NSS
Neutrosophic Sets and Systems, Vol. 85, 2025
University of New Mexico

TOPSIS method-based decision-making model for bipolar quadripartitioned neutrosophic environment

G. Muhiuddin[1*], Mohamed E. Elnair[1,2], Satham Hussain S[3], Durga Nagarajan[3]
[1]Department of Mathematics, University of Tabuk, Tabuk 71491, Saudi Arabia; chishtygm@gmail.com
[2]Department of Mathematics and Physics, Gezira University, P. O. Box 20, Sudan; abomunzir124@gmail.com
[3]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Chennai-600 127, Tamil Nadu, India; durga.nagarajan@vit.ac.in, sathamhussain5592@gmail.com
[*]Correspondence: chishtygm@gmail.com

Abstract. In the domain of renewable energy, selecting the most suitable energy source involves navigating complex decision-making processes influenced by multiple criteria and inherent uncertainties. This study proposes a novel approach using the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) and ELECTRE-I (Elimination and Choice Translating Reality) methods within a Bipolar Quadripartitioned Neutrosophic (BQN) environment to address these challenges. The BQN framework integrates truth, contradiction, ignorance, and falsity membership functions, allowing for a comprehensive evaluation of renewable energy sources. Criteria such as energy efficiency, environmental impact, cost and resource availability are considered, each characterized by its respective membership function. Numerical examples and comparative analyses demonstrate the efficacy of the proposed approach, highlighting its applicability in enhancing decision-making reliability and robustness in renewable energy selection scenarios.

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1. Introduction

A bipolar neutrosophic set is an extension of the neutrosophic set that considers both positive and negative membership degrees, providing a more comprehensive framework for handling uncertainty and imprecision in decision-making problems. It incorporates truth-membership, indeterminacy-membership and falsity-membership functions for both positive and negative aspects. Building on this, the bipolar quadripartitioned neutrosophic set further refines the framework by dividing each membership function into four parts: truth, contradiction, ignorance and falsity. This enhanced model allows for a more detailed assessment of criteria and alternatives, making it particularly useful in complex decision-making scenarios like renewable energy source selection, where multiple conflicting and uncertain criteria need to be evaluated systematically. Decision-making approaches have gained popularity due to their extensive use in a wide range of fields, including waste management [2–4], the medical sciences [5, 6], commercial investing [10–12], and many other areas of research and technology. The quadripartitioned neutrosophic set advanced framework allows for a more comprehensive representation of uncertain, imprecise and inconsistent information in decision-making processes [25–28, 30]. Our bipolar quadripartitioned neutrosophic TOPSIS method utilizes these elements to deliver a delicate ranking of alternatives, taking into account both positive and negative ideal solutions. Likewise, the bipolar quadripartitioned neutrosophic ELECTRE-I method integrates this framework to improve the precision of the outranking relations among the alternatives.

A comprehensive review of MCDM issues with healthcare applications can be found in the paper [13]. The outranking methodology of the ELECTRE method for MCDM issues in the interval-valued neutrosophic region is discussed in the article [14].A dynamic single-valued neutrosophic multiset was presented by the author in [15] as an improved technique for expressing dynamic information of dynamic issues that represent dynamic information gathered from various time intervals. The authors in [16] define a dynamic interval-valued neutrosophic set as a way to characterise time-dependent real-world data based on neutrosophic sets. The paper [17] discusses information measures based on similarity in an MCDM problem and a neutrosophic fuzzy environment. [18] establishes the MCDM model utilising trigonometric aggregation operations of single-valued neutrosophic credibility numbers. The work [20] establishes an integrated SWARA-CODAS decision-making method with spherical fuzzy information. A recent study in [21] examines the application of ranking approach to aggregation operators for complex intuitionistic fuzzy sets and the MCDM problem. The coupling of a truthful-distance measure with the TOPSIS framework for neutrosophic soft sets is developed in [22]. The selection procedure based on new building construction job is established in [23] by employing square root ambiguous sets and their aggregated operators. Aggregation operators of quadripartitioned single-valued neutrosophic Z-numbers are investigated by the authors in [24], with applicability to various COVID-19 scenarios. A novel stability analysis of functional equation in neutrosophic normed spaces are determined in [29]. The significance of selecting the best renewable energy source within the Bipolar Quadripartitioned Neutrosophic framework cannot be overstated. As global energy demand continues to rise, alongside mounting environmental concerns, identifying energy solutions that balance efficiency, environmental impact, cost-effectiveness and resource availability is critical. Each renewable energy option, whether solar, wind, hydropower, or biomass, offers unique advantages and challenges that must be carefully weighed against these criteria. Moreover, the BQNF approach acknowledges and quantifies uncertainties and incomplete information inherent in these assessments, ensuring that decisions are robust and defensible. By applying this advanced decision-making methodology, stakeholders can strategically invest in renewable energy infrastructure that maximizes benefits while minimizing environmental and economic risks, thereby fostering a sustainable and resilient energy future for generations to come [35–39]. The following are the contribution of the present work:

(i) This paper introduces the bipolar quadripartitioned neutrosophic TOPSIS and the bipolar quadripartitioned neutrosophic ELECTRE-I algorithms

(ii) We formulated the decision problem using bipolar quadripartitioned neutrosophic set, where each alternative material was evaluated against four attributes.

(iii) The decision matrix is constructed based on bipolar quadripartitioned neutrosophic information provided by domain experts. The weights of the attributes were determined using the maximizing deviation method, reflecting their relative importance in the decision process.

(iv) For both benefit and cost type attributes, bipolar quadripartitioned neutrosophic relative positive ideal solution and bipolar quadripartitioned neutrosophic relative negative ideal solution are derived.

(v) Calculated pairwise comparisons and aggregating outranking relations are computationally intensive and quite challenging, particularly as the number of alternatives and criteria increases.

(vi) Based on the calculated distances, the application model is finalized according to their inferior ratio values for better suitability for renewable energy source. This work is the generalization of the existing work [40–42].

The structure of the work is provided as follows. Section 2 describes the quadripartitioned neutrosophic TOPSIS method with an application. Section 3 gives the quadripartitioned neutrosophic ELECTRE-I method with an application to the renewable energy source selection. Finally, the comparison of the proposed methods are given in Section 4.

In the application of selecting the best renewable energy source using the Bipolar Quadripartitioned Neutrosophic Field, we consider multiple evaluation criteria and potential alternatives to ensure a comprehensive decision-making process.

Criteria: 1. Energy Efficiency ($\mathbb{T}_1$) :

This criterion assesses how effectively an energy source converts input energy (like sunlight, wind, water flow, or biomass) into usable electrical energy. High energy efficiency means less energy waste, leading to more sustainable and cost-effective power generation. Energy efficiency is typically measured as a percentage or ratio of output energy to input energy.

2. Environmental Impact ($\mathbb{T}_2$) :

This criterion evaluates the ecological footprint of the energy source, including emissions, pollution, habitat disruption and resource depletion. A lower environmental impact is crucial for maintaining ecological balance and adhering to environmental regulations. Environmental impact can be measured through various indicators such as carbon footprint, emission levels and effects on biodiversity.

3. Cost ($\mathbb{T}_3$) :

This criterion considers the overall expenses associated with deploying, operating and maintaining the energy source. Cost is a critical factor for economic feasibility and budget planning. Cost is measured in terms of capital expenditure, operational expenditure and life cycle costs.

4. Resource Availability ($\mathbb{T}_4$) :

This criterion assesses the accessibility and abundance of the resources needed for the energy source (e.g., sunlight for solar power, wind for wind power). High resource availability ensures a stable and reliable energy supply. Resource availability can be quantified by the potential energy yield based on geographic and climatic conditions.

Alternatives

1. Solar Power (Υ_1) : Solar power produces energy from the sun using photovoltaic cells or solar thermal systems. Strengths are renewable, abundant and low operational costs. Intermittent energy supply is the challenge due to weather and time of day, high initial setup costs.

2. Wind Power (Υ_2) : Wind power captures kinetic energy from wind using turbines. Strengths are renewable, low emissions and scalable. Challenges are variable wind speeds, noise and visual impact.

3. Hydropower (Υ_3) : Hydropower generates electricity by using water flow through dams or run-of-the-river systems. Strengths are consistent energy supply, low emissions and potential for energy storage (pumped storage). Challenges are environmental and ecological impacts on aquatic systems, high initial infrastructure costs.

4. Biomass Energy (Υ_4) : Biomass energy derives from organic materials such as plant and animal waste, which are burned or converted into biofuels. Strengths are utilizes waste materials, can be continuously produced and reduces landfill use. Challenges are emissions from combustion, competition with food production for resources.

Decision-Making Process

To select the optimal renewable energy source, the bipolar quadripartitioned neutrosophic field approach is employed. This involves:

1. Defining Membership Functions : Establishing truth, contradiction, ignorance and falsity membership functions for each criterion and alternative.

2. Evaluating Alternatives : Using the defined membership functions to assess each alternative against the criteria.

3. Applying Decision-Making Methods : Implementing TOPSIS and ELECTRE-I methods to rank and select the best alternative based on the evaluation.

This approach allows for a delicate consideration of the uncertainties and complexities inherent in renewable energy evaluation, providing a robust framework for making informed decisions.

2. Bipolar Quadripartitioned Neutrosophic TOPSIS Method

Definition 2.1. A Bipolar Quadripartitioned Neutrosophic set (BQNs) $\mathbb{B}$ on a non empty set $\mathbb{C}$ is defined as: $\mathbb{B} = \{g, \langle \mathbb{T}_{\mathbb{B}}^{p+}(g), \mathbb{C}_{\mathbb{B}}^{p+}(g), \mathbb{U}_{\mathbb{B}}^{p+}(g), \mathbb{F}_{\mathbb{B}}^{p+}(g), \mathbb{T}_{\mathbb{B}}^{n-}(g), \mathbb{C}_{\mathbb{B}}^{n-}(g), \mathbb{U}_{\mathbb{B}}^{n-}(g), \mathbb{F}_{\mathbb{B}}^{n-}(g)\rangle | g \in \mathbb{G}\}$, where, $\mathbb{T}_{\mathbb{B}}^{p+}(g), \mathbb{C}_{\mathbb{B}}^{p+}(g), \mathbb{U}_{\mathbb{B}}^{p+}(g), \mathbb{F}_{\mathbb{B}}^{p+}(g) : \mathbb{G} \to [0,1]$ and $\mathbb{T}_{\mathbb{B}}^{n-}(g), \mathbb{C}_{\mathbb{B}}^{n-}(g), \mathbb{U}_{\mathbb{B}}^{n-}(g), \mathbb{F}_{\mathbb{B}}^{n-}(g) : \mathbb{G} \to [-1,0]$

We now provide our suggested Bipolar Quadripartitioned TOPSIS approach for neutrosophic analysis.

Let $\mathbb{T} = \{\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3, ..., \mathbb{T}_q\}$ be a collection of q attributes and let $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_p\}$ be a collection of p favorable alternative. To ensure that $0 \leq \omega_k \leq 1$, $\mathbb{W} = [\omega_1, \omega_2, \omega_3, ..., \omega_q]^T$ be the weight vector and $\sum\limits_{k=1}^{q} \omega_k = 1$. Assume that the decision maker provides the rating value of each alternative Υ_l, $(l = 1, 2, ..., p)$ in the form of BQNSs, with respect to the attributes $\mathbb{T}_k$, $(k = 1, 2, 3, ..., q)$.

The following describes the steps of the Bipolar Quadripartitioned Neutrosophic TOPSIS method:

(i) The q criteria is used to estimate each alternative value. The BQNSs provide the value of each alternative under each criterion and they can be stated in the decision matrix as

$$H = [h_{lk}]_{p\times q} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1q} \\ h_{21} & h_{22} & \dots & h_{2q} \\ . & . & \dots & . \\ . & . & \dots & . \\ h_{p1} & h_{p2} & \dots & h_{pq} \end{bmatrix}$$

Each entry $h_{lk} = (\mathbb{T}_{lk}^{p+}, \mathbb{C}_{lk}^{p+}, \mathbb{U}_{lk}^{p+}, \mathbb{F}_{lk}^{p+}, \mathbb{T}_{lk}^{n-}, \mathbb{C}_{lk}^{n-}, \mathbb{U}_{lk}^{n-}, \mathbb{F}_{lk}^{n-})$, where, $\mathbb{T}_{lk}^{p+}, \mathbb{C}_{lk}^{p+}, \mathbb{U}_{lk}^{p+}$ and $\mathbb{F}_{lk}^{p+}$ represent the degree of positive truth, positive contradiction, positive ignorance and positive false membership degree and $\mathbb{T}_{lk}^{n-}, \mathbb{C}_{lk}^{n-}, \mathbb{U}_{lk}^{n-}$ and $\mathbb{F}_{lk}^{n-}$ represent the degree of negative truth, negative contradiction, negative ignorance and negative false membership degree respectively, such that $\mathbb{T}_{lk}^{p+}, \mathbb{C}_{lk}^{p+}, \mathbb{U}_{lk}^{p+}, \mathbb{F}_{lk}^{p+} \in [0,1]$.

Also, $\mathbb{T}_{lk}^{n-}, \mathbb{C}_{lk}^{n-}, \mathbb{U}_{lk}^{n-}, \mathbb{F}_{lk}^{n-} \in [-1,0]$ and $0 \leq \mathbb{T}_{lk}^{p+} + \mathbb{C}_{lk}^{p+} + \mathbb{U}_{lk}^{p+} + \mathbb{F}_{lk}^{p+} - \mathbb{T}_{lk}^{n-} - \mathbb{C}_{lk}^{n-} - \mathbb{U}_{lk}^{n-} - \mathbb{F}_{lk}^{n-} \leq 8$, $l = 1, 2, \dots, p, \quad k = 1, 2, \dots, q$.

(ii) Assume the decision maker does not know the weights of the criterion and they are not distributed equally. To get the undefined weights of the criteria, we apply the maximising deviation approach [1]. Consequently, $\mathbb{T}_k$'s weight is provided as

$$\omega_k = \frac{\sum_{l=1}^{p}\sum_{r=1}^{p}|h_{lk}-h_{rk}|}{\sqrt{\sum_{k=1}^{q}(\sum_{l=1}^{p}\sum_{r=1}^{p}|h_{lk}-h_{rk}|)^2}}$$

and the normalized weight of the attributes $\mathbb{T}_k$ is given as

$$\omega_k^* = \frac{\sum_{l=1}^{p}\sum_{r=1}^{p}|h_{lk}-h_{rk}|}{\sqrt{\sum_{k=1}^{q}(\sum_{l=1}^{p}\sum_{r=1}^{p}|h_{lk}-h_{rk}|)}}$$

(iii) The weights of the attributes of the aggregated decision matrix are multiplied as follows to calculate the accumulated weighted bipolar quadripartitioned neutrosophic decision matrix:

$$H \boxtimes \omega = [h_{lk}^{\omega_k}]_{p\times q} = \begin{bmatrix} h_{11}^{\omega_1} & h_{12}^{\omega_2} & \dots & h_{1q}^{\omega_q} \\ h_{21}^{\omega_1} & h_{22}^{\omega_2} & \dots & h_{2q}^{\omega_q} \\ . & . & \dots & . \\ . & . & \dots & . \\ h_{p1}^{\omega_1} & h_{p2}^{\omega_2} & \dots & h_{pq}^{\omega_p} \end{bmatrix}$$

where

$$h_{lk}^{\omega_k} = (\mathbb{T}_{lk}^{\omega_k p+}, \mathbb{C}_{lk}^{\omega_k p+}, \mathbb{U}_{lk}^{\omega_k p+}, \mathbb{F}_{lk}^{\omega_k p+}, \mathbb{T}_{lk}^{\omega_k n-}, \mathbb{C}_{lk}^{\omega_k n-}, \mathbb{U}_{lk}^{\omega_k n-}, \mathbb{F}_{lk}^{\omega_k n-}) = (1-(1-\mathbb{T}_{lk}^{p+})^{\omega_k}, (\mathbb{C}_{lk}^{p+})^{\omega_k}, (\mathbb{U}_{lk}^{p+})^{\omega_k}, (\mathbb{F}_{lk}^{p+})^{\omega_k},$$
$$(1-(1-\mathbb{T}_{lk}^{n-})^{\omega_k}, (\mathbb{C}_{lk}^{n-})^{\omega_k}, (\mathbb{U}_{lk}^{n-})^{\omega_k}, (\mathbb{F}_{lk}^{n-})^{\omega_k}$$

(iv) When making decisions in real life, two sorts of qualities are most useful: benefit-type attributes and cost-type attributes. For both kinds of qualities, the following definitions apply to the bipolar quadripartitioned neutrosophic relative positive ideal solution (BQNRPIS) and bipolar quadripartitioned neutrosophic relative negative ideal solution (BQNRNIS):

$$\begin{aligned} BQNRPIS =& (\langle {}^{p+}\mathbb{T}_1^{\omega_1+}, {}^{p+}\mathbb{C}_1^{\omega_1+}, {}^{p+}\mathbb{U}_1^{\omega_1+}, {}^{p+}\mathbb{F}_1^{\omega_1+}, {}^{p+}\mathbb{T}_1^{\omega_1-}, {}^{p+}\mathbb{C}_1^{\omega_1-}, {}^{p+}\mathbb{U}_1^{\omega_1-}, {}^{p+}\mathbb{F}_1^{\omega_1-}\rangle, \\ & \langle {}^{p+}\mathbb{T}_2^{\omega_2+}, {}^{p+}\mathbb{C}_2^{\omega_2+}, {}^{p+}\mathbb{U}_2^{\omega_2+}, {}^{p+}\mathbb{F}_2^{\omega_2+}, {}^{p+}\mathbb{T}_2^{\omega_2-}, {}^{p+}\mathbb{C}_2^{\omega_2-}, {}^{p+}\mathbb{U}_2^{\omega_2-}, {}^{p+}\mathbb{F}_2^{\omega_2-}\rangle, \dots, \\ & {}^{p+}\mathbb{T}_q^{\omega_q+}, {}^{p+}\mathbb{C}_q^{\omega_q+}, {}^{p+}\mathbb{U}_q^{\omega_q+}, {}^{p+}\mathbb{F}_q^{\omega_q+}, {}^{p+}\mathbb{T}_q^{\omega_q-}, {}^{p+}\mathbb{C}_q^{\omega_q-}, {}^{p+}\mathbb{U}_q^{\omega_q-}, {}^{p+}\mathbb{F}_q^{\omega_q-}\rangle \\ BQNRNIS =& (\langle {}^{n-}\mathbb{T}_1^{\omega_1+}, {}^{n-}\mathbb{C}_1^{\omega_1+}, {}^{n-}\mathbb{U}_1^{\omega_1+}, {}^{n-}\mathbb{F}_1^{\omega_1+}, {}^{n-}\mathbb{T}_1^{\omega_1-}, {}^{n-}\mathbb{C}_1^{\omega_1-}, {}^{n-}\mathbb{U}_1^{\omega_1-}, {}^{n-}\mathbb{F}_1^{\omega_1-}\rangle, \\ & \langle {}^{n-}\mathbb{T}_2^{\omega_2+}, {}^{n-}\mathbb{C}_2^{\omega_2+}, {}^{n-}\mathbb{U}_2^{\omega_2+}, {}^{n-}\mathbb{F}_2^{\omega_2+}, {}^{n-}\mathbb{T}_2^{\omega_2-}, {}^{n-}\mathbb{C}_2^{\omega_2-}, {}^{n-}\mathbb{U}_2^{\omega_2-}, {}^{n-}\mathbb{F}_2^{\omega_2-}\rangle, \dots, \\ & {}^{n-}\mathbb{T}_q^{\omega_q+}, {}^{n-}\mathbb{C}_q^{\omega_q+}, {}^{n-}\mathbb{U}_q^{\omega_q+}, {}^{n-}\mathbb{F}_q^{\omega_q+}, {}^{n-}\mathbb{T}_q^{\omega_q-}, {}^{n-}\mathbb{C}_q^{\omega_q-}, {}^{n-}\mathbb{U}_q^{\omega_q-}, {}^{n-}\mathbb{F}_q^{\omega_q-}\rangle \end{aligned}$$

with regarding benefit type criteria, $k = 1, 2, \dots q$.

$$({}^{p+}\mathbb{T}_k^{\omega_k+}, {}^{p+}\mathbb{C}_k^{\omega_k+}, {}^{p+}\mathbb{U}_k^{\omega_k+}, {}^{p+}\mathbb{F}_k^{\omega_k+}, {}^{p+}\mathbb{T}_k^{\omega_k-}, {}^{p+}\mathbb{C}_k^{\omega_k-}, {}^{p+}\mathbb{U}_k^{\omega_k-}, {}^{p+}\mathbb{F}_k^{\omega_k-}) = (\max(\mathbb{T}_{lk}^{\omega_k+}), \min(\mathbb{C}_{lk}^{\omega_k+}), \min(\mathbb{U}_{lk}^{\omega_k+}), \min(\mathbb{F}_{lk}^{\omega_k+}),$$
$$\min(\mathbb{T}_{lk}^{\omega_k-}), \max(\mathbb{C}_{lk}^{\omega_k-}), \max(\mathbb{U}_{lk}^{\omega_k-}), \max(\mathbb{F}_{lk}^{\omega_k-}))$$

$$({}^{n-}\mathbb{T}_k^{\omega_k+},\ {}^{n-}\mathbb{C}_k^{\omega_k+},\ {}^{n-}\mathbb{U}_k^{\omega_k+},\ {}^{n-}\mathbb{F}_k^{\omega_k+}, {}^{n-}\mathbb{T}_k^{\omega_k-},\ {}^{n-}\mathbb{C}_k^{\omega_k-},\ {}^{n-}\mathbb{U}_k^{\omega_k-},\ {}^{n-}\mathbb{F}_k^{\omega_k-}) = (\min(\mathbb{T}_{lk}^{\omega_k+}), \max(\mathbb{C}_{lk}^{\omega_k+}), \max(\mathbb{U}_{lk}^{\omega_k+}), \max(\mathbb{F}_{lk}^{\omega_k+}), \max(\mathbb{T}_{lk}^{\omega_k-}), \min(\mathbb{C}_{lk}^{\omega_k-}), \min(\mathbb{U}_{lk}^{\omega_k-}), \min(\mathbb{F}_{lk}^{\omega_k-}))$$

Likewise, for cost type criteria, $k = 1, 2, \dots q$.

$$({}^{p+}\mathbb{T}_k^{\omega_k+},\ {}^{p+}\mathbb{C}_k^{\omega_k+},\ {}^{p+}\mathbb{U}_k^{\omega_k+},\ {}^{p+}\mathbb{F}_k^{\omega_k+}, {}^{p+}\mathbb{T}_k^{\omega_k-},\ {}^{p+}\mathbb{C}_k^{\omega_k-},\ {}^{p+}\mathbb{U}_k^{\omega_k-},\ {}^{p+}\mathbb{F}_k^{\omega_k-}) = (\min(\mathbb{T}_{lk}^{\omega_k+}), \max(\mathbb{C}_{lk}^{\omega_k+}), \max(\mathbb{U}_{lk}^{\omega_k+}), \max(\mathbb{F}_{lk}^{\omega_k+}), \max(\mathbb{T}_{lk}^{\omega_k-}), \min(\mathbb{C}_{lk}^{\omega_k-}), \min(\mathbb{U}_{lk}^{\omega_k-}), \min(\mathbb{F}_{lk}^{\omega_k-}))$$

$$({}^{n-}\mathbb{T}_k^{\omega_k+},\ {}^{n-}\mathbb{C}_k^{\omega_k+},\ {}^{n-}\mathbb{U}_k^{\omega_k+},\ {}^{n-}\mathbb{F}_k^{\omega_k+}, {}^{n-}\mathbb{T}_k^{\omega_k-},\ {}^{n-}\mathbb{C}_k^{\omega_k-},\ {}^{n-}\mathbb{U}_k^{\omega_k-},\ {}^{n-}\mathbb{F}_k^{\omega_k-}) = (\max(\mathbb{T}_{lk}^{\omega_k+}), \min(\mathbb{C}_{lk}^{\omega_k+}), \min(\mathbb{U}_{lk}^{\omega_k+}), \min(\mathbb{F}_{lk}^{\omega_k+}), \min(\mathbb{T}_{lk}^{\omega_k-}), \max(\mathbb{C}_{lk}^{\omega_k-}), \max(\mathbb{U}_{lk}^{\omega_k-}), \max(\mathbb{F}_{lk}^{\omega_k-}))$$

(v) The normalized Euclidean distance for all alternatives $(\mathbb{T}_{lk}^{\omega_k+}, \mathbb{C}_{lk}^{\omega_k+}, \mathbb{U}_{lk}^{\omega_k+}, \mathbb{F}_{lk}^{\omega_k+}, \mathbb{T}_{lk}^{\omega_k-}, \mathbb{C}_{lk}^{\omega_k-}, \mathbb{U}_{lk}^{\omega_k-}, \mathbb{F}_{lk}^{\omega_k-})$ from QNRPIS $({}^{p+}\mathbb{T}_k^{\omega_k+},\ {}^{p+}\mathbb{C}_k^{\omega_k+},\ {}^{p+}\mathbb{U}_k^{\omega_k+},\ {}^{p+}\mathbb{F}_k^{\omega_k+}, {}^{p+}\mathbb{T}_k^{\omega_k-},\ {}^{p+}\mathbb{C}_k^{\omega_k-},\ {}^{p+}\mathbb{U}_k^{\omega_k-},\ {}^{p+}\mathbb{F}_k^{\omega_k-})$ is evaluated as

$$d_N(\Upsilon_l, BQNRPIS) = \sqrt{\begin{array}{l}\dfrac{1}{8q}\displaystyle\sum_{k=1}^{q}\Big\{(\mathbb{T}_{lk}^{\omega_k+} - {}^{p+}\mathbb{T}_k^{\omega_k+})^2 + (\mathbb{C}_{lk}^{\omega_k+} - {}^{p+}\mathbb{C}_k^{\omega_k+})^2 + (\mathbb{U}_{lk}^{\omega_k+} - {}^{p+}\mathbb{U}_k^{\omega_k+})^2 + (\mathbb{F}_{lk}^{\omega_k+} - {}^{p+}\mathbb{F}_k^{\omega_k+})^2 \\ + (\mathbb{T}_{lk}^{\omega_k-} - {}^{p+}\mathbb{T}_k^{\omega_k-})^2 + (\mathbb{C}_{lk}^{\omega_k-} - {}^{p+}\mathbb{C}_k^{\omega_k-})^2 + (\mathbb{U}_{lk}^{\omega_k-} - {}^{p+}\mathbb{U}_k^{\omega_k-})^2 + (\mathbb{F}_{lk}^{\omega_k-} - {}^{p+}\mathbb{F}_k^{\omega_k-})^2\Big\}\end{array}}$$

and the normalized Euclidean distance of all alternative $(\mathbb{T}_{lk}^{\omega_k+}, \mathbb{C}_{lk}^{\omega_k+}, \mathbb{U}_{lk}^{\omega_k+}, \mathbb{F}_{lk}^{\omega_k+}, \mathbb{T}_{lk}^{\omega_k-}, \mathbb{C}_{lk}^{\omega_k-}, \mathbb{U}_{lk}^{\omega_k-}, \mathbb{F}_{lk}^{\omega_k-})$ from BQNRNIS $({}^{n-}\mathbb{T}_k^{\omega_k+},\ {}^{n-}\mathbb{C}_k^{\omega_k+},\ {}^{n-}\mathbb{U}_k^{\omega_k+},\ {}^{n-}\mathbb{F}_k^{\omega_k+}, {}^{n-}\mathbb{T}_k^{\omega_k-},\ {}^{n-}\mathbb{C}_k^{\omega_k-},\ {}^{n-}\mathbb{U}_k^{\omega_k-},\ {}^{n-}\mathbb{F}_k^{\omega_k-})$ is evaluated as

$$d_N(\Upsilon_l, BQNRNIS) = \sqrt{\begin{array}{l}\dfrac{1}{8q}\displaystyle\sum_{k=1}^{q}\{(\mathbb{T}_{lk}^{\omega_k+} - {}^{n-}\mathbb{T}_k^{\omega_k+})^2 + (\mathbb{C}_{lk}^{\omega_k+} - {}^{n-}\mathbb{C}_k^{\omega_k+})^2 + (\mathbb{U}_{lk}^{\omega_k+} - {}^{n-}\mathbb{U}_k^{\omega_k+})^2 + (\mathbb{F}_{lk}^{\omega_k+} - {}^{n-}\mathbb{F}_k^{\omega_k+})^2 \\ + (\mathbb{T}_{lk}^{\omega_k-} - {}^{n-}\mathbb{T}_k^{\omega_k-})^2 + (\mathbb{C}_{lk}^{\omega_k-} - {}^{n-}\mathbb{C}_k^{\omega_k-})^2 + (\mathbb{U}_{lk}^{\omega_k-} - {}^{n-}\mathbb{U}_k^{\omega_k-})^2 + (\mathbb{F}_{lk}^{\omega_k-} - {}^{n-}\mathbb{F}_k^{\omega_k-})^2\}\end{array}}$$

(vi)The revised closeness degree of all alternative to the BQNRPIS, is computed using a formula.

$$\Psi(\Upsilon_l) = \frac{d_N(\Upsilon_l, BQNRNIS)}{\max\{d_N(\Upsilon_l, BQNRNIS)\}} - \frac{d_N(\Upsilon_l, BQNRPIS)}{\min\{d_N(\Upsilon_l, BQNRPIS)\}}, \quad l = 1, 2, \dots, p.$$

(vii) The inferior ratio to each choice is ascertained by using the updated proximity degrees, which are as follows:

$$\mathcal{IR}(l) = \frac{\Psi(\Upsilon_l)}{\min\limits_{1 \le l \le p}(\Psi(\Upsilon_l))}.$$

It is evident that the closed unit interval [0,1] contains all values of $\mathcal{IR}(l)$.

(viii) The alternatives are arranged in ascending order of inferior ratio values and the option with the lowest choice value is selected as the best one.

2.1. *Application of the Proposed Method - Selection of the Best Renewable Energy Source using Bipolar Quadripartitioned Neutrosophic Field*

In the pursuit of sustainable development, the selection of an appropriate renewable energy source plays a pivotal role in shaping our environmental footprint and energy future. The integration of renewable energy technologies not only addresses the imperative of reducing carbon emissions but also fosters energy security and economic resilience. However, choosing the optimal renewable energy source amidst a lot of alternatives involves navigating through complexities such as varying efficiencies, environmental impacts, costs and resource availabilities. These decisions are further compounded by uncertainties and ambiguities inherent in evaluating diverse criteria. In this context, the BQNFs offer a sophisticated framework that accommodates delicate assessments, considering truth, contradiction, ignorance and falsity membership functions. This approach provides a structured methodology to systematically evaluate and rank renewable energy sources based on comprehensive criteria, thereby facilitating informed decision-making towards a sustainable energy landscape. In the renewable energy, selecting the optimal energy source involves evaluating multiple criteria under conditions of uncertainty and imprecision. The decision-making process benefits from the Bipolar Quadripartitioned Neutrosophic approach, which provides a more delicate assessment by considering the truth, contradiction, ignorance and falsity membership functions.

Criteria:

(T1): How efficiently the energy source converts input into usable energy. (T2): The ecological footprint, including emissions and resource depletion. (T3): The overall expense of deploying and maintaining the energy source. (T4): The accessibility and abundance of the energy source. Energy Sources:

Steps in the Decision-Making Process Define the Criteria and Alternatives:

List the criteria (energy efficiency, environmental impact, cost, resource availability). Identify the alternatives (solar power, wind power, hydropower, biomass energy).

This application focuses on the use of bipolar quadripartitioned neutrosophic sets to handle the uncertainties and imprecisions associated with the evaluation criteria.

Energy Efficiency = $\mathbb{T}_1$, Environmental Impact = $\mathbb{T}_2$, Cost = $\mathbb{T}_3$, Resource Availability = $\mathbb{T}_4$ are considered. Four attributes, Υ_1 = Solar Power, Υ_2 = Wind Power, Υ_3 = Hydropower and Υ_4 = Biomass Energy, are made to select the optimal option.

Step 1. Table 1 presents the decision matrix as bipolar quadripartitioned neutrosophic information.

Step 2 : The maximising deviation approach is used to obtain the normalised weights of the criteria, as follows: $\omega_1 = 0.2221$, $\omega_2 = 0.1665$, $\omega_3 = 0.2774$, $\omega_4 = 0.3326$

Step 3: The weighted bipolar quadripartitioned neutrosophic decision matrix is prepared by

$\Upsilon \setminus \mathbb{T}$	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$
Υ_1	(0.5,0.6,0.5,0.4) (-0.4,-0.7,-0.4,-0.6)	(0.7,0.8,0.4,0.3) (-0.4,-0.6,-0.4,-0.7)	(0.1,0.3,0.4,0.6) (-0.2,-0.7,-0.4,-0.1)	(0.4,0.4,0.2,0.3) (-0.5,-0.5,-0.3,-0.3)
Υ_2	(0.4,0.5,0.7,0.4) (-0.7,-0.6,-0.4,-0.1)	(0.6,0.4,0.3,0.2) (-0.2,-0.4,-0.4,-0.1)	(0.8,0.6,0.4,0.6) (-0.9,-0.4,-0.4,-0.3)	(0.9,0.7,0.6,0.1) (-0.4,-0.4,-0.3,-0.4)
Υ_3	(0.7,0.4,0.4,0.2) (-0.4,-0.4,-0.3,-0.2))	(0.8,0.6,0.4,0.6) (-0.6,-0.7,-0.4,-0.2)	(0.6,0.5,0.4,0.8) (-0.4,-0.5,-0.6,-0.1)	(0.4,0.3,0.6,0.1) (-0.1,-0.2,-0.3,-0.4)
Υ_4	(0.8,0.6,0.4,0.4) (-0.6,-0.4,-0.3,-0.6)	(0.6,0.7,0.5,0.4) (-0.6,-0.4,-0.4,-0.4)	(0.6,0.7,0.8,0.9) (-0.8,-0.7,-0.6,-0.5)	(0.8,0.7,0.6,0.5) (-0.7,-0.6,-0.4,-0.4)

TABLE 1. Bipolar quadripartitioned neutrosophic decision matrix

multiplying the weights by the decision matrix, as indicated in Table 2.

$\Upsilon \setminus \mathbb{T}$	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$
Υ_1	(0.142,0.899,0.857,0.599) (-0.815,- 0.923,-0.815,-0.184)	(0.181,0.963,0.858,0.818) (-0.858,-0.918,-0.858,-0.181)	(0.028,0.716,0.775,0.867) (-0.639,-0.905,-0.775,-0.028)	(0.262,0.737,0.585,0.670) (-0.794,-0.794,-0.670,-0.111)
Υ_2	(0.107,0.857,0.924,0.816) (-0.924,-0.893,-0.816,-0.023)	(0.141,0.858,0.818,0.764) (-0.764,-0.858,-0.858,-0.017)	(0.360,0.867,0.775,0.867) (-0.971,-0.775,-0.775,-0.716)	(0.535,0.888,0.843,0.464) (-0.737,-0.734,-0.670,-0.156)
Υ_3	(0.234,0.815,0.815,0.699) (-0.815,-0.815,-0.765,-0.048)	(0.235,0.918,0.858,0.918) (-0.918,-0.942,-0.858,-0.036)	(0.224,0.825,0.775,0.939) (-0.775,-0.825,-0.867,-0.028)	(0.156,0.670,0.843,0.464) (-0.464,-0.585,-0.670,-0.156)
Υ_4	(0.300,0.892,0.815,0.815) (-0.892,-0.815,-0.765,-0.048)	(0.141,0.942,0.858,0.918) (-0.918,-0.942,-0.858,-0.036)	(0.224,0.905,0.939,0.971) (-0.939,-0.905,-0.867,-0.825)	(0.414,0.888,0.843,0.794) (-0.888,-0.843,-0.737,-0.156)

TABLE 2. weighted bipolar quadripartitioned neutrosophic decision matrix

Step 4. The BQNRPIS and BQNRNIS are given by

$$\begin{aligned}\text{BQNRPIS} = \{&(0.300, 0.815, 0.815, 0.599, -0.924, -0.815, -0.765, -0.023),\\ &(0.235, 0.858, 0.818, 0.764, -0.918, -0.858, -0.858, -0.017),\\ &(0.360, 0.716, 0.775, 0.867, -0.971, -0.825, -0.775, -0.028),\\ &(0.535, 0.670, 0.585, 0.464, -0.888, -0.585, -0.670, -0.111)\}.\end{aligned}$$

$$\begin{aligned}\text{BQNRNIS} = \{&(0.107, 0.899, 0.924, 0.816, -0.815, -0.923, -0.816, -0.184),\\ &(0.141, 0.963, 0.858, 0.918, -0.764, -0.918, -0.942, -0.181),\\ &(0.028, 0.905, 0.939, 0.971, -0.639, -0.905, -0.867, -0.825),\\ &(0.156, 0.888, 0.843, 0.794, -0.464, -0.843, -0.737, -0.156)\}.\end{aligned}$$

Step 5. The following are the normalised Euclidean distances of each alternative from the BQNRPISs and the BQNRNISs.

$$d_N(\Upsilon_1, BQNRPIS) = 0.1187, \quad d_N(\Upsilon_1, BQNRNIS) = (0.1296)$$
$$d_N(\Upsilon_2, BQNRPIS) = 0.1224, \quad d_N(\Upsilon_2, BQNRNIS) = (0.1523)$$
$$d_N(\Upsilon_3, BQNRPIS) = 0.08, \quad d_N(\Upsilon_3, BQNRNIS) = (0.1423)$$
$$d_N(\Upsilon_4, BQNRPIS) = 0.01714, \quad d_N(\Upsilon_4, BQNRNIS) = (0.0989).$$

Step 6. Each alternative's updated closeness degree is provided as:

$$\Psi(\Upsilon_1) = (-0.637), \Psi(\Upsilon_2) = (-0.53), \Psi(\Upsilon_3) = (-0.0657), \Psi(\Upsilon_4) = (-0.3546),$$

Step 7. The inferior ratio to all alternative is provided as:

$$\mathcal{IR}(1) = 1, \mathcal{IR}(2) = 0.8320, \mathcal{IR}(3) = 0.1031, \mathcal{IR}(4) = 0.5566.$$

Step 8. The following is the result one get while we organise the power source in increasing order of alternatives: $\Upsilon_3 < \Upsilon_4 < \Upsilon_2 < \Upsilon_1$. Hence Υ_3 = Hydro Power is the best energy source. Hydro Power is the closest to the ideal solution according to TOPSIS scores.

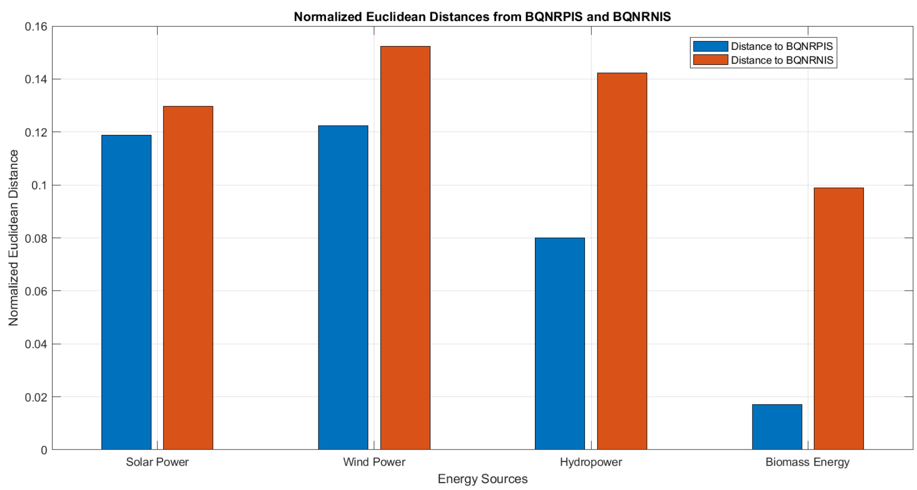

FIGURE 1. Normalised Euclidean Distances

3. Quadripartitioned Neutrsophic ELECTRE-I Method

In this section, we propose to apply the bipolar quadripartitioned neutrosophic ELECTRE-I technique to address MCDM problems.

Let the set of alternatives be represented as $\Upsilon = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \ldots, \Upsilon_p\}$ and the set of criteria, defined as $\mathbb{T} = \{\mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3, \ldots, \mathbb{T}_q\}$ that are utilized to calculate all alternatives.

$(i - iii)$ Similar to the bipolar quadripartitioned neutrosophic TOPSIS section, the options' rating values in relation to the criteria are represented as a matrix $[h_{lk}]_{p\times q}$. By using the maximising deviation technique, the weights ω_k of the criterion $\mathbb{T}_k$ are determined with weighted bipolar quadripartitioned neutrosophic decision matrix $[h_{lk}]_{p\times q}$ is created.

(iv) The bipolar quadripartitioned neutrosophic discordance sets $\mathbb{F}_{ab}$ and concordance sets $\mathbb{E}_{ab}$ are defined as follows:

$$\mathbb{E}_{ab} = \{1 \leq l \leq q | \Psi_{ak} \geq \Psi_{bk}\}, \quad a, b = 1, 2, \ldots, p, \quad a \neq b.$$

$$\mathbb{F}_{ab} = \{1 \leq l \leq q | \Psi_{ak} \leq \Psi_{bk}\}, \quad a, b = 1, 2, \ldots, p, \quad a \neq b.$$

where $\quad \Psi = \mathbb{T}_{lk}^{p+} + \mathbb{C}_{lk}^{p+} + \mathbb{U}_{lk}^{p+} + \mathbb{F}_{lk}^{p+} + \mathbb{T}_{lk}^{n-} + \mathbb{C}_{lk}^{n-} + \mathbb{U}_{lk}^{n-} + \mathbb{F}_{lk}^{n-}, \quad l = 1, 2, \ldots, p, \quad k = 1, 2, \ldots, q.$

(v) The following is the construction of the bipolar quadripartitioned neutrosophic concordance matrix $\mathbb{E}$:

$$\mathbb{E} = \begin{bmatrix} - & \mathtt{e}_{12} & \ldots & \mathtt{e}_{1p} \\ \mathtt{e}_{21} & - & \ldots & \mathtt{e}_{2p} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ \mathtt{e}_{p1} & \mathtt{e}_{p2} & \ldots & - \end{bmatrix}$$

wherein $\mathtt{e}_{ab}$, the bipolar quadripartitioned neutrosophic concordance indices, are calculated as

$$\mathtt{e}_{ab} = \sum_{k \in \mathbb{E}_{ab}} \omega_k$$

(vi) Here is how to construct the bipolar quadripartitioned neutrosophic disconcordance matrix $\mathbb{F}$.

$$\mathbb{F} = \begin{bmatrix} - & \mathtt{f}_{12} & \ldots & \mathtt{f}_{1p} \\ \mathtt{f}_{21} & - & \ldots & \mathtt{f}_{2p} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ \mathtt{f}_{p1} & \mathtt{f}_{p2} & \ldots & - \end{bmatrix}$$

wherein $\mathtt{f}_{ab}$, the bipolar quadripartitioned neutrosophic disconcordance indices, are calculated as

$$\mathtt{f}_{ab} = \frac{\max_{k \in \mathbb{F}_{ab}} \sqrt{\begin{aligned} &\frac{1}{8q}\Big\{(\mathbb{T}_{ak}^{\omega_k +} - \mathbb{T}_{bk}^{\omega_k +})^2 + (\mathbb{C}_{ak}^{\omega_k +} - \mathbb{C}_{bk}^{\omega_k +})^2 + (\mathbb{U}_{ak}^{\omega_k +} - \mathbb{U}_{bk}^{\omega_k +})^2 + (\mathbb{F}_{ak}^{\omega_k +} - \mathbb{F}_{bk}^{\omega_k +})^2 \\ &+ (\mathbb{T}_{ak}^{\omega_k -} - \mathbb{T}_{bk}^{\omega_k -})^2 + (\mathbb{C}_{ak}^{\omega_k -} - \mathbb{C}_{bk}^{\omega_k -})^2 + (\mathbb{U}_{ak}^{\omega_k -} - \mathbb{U}_{bk}^{\omega_k -})^2 + (\mathbb{F}_{ak}^{\omega_k -} - \mathbb{F}_{bk}^{\omega_k -})^2\Big\}\end{aligned}}}{\max_{k} \sqrt{\begin{aligned} &\frac{1}{8q}\Big\{(\mathbb{T}_{ak}^{\omega_k +} - \mathbb{T}_{bk}^{\omega_k +})^2 + (\mathbb{C}_{ak}^{\omega_k +} - \mathbb{C}_{bk}^{\omega_k +})^2 + (\mathbb{U}_{ak}^{\omega_k +} - \mathbb{U}_{bk}^{\omega_k +})^2 + (\mathbb{F}_{ak}^{\omega_k +} - \mathbb{F}_{bk}^{\omega_k +})^2 \\ &+ (\mathbb{T}_{ak}^{\omega_k -} - \mathbb{T}_{bk}^{\omega_k -})^2 + (\mathbb{C}_{ak}^{\omega_k -} - \mathbb{C}_{bk}^{\omega_k -})^2 + (\mathbb{U}_{ak}^{\omega_k -} - \mathbb{U}_{bk}^{\omega_k -})^2 + (\mathbb{F}_{ak}^{\omega_k -} - \mathbb{F}_{bk}^{\omega_k -})^2\Big\}\end{aligned}}}$$

(vii) To evaluate alternatives, the levels of concordance and discordance are calculated. The average value of the bipolar quadripartitioned neutrosophic concordance index as $\bar{\mathtt{e}}$ defines the bipolar quadripartitioned neutrosophic concordance level.

$$\bar{\mathtt{e}} = \frac{1}{p(p-1)} \sum_{a=1, b\neq a}^{m} \sum_{b=1, a\neq b}^{m} \mathtt{e}_{ab}.$$

In a similar way the average value of the bipolar quadripartitioned neutrosophic discordance indices as $\bar{\mathtt{f}}$ defines the bipolar quadripartitioned neutrosophic discordance level.

$$\bar{\mathtt{f}} = \frac{1}{p(p-1)} \sum_{a=1, b\neq a}^{m} \sum_{b=1, a\neq b}^{m} \mathtt{f}_{ab}$$

$(viii)$ Based on $\bar{\mathtt{e}}$, the bipolar quadripartitioned neutrosophic concordance dominance matrix Φ is calculated as follows:

$$\Phi = \begin{bmatrix} - & \Phi_{12} & \dots & \Phi_{1p} \\ \Phi_{21} & - & \dots & \Phi_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \Phi_{p1} & \Phi_{p2} & \dots & - \end{bmatrix}$$

where Φ_{ab}

$$\Phi_{ab} = \begin{cases} 1, \text{if} & \mathtt{e}_{ab} \geq \bar{\mathtt{e}} \\ 0, \text{if} & \mathtt{e}_{ab} < \bar{\mathtt{e}} \end{cases}$$

(ix) Based on $\bar{\mathtt{f}}$, the bipolar quadripartitioned neutrosophic discordance dominance matrix Ψ can be established as follows:

$$\Psi = \begin{bmatrix} - & \Psi_{12} & \dots & \Psi_{1p} \\ \Psi_{21} & - & \dots & \Psi_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \Psi_{p1} & \Psi_{p2} & \dots & - \end{bmatrix}$$

where Ψ_{ab}

$$\Psi_{ab} = \begin{cases} 1, \text{if} & \mathtt{f}_{ab} \leq \bar{\mathtt{f}} \\ 0, \text{if} & \mathtt{f}_{ab} > \bar{\mathtt{f}}. \end{cases}$$

(x) Thus, by multiplying the appropriate elements of Φ and Ψ, the bipolar quadripartitioned neutrosophic aggregated dominance matrix Ψ is determined, that is

$$\Pi = \begin{bmatrix} - & \Pi_{12} & \dots & \Pi_{1p} \\ \Pi_{21} & - & \dots & \Pi_{2p} \\ . & . & \dots & . \\ . & . & \dots & . \\ \Pi_{p1} & \Pi_{p2} & \dots & - \end{bmatrix}$$

where, Π_{ab} is defined as

$$\Pi_{ab} = \Phi_{ab}\Psi_{ab}$$

(xi) Lastly, a ranking based on the outranking values Π_{ab} is applied to the alternatives. In other words, an arrow from Υ_a to Υ_b exists for any pair of alternatives Υ_a and Υ_b if and only if $\Pi_{ab} = 1$. Thus, we have the following three scenarios:

(A) A distinct arrow departs from Υ_a and enters Υ_b

(B) Between Υ_a and Υ_b there are two possible arrows

(C) There is no arrow from Υ_a and Υ_b

We conclude that Υ_a is preferable than Υ_b for case A. In case C, Υ_a and Υ_b are incomparable, whereas in case B, Υ_a and Υ_b are indifferent.

3.1. *Application of the Proposed Method*

In Section 2, the bipolar quadripartitioned neutrosophic TOPSIS approach is used to present MCDM problems. In this section, we choose the optimal renewable energy source to compare these two MCDM approaches on using our suggested bipolar quadripartitioned neutrosophic ELECTRE-I method. In Section 2.1, steps (1-3) have already been completed. Thus, we proceed to Step 4.

Step 4. Table 3 presents the bipolar quadripartitioned neutrosophic concordance sets $\mathtt{E}_{ab}$.

$\mathtt{E}_{ab} \setminus b$	1	2	3	4
$\mathtt{E}_{1b}$	-	{3}	{1}	{3}
$\mathtt{E}_{2b}$	{1,2,4}	-	{1,4}	{3,4}
$\mathtt{E}_{3b}$	{1,2,3,4}	{1,2,3}	-	{2,3}
$\mathtt{E}_{4b}$	{1,2,3,4}	{1,2}	{1,4}	-

TABLE 3. bipolar quadripartitioned neutrosophic concordance set

Step 5. The $\mathtt{F}_{ab}$ bipolar quadripartitioned neutrosophic discordance sets are provided as Table 4.

$\mathtt{F}_{ab} \setminus b$	1	2	3	4
$\mathtt{F}_{1b}$	-	{1,2,4}	{2,3,4}	{1,2,4}
$\mathtt{F}_{2b}$	{3}	-	{2,3}	{1,2}
$\mathtt{F}_{3b}$	{}	{4}	-	{1,4}
$\mathtt{F}_{4b}$	{}	{3,4}	{2,3}	-

TABLE 4. bipolar quadripartitioned neutrosophic discordance sets

Step 6. Here is the computation of the bipolar quadripartitioned neutrosophic concordance matrix $\mathtt{E}$

$$\mathtt{E} = \begin{bmatrix} - & (0.2774) & (0.2221) & (0.2774) \\ (0.7212) & - & (0.5547) & (0.61) \\ (1) & (0.6674) & - & (0.4439) \\ (1) & (0.3886) & (0.5547) & - \end{bmatrix}$$

Step 7. The following formula is used to compute the bipolar quadripartitioned neutrosophic disconcordance matrix $\mathtt{F}$.

$$\mathtt{F} = \begin{bmatrix} - & (1) & (1) & (1) \\ (0.5844) & - & (0.9634) & (0.6524) \\ (0) & (1) & - & (0.5725) \\ (0) & (1) & (0.5423) & - \end{bmatrix}$$

Step 8. Now, $\bar{\mathtt{e}} = 0.5507$ is the bipolar quadripartitioned neutrosophic concordance level, while $\bar{\mathtt{f}} = 0.7179$ is the bipolar quadripartitioned discordance level. The dominance matrices Φ and Ψ, which represent the bipolar quadripartitioned neutrosophic concordance and discordance, respectively, are presented below.

$$\Phi = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 1 & 1 \\ 1 & 1 & - & 0 \\ 1 & 0 & 1 & - \end{bmatrix}$$

$$\Psi = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 1 \\ 0 & 0 & - & 1 \\ 0 & 0 & 1 & - \end{bmatrix}$$

Step 9. It is calculated that the quadripartitioned neutrosophic aggregated dominance matrix Π

$$\Pi = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 1 \\ 0 & 0 & - & 0 \\ 0 & 0 & 1 & - \end{bmatrix}$$

Based on the Π matrix, the decision-making process is as follows:

- Υ_2 is the best alternative among the four, as it has the most dominance relationships (i.e., it dominates Υ_1, Υ_3 and Υ_4).
- Υ_1, Υ_3 and Υ_4 are less preferable compared to Υ_2.

In summary, Υ_2 (Wind Power) is ranked as the optimal renewable energy source based on the bipolar quadripartitioned neutrosophic ELECTRE-I method.

	Υ_1	Υ_2	Υ_3	Υ_4
Υ_1	-	0	0	0
Υ_2	1	-	0	1
Υ_3	0	0	-	0
Υ_4	0	0	1	-

Table 5. Bipolar Quadripartitioned Neutrosophic Aggregated Dominance Matrix Π

Interpretation of the Matrix:

- $\Pi_{21} = 1$: Υ_2 dominates Υ_1.
- $\Pi_{31} = 0$: Υ_3 does not dominate Υ_1.
- $\Pi_{41} = 0$: Υ_4 does not dominate Υ_1.
- $\Pi_{32} = 0$: Υ_3 does not dominate Υ_2.
- $\Pi_{42} = 0$: Υ_4 does not dominate Υ_2.
- $\Pi_{34} = 1$: Υ_3 dominates Υ_4.

Therefore, Υ_2 (Wind Power) is the preferred alternative among the options considered. Wind Power dominates all other alternatives based on the concordance and discordance relations.

Decision: Υ_2 (Wind Power) is ranked as the optimal renewable energy source based on the bipolar quadripartitioned neutrosophic ELECTRE-I method.

4. Comparison of the proposed methods

Quadripartitioned neutrosophic TOPSIS method is suitable when the emphasis is on finding alternatives that are closest to an ideal solution and farthest from a negative ideal, making it useful in scenarios where precise closeness to ideal values is crucial. In contrast, quadripartitioned neutrosophic ELECTRE-I method provides a broader perspective by considering pairwise comparisons and collective outranking, making it more adaptable to contexts where relative performance and comparative evaluations among alternatives are paramount. Both methods effectively handle neutrosophic information, but quadripartitioned neutrosophic

TOPSIS method directly quantifies closeness and fairness to ideal solutions through distance calculations, while quadripartitioned neutrosophic ELECTRE-I method employs pairwise comparisons and threshold-based indices to establish outranking relationships. Quadripartitioned neutrosophic TOPSIS method faces challenges in defining and normalizing neutrosophic values accurately, whereas quadripartitioned neutrosophic ELECTRE-I method's reliance on thresholds introduces subjectivity in determining these values. Additionally, quadripartitioned neutrosophic ELECTRE-I method's computational complexity increases with the number of alternatives and criteria due to the pairwise comparison nature. Both models are extending traditional MCDM methods to handle quadripartitioned neutrosophic information, their application and suitability depend on the specific decision context, the preference for precise closeness or relative comparisons and the complexity of neutrosophic value handling and computation. According to our analysis, in the renewable energy source selection, the TOPSIS and ELECTRE-I methods provide distinct perspectives. TOPSIS ranks Υ_3 (Hydro Power) as the best alternative based on its proximity to the ideal solution, which emphasizes the quantitative performance metrics of the alternatives. It evaluates each option against a positive ideal solution and a negative ideal solution, focusing on minimizing the distance from the ideal and maximizing the distance from the non-ideal. In contrast, ELECTRE-I ranks Υ_2 (Wind Power) as the best choice by examining the dominance relationships through concordance and discordance matrices. This method takes into account both positive and negative evaluations, providing a broader and more delicate comparison of alternatives based on a consensus of criteria rather than a single performance measure. Therefore, while TOPSIS emphasizes the ideal performance of alternatives, ELECTRE-I offers a strategic evaluation of how each alternative compares to others in a more qualitative sense.

Conclusion

In this study, we introduced a novel approach, the bipolar Quadripartitioned Neutrosophic TOPSIS and ELECTRE-I methods have studied. The use of bipolar quadripartitioned neutrosophic sets allowed to effectively handle uncertainties and imprecisions inherent in the decision-making process. The divergence in results from these methods highlights the importance of adopting a multifaceted approach in decision-making processes, where both quantitative performance and qualitative dominance factors are considered. By integrating these two methods, the study provides a robust framework for selecting the most suitable renewable energy source based on a balanced evaluation of all relevant criteria. Ultimately, the combination of TOPSIS and ELECTRE-I offers a delicate decision-support tool that can guide policymakers and stakeholders in making informed and strategic choices for sustainable energy solutions. In future, we will investigate additional criteria or attributes that can further refine energy source

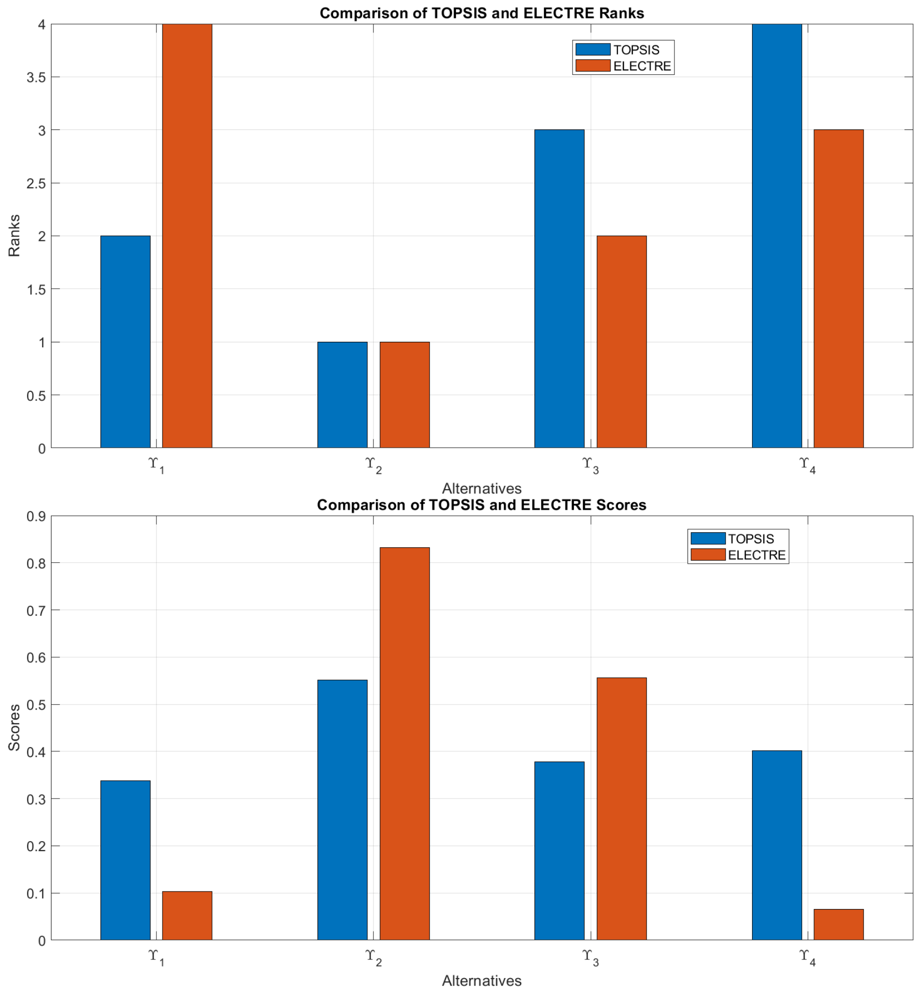

Figure 2. Comparison of the methods

selection decisions. Further, exploring extensions or modifications of the TOPSIS method to address specific industry requirements or constraints. Also, we have planned to conduct comparative studies with other multi-criteria decision-making methods to validate the effectiveness and efficiency of the developed approaches in different bipolar quadripartitioned neutrosophic domains.

Compliance with ethical standards

Data availability statements Not applicable.

Conflict of interest: The authors declare that they have no conflict of interest.

Funding: No funding received.

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Received: Nov. 15, 2024. Accepted: May 12, 2025