theoretical landscape of neutrosophic logic but also provides actionable insights for educators navigating the complexities of digital transformation in basic education.

2. Spherical Neutrosophic Numbers: Definitions and Structure

We begin by defining the core mathematical space in which our model operates.

Definition 2.1: Spherical Neutrosophic Number (SNN)

A Spherical Neutrosophic Number is a triplet:

$$(t, i, f) \in [0, 3]^3$$

that satisfies the constraint:

$$t^2 + i^2 + f^2 \leq 3$$

This defines a closed spherical region $\mathbb{S}_3 \subseteq \mathbb{R}^3$. All valid neutrosophic triplets lie inside or on the surface of a sphere with radius $\sqrt{3}$, centered at the origin.

Definition 2.2: Spherical Neutrosophic Space

We denote the space of all SNNs as:

$$\mathbb{S}_3 := \{(t, i, f) \in [0, 3]^3 \mid t^2 + i^2 + f^2 \leq 3\}$$

This space is:

Bounded (because all values are ≤ 3),

Continuous (no discrete jumps),

Closed under certain nonlinear mappings (as we will show).

Example 2.1: Valid and Invalid SNNs

(t, i, f)	$t^2 + i^2 + f^2$	Valid SNN?
$(1, 1, 1)$	3	Yes
$(0.5, 0.5, 0.5)$	0.75	Yes
$(2, 2, 2)$	12	No
$(1.2, 1.2, 0.6)$	3.24	No
$(1, 0.5, 0.5)$	1.5	Yes

Property 2.1: Symmetry

If $(t, i, f) \in \mathbb{S}_3$, then any permutation (f, t, i) , (i, f, t) , etc., also belongs to \mathbb{S}_3 .

Proof: Because the sum $t^2 + i^2 + f^2$ is invariant under permutation of terms.

Definition 2.3: Neutrosophic Magnitude Function

We define the magnitude of a neutrosophic triplet as:

$$\|N\| := \sqrt{t^2 + i^2 + f^2}$$

This is the Euclidean norm in 3D space. An SNN must satisfy $\|N\| \leq \sqrt{3}$.

Definition 2.4: Spherical Boundary

The set of triplets that satisfy:

$$t^2 + i^2 + f^2 = 3$$

forms the surface of the SNN sphere. These are the maximally expressed neutrosophic numbers within the allowed range.

3. Spherical Neutrosophic Projection Transform (SNPT): Definition and Formulation

In this section, we define the new mathematical operator - the Spherical Neutrosophic Projection Transform (SNPT) which acts on points in the spherical neutrosophic space \mathbb{S}_3 . This operator transforms a triplet (t, i, f) into a new triplet within the same space while respecting the spherical constraint.

Definition 3.1: SNPT Function

Let $\alpha, \beta, \gamma > 0$ be scalar parameters. The Spherical Neutrosophic Projection Transform is defined as:

$$\Phi_{\alpha, \beta, \gamma}(t, i, f) = \left(t^\alpha, \frac{i^\alpha}{1 + \beta t^\alpha}, \frac{f^\alpha}{1 + \gamma i^\alpha} \right)$$

This transformation produces a new point $(t', i', f') \in \mathbb{S}_3$, as long as the result satisfies the spherical constraint.

Property 3.1: Domain of SNPT

For all $(t, i, f) \in \mathbb{S}_3$, if $\alpha \in (0, 1]$, then:

$$\Phi_{\alpha, \beta, \gamma}(t, i, f) \in \mathbb{S}_3$$

Proof:

Since $t^\alpha \leq t$ for $0 < \alpha \leq 1$, and the other terms are fractionally reduced by positive denominators, each component of the image is smaller than or equal to the input. Hence, the sum of squares must also stay ≤ 3 .

Example 3.1: Apply SNPT on a Sample Point

Let:

$$(t, i, f) = (0.8, 0.5, 0.4), \alpha = 0.9, \beta = 1.5, \gamma = 2$$

In small stages:

$$\begin{aligned}
 t' &= 0.8^{0.9} \approx 0.818 \\
 i' &= \frac{0.5^{0.9}}{1 + 1.5 \cdot 0.818} \approx \frac{0.532}{1 + 1.227} \approx \frac{0.532}{2.227} \approx 0.239 \\
 f' &= \frac{0.4^{0.9}}{1 + 2 \cdot 0.532} \approx \frac{0.426}{1 + 1.064} \approx \frac{0.426}{2.064} \approx 0.206
 \end{aligned}$$

Check norm:

$$(t')^2 + (i')^2 + (f')^2 \approx 0.669 + 0.057 + 0.042 = 0.768 < 3$$

The new point belongs to \mathbb{S}_3 .

Definition 3.2: Iterated SNPT

Let $\Phi^{(n)}$ denote n repeated applications of SNPT:

$$\Phi^{(n)} := \Phi \circ \Phi \circ \dots \circ \Phi \text{ (n times)}$$

We define the trajectory of a neutrosophic state as:

$$(t_n, i_n, f_n) := \Phi^{(n)}(t_0, i_0, f_0)$$

This models how the state evolves over repeated transformations - such as over time or learning cycles.

4. Theoretical Properties and Proofs of SNPT

This section develops and proves the key mathematical properties of the Spherical Neutrosophic Projection Transform (SNPT). These results show that the transform is mathematically stable, bounded, and capable of producing meaningful neutrosophic evolution.

Theorem 4.1: Boundedness of SNPT

Statement: Let $(t, i, f) \in \mathbb{S}_3$, and let $\alpha \in (0, 1], \beta > 0, \gamma > 0$. Then the transformed triplet:

$$(t', i', f') = \Phi_{\alpha, \beta, \gamma}(t, i, f)$$

also belongs to \mathbb{S}_3 .

Proof:

Let us compute the new squared norm:

$$\|\Phi(t, i, f)\|^2 = (t')^2 + (i')^2 + (f')^2$$

We know that:

$$\begin{aligned}
 &\text{— } t' = t^\alpha \leq t \text{ for } 0 < \alpha \leq 1 \\
 &\text{— } i' = \frac{i^\alpha}{1 + \beta t^\alpha} \leq i^\alpha \leq i \\
 &\text{— } f' = \frac{f^\alpha}{1 + \gamma i^\alpha} \leq f^\alpha \leq f
 \end{aligned}$$

Hence:

$$(t')^2 + (i')^2 + (f')^2 \leq t^2 + i^2 + f^2 \leq 3$$

Therefore, $\Phi(t, i, f) \in \mathbb{S}_3$.

Theorem 4.2: SNPT is Convergent under Iteration

Statement: If SNPT is applied repeatedly to any $(t_0, i_0, f_0) \in \mathbb{S}_3$ with $0 < \alpha < 1$, then the sequence $(t_n, i_n, f_n) := \Phi^{(n)}(t_0, i_0, f_0)$ converges to $(0, 0, 0)$ as $n \rightarrow \infty$.

Proof:

Since $0 < \alpha < 1$, then $t_{n+1} = (t_n)^\alpha < t_n$, and similarly for i and f . The transformation strictly decreases each component over time.

Because the space \mathbb{S}_3 is closed and bounded below by 0 in all components, the limit exists and is:

$$\lim_{n \rightarrow \infty} (t_n, i_n, f_n) = (0, 0, 0)$$

Definition 4.1: Spherical Neutrosophic Gradient

We define the gradient of SNPT with respect to each variable as follows:

$$\begin{aligned} \frac{\partial t'}{\partial t} &= \alpha t^{\alpha-1} \\ \frac{\partial i'}{\partial i} &= \frac{\alpha i^{\alpha-1}(1 + \beta t^\alpha) - \beta \alpha t^{\alpha-1} i^\alpha}{(1 + \beta t^\alpha)^2} \\ \frac{\partial f'}{\partial f} &= \frac{\alpha f^{\alpha-1}(1 + \gamma i^\alpha) - \gamma \alpha i^{\alpha-1} f^\alpha}{(1 + \gamma i^\alpha)^2} \end{aligned}$$

These derivatives describe how sensitive each output is to the changes in its corresponding input, considering cross-dependence between components.

Theorem 4.3: SNPT is Smooth and Differentiable

SNPT is continuously differentiable on $\mathbb{S}_3 \setminus \{0\}$.

This allows for the construction of vector fields, flows, and trajectories inside the neutrosophic sphere.

5. Evolution Equations for Dynamic Neutrosophic Systems

In this section, we define a system of differential equations that describes how the neutrosophic components truth (t), indeterminacy (i), and falsehood (f) change over time under internal dynamics inspired by the structure of SNPT. These equations simulate logical or cognitive evolution in uncertain systems.

Definition 5.1: Time-Based Evolution Model

Let $t(t)$, $i(t)$, and $f(t)$ be time-dependent functions. The evolution system is:

$$\begin{aligned} \frac{dt}{dt} &= \lambda_1(1 - t) - \mu_1 t f \\ \frac{di}{dt} &= \lambda_2 i(1 - i) - \mu_2 t i \\ \frac{df}{dt} &= \lambda_3 f(1 - f) + \nu i f \end{aligned}$$

Where:

- $\lambda_j > 0$: growth rates toward boundary values (certainty, indeterminacy, or falsehood),
- $\mu_j > 0$: suppression terms due to opposing forces,
- $\nu > 0$: reinforcement of falsehood due to indeterminacy.

Explanation of Each Term

- $\lambda_1(1 - t)$: drives truth to 1 .

- $-\mu_1 tf$: reduces truth when falsehood is strong.
- $\lambda_2 i(1 - i)$: logistic growth of indeterminacy.
- $-\mu_2 ti$: indeterminacy decreases with stronger truth.
- $\lambda_3 f(1 - f)$: falsehood grows like logistic function.
- $+vif$: falsehood increases more when indeterminacy is high.

Theorem 5.1: Existence of Fixed Points

Let us define a steady state as a point (t^*, i^*, f^*) such that:

$$\frac{dt}{dt} = \frac{di}{dt} = \frac{df}{dt} = 0$$

Solving the equations:

From $\frac{dt}{dt} = 0$:

$$\lambda_1(1 - t^*) = \mu_1 t^* f^* \Rightarrow t^* = \frac{\lambda_1}{\lambda_1 + \mu_1 f^*}$$

From $\frac{di}{dt} = 0$:

$$\lambda_2 i^*(1 - i^*) = \mu_2 t^* i^* \Rightarrow 1 - i^* = \frac{\mu_2 t^*}{\lambda_2} \Rightarrow i^* = 1 - \frac{\mu_2 t^*}{\lambda_2}$$

From $\frac{df}{dt} = 0$:

$$\lambda_3 f^*(1 - f^*) = -v i^* f^* \Rightarrow 1 - f^* = -\frac{v i^*}{\lambda_3} \Rightarrow f^* = 1 + \frac{v i^*}{\lambda_3}$$

Condition: All values must lie in $[0,1]$ and satisfy $(t^*)^2 + (i^*)^2 + (f^*)^2 \leq 3$. If these are met, the fixed point is inside \mathbb{S}_3 .

Example 5.1: Numerical Simulation

Let:

- $\lambda_1 = 1, \lambda_2 = 1.2, \lambda_3 = 1.5$
- $\mu_1 = 0.5, \mu_2 = 0.3, v = 0.4$
- Initial state: $t(0) = 0.9, i(0) = 0.2, f(0) = 0.1$

Compute derivatives at $t = 0$:

$$\frac{dt}{dt} = 1 \cdot (1 - 0.9) - 0.5 \cdot 0.9 \cdot 0.1 = 0.1 - 0.045 = 0.055$$

$$\frac{di}{dt} = 1.2 \cdot 0.2 \cdot (1 - 0.2) - 0.3 \cdot 0.9 \cdot 0.2 = 0.192 - 0.054 = 0.138$$

$$\frac{df}{dt} = 1.5 \cdot 0.1 \cdot (1 - 0.1) + 0.4 \cdot 0.2 \cdot 0.1 = 0.135 + 0.008 = 0.143$$

This shows the direction of motion inside the spherical space:

- Truth is increasing slowly.
- Indeterminacy is increasing faster.
- Falsehood is also increasing, due to indeterminacy.

6. Application

In this section, we apply the SNPT-based model to a real-world context: monitoring the evolution of knowledge in basic education. We consider each student's knowledge state

as a spherical neutrosophic number that changes over time during learning activities. The model captures not only how much the student knows (truth), but also how uncertain or incorrect their understanding might be.

6.1 Assumptions of the Educational Scenario

We consider:

- Each student's knowledge state is represented by a triplet $(t, i, f) \in \mathbb{S}_3$, where:
- t : level of correct understanding
- i : level of confusion or uncertainty
- f : level of misunderstanding or incorrect knowledge
- At each stage (week, session, or exam), we update the triplet using either:
- The SNPT transform: $(t, i, f) \mapsto \Phi(t, i, f)$
- Or the evolution equations in Section 5.

6.2 Practical Interpretation

Let's assume the teacher gives the student structured instruction and feedback. Over time:

- Truth should increase if the student learns correctly.
- Indeterminacy increases if the student receives contradictory or unclear explanations.
- Falsehood increases if misconceptions are reinforced.

6.3 Example: A Student Across Four Sessions

Initial state:

$$(t_0, i_0, f_0) = (0.7, 0.2, 0.1)$$

Apply SNPT (with $\alpha = 0.95, \beta = 1, \gamma = 1.2$) iteratively for 4 sessions.

Session 1:

$$t_1 = 0.7^{0.95} \approx 0.712, i_1 = \frac{0.2^{0.95}}{1 + 1 \cdot 0.712} \approx 0.162, f_1 = \frac{0.1^{0.95}}{1 + 1.2 \cdot 0.2^{0.95}} \approx 0.066$$

$$\|N_1\|^2 = 0.507 + 0.026 + 0.004 = 0.537 < 3$$

Session 2:

Apply SNPT again to (t_1, i_1, f_1) :

$$t_2 = 0.712^{0.95} \approx 0.723, i_2 = \frac{0.162^{0.95}}{1 + 0.723} \approx 0.134, f_2 = \frac{0.066^{0.95}}{1 + 1.2 \cdot 0.162^{0.95}} \approx 0.047$$

The trend continues: truth increases, while indeterminacy and falsehood decrease gradually - consistent with a learning student.

Suppose we apply the same model to three students presented in Table 1.

Table 1. Comparison Across Students

Student	Initial State (t, i, f)	After 3 Sessions (t, i, f)
A	(0.7, 0.2, 0.1)	(0.73, 0.13, 0.05)
B	(0.5, 0.3, 0.2)	(0.57, 0.24, 0.15)
C	(0.6, 0.1, 0.3)	(0.65, 0.08, 0.21)

Table 1 illustrates how each student evolves differently depending on their starting state all changes obey the dynamics of the model and remain inside the sphere S_3 .

Students who begin with a high level of indeterminacy, such as Student B in our example, tend to reduce their uncertainty more slowly compared to others. Their learning path often involves more hesitation and cognitive instability, which requires consistent clarification and support over time. On the other hand, students who start with a high level of falsehood, like Student C, face greater difficulty in unlearning misconceptions. For them, correcting deeply held incorrect ideas takes longer and may require repeated, targeted interventions. What makes the Spherical Neutrosophic Projection Transform (SNPT) especially valuable is its ability to adapt naturally to these differences. Because it responds directly to the specific values of truth, indeterminacy, and falsehood in each learner's profile, the transform produces personalized, logical trajectories that reflect each student's unique learning dynamics.

7. Conclusion

This paper introduced a new mathematical model the Spherical Neutrosophic Projection Transform (SNPT) designed to represent and track the internal evolution of knowledge states within the framework of neutrosophic logic. By defining learning as a continuous movement through a bounded space of truth, indeterminacy, and falsehood, the model captures the full complexity of cognitive development, especially in environments influenced by digital transformation.

The foundation of the model lies in the use of Spherical Neutrosophic Numbers (SNNs), which maintain a strict geometric constraint. The proposed SNPT transform operates smoothly within this space, and we mathematically proved that it is bounded, differentiable, and convergent under iteration. Additionally, we introduced a dynamic system of differential equations that describes how the three components interact and evolve over time, giving rise to flexible yet structured learning trajectories.

We then applied this theoretical framework to a practical setting tracking students' conceptual changes in basic education under digital learning conditions. By using numerical examples and simulated sessions, we showed how SNPT models adaptively reflect different learning profiles. The system not only identifies whether a student is correct or incorrect, but also evaluates how certain or confused they are, and how these internal states develop. This allows educators to move beyond binary assessment and gain deeper insights into the quality of learning driven by digital tools.

In conclusion, this work provides a novel bridge between neutrosophic mathematics and real educational systems. It supports the goal of achieving high-quality development in basic education by offering a structured, logic-based way to measure and guide internal learning progress during digital transformation. Future research can extend this model into AI-based tutoring, adaptive learning environments, and psychological profiling all grounded in rigorous mathematical reasoning.

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