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Navigating Economic and Agricultural Realms with Interval-Valued Complex Neutrosophic Fuzzy Sets.

Velan Kalaiyarasan[1] and Krishnan Muthunagai[2,*]

[1]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology Chennai, Vandaloor to Kelambakkam Road, Chennai, 600127, Tamil Nadu, India; E.mail: kalaiyarasan.v@vitstudent.ac.in

[2]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology Chennai Vandaloor to Kelambakkam Road, Chennai, 600127, Tamil Nadu, India; E.mail: muthunagai@vit.ac.in

[*]Correspondence: e-mail: muthunagai@vit.ac.in

Abstract. This article introduces the concept of the interval-valued complex neutrosophic fuzzy set, which extends traditional neutrosophic sets to address challenges like uncertainty, inconsistency, and indeterminate information. This specialized set blends complex fuzzy functions and real-valued fuzzy terms with phase elements. This article also discusses Hamming and Euclidean distances and proposes similarity measures based on these distances. These measures are applied in a multi-criteria decision-making method within an interval-valued complex neutrosophic fuzzy structure. Practical applications include ranking alternatives and identifying optimal choices. Illustrative examples demonstrate its effectiveness in handling interval-valued complex neutrosophic fuzzy sets.

Keywords: Interval-valued fuzzy set, Complex fuzzy set, Neutrosophic set, Neutrosophic fuzzy set, Complex neutrosophic fuzzy set, Interval neutrosophic set, Hamming distance, Eulidean distance, Similarity measure, Multicriteria decision-making fuzzy set.

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1. Introduction

The paper begins with a historical context, acknowledging the seminal contributions of Cardano (1501-1576), Euler (1707-1783), Gauss (1777-1855) and Caspar Wessel (1745-1818) to the development of complex numbers and their representation. Then it talks about the evolution of the classical set theory into the fuzzy set theory [1] marked by elements that possess membership degrees. The subsequent introduction of non-membership functions was carried out by Krassimir T. Atanassov and S.Stoeva [2] in the form of "Intuitionistic fuzzy sets".

Despite these advancements, the paper notes the persistence of challenges in handling indeterminate and inconsistent information, which led to the emergence of Neutrosophic sets [3], characterized by truth, indeterminacy, and false membership functions. The limitations of fuzzy sets and intuitionistic fuzzy sets in representing both attributes concurrently prompted the development of Neutrosophic fuzzy sets in 2020 by Sujit Das et al [4]. For more details, see [5], [6], [7].

Further complexities in dealing with periodicity within uncertain data were addressed with Complex fuzzy sets [8] in 2002, followed by Complex intuitionistic fuzzy sets [9], [10] in 2012, extending these concepts to incorporate complex-valued non-membership grades. In 2015, Complex neutrosophic sets introduced in [11], [12], addressed the challenge of handling indeterminate and inconsistent periodic information. Greenfield et al. [13] introduced a combination of interval-valued fuzzy sets and complex fuzzy sets and developed interval-valued complex fuzzy sets. Mumtaz Ali et al. [14] worked on the complex neutrosophic interval set.

The need for IVCNFS came from the necessity to effectively represent and manage complex uncertainty in real-world scenarios. IVCNFS offers a powerful tool for this purpose by incorporating several key features. First and foremost they allow for more refined representation of membership degrees through interval-valued membership, accommodating situations where precise values are uncertain. Next, they enable the modeling of periodic or oscillatory phenomena, which are common in real-world systems, through complex membership.

In many real-world scenarios, environmental and other natural situations exhibit dynamic and uncertain characteristics. Consequently, the truth, falsity, and indeterminacy associated with statements may be more accurately represented by intervals, rather than single values. To address this, Wang et al. [15] introduced the concept of Interval Neutrosophic Sets (INS), where degrees of truth, falsehood, and indeterminacy belong to intervals instead of assuming specific numbers. Developing on this, Ye [16] defined the Hamming and Euclidean distances between INS and developed similarity measures based on these distances. Furthermore, Tian et al. [17] proposed a multi-criteria decision-making method (MCDM) that uses cross entropy with INS.

Several studies explored multicriteria decision making (MCDM) using neutrosophic sets and their alternatives. In [18], two distinct MCDM methods were developed and applied to an investment problem using interval neutrosophic vague sets (INVS) and entropy measures by Hashim et.al. Further research has focused on supplier evaluation, with Yazdani et. al. [19]

proposing an enhanced sustainable supplier evaluation structure using multiple criteria and a fuzzy neutrosophic model valued at intervals (IVFN). The interval neutrosophic optimization and the sets of neutrosophic values (IVNS) were investigated in [20] by Khalil et al. The application of these techniques extends to various fields, as demonstrated in [21] by Torkayesh et. al., which examined the development of sustainable municipal waste management systems by analyzing indicators of economic, environmental and social sustainability.

Furthermore, in [22], the authors explored the design of acceptance sampling plans based on neutrosophic sets with interval values. In addition, in [23], Manshath et. al. introduced operators on fuzzy neutrosophic sets with interval values and discussed their properties with numerical examples. Distance measures for neutrosophic sets in intervals and their application in ecological management were also introduced. The article emphasizes the importance of using normalized distance measures for effective management decisions.

Notably, this paper highlights the absence of literature addressing similarity measures between Interval Neutrosophic Sets (INS) and decision-making problems. In response to the challenge of obtaining precise neutrosophic membership degrees in real-life scenarios dominated by vague information, the paper presents Interval-Valued Complex Neutrosophic Fuzzy Sets (IVCNFS) as a versatile and adaptable structure. This concept is positioned as an enhanced alternative to Single-Valued Complex Neutrosophic Sets (SVCNS) and (INS), particularly in decision-making scenarios where precision and accuracy are paramount.

Section 2 provides the basic definitions required for the present work. The definition of the Interval-Valued Complex Neutrosophic Fuzzy Set, basic operations on it, and its properties are discussed in Section 3. Section 4 introduces and analyzes similarity and distance measures, along with their properties. An approach to addressing multi-criteria decision-making problems using these similarity measures with respect to the Interval-Valued Complex Neutrosophic Fuzzy Set is discussed in Section 5. Section 6 covers the computation of the similarity measure between the ideal alternatives and individual options within specified time periods and suggests the preferred options. The advantages and limitations of the proposed method are discussed in Section 7.

2. **Preliminaries**

This section carries the definitions and statements of several theorems that are essential for our research work.

2.1. *Neutrosophic set [3]*

Let U be a space of points and $l \in U$. A neutrosophic set A in U is characterized by the truth, indeterminacy, and falsity membership functions, which are denoted using $T_A(l), I_A(l), F_A(l)$ and defined by

$$A = \{(l, T_A(l), I_A(l), F_A(l)) : l \in U\}.$$

The ranges of $T_A(l), I_A(l)$ and $F_A(l)$ are real standard or non-standard subsets of $]^-0, 1^+[$

$$T_A(l) : l \rightarrow]^-0, 1^+[,$$
$$I_A(l) : l \rightarrow]^-0, 1^+[,$$
$$F_A(l) : l \rightarrow]^-0, 1^+[,$$

There is no restriction on the sum of $T_A(l), I_A(l)$ and $F_A(l)$, but
$0^- \leq \sup T_A(l) + \sup I_A(l) + \sup F_A(l) \leq 3^+$.

2.2. *Complement of neutrosophic set [4]*

The complement of a neutrosophic set A is denoted by A^c and is defined as

$$A^c = \{(l \in U, T^c_A(l), I^c_A(l), F^c_A(l))\}.$$

where $T^c_A(l) = 1 - T_A(l) = F_A(l), I^c_A(l) = 1 - I_A(l), F^c_A(l) = 1 - F_A(l) = T_A(l)$.

2.3. *Subset of a neutrosophic set [4]*

A neutrosophic set A is said to be a subset of a neutrosophic set B if and only if $\inf T_A(l) \leq \inf T_B(l)$, $\sup T_A(l) \leq \sup T_B(l)$, $\inf I_A(l) \geq \inf I_B(l)$, $\sup I_A(l) \geq \sup I_B(l)$, $\inf F_A(l) \geq \inf F_B(l)$ and $\sup F_A(l) \geq \sup F_B(l)$ $\forall\ l \in U$

2.4. *Neutrosophic fuzzy set [4]*

To express uncertain, indeterminate and inconsistent information, an additional component called indeterminacy with $0^- \leq \sup \mu_{T_A}(v) + \sup \mu_{I_A}(l) + \sup \mu_{F_A}(l) \leq 3^+, \quad \forall\ l \in U$ was proposed. Thus the Intutionistic Fuzzy set was made Neutrosophic by adding the component, indeterminacy and it is denoted as

$$A = \{\mu_A(l), T_A(l, \mu), I_A(l, \mu), F_A(l, \mu) \in [0, 1], \forall\ l \in U\}.$$

2.5. *Complex neutrosophic Set [11]*

By adding a complex-valued indeterminacy membership grade $I_A(l)$, to a complex intuitionistic set, it is made complex neutrosophic set. Here $T_A(l) = r_A(l).e^{i\mu_A(l)}, I_A(l) = s_A(l).e^{i\nu_A(l)}, F_A(l) = t_A(l).e^{i\omega_A(l)}$ where $r_A(l), s_A(l), t_A(l) \in [0, 1]$ such that

$$0^- \leq r_A(l) + s_A(l) + t_A(l) \leq 3^+$$

and

$$|T_A(l) + I_A(l) + F_A(l)| \leq 3.$$

2.6. *Complex neutrosophic fuzzy set [24]*

A complex neutrosophic fuzzy set A defined on a universe of discourse U is the one which is characterized by a complex-valued fuzzy membership, truth, indeterminacy, and falsity functions which are respectively denoted using $\mu_A(l), T_A(l,\mu), I_A(l,\mu)$ and $F_A(l,\mu), \forall\ l \in U$. They are defined by $\mu_A(l) = q_A(l).e^{i\omega_{\mu_A}(l)}$, $T_A(l) = r_A(l).e^{i\omega_{T_A}(l)}$, $I_A(l) = s_A(l).e^{i\omega_{I_A}(l),}$ and $F_A(l) = t_A(l).e^{i\omega_{F_A}(l)}$, where the complex membership values of the amplitude terms $\mu_A(l), r_A(l), s_A(l)$ and $t_A(l)$ and the phase terms $\omega_{\mu_A}(l), \omega_{T_A}(l), \omega_{I_A}(l), \omega_{F_A}(l)$ are real valued and $q_A(l), r_A(l), s_A(l), t_A(l) \in [0,1]$

$$\ni 0^- \leq q_A(l) + r_A(l) + s_A(l) + t_A(l) \leq 4^+.$$

Thus the complex neutrosophic fuzzy set A is given by

$$A = \{(l, \mu_A(l) = z_\mu, T_A(l,\mu) = z_T, I_A(l,\mu) = z_I, F_A(l,\mu) = z_F) : l \in U\}$$

$$|\mu_A(l) + T_A(l,\mu) + I_A(l,\mu) + F_A(l,\mu)| \leq 4$$

2.7. *Interval neutrosophic set [15]*

Let us consider a space of points denoted by U, where the generic element in U is represented using l. Within this context, an Interval Neutrosophic Set (INS) A in U is defined by three essential components: a truth-membership function $T_A(l)$, an indeterminacy-membership function $I_A(l)$, and a falsity-membership function $F_A(l)$, where

$T_A(l) = [\inf T_A(l), \sup T_A(l)]$, $I_A(l) = [\inf I_A(l), \sup I_A(l)]$, $F_A(l) = [\inf F_A(l), \sup F_A(l)] \subseteq [0,1]$ and $0^- \leq \sup \mu_{T_A}(l) + \sup \mu_{I_A}(l) + \sup \mu_{F_A}(l) \leq 3^+, \quad \forall\ l \in U.$

3. **Interval-Valued Complex Neutrosophic Fuzzy Set and Set Operations**

3.1. *Interval-Valued Complex Neutrosophic Fuzzy Set:*

An interval-valued complex neutrosophic fuzzy set A defined in a universe of discourse U is characterized by interval-valued complex fuzzy membership, truth, indeterminacy and falsity functions:

$\mu_A(l) = [q_{\mathcal{L}}(l), q_{\mathcal{U}}(l)].e^{i(\omega_{\mu_{\mathcal{L}}}(l),\ \omega_{\mu_{\mathcal{U}}}(l))}$, $T_A(l,\mu) = [r_{\mathcal{L}}(l), r_{\mathcal{U}}(l)].\ e^{i(\omega_{T_{\mathcal{L}}}(l),\ \omega_{T_{\mathcal{U}}}(l))}$, $I_A(l,\mu) = [s_{\mathcal{L}}(l), s_{\mathcal{U}}(l)].e^{i(\omega_{I_{\mathcal{L}}}(l),\ \omega_{I_{\mathcal{U}}}(l))}$, $F_A(l,\mu) = [t_{\mathcal{L}}(l), t_{\mathcal{U}}(l)].e^{i(\omega_{F_{\mathcal{L}}}(l),\ \omega_{F_{\mathcal{U}}}(l))}, \forall\ l \in U.$

These functions include both amplitude terms

$[q_{\mathcal{L}}(l), q_{\mathcal{U}}(l)], [r_{\mathcal{L}}(l), r_{\mathcal{U}}(l)], [s_{\mathcal{L}}(l), s_{\mathcal{U}}(l)], [t_{\mathcal{L}}(l), t_{\mathcal{U}}(l)] \in [0,1]$ and phase terms $[\omega_{\mu_{\mathcal{L}}}(l), \omega_{\mu_{\mathcal{U}}}(l)], [\omega_{T_{\mathcal{L}}}(l), \omega_{T_{\mathcal{U}}}(l)], [\omega_{I_{\mathcal{L}}}(l), \omega_{I_{\mathcal{U}}}(l)],\ [\omega_{F_{\mathcal{L}}}(l), \omega_{F_{\mathcal{U}}}(l)] \in [0, 2\pi]$

with their component constraint that

$$^{-}0 \le |\mu_{A}(l) + T_{A}(l,\mu) + I_{A}(l,\mu) + F_{A}(l,\mu)| \le 4^{+}.$$

This representation captures the intricate properties of the complex neutrosophic fuzzy set with interval values A.

$\mu_A(l) : U \to \{Z_{\mu^{\mathcal{L}}}, Z_{\mu^{\mathcal{U}}} : Z_{\mu^{\mathcal{L}}}, Z_{\mu^{\mathcal{U}}} \in C, |\frac{Z_{\mu^{\mathcal{L}}}+Z_{\mu^{\mathcal{U}}}}{2}| \le 1\}$

$T_A(l,\mu) : U \to \{Z_{T^{\mathcal{L}}}, Z_{T^{\mathcal{U}}} : Z_{T^{\mathcal{L}}}, Z_{T^{\mathcal{U}}} \in C, |\frac{Z_{T^{\mathcal{L}}}+Z_{T^{\mathcal{U}}}}{2}| \le 1\}$

$I_A(l,\mu) : U \to \{Z_{I^{\mathcal{L}}}, Z_{I^{\mathcal{U}}} : Z_{I^{\mathcal{L}}}, Z_{I^{\mathcal{U}}} \in C, |\frac{Z_{I^{\mathcal{L}}}+Z_{I^{\mathcal{U}}}}{2}| \le 1\}$

$F_A(l,\mu) : U \to \{Z_{F^{\mathcal{L}}}, Z_{F^{\mathcal{U}}} : Z_{F^{\mathcal{L}}}, Z_{F^{\mathcal{U}}} \in C, |\frac{Z_{F^{\mathcal{L}}}+Z_{F^{\mathcal{U}}}}{2}| \le 1\}$ and

$$\left|\frac{Z_{\mu^{\mathcal{L}}}+Z_{\mu^{\mathcal{U}}}}{2} + \frac{Z_{T^{\mathcal{L}}}+Z_{T^{\mathcal{U}}}}{2} + \frac{Z_{I^{\mathcal{L}}}+Z_{I^{\mathcal{U}}}}{2} + \frac{Z_{F^{\mathcal{L}}}+Z_{F^{\mathcal{U}}}}{2}\right| \le 4$$
(or)
$$|\mu_{A}(l) + T_{A}(l,\mu) + I_{A}(l,\mu) + F_{A}(l,\mu)| \le 4$$

Example 3.1. Let $U = \{u_1, u_2, u_3, u_4\}$ be the Universe of discourse. Then, A be an interval-Valued Complex Neutrosophic Fuzzy Set in U as given below.

$$A = \frac{\begin{pmatrix} [0.23, 0.3].e^{i[0.93,0.94]}, [0.5, 0.6].e^{i[0.67,0.69]} \\ , [0.5, 0.7].e^{i[0.5,0.54]}, [0.3, 0.4].e^{i[0.56,0.58]} \end{pmatrix}}{y_1} + \frac{\begin{pmatrix} [0.3, 0.4].e^{i[0.3,0.32}, [0.2, 0.3].e^{i[0.77,0.79]} \\ , [0.6, 0.7].e^{i[0.45,0.49]}, [0.12, 0.3].e^{i[0.12,0.4]} \end{pmatrix}}{y_2} + \frac{\begin{pmatrix} [0.48, 0.87].e^{i[0.65,0.77}, [0.3, 0.3].e^{i[0.0,0.32]} \\ , [0.8, 0.9].e^{i[0.11,0.17]}, [0.55, 0.6].e^{i[0.54,0.71]} \end{pmatrix}}{y_3} + \frac{\begin{pmatrix} [0.6, 0.61].e^{i[0.5,0.72}, [0.14, 0.15].e^{i[0.1,0.15]} \\ , [0.54, 0.6].e^{i[0.39,0.88]}, [0.6, 0.5].e^{i[0.79,0.81]} \end{pmatrix}}{y_4}$$

3.2. *Set operations on Interval-Valued Complex Neutrosophic Fuzzy Sets*

3.2.1. *Union and Intersection*

Let A_1 and A_2 be two Interval-Valued Complex Neutrosophic Fuzzy Sets. These are represented by

$A_1 = \{(l, \mu_{A_1}(l) = [Z_{\mu^{\mathcal{L}_1}}, Z_{\mu^{\mathcal{U}_1}}],\ T_{A_1}(l,\mu) = [Z_{T^{\mathcal{L}_1}}, Z_{T^{\mathcal{U}_1}}],\ I_{A_1}(l,\mu) = [Z_{I^{\mathcal{L}_1}}, Z_{I^{\mathcal{U}_1}}],\ F_{A_1}(l\mu) = [Z_{F^{\mathcal{L}_1}}, Z_{F^{\mathcal{U}_1}}]) : l \in U\}$ and

$A_2 = \{(l, \mu_{A_2}(l) = [Z_{\mu^{\mathcal{L}_2}}, Z_{\mu^{\mathcal{U}_2}}],\ T_{A_2}(l,\mu) = [Z_{T^{\mathcal{L}_2}}, Z_{T^{\mathcal{U}_2}}],\ I_{A_2}(l,\mu) = [Z_{I^{\mathcal{L}_2}}, Z_{I^{\mathcal{U}_2}}],\ F_{A_2}(l,\mu) = [Z_{F^{\mathcal{L}_2}}, Z_{F^{\mathcal{U}_2}}]) : l \in U\}$

Their union and intersection are given by

$$A_1 \cup A_2 = \{(l, [\mu_{a^*\mathcal{L}}(l), \mu_{a^*\mathcal{U}}(l)], [T_{a^*\mathcal{L}}(l), T_{a^*\mathcal{U}}(l)], [I_{a^*\mathcal{L}}(l), I_{a^*\mathcal{U}}(l)], [F_{a^*\mathcal{L}}(l), F_{a^*\mathcal{U}}(l)]) : l \in U\}$$

$$A_1 \cap A_2 = \{(l, [\mu_{b^*\mathcal{L}}(l), \mu_{b^*\mathcal{U}}(l)], [T_{b^*\mathcal{L}}(l), T_{b^*\mathcal{L}}(l)], [I_{b^*\mathcal{U}}(l), I_{b^*\mathcal{L}}(vl)], [F_{b^*\mathcal{L}}(l), F_{b^*\mathcal{U}}(l)]) : l \in U\}$$

where $a^{*\mathcal{L}} = \mathcal{L}_1 \cup \mathcal{L}_2,\ a^{*\mathcal{U}} = \mathcal{U}_1 \cup \mathcal{U}_2,\ b^{*\mathcal{L}} = \mathcal{L}_1 \cap \mathcal{L}_2,\ b^{*\mathcal{U}} = \mathcal{U}_1 \cap \mathcal{U}_2$. The Interval-valued complex neutrosophic fuzzy membership function, truth, indeterminacy and falsity functions of union and intersection are given by

$$\mu_{a^*\mathcal{L}}(l) = \mu_{\mathcal{L}_1 \cup \mathcal{L}_2}(l) = [q_{\mathcal{L}_1}(l) \vee q_{\mathcal{L}_2}(l)].\ e^{i(\mu_1{}^{\mathcal{L}}(l)\ \vee\ \mu_2{}^{\mathcal{L}}(l))},$$

$$\mu_{a^*\mathcal{U}}(l) = \mu_{\mathcal{U}_1 \cup \mathcal{U}_2}(l) = [q_{\mathcal{U}_1}(l) \vee q_{\mathcal{U}_2}(l)].\ e^{i(\mu_1{}^{\mathcal{U}}(l)\ \vee\ \mu_2{}^{\mathcal{U}}(l))},$$

$$T_{a^*\mathcal{L}}(l) = T_{\mathcal{L}_1 \cup \mathcal{L}_2}(l) = [r_{\mathcal{L}_1}(l) \vee r_{\mathcal{L}_2}(l)].\ e^{i(T_1{}^{\mathcal{L}}(l)\ \vee\ T_2{}^{\mathcal{L}}(l))},$$

$$T_{a^*\mathcal{U}}(l) = T_{\mathcal{U}_1 \cup \mathcal{U}_2}(l) = [r_{\mathcal{U}_1}(l) \vee r_{\mathcal{U}_2}(l)].\ e^{i(T_1{}^{\mathcal{U}}(l)\ \vee\ T_2{}^{\mathcal{U}}(l))},$$

$$I_{a^*\mathcal{L}}(l) = I_{\mathcal{L}_1 \cup \mathcal{L}_2}(l) = [s_{\mathcal{L}_1}(l) \vee s_{\mathcal{L}_2}(l)].\ e^{i(I_1{}^{\mathcal{L}}(l)\ \vee\ I_2{}^{\mathcal{L}}(l))},$$

$$I_{a^*\mathcal{U}}(l) = I_{\mathcal{U}_1 \cup \mathcal{U}_2}(l) = [s_{\mathcal{U}_1}(l) \vee s_{\mathcal{U}_2}(l)].\ e^{i(I_1{}^{\mathcal{U}}(l)\ \vee\ I_2{}^{\mathcal{U}}(l))},$$

$$F_{a^*\mathcal{L}}(l) = F_{\mathcal{L}_1 \cup \mathcal{L}_2}(l) = [t_{\mathcal{L}_1}(l) \vee t_{\mathcal{L}_2}(l)].\ e^{i(F_1{}^{\mathcal{L}}(l)\ \vee\ F_2{}^{\mathcal{L}}(l))},$$

$$F_{a^*\mathcal{U}}(l) = F_{\mathcal{U}_1 \cup \mathcal{U}_2}(l) = [t_{\mathcal{U}_1}(l) \vee t_{\mathcal{U}_2}(l)].\ e^{i(F_1{}^{\mathcal{U}}(l)\ \vee\ F_2{}^{\mathcal{U}}(l))},$$

$$\mu_{b^*\mathcal{L}}(l) = \mu_{\mathcal{L}_1 \cap \mathcal{L}_2}(l) = [q_{\mathcal{L}_1}(l) \wedge q_{\mathcal{L}_2}(l)].\ e^{i(\mu_1{}^{\mathcal{L}}(l)\ \wedge\ \mu_2{}^{\mathcal{L}}(l))},$$

$$\mu_{b^*\mathcal{U}}(l) = \mu_{\mathcal{U}_1 \cap \mathcal{U}_2}(l) = [q_{\mathcal{U}_1}(l) \wedge q_{\mathcal{U}_2}(l)].\ e^{i(\mu_1{}^{\mathcal{U}}(l)\ \wedge\ \mu_2{}^{\mathcal{U}}(l))},$$

$$T_{b^*\mathcal{L}}(l) = T_{\mathcal{L}_1 \cap \mathcal{L}_2}(l) = [r_{\mathcal{L}_1}(l) \wedge r_{\mathcal{L}_2}(l)].\ e^{i(T_1{}^{\mathcal{L}}(l)\ \wedge\ T_2{}^{\mathcal{L}}(l))},$$

$$T_{b^*\mathcal{U}}(l) = T_{\mathcal{U}_1 \cap \mathcal{U}_2}(l) = [r_{\mathcal{U}_1}(l) \wedge r_{\mathcal{U}_2}(l)].\ e^{i(T_1{}^{\mathcal{U}}{}_l)\ \wedge\ T_2{}^{\mathcal{U}}(l))},$$

$$I_{b^*\mathcal{L}}(l) = I_{\mathcal{L}_1 \cap \mathcal{L}_2}(l) = [s_{\mathcal{L}_1}(l) \wedge s_{\mathcal{L}_2}(l)].\ e^{i(I_1{}^{\mathcal{L}}(l)\ \wedge\ I_2{}^{\mathcal{L}}(l))},$$

$$I_{b^*\mathcal{U}}(l) = I_{\mathcal{U}_1 \cap \mathcal{U}_2}(l) = [s_{\mathcal{U}_1}(l) \wedge s_{\mathcal{U}_2}(l)].\ e^{i(I_1{}^{\mathcal{U}}(l)\ \wedge\ I_2{}^{\mathcal{U}}(l))},$$

$$F_{b^*\mathcal{L}}(l) = F_{\mathcal{L}_1 \cap \mathcal{L}_2}(l) = [t_{\mathcal{L}_1}(l) \wedge t_{\mathcal{L}_2}(l)].\ e^{i(F_1{}^{\mathcal{L}}(l)\ \wedge\ F_2{}^{\mathcal{L}}(l))},$$

$$F_{b^*\mathcal{U}}(l) = F_{\mathcal{U}_1 \cap \mathcal{U}_2}(l) = [t_{\mathcal{U}_1}(l) \wedge t_{\mathcal{U}_2}(l)].\ e^{i(F_1{}^{\mathcal{U}}(l)\ \wedge\ F_2{}^{\mathcal{U}}(l))},$$

where

$$\mu_1{}^{\mathcal{L}} = \omega_{\mu_{\mathcal{L}_1}}, \quad \mu_1{}^{\mathcal{U}} = \omega_{\mu_{\mathcal{U}_1}}, \quad \mu_2{}^{\mathcal{L}} = \omega_{\mu_{\mathcal{L}_2}}, \quad \mu_2{}^{\mathcal{U}} = \omega_{\mu_{\mathcal{U}_2}},$$
$$T_1{}^{\mathcal{L}} = \omega_{T_{\mathcal{L}_1}}, \quad T_1{}^{\mathcal{U}} = \omega_{T_{\mathcal{U}_1}}, \quad T_2{}^{\mathcal{L}} = \omega_{T_{\mathcal{L}_2}}, \quad T_2{}^{\mathcal{U}} = \omega_{T_{\mathcal{U}_2}},$$
$$I_1{}^{\mathcal{L}} = \omega_{I_{\mathcal{L}_1}}, \quad I_1{}^{\mathcal{U}} = \omega_{I_{\mathcal{U}_1}}, \quad I_2{}^{\mathcal{L}} = \omega_{I_{\mathcal{L}_2}}, \quad I_2{}^{\mathcal{U}} = \omega_{I_{\mathcal{U}_2}},$$
$$F_1{}^{\mathcal{L}} = \omega_{F_{\mathcal{L}_1}}, \quad F_1{}^{\mathcal{U}} = \omega_{F_{\mathcal{U}_1}}, \quad F_2{}^{\mathcal{L}} = \omega_{F_{\mathcal{L}_2}}, \quad F_2{}^{\mathcal{U}} = \omega_{F_{\mathcal{U}_2}},$$

and

$$e^{i(\mu_1{}^{\mathcal{L}}(l) \,\vee\, \mu_2{}^{\mathcal{L}}(l))}, \; e^{i(\mu_1{}^{\mathcal{U}}(l) \,\vee\, \mu_2{}^{\mathcal{U}}(l))}, \; e^{i(T_1{}^{\mathcal{L}}(l) \,\vee\, T_2{}^{\mathcal{L}}(l))}, \; e^{i(T_1{}^{\mathcal{U}}(l) \,\vee\, T_2{}^{\mathcal{U}}(l))},$$
$$e^{i(I_1{}^{\mathcal{L}}(l) \,\vee\, I_2{}^{\mathcal{L}}(l))}, \; e^{i(I_1{}^{\mathcal{U}}(l) \,\vee\, I_2{}^{\mathcal{U}}(l))}, \; e^{i(F_1{}^{\mathcal{L}}(l) \,\vee\, F_2{}^{\mathcal{L}}(l))}, \; e^{i(F_1{}^{\mathcal{U}}(l) \,\vee\, F_2{}^{\mathcal{U}}(l))},$$
$$e^{i(\mu_1{}^{\mathcal{L}}(l) \,\wedge\, \mu_2{}^{\mathcal{L}}(l))}, \; e^{i(\mu_1{}^{\mathcal{U}}(l) \,\wedge\, \mu_2{}^{\mathcal{U}}(l))}, \; e^{i(T_1{}^{\mathcal{L}}(l) \,\wedge\, T_2{}^{\mathcal{L}}(l))}, \; e^{i(T_1{}^{\mathcal{U}}(l) \,\wedge\, T_2{}^{\mathcal{U}}(l))},$$
$$e^{i(I_1{}^{\mathcal{L}}(l) \,\wedge\, I_2{}^{\mathcal{L}}(l))}, \; e^{i(I_1{}^{\mathcal{U}}(l) \,\wedge\, I_2{}^{\mathcal{U}}(l))}, \; e^{i(F_1{}^{\mathcal{L}}(l) \,\wedge\, F_2{}^{\mathcal{L}}(l))}, \; e^{i(F_1{}^{\mathcal{U}}(l) \,\wedge\, F_2{}^{\mathcal{U}}(l))}.$$

are the reformulated versions of the amplitude and the phase terms respectively, $\vee$ and $\wedge$ signify the maximum and minimum operators.

3.2.2. *Properties*

Let A_1 and A_2 be two Interval-Valued Complex Neutrosophic Fuzzy Sets in U with complex-valued interval fuzzy membership, truth, indeterminacy and falsity functions
$A_1 = \{l, [\mu_{\mathcal{L}_1}(l), \mu_{\mathcal{U}_1}(l)], [T_{\mathcal{L}_1}(l), T_{\mathcal{U}_1}(l)], [I_{\mathcal{L}_1}(l), I_{\mathcal{U}_1}(l)], [F_{\mathcal{L}_{11}}(l), F_{\mathcal{U}_1}(l)] : l \in U\}$,
and
$A_2 = \{l, [\mu_{\mathcal{L}_2}(l), \mu_{\mathcal{U}_2}(l)], [T_{\mathcal{L}_2}(l), T_{\mathcal{U}_2}(l)], [I_{\mathcal{L}_2}(l), I_{\mathcal{U}_2}(l)], [F_{\mathcal{L}_2}(l), F_{\mathcal{U}_2}(l)] : l \in U\}$,
respectively. The Union and Intersection of the Interval-Valued Complex Neutrosophic Fuzzy Sets A_1 and A_2 denoted by $A_1 \cup A_2$ and $A_1 \cap A_2$ can be expressed in terms of their associated function:

$$\varphi : \{\big((a^{\mathcal{L}}, a^{\mathcal{U}}), (a_T^{\mathcal{L}}, a_T^{\mathcal{U}}), (a_I^{\mathcal{L}}, a_I^{\mathcal{U}}), (a_F^{\mathcal{L}}, a_F^{\mathcal{U}})\big) : a^{\mathcal{L}}, a^{\mathcal{U}}, a_T^{\mathcal{L}}, a_T^{\mathcal{U}}, a_I^{\mathcal{L}}, a_I^{\mathcal{U}}, a_F^{\mathcal{L}}, a_F^{\mathcal{U}} \in$$
$$\mathbb{C}, \left|\frac{(a^{\mathcal{L}}+a_T^{\mathcal{L}}+a_I^{\mathcal{L}}+a_F^{\mathcal{L}})+(a^{\mathcal{U}}+a_T^{\mathcal{U}}+a_I^{\mathcal{U}}+a_F^{\mathcal{U}})}{2}\right| \leq 4, |a^{\mathcal{L}}|, |a_T^{\mathcal{L}}|, |a_I^{\mathcal{L}}|, |a_F^{\mathcal{L}}|, |a^{\mathcal{U}}|, |a_T^{\mathcal{U}}|, |a_I^{\mathcal{U}}|, |a_F^{\mathcal{U}}| \leq 1\}$$
$$\times\{\big((b^{\mathcal{L}}, b^{\mathcal{U}}), (b_T^{\mathcal{L}}, b_T^{\mathcal{U}}), (b_I^{\mathcal{L}}, b_I^{\mathcal{U}}), (b_F^{\mathcal{L}}, b_F^{\mathcal{U}})\big) : b^{\mathcal{U}}, b^{\mathcal{U}}, b_T^{\mathcal{L}}, b_T^{\mathcal{U}}, b_I^{\mathcal{L}}, b_I^{\mathcal{U}}, b_F^{\mathcal{L}}, b_F^{\mathcal{U}} \in \mathbb{C},$$
$$\left|\frac{(b^{\mathcal{L}}+b_T^{\mathcal{L}}+b_I^{\mathcal{L}}+b_F^{\mathcal{L}})+(b^{\mathcal{U}}+b_T^{\mathcal{U}}+b_I^{\mathcal{U}}+b_F^{\mathcal{U}})}{2}\right| \leq 4, |b^{\mathcal{L}}|, |b_T^{\mathcal{L}}|, |b_I^{\mathcal{L}}|, |b_F^{\mathcal{L}}|, |b^{\mathcal{U}}|, |b_T^{\mathcal{U}}|, |b_I^{\mathcal{U}}|, |b_F^{\mathcal{U}}| \leq 1\} \to$$
$$\{\big((c^{\mathcal{L}}, c^{\mathcal{U}}), (c_T^{\mathcal{L}}, c_T^{\mathcal{U}}), (c_I^{\mathcal{L}}, c_I^{\mathcal{U}}), (c_F^{\mathcal{L}}, c_F^{\mathcal{U}})\big) : c^{\mathcal{L}}, c^{\mathcal{U}}, c_T^{\mathcal{L}}, c_T^{\mathcal{U}}, c_I^{\mathcal{L}}, c_I^{\mathcal{U}}, c_F^{\mathcal{L}}, c_F^{\mathcal{U}} \in \mathbb{C},$$
$$\left|\frac{(c^{\mathcal{L}}+c_T^{\mathcal{L}}+c_I^{\mathcal{L}}+c_F^{\mathcal{L}})+(c^{\mathcal{U}}+c_T^{\mathcal{U}}+c_I^{\mathcal{U}}+c_F^{\mathcal{U}})}{2}\right| \leq 4, |c^{\mathcal{L}}|, |c_T^{\mathcal{L}}|, |c_I^{\mathcal{L}}|, |c_F^{\mathcal{L}}|, |c^{\mathcal{U}}|, |c_T^{\mathcal{U}}|, |c_I^{\mathcal{U}}|, |c_F^{\mathcal{U}}| \leq 1\}$$

where $(a, b, c, d)^{\mathcal{L},\mathcal{U}}, (a', b', c', d')^{\mathcal{L},\mathcal{U}}, (a''b'', c'', d'')^{\mathcal{L},\mathcal{U}}, (a''', b''', c''', d''')^{\mathcal{L},\mathcal{U}}$ are the complex-valued interval fuzzy membership, truth, indeterminacy and falsity membership functions of

$A_1, A_2,\ A_1 \cup A_2$ and $A_1 \cap A_2$ respectively. Hence by assigning complex values to φ, we have $\forall\ l \in U$.

$$\varphi\left([\mu_{\mathcal{L}_1}(l), \mu_{\mathcal{U}_1}(l)], [\mu_{\mathcal{L}_2}(l), \mu_{\mathcal{U}_2}(l)]\right) = [\mu_{a^*\mathcal{L}}(l), \mu_{a^*\mathcal{U}}(l)] = [z_{\mu^{\mathcal{L}}}, z_{\mu^{\mathcal{U}}}],$$

$$\varphi\left([T_{\mathcal{L}_1}(l), T_{A^{\mathcal{U}}_1}(l)], [T_{\mathcal{L}_2}(l), T_{\mathcal{U}_2}(l)]\right) = [T_{a^*\mathcal{L}}(l), T_{a^*\mathcal{U}}(l)] = [z_{T^{\mathcal{L}}}, z_{T^{\mathcal{U}}}],$$

$$\varphi\left([I_{\mathcal{L}_1}(l), I_{\mathcal{U}_1}(l)], [I_{\mathcal{L}_2}(l), I_{\mathcal{U}_2}(l)]\right) = [I_{a^*\mathcal{L}}(l), I_{a^*\mathcal{U}}(l)] = [z_{I^{\mathcal{L}}}, z_{I^{\mathcal{U}}}],$$

$$\varphi\left([F_{\mathcal{L}_1}(l), F_{\mathcal{U}_1}(l)], [F_{\mathcal{L}_2}(l), F_{\mathcal{U}_2}(l)]\right) = [F_{a^*\mathcal{L}}(l), F_{a^*\mathcal{U}}(l)] = [z_{F^{\mathcal{L}}}, z_{F^{\mathcal{U}}}],$$

and

$$\varphi\left([\mu_{\mathcal{L}_1}(l), \mu_{\mathcal{U}_1}(l)], [\mu_{\mathcal{L}_2}(l), \mu_{\mathcal{U}_2}(l)]\right) = [\mu_{b^*\mathcal{L}}(l), \mu_{b^*\mathcal{U}}(l)] = [z_{\mu}^{\mathcal{L}}, z_{\mu}^{\mathcal{U}}],$$

$$\varphi\left([T_{\mathcal{L}_1}(l), T_{\mathcal{U}_1}(l)], [T_{\mathcal{L}_2}(l), T_{\mathcal{U}_2}(l)]\right) = [T_{b^*\mathcal{L}}(l), T_{b^*\mathcal{U}}(l)] = [z_{T^{\mathcal{L}}}, z_{T^{\mathcal{U}}}],$$

$$\varphi\left([I_{\mathcal{L}_1}(l), I_{\mathcal{U}_1}(l)], [I_{\mathcal{L}_2}(l), I_{\mathcal{U}_2}(l)]\right) = [I_{b^*\mathcal{L}}(l), I_{b^*\mathcal{U}}(l)] = [z_{I^{\mathcal{L}}}, z_{I^{\mathcal{U}}}],$$

$$\varphi\left([F_{\mathcal{L}_1}(l), F_{\mathcal{U}_1}(l)], [F_{\mathcal{L}_2}(l), F_{\mathcal{U}_2}(l)]\right) = [F_{b^*\mathcal{L}}(l), F_{b^*\mathcal{U}}(l)] = [z_{F^{\mathcal{L}}}, z_{F^{\mathcal{U}}}],$$

Here the function must be following axiomatic condition in [11]

3.2.3. *Complement*

Let $A = \{(l, [\mu_{\mathcal{L}}(l), \mu_{\mathcal{U}}(l)], [T_{\mathcal{L}}(l), T_{\mathcal{U}}(l)], [I_{\mathcal{L}}(l), I_{\mathcal{U}}(l)],$ $[F_{A^{\mathcal{L}}}(l), F_{A^{\mathcal{U}}}(l)] : l \in U\}$ be an Interval-Valued Complex Neutrosophic Fuzzy Set in U. The complement of the Interval-Valued Complex Neutrosophic Fuzzy Set A denoted by $c(A)$, can be expressed as follows.

$c(A) = \{(l, [1 - \mu_{\mathcal{L}}(l), 1 - \mu_{\mathcal{U}}(l)], [F_{\mathcal{L}}(l), F_{\mathcal{U}}(l)], [1 - I_{\mathcal{L}}(l), 1 - I_{\mathcal{U}}(l)],$

$[T_{A^{\mathcal{L}}}(l), T_{A^{U}}(l)] : l \in U\},$

where

$c[\mu_{\mathcal{L}}(l), \mu_{\mathcal{U}}(l)] = [1 - \mu_{\mathcal{L}}(l), 1 - \mu_{\mathcal{U}}(l)], c[T_{\mathcal{L}}(l), T_{\mathcal{U}}(l)] = [F_{\mathcal{L}}(l), F_{\mathcal{U}}(l)],$

$c[I_{\mathcal{L}}(l), I_{\mathcal{U}}(l)] = [1 - I_{\mathcal{L}}(l), 1 - I_{\mathcal{U}}(l)], c[F_{\mathcal{L}}(l), F_{U}(l)] = [T_{\mathcal{L}}(l), T_{\mathcal{U}}(l)].$

The phase terms are

$$c[\omega_{\mu_{\mathcal{L}}}(l),\ \omega_{\mu_{\mathcal{U}}}(l)] = (2\pi - \omega_{\mu_{\mathcal{L}}}(l),\ 2\pi - \omega_{\mu_{\mathcal{U}}}(l)), (\omega_{\mu_{\mathcal{L}}}(l),\ \omega_{\mu_{\mathcal{U}}}(l))\ (or)\ [\omega_{\mu_{\mathcal{L}}}(l) + \pi,\ \omega_{\mu_{\mathcal{U}}}(l) + \pi]$$
$$c[(\omega_{T_{\mathcal{L}}}(l),\ \omega_{T_{\mathcal{U}}}(l)] = (2\pi - \omega_{T_{\mathcal{L}}}(l), 2\pi - \omega_{T_{\mathcal{U}}}(l)), (\omega_{T_{\mathcal{L}}}(l),\ \omega_{T_{\mathcal{U}}}(l))\ (or)\ [\omega_{T_{\mathcal{L}}}(l) + \pi,\ \omega_{T_{\mathcal{U}}}(l) + \pi]$$
$$c[(\omega_{I_{\mathcal{L}}}(l),\ \omega_{I_{\mathcal{U}}}(l)] = (2\pi - \omega_{I_{\mathcal{L}}}(l),\ 2\pi - \omega_{I_{\mathcal{U}}}(l)), (\omega_{I_{\mathcal{L}}}(l),\ \omega_{I_{\mathcal{U}}}(l))\ (or)\ [\omega_{I_{\mathcal{L}}}(l) + \pi,\ \omega_{I_{\mathcal{U}}}(l) + \pi]$$
$$c[(\omega_{F_{\mathcal{L}}}(l),\ \omega_{F_{\mathcal{U}}}(l)] = (2\pi - \omega_{F_{\mathcal{L}}}(l), 2\pi - \omega_{F_{\mathcal{U}}}(l)), (\omega_{F_{\mathcal{L}}}(l),\ \omega_{F_{\mathcal{U}}}(l))\ (or)\ [\omega_{F_{\mathcal{L}}}(l) + \pi,\ \omega_{F_{\mathcal{U}}}(l) + \pi].$$

3.2.4. *Composition*

Let us consider two Interval-Valued Complex Neutrosophic Fuzzy Sets A_1 and A_2 on U with

$[\mu_{\mathcal{L}_1}(l), \mu_{\mathcal{U}_1}(l)] = [q_{\mathcal{L}_1}(l), q_{\mathcal{U}_1}(l)].e^{i(\mu_1{}^{\mathcal{L}}(l), \mu_1{}^{\mathcal{U}}(l))}$,

$[T_{\mathcal{L}_1}(l), T_{\mathcal{U}_1}(l)] = [r_{\mathcal{L}_1}(l), r_{\mathcal{U}_1}(l)].e^{i(T_1{}^{\mathcal{L}}(l), T_1{}^{\mathcal{U}}(l))}$,

$[I_{\mathcal{L}_1}(l), I_{\mathcal{U}_1}(l)] = [s_{\mathcal{L}_1}(l), s_{\mathcal{U}_1}(l)].e^{i(I_1{}^{\mathcal{L}}(l), I_1{}^{\mathcal{U}}(l))}$,

$[F_{\mathcal{L}_1}(l), F_{\mathcal{U}_1}(l)] = [t_{\mathcal{L}_1}(l), t_{\mathcal{U}_1}(l)].e^{i(F_1{}^{\mathcal{L}}(l), F_1{}^{\mathcal{U}}(l))}$.

and

$[\mu_{\mathcal{L}_2}(l), \mu_{\mathcal{U}_2}(l)] = [q_{\mathcal{L}_2}(l), q_{\mathcal{U}_2}(l)].e^{i(\mu_2{}^{\mathcal{L}}(l), \mu_2{}^{\mathcal{U}}(l))}$,

$[T_{\mathcal{L}_2}(l), T_{\mathcal{U}_2}(l)] = [r_{\mathcal{L}_2}(l), r_{\mathcal{U}_2}(l)].e^{i(T_2{}^{\mathcal{L}}(l), T_2{}^{\mathcal{U}}(l))}$,

$[I_{\mathcal{L}_2}(l), I_{\mathcal{U}_2}(l)] = [s_{\mathcal{L}_2}(l), s_{\mathcal{U}_2}(l)].e^{i(I_2{}^{\mathcal{L}}(l), I_2{}^{\mathcal{U}}(l))}$,

$[F_{\mathcal{L}_2}(l), F_{\mathcal{U}_2}(l)] = [t_{\mathcal{L}_2}(l), t_{\mathcal{U}_2}(l)].e^{i(F_2{}^{\mathcal{L}}(l), F_2{}^{\mathcal{U}}(l))}$.

The complex interval-valued fuzzy membership, truth membership, indeterminacy membership and falsehood membership functions of the product of the interval-valued complex neutrosophic fuzzy sets A_1 and A_2 denoted by $A_1 \circ A_2$ are

$$[\mu_{\mathcal{L}_1\circ\mathcal{L}_2}(l), \mu_{\mathcal{U}_1\circ\mathcal{U}_2}(l)] = \Big\{ \Big([q_{\mathcal{L}_1}(l)\cdot q_{\mathcal{L}_2}(l)], [q_{\mathcal{U}_1}(l)\cdot q_{\mathcal{U}_2}(l)]\Big) \cdot e^{i\left((\mu_1^{*\mathcal{L}}(l)\cdot\mu_2^{*\mathcal{L}}(l)), (\mu_1^{*\mathcal{U}}(l)\cdot\mu_2^{*\mathcal{U}}(l))\right)} \Big\}$$

$$[T_{\mathcal{L}_1\circ\mathcal{L}_2}(l), T_{\mathcal{U}_1\circ\mathcal{U}_2}(l)] = \Big\{ ([r_{\mathcal{L}_1}(l)\cdot r_{\mathcal{L}_2}(l)], [r_{\mathcal{U}_1}(l)\cdot r_{\mathcal{U}_2}(l)]) \cdot e^{i\left((T_1^{*\mathcal{L}}(l)\cdot T_2^{*\mathcal{L}}(l)), (T_1^{*\mathcal{U}}(l)\cdot T_2^{*\mathcal{U}}(l))\right)} \Big\}$$

$$[I_{\mathcal{L}_1\circ\mathcal{L}_2}(l), I_{\mathcal{U}_1\circ\mathcal{U}_2}(l)] = \Big\{ ([s_{\mathcal{L}_1}(l)\cdot s_{\mathcal{L}_2}(l)], [s_{\mathcal{U}_1}(l)\cdot s_{\mathcal{U}_2}(l)]) \cdot e^{i\left((I_1^{*\mathcal{L}}(l)\cdot I_2^{*\mathcal{L}}(l)), (I_1^{*\mathcal{U}}(l)\cdot I_2^{*\mathcal{U}}(l))\right)} \Big\}$$

$$[F_{\mathcal{L}_1\circ\mathcal{L}_2}(l), F_{\mathcal{U}_1\circ\mathcal{U}_2}(l)] = \Big\{ ([t_{\mathcal{L}_1}(l)\cdot t_{\mathcal{L}_2}(l)], [t_{\mathcal{U}_1}(l)\cdot t_{\mathcal{U}_2}(l)]) \cdot e^{i\left((F_1^{*\mathcal{L}}(l)\cdot F_2^{*\mathcal{L}}(l)), (F_1^{*\mathcal{U}}(l)\cdot F_2^{*\mathcal{U}}(l))\right)} \Big\}$$

where

$$\mu_1^{*\mathcal{L}} = \omega_{\mu_{\mathcal{L}_1/2\pi}}, \quad \mu_1^{*\mathcal{U}} = \omega_{\mu_{\mathcal{U}_1/2\pi}}, \quad \mu_2^{*\mathcal{L}} = \omega_{\mu_{\mathcal{L}_2/2\pi}}, \quad \mu_2^{*\mathcal{U}} = \omega_{\mu_{\mathcal{U}_2/2\pi}},$$
$$T_1^{*\mathcal{L}} = \omega_{T_{\mathcal{L}_1/2\pi}}, \quad T_1^{*\mathcal{U}} = \omega_{T_{\mathcal{U}_1/2\pi}}, \quad T_2^{*\mathcal{L}} = \omega_{T_{\mathcal{L}_2/2\pi}}, \quad T_2^{*\mathcal{U}} = \omega_{T_{\mathcal{U}_2/2\pi}},$$
$$I_1^{*\mathcal{L}} = \omega_{I_{\mathcal{L}_1/2\pi}}, \quad I_1^{*\mathcal{U}} = \omega_{I_{\mathcal{U}_1/2\pi}}, \quad I_2^{*\mathcal{L}} = \omega_{I_{\mathcal{L}_2/2\pi}}, \quad I_2^{*\mathcal{U}} = \omega_{I_{\mathcal{U}_2/2\pi}},$$
$$F_1^{*\mathcal{L}} = \omega_{F_{\mathcal{L}_1/2\pi}}, \quad F_1^{*\mathcal{U}} = \omega_{F_{\mathcal{U}_1/2\pi}}, \quad F_2^{*\mathcal{L}} = \omega_{F_{\mathcal{L}_2/2\pi}}, \quad F_2^{*\mathcal{U}} = \omega_{F_{\mathcal{U}_2/2\pi}}.$$

Propositions:

• Let B_1, B_2, B_3 be Interval-Valued Complex Neutrosophic Fuzzy Sets on U. Then

(1) $(B_1 \cup B_2) = (B_2 \cup B_1)$,

(2) $(B_1 \cap B_2) = (B_2 \cap B_1)$,

(3) $(B_1 \cup B_1) = B_1$,

(4) $(B_1 \cap B_1) = B$,

(5) $B_1 \cup (B_2 \cup B_3) = (B_1 \cup B_2) \cup B_3$,

(6) $B_1 \cap (B_2 \cap B_3) = (B_1 \cap B_2) \cap B_3$,

(7) $B_1 \cup (B_2 \cap B_3) = (B_1 \cup B_2) \cap (B_1 \cup B_3)$,

(8) $B_1 \cap (B_2 \cup B_3) = (B_1 \cap B_2) \cup (B_1 \cap B_3)$,

(9) $B_1 \cup (B_1 \cap B_2) = B_1$,

(10) $B_1 \cap (B_1 \cup B_2) = B_1$,

(11) $(B_1 \cup B_2)^c = B_1^c \cap B_2^c$,

(12) $(B_1 \cap B_2)^c = B_1^c \cup B_2^c$,

(13) $(B_1^c)^c = B_1$.

• The Interval-Valued Complex Neutrosophic Fuzzy Set $B_1 \cup B_2$ is the maximal set that encompasses both B_1 and B_2.

• The Interval-Valued Complex Neutrosophic Fuzzy Sets $B_1 \cap B_2$ is the minimal set that encompasses both B_1 and B_2.

• Let B_1 and B_2 be the Interval-Valued Complex Neutrosophic Fuzzy Sets defined on U. Then $B_1 \subseteq B_2 \iff B_2^c \subseteq B_1^c$.

4. Application

Expanding on this, we aim to demonstrate the practical applications of the defined Interval-Valued Complex Neutrosophic Fuzzy Sets (IVCNFS). We intend to illustrate how these fundamental properties of IVCNFS can be effectively used in addressing real-world challenges and integrating this concept into forthcoming problem-solving scenarios.

4.1. *Similarity Measure on IVCNFS*

In this section, we apply the proposed Multiple Criteria Group Decision-Making Method(MCGDM) discussed in [4], [16]. This section addresses the topics of Hamming distance, normalized Hamming distance, Euclidean distance, normalized Euclidean distance, and similarity measures within the context of interval-valued complex neutrosophic fuzzy sets.
Let the three Interval-Valued Complex Neutrosophic Fuzzy Sets

$$\begin{aligned}
&B_1 = \{l, [\mu_{\mathcal{L}_1}(l_i), \mu_{\mathcal{U}_1}(l_i)], [T_{\mathcal{L}_1}(l_i), T_{\mathcal{U}_1}(l_i)], [I_{\mathcal{L}_1}(l_i), I_{\mathcal{U}_1}(l_i)],\\
&[F_{\mathcal{L}_1}(l_i), F_{\mathcal{U}_1}(l_i)] : l \in U\},\\
&B_2 = \{l, [\mu_{\mathcal{L}_2}(l_i), \mu_{\mathcal{U}_2}(l_i)], [T_{\mathcal{L}_2}(l_i), T_{\mathcal{U}_2}(l_i)], [I_{\mathcal{L}_2}(l_i), I_{\mathcal{U}_2}(l_i)],\\
&[F_{\mathcal{L}_2}(l_i), F_{\mathcal{U}_2}(l_i)] : l \in U\},\\
&B_3 = \{l, [\mu_{\mathcal{L}_3}(l_i), \mu_{\mathcal{U}_3}(l_i)], [T_{\mathcal{L}_3}(l_i), T_{\mathcal{U}_3}(l_i)], [I_{\mathcal{L}_3}(l_i), I_{\mathcal{U}_3}(l_i)],\\
&[F_{\mathcal{L}_3}(l_i), F_{\mathcal{U}_3}(l_i)] : l \in U\},
\end{aligned}$$

be defined over the universe $U = \{l_1, l_2, l_3, ...l_n\}$. The following definitions describe several distance measures between B_1 and B_2.

Definition 1. Let B_1 and B_2 be two Interval-Valued Complex Neutrosophic Fuzzy Sets. The Hamming distance between B_1 and B_2 is defined by

$$\begin{aligned}
d_1(B_1, B_2) = \frac{1}{8}\sum_{l_i \in U}\Big[&|\mu_{\mathcal{L}_1}(l_i) - \mu_{\mathcal{L}_2}(l_i)| + |\mu_{\mathcal{U}_1}(l_i) - \mu_{\mathcal{U}_2}(l_i)| + |T_{\mathcal{L}_1}(l_i) - T_{\mathcal{L}_2}(l_i)| + |T_{\mathcal{U}_1}(l_i)\\
&-T_{\mathcal{U}_2}(l_i)| + |I_{\mathcal{L}_1}(l_i) - I_{\mathcal{L}_2}(l_i)| + |I_{\mathcal{U}_1}(l_i) - I_{\mathcal{U}_2}(l_i)| + |F_{\mathcal{L}_1}(l_i) - F_{\mathcal{L}_2}(l_i)|\\
&+|F_{\mathcal{U}_1}(l_i) - F_{\mathcal{U}_2}(l_i)|\Big] \qquad (1)
\end{aligned}$$

Definition 2. The normalized Hamming distance between B_1 and B_2 is defined by

$$\begin{aligned}
d_2(B_1, B_2) = \frac{1}{8n}\sum_{l_i \in U}\Big[&|\mu_{\mathcal{L}_1}(l_i) - \mu_{\mathcal{L}_2}(l_i)| + |\mu_{\mathcal{U}_1}(l_i) - \mu_{\mathcal{U}_2}(l_i)| + |T_{\mathcal{L}_1}(l_i) - T_{\mathcal{L}_2}(l_i)|\\
&+|T_{\mathcal{U}_1}(l_i) - T_{\mathcal{U}_2}(l_i)| + |I_{\mathcal{L}_1}(l_i) - I_{\mathcal{L}_2}(l_i)| + |I_{\mathcal{U}_1}(l_i) - I_{\mathcal{U}_2}(l_i)| + |F_{\mathcal{L}_1}(l_i) - F_{\mathcal{L}_2}(l_i)|
\end{aligned}$$

$$+|F_{\mathcal{U}_1}(l_i) - F_{\mathcal{U}_2}(l_i)|\Big] \tag{2}$$

Definition 3. The Euclidean distance measures between B_1 and B_2 is defined to be

$$d_3(B_1, B_2) = \Bigg[\frac{1}{8}\sum_{l_i \in U}\Big[|\mu_{\mathcal{L}_1}(l_i) - \mu_{\mathcal{L}_2}(l_i)|^2 + |\mu_{\mathcal{U}_1}(l_i) - \mu_{\mathcal{U}_2}(l_i)|^2 + |T_{\mathcal{L}_1}(l_i) - T_{\mathcal{L}_2}(l_i)|^2$$

$$+|T_{\mathcal{U}_1}(l_i) - T_{\mathcal{U}_2}(l_i)|^2 + |I_{\mathcal{L}_1}(l_i) - I_{\mathcal{L}_2}(l_i)|^2 + |I_{\mathcal{U}_1}(l_i) - I_{\mathcal{U}_2}(l_i)|^2 + |F_{\mathcal{L}_1}(l_i) - F_{\mathcal{L}_2}(l_i)|^2$$

$$+|F_{\mathcal{U}_1}(l_i) - F_{\mathcal{U}_2}(l_i)|^2\Big]\Bigg]^{1/2} \tag{3}$$

Definition 4. The normalized Euclidean distance measures between B_1 and B_2 is given by

$$d_4(B_1, B_2) = \Bigg[\frac{1}{8n}\sum_{l_i \in U}\Big[|\mu_{\mathcal{L}_1}(l_i) - \mu_{\mathcal{L}_2}(l_i)|^2 + |\mu_{\mathcal{U}_1}(l_i) - \mu_{\mathcal{U}_2}(l_i)|^2 + |T_{\mathcal{L}_1}(l_i) - T_{\mathcal{L}_2}(l_i)|^2$$

$$+|T_{\mathcal{U}_1}(l_i) - T_{\mathcal{U}_2}(l_i)|^2 + |I_{\mathcal{L}_1}(l_i) - I_{\mathcal{L}_2}(l_i)|^2 + |I_{\mathcal{U}_1}(l_i) - I_{\mathcal{U}_2}(l_i)|^2 + |F_{\mathcal{L}_1}(l_i) - F_{\mathcal{L}_2}(l_i)|^2$$

$$+|F_{\mathcal{U}_1}(l_i) - F_{\mathcal{U}_2}(l_i)|^2\Big]\Bigg]^{1/2} \tag{4}$$

4.2. *Properties*

The distances d_k $(k = 1, 2, 3, 4)$ defined above between interval-valued complex neutrosophic fuzzy sets B_1 ,B_2 and B_3 satisfy the following properties (D1-D4).

(D_1) $d_k(B_1^{\mathcal{L}}, B_2^{\mathcal{L}}) \geq 0,\ d_k(B_1^{\mathcal{U}}, B_2^{\mathcal{U}}) \geq 0;$

(D_2) $d_k(B_1^{\mathcal{L}}, B_2^{\mathcal{L}}) = 0 \iff B_1^{\mathcal{L}} = B_2^{\mathcal{L}},\ d_k(B_1^{\mathcal{U}}, B_2^{\mathcal{U}}) = 0 \iff B_1^{\mathcal{U}} = B_2^{\mathcal{U}}$

(D_3) $d_k(B_1^{\mathcal{L}}, B_2^{\mathcal{L}}) = d_k(B_2^{\mathcal{L}}, B_1^{\mathcal{L}})$ and $d_k(B_1^{\mathcal{U}}, B_2^{\mathcal{U}}) = d_k(B_2^{\mathcal{U}}, B_1^{\mathcal{U}});$

(D_4) If $B_1^{\mathcal{L}} \subseteq B_2^{\mathcal{L}} \subseteq B_3^{\mathcal{L}}$ and $B_1^{\mathcal{U}} \subseteq B_2^{\mathcal{U}} \subseteq B_3^{\mathcal{U}}$ where $B_3^{\mathcal{L}}$ and $B_3^{\mathcal{U}}$ are IVCNFS in U.

Then $d_k(B_1^{\mathcal{L}}, B_3^{\mathcal{L}}) \geq d_k(B_1^{\mathcal{L}}, B_2^{\mathcal{L}}),\ d_k(B_1^{\mathcal{L}}, B_3^{\mathcal{L}}) \geq d_k(B_2^{\mathcal{L}}, B_3^{\mathcal{L}})$ and

$d_k(B_1^{\mathcal{U}}, B_3^{\mathcal{U}}) \geq d_k(B_1^{\mathcal{U}}, B_2^{\mathcal{U}}),\ d_k(B_1^{\mathcal{U}}, B_3^{\mathcal{U}}) \geq d_k(B_2^{\mathcal{U}}, B_3^{\mathcal{U}}).$

(or)

(1) $d_K(B_1, B_2) \leq d_k(B_1, B_3)$

(2) $d_K(B_2, B_3) \leq d_k(B_1, B_3)$.

It is a well-established fact that similarity measures can be generated from distance measures, as documented in references [25], [26], [27]. Hence, we can utilize the proposed distance measures to formulate similarity measures. The inherent relationship between similarity measures $\mathfrak{S}_1(B_1, B_2)$ and $\mathfrak{S}_2(B_1, B_2)$ and Interval-valued complex neutrosophic fuzzy sets B_1,B_2 and B_3 are as follows:

(1) $0 \leq \mathfrak{S}_k(B_1, B_2) \leq 1$
(2) $\mathfrak{S}_k(B_1, B_2) = 1 \iff B_1 = B_2$
(3) $\mathfrak{S}_k(B_1, B_2) = \mathfrak{S}_k(B_2, B_1)$
(4) If $B_1 \subseteq B_2 \subseteq B_3, B_3$ is IVCNFS in U, then $\mathfrak{S}_k(B_1, B_3) \leq \mathfrak{S}_k(B_1, B_2)$ and $\mathfrak{S}_k(B_1, B_3) \leq \mathfrak{S}_k(B_2, B_3)$, where $[B_1^{\mathcal{L}}, B_1^{\mathcal{U}}] \in B_1, [B_2^{\mathcal{L}}, B_2^{\mathcal{U}}] \in B_2$

Clearly, $\mathfrak{S}_k(B_1, B_2)$ (for $k = 1, 2$) increases the degree of similarity between B_1 and B_2.

5. Multi Criteria Decision-Making

In this section, using the similarity measure to the MCDM method, we present an approach to addressing multi-criteria decision-making problems with respect to an Interval-Valued Complex Neutrosophic Fuzzy Set using these similarity measures.

Let $B = \{B_1, B_2, ...B_m\}$ represent a set of alternatives, and $L = \{l_1, l_2, ...l_n\}$ represent a set of criteria. Assume that the decision-maker considers the criteria L_j(for $j = 1, 2, 3, ...n$), and alternative B_i (for $i = 1, 2, 3, ...m$) characterized by the following Interval-Valued Complex Ntrosophic Fuzzy set (IVCNFS).

$$\begin{aligned}
B_i =&\{l_j, \mu_{B_i}(l_j), T_{B_i}(l_j), I_{B_i}(l_j), F_{B_i}(l_j)\ |l_j \in U\} \\
=&\{l_j, [|\mu_{\mathcal{L}_i}(l_j)|, |\mu_{\mathcal{U}_i}(l_j)|], [|T_{\mathcal{L}_i}(l_j)|, |T_{\mathcal{U}_i}(l_j)|], [|I_{\mathcal{L}_i}(l_j)|, |I_{\mathcal{U}_i}(l_j)|], [|F_{\mathcal{L}_i}(l_j)|, \\
&|F_{\mathcal{U}_i}(l_j)|] : l_j \in U\}
\end{aligned}$$

where

$\{\mu_{B_i}(l_j) \in [|\mu_{\mathcal{L}_i}(l_j)|, |\mu_{\mathcal{U}_i}(l_j)|],\ T_{B_i}(l_j) \in [|T_{\mathcal{L}_i}(l_j)|, |T_{\mathcal{U}_i}(l_j)|],\ I_{B_i}(l_j) \in [|I_{\mathcal{L}_i}(l_j)|,$

$|I_{\mathcal{U}_i}(l_j)|],\ F_{B_i}(l_j) \in [|F_{\mathcal{L}_i}(l_j)|, |F_{\mathcal{U}_i}(l_j)|]\} \subset [0, 1]$,

$$^{-}0 \leq \mu_{B_i}(l_j) + T_{B_i}(l_j) + I_{B_i}(l_j) + F_{B_i}(l_j) \leq 4^{+}$$
$$\text{(or)}$$
$$|\mu_{B_i}(l_j) + T_{B_i}(l_j) + I_{B_i}(l_j) + F_{B_i}(l_j)| \leq 4$$

$\forall\ l_j \in U, i = 1, 2, 3, ..n$ and $j = 1, 2, 3, ...m$. The Interval pairs $|\mu_{B_i}(l_j)| \in [|\mu_{\mathcal{L}_i}(l_j)|, |\mu_{\mathcal{U}_i}(l_j)|]$, $|T_{B_i}(l_j)| \in [|T_{\mathcal{L}_i}(l_j)|, |T_{\mathcal{U}_i}(l_j)|]$, $|I_{B_i}(l_j)| \in [|I_{\mathcal{L}_i}(l_j)|, |I_{\mathcal{U}_i}(l_j)|]$, $|F_{B_i}(l_j)| \in [|F_{\mathcal{L}_i}(l_i)|, |F_{\mathcal{U}_i}(l_j)|]$ are denoted by an Interval-Valued Complex Neutrosophic Fuzzy Value (IVCNFV) γ_{ij}. These values are usually derived from evaluating an alternative B_i with respect to a criterion L_j using a scoring method and practical data processing. Consequently, we can construct an Interval-Valued Complex Neutrosophic Fuzzy Decision Matrix

$$D = [\gamma_{ij}]_{m\times n}.$$

The ideal point is to determine the optimal alternative in multi-criteria decision making environments. Although the ideal alternative doesn't manifest in the real world, this concept provides a variable theoretical framework for the evaluation and comparison of alternatives.
In general, evaluation can be broadly classified into two categories: benefit criteria and cost criteria. Let X represent the set of benefit criteria, and Y represent the set of cost criteria. In the decision-making method presented here, the ideal alternative can be ascertained by employing a maximum operator for the benefit criteria and minimum operator for the cost criteria, which helps us to determine the optimal value for each criteria across all alternatives. We define an ideal Interval-Valued Complex Neutrosophic Fuzzy Value (IVCNFV) for benefit criterion within the ideal alternative B^* as follows:

$$\begin{aligned}\gamma^*_{1j} =&\big([|\mu_j^{*\mathcal{L}}|, |\mu_j^{*\mathcal{U}}|], [|T_j^{*\mathcal{L}}|, |T_j^{*\mathcal{U}}|], [|I_j^{*\mathcal{L}}|, |I_j^{*\mathcal{U}}|], [|F_j^{*\mathcal{L}}|, |F_j^{*\mathcal{U}}|]\big)\\ =&\Big\{[\max_i(|\mu_{ij}^{*\mathcal{L}}|), \max_i(|\mu_{ij}^{*\mathcal{U}}|)], [\max_i(|T_{ij}^{*\mathcal{L}}|), \max_i(|T_{ij}^{*\mathcal{U}}|)], [\min_i(|I_{ij}^{*\mathcal{L}}|), \min_i(|I_{ij}^{*\mathcal{U}}|)],\\ &[\min_i(|F_{ij}^{*\mathcal{L}}|), \min_i(|F_{ij}^{*\mathcal{U}}|)]\Big\}\forall\ j \in X,\end{aligned}$$

while for a cost criterion, we define an ideal (IVCNFV) in B^* as

$$\begin{aligned}\gamma^*_{2j} =&\big([|\mu_j^{*\mathcal{L}}|, |\mu_j^{*\mathcal{U}}|], [|T_j^{*\mathcal{L}}|, |T_j^{*\mathcal{U}}|], [|I_j^{*\mathcal{L}}|, |I_j^{*\mathcal{U}}|], [|F_j^{*\mathcal{L}}|, |F_j^{*\mathcal{U}}|]\big)\\ =&\Big\{[\min_i(|\mu_{ij}^{*\mathcal{L}}|), \min_i(|\mu_{ij}^{*\mathcal{U}}|)], [\min_i(|T_{ij}^{*\mathcal{L}}|), \min_i(|T_{ij}^{*\mathcal{U}}|)], [\max_i(|I_{ij}^{*\mathcal{L}}|), \max_i(|I_{ij}^{*\mathcal{U}}|)],\\ &[\max_i(|F_{ij}^{*\mathcal{L}}|), \max_i(|F_{ij}^{*\mathcal{U}}|)]\Big\}\forall\ j \in Y.\end{aligned}$$

Therefore, utilizing equations (2) and (4), we define two similarity measures between an alternative B_i and the ideal alternative B^* as follows:

$$\begin{aligned}\mathfrak{S}_1(B^*, B_i) = 1 - \frac{1}{8n}\sum_{j=1}^{n}\Big[&|\mu_j^{*\mathcal{L}} - \mu_{ij}^{\mathcal{L}}| + |\mu_j^{*\mathcal{U}} - \mu_{ij}^{\mathcal{U}}| + |T_j^{*\mathcal{L}} - T_{ij}^{\mathcal{L}}| + |T_j^{*\mathcal{U}} - T_{ij}^{\mathcal{U}}| + |I_j^{*\mathcal{L}} - I_{ij}^{\mathcal{L}}|\\ &+|I_j^{*\mathcal{U}} - I_{ij}^{\mathcal{U}}| + |F_j^{*\mathcal{L}} - F_{ij}^{\mathcal{L}}| + |F_j^{*\mathcal{U}} - F_{ij}^{\mathcal{U}}|\Big]\end{aligned} \tag{5}$$

$$\begin{aligned}\mathfrak{S}_2(B^*, B_i) = 1 - \Bigg[\frac{1}{8n}\sum_{j=1}^{n}\Big[&|\mu_j^{*\mathcal{L}} - \mu_{ij}^{\mathcal{L}}|^2 + |\mu_j^{*\mathcal{U}} - \mu_{ij}^{\mathcal{U}}|^2 + |T_j^{*\mathcal{L}} - T_{ij}^{\mathcal{L}}|^2 + |T_j^{*\mathcal{U}} - T_{ij}^{\mathcal{U}}|^2 + |I_j^{*\mathcal{L}} - I_{ij}^{\mathcal{L}}|^2\\ &+|I_j^{*\mathcal{U}} - I_{ij}^{\mathcal{U}}|^2 + |F_j^{*\mathcal{L}} - F_{ij}^{\mathcal{L}}|^2 + |F_j^{*\mathcal{U}} - F_{ij}^{\mathcal{U}}|^2\Big]\Bigg]^{1/2}\end{aligned} \tag{6}$$

To assess each alternative in comparison with the ideal alternative, we can establish a ranking order for all alternatives and identify the best one by utilizing the similarity measures $\mathfrak{S}_1(B^*, B_i)$ and $\mathfrak{S}_2(B^*, B_i), (i = 1, 2, ...m)$.

6. Illustrative Examples

Two instances have been covered in this section. Example (6.2) recommends a seed selection that helps the farmer consistently make money, whereas Example (6.1) assists the investor in identifying the preferable option.
In the example below, we have computed the similarity measure between the ideal alternatives and individual options, and we have, within specified time periods, suggested the investor's preferred option.

Example 6.1. Let's consider an investor who intends to invest in a corporation. Initially, the investor has chosen five corporations based on their periodic profit and loss trends. The investor's selection depends on specific criteria, namely: Risk (C_1), availability of green materials (C_2), availability of labor (C_3), market demand (C_4) and production quality (C_5). These criteria are applied to five different industries:
1. Automobile industry (I_1), which exhibits good profits for three to four months.
2. The food manufacturing industry (I_2), which shows good profits for two to five months.
3. Electronic manufacturing industry (I_3), with good profits occurring for six to seven months.
4. Oil industry (I_4), where good profits are observed for six to eight months.
5. Pharmaceutical industry (I_5), which experiences good profits for five to nine months.

The company evaluation and criteria data were collected from Ref [4].
Tables 1,2,3,4,5 contain the similarity measures between five industries and five criteria.
The ideal alternative (company) is computed from Tables 1,2,3,4,5 as

b*= ([0.0631, 0.0794], [0.4419, 0.0978], [0.0000, 0.0109], [0.0000, 0.0109]), ([0.1263, 0.0652] ,[0.0631, 0.0326], [0.0084, 0.0362], [0.0126, 0.0223]), ([0.1578, 0.0936], [0.0758, 0.1114], [0.0000, 0.0000] ,[0.0000, 0.0000]), ([0.2904, 0.0826] [0.2525 ,0.0783], [0.0342 , 0.0794], [0.0105, 0.0326]), ([0.1263, 0.0908], [0.1263, 0.0794], [0.0100, 0.0227], [0.0151, 0.0340]).

The similarity measures between the ideal alternative and each individual alternative are computed as follows.
$\mathfrak{S}_1(b*, I_1) = 0.94466, \mathfrak{S}_1(b*, I_2) = 0.94395, \mathfrak{S}_1(b*, I_3) = 0.95438, \mathfrak{S}_1(b*, I_4) = 0.94521, \mathfrak{S}_1(b*, I_5) = 0.94776,$

$\mathfrak{S}_2(b*, I_1) = 0.90251, \mathfrak{S}_2(b*, I_2) = 0.90043, \mathfrak{S}_2(b*, I_3) = 0.91961, \mathfrak{S}_2(b*, I_4) = 0.90454, \mathfrak{S}_2(b*, I_5) = 0.90354.$

TABLE 1. Evaluation of Companies in accordance with the criteria

	C_1
I_1	([0.017, 0.038], [0.042, 0.066], [0.017, 0.038], [0.008, 0.028])
I_2	([0.020, 0.060], [0.020, 0.050], [0.027, 0.080], [0.008, 0.020])
I_3	([0.063, 0.033], [0.442, 0.098], [0.000, 0.011], [0.000, 0.011])
I_4	([0.021, 0.056], [0.053, 0.089], [0.021, 0.067], [0.011, 0.033])
I_5	([0.050, 0.079], [0.035, 0.074], [0.010, 0.023], [0.030, 0.091])

TABLE 2. Evaluation of Companies in accordance with the criteria

	C_2
I_1	([0.008, 0.028], [0.008, 0.025], [0.008, 0.038], [0.025, 0.066])
I_2	([0.000 ,0.025], [0.000, 0.015], [0.011, 0.036], [0.028, 0.078])
I_3	([0.126, 0.065], [0.063, 0.033], [0.189, 0.065], [0.189, 0.098])
I_4	([0.000, 0.011], [0.008, 0.011], [0.042, 0.089], [0.013, 0.022])
I_5	([0.012, 0.023], [0.010, 0.023], [0.025, 0.051], [0.024, 0.074])

TABLE 3. Evaluation of Companies in accordance with the criteria

	C_3
I_1	([0.025, 0.066], [0.017, 0.047], [0.004, 0.009], [0.007, 0.019])
I_2	([0.020, 0.080], [0.027, 0.090], [0.000, 0.000], [0.010, 0.020])
I_3	([0.158, 0.071], [0.076, 0.027], [0.063, 0.043], [0.126, 0.054])
I_4	([0.051, 0.094], [0.074, 0.111], [0.021, 0.056], [0.000, 0.000])
I_5	([0.020, 0.057], [0.030, 0.079], [0.025, 0.045], [0.020, 0.068])

TABLE 4. Evaluation of Companies in accordance with the criteria

	C_4
I_1	([0.025, 0.056], [0.017, 0.047], [0.042, 0.084], [0.017 0.038])
I_2	([0.027, 0.070], [0.020, 0.060], [0.044, 0.096], [0.034, 0.045])
I_3	([0.290, 0.083], [0.253, 0.078], [0.265, 0.092], [0.158, 0.033])
I_4	([0.040, 0.072], [0.021, 0.063], [0.057, 0.089], [0.011, 0.039])
I_5	([0.020, 0.057], [0.015, 0.045], [0.034, 0.079], [0.035, 0.068])

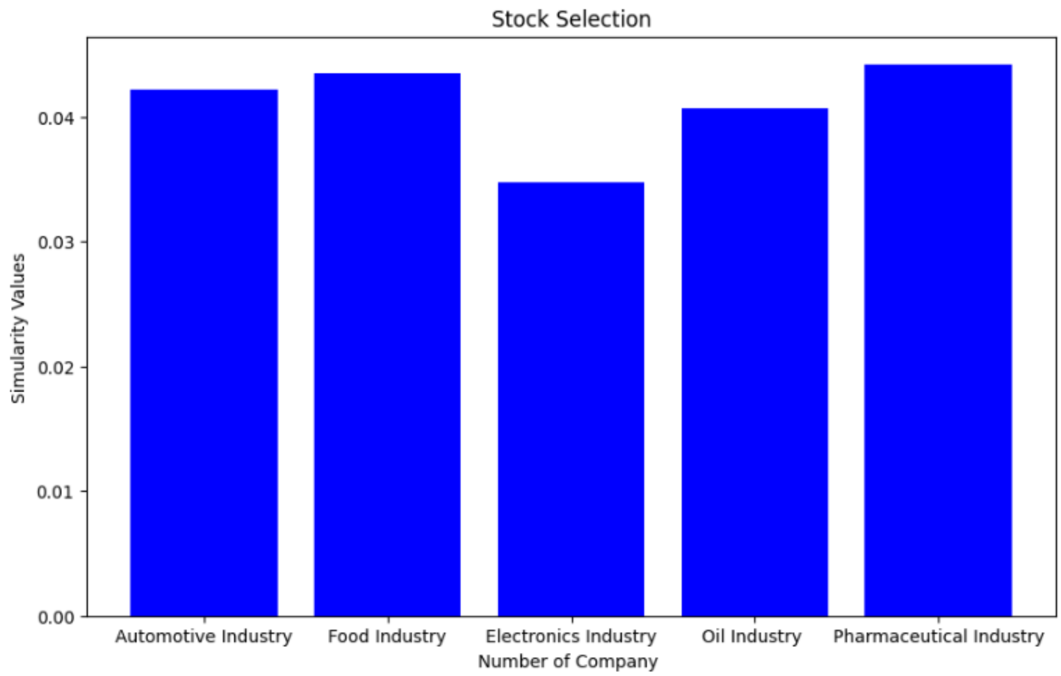

FIGURE 1. Best Stock Selection (Variation of Similarity Values with period)

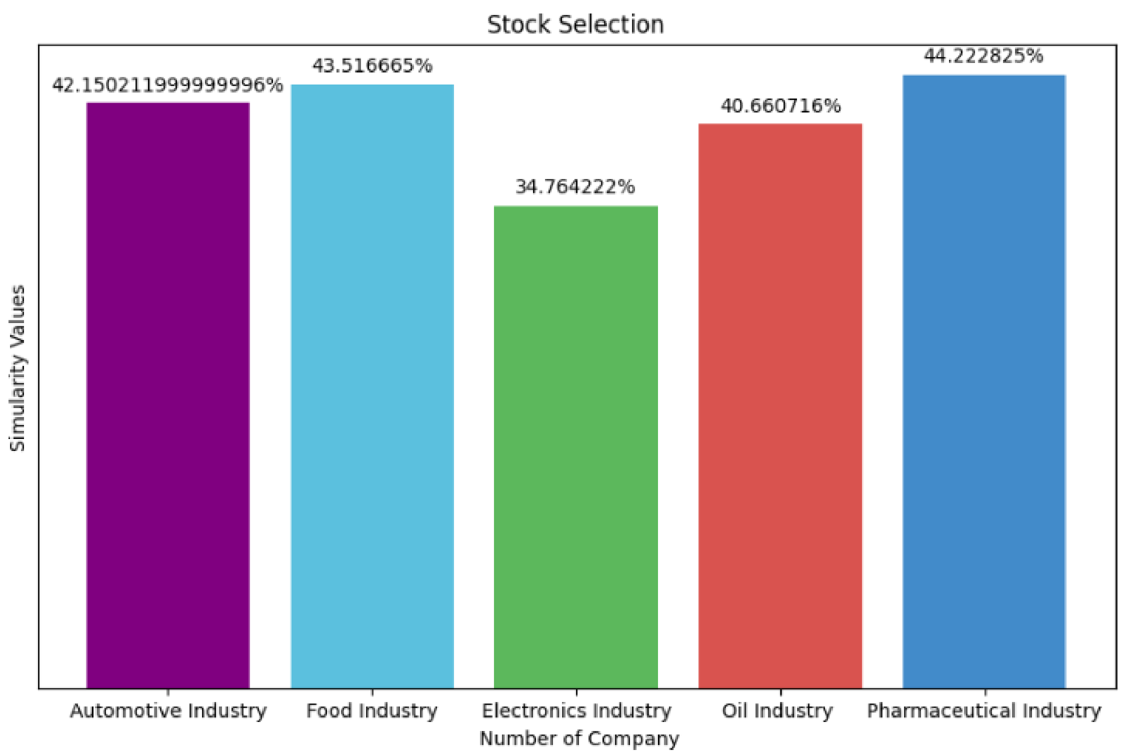

FIGURE 2. Best Stock Selection (percentage of Similarity Values with period)

Table 5. Evaluation of Companies in accordance with the criteria

	C_5
I_1	([0.008, 0.019], [0.008, 0.019], [0.013, 0.028], [0.042, 0.075])
I_2	([0.010, 0.030], [0.010, 0.030], [0.027, 0.050], [0.024, 0.075])
I_3	([0.126, 0.038], [0.126, 0.033], [0.126, 0.049], [0.253, 0.085])
I_4	([0.005, 0.011], [0.005, 0.011], [0.053, 0.089], [0.026, 0.067])
I_5	([0.050, 0.091], [0.040, 0.079], [0.010, 0.023], [0.015, 0.034])

Consequently, in both similarity measures, the **"Pharmaceutical industry"** seems to be the preferred choice for investment within the specified time periods. Figures (1) and (2) illustrate the results obtained in Example (6.1).

Example 6.2. Consider a farmer who intends to select a seed for farming within a specific period. The farmer has initially chosen six seeds, taking into account their periodic profit and loss, which is influenced by the prevailing environmental conditions. Additionally, the farmer has identified nine criteria for seed selection: Sunlight (C_1), Soil fertility (C_2), Humidity (C_3), Fertilizer (C_4), Pesticides (C_5), Pollination (C_6), Atmosphere (C_7), Genotype (C_8) and Weed control (C_9). The six selected seeds are Flax Seed (s_1), Chia Seed (s_2), Ground Seed (s_3), Sesame Seed (s_4), Pumpkin Seed (s_5) and Sunflower Seed (s_6). Tables 6,7,8,9,10,11,12,13,14 provide the similarity measures between these six seeds and the nine criteria. The data employed for calculating similarity measures between the six seeds and nine criteria have been sourced from: `https://www.agrifarming.in`, `https://greenplanet.eolss.net` and some of articles Ref[[28], [29]].

The ideal alternative (seeds) is obtained from the tables 6,7,8,9,10,11,12,13,14 as
b*= ([0.0003, 0.0389], [0.0003, 0.0379], [0.0003, 0.0316], [0.0006, 0.0526]), ([0.0010, 0.0905], [0.0011, 0.0894], [0.0002, 0.0263] [0.0001, 0.0158]), ([0.0010, 0.0874], [0.0010, 0.0854], [0.0003, 0.0301], [0.0001, 0.0151]), ([0.0399, 0.0838], [0.0416, 0.0828], [0.0133, 0.0341], [0.0055, 0.0195]), ([0.0472, 0.0842], [0.0472, 0.0947], [0.0101, 0.0263], [0.0034, 0.0105]), ([0.0597, 0.0840], [0.0615, 0.0821], [0.0103, 0.0241], [0.0080, 0.0193]), ([0.0003, 0.0316], [0.0002, 0.0263], [0.0000, 0.0032], [0.0001, 0.0473]), ([0.0012, 0.0968], [0.0012, 0.0989], [0.0000, 0.0053], [0.0000, 0.0053]), ([0.0539, 0.0947], [0.0532, 0.0979], [0.0168, 0.0368], [0.0007, 0.0074])).

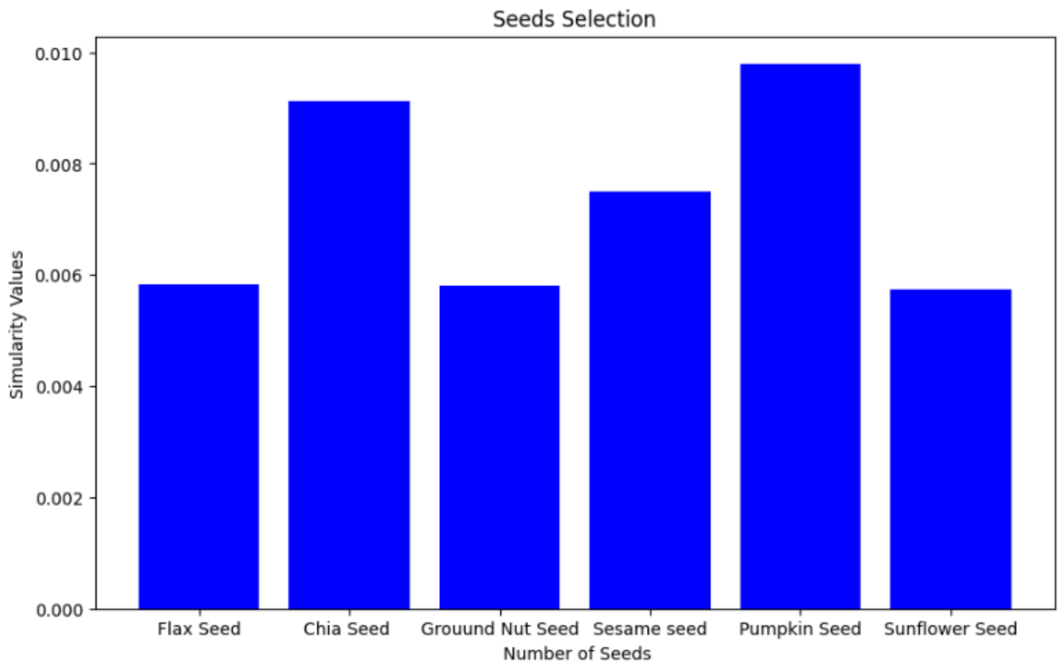

FIGURE 3. Best Seed Selection (Variation of Similarity Values with period)

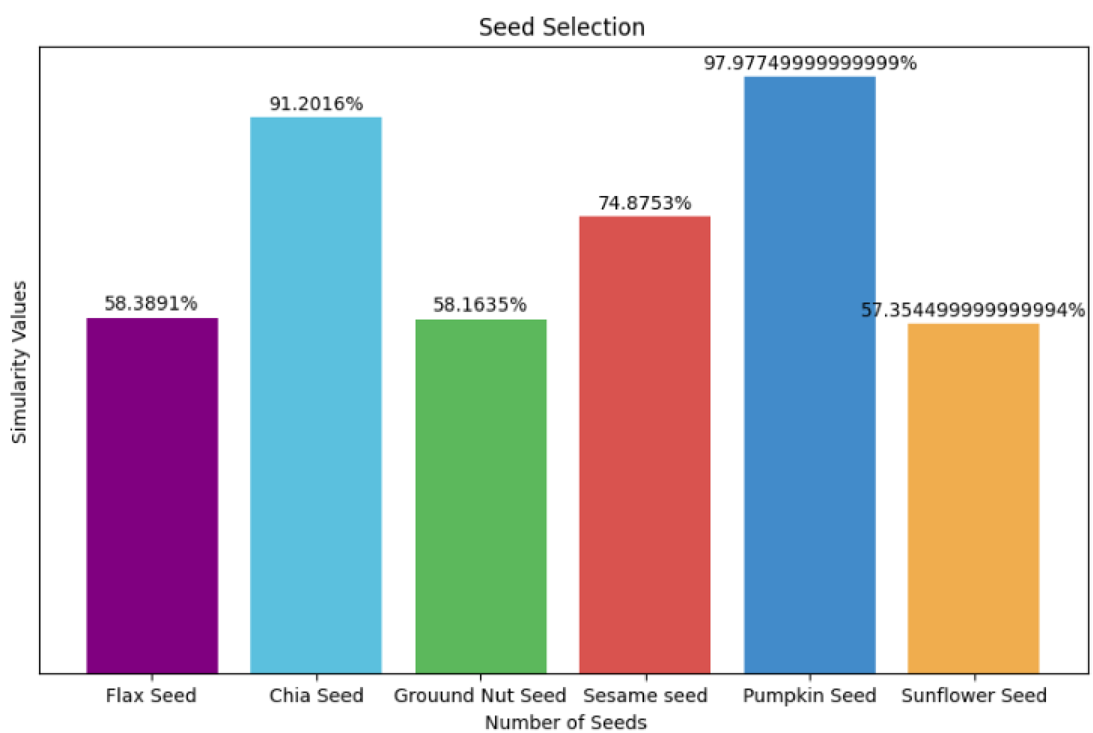

FIGURE 4. Best Seed Selection (percentage of Similarity Values with period)

Table 6. Evaluation of Seeds in accordance with the criteria

	C_1
I_1	([0.00017, 0.02630] [0.00017, 0.02841] [0.00026, 0.03157] [0.00081, 0.07681])
I_2	([0.00031, 0.03893] [0.00031, 0.03788] [0.00044, 0.04209] [0.00067, 0.06734])
I_3	([0.00015, 0.02841] [0.00023, 0.03157] [0.00073, 0.06313] [0.00055, 0.05261])
I_4	([0.00022, 0.02946] [0.00019, 0.02736] [0.00058, 0.05261] [0.00073, 0.06313])
I_5	([0.00031, 0.03683] [0.00031, 0.03157] [0.00087, 0.07365] [0.00102, 0.08418])
I_6	([0.00028, 0.03157] [0.00026, 0.03051] [0.00044, 0.05261] [0.00073, 0.06313])

Table 7. Evaluation of Seeds in accordance with the criteria

	C_2
I_1	([0.00070, 0.06734] [0.00084, 0.06839] [0.00047, 0.04524] [0.00016, 0.02630])
I_2	([0.00073, 0.07365] [0.00090, 0.07786] [0.00076, 0.06734] [0.00015, 0.01578])
I_3	([0.00097, 0.09049] [0.00109, 0.08944] [0.00017, 0.02630] [0.00045, 0.04735])
I_4	([0.00073, 0.07365] [0.00080, 0.06839] [0.00077, 0.06839] [0.00031, 0.03683])
I_5	([0.00093, 0.07891] [0.00092, 0.07681] [0.00049, 0.04524] [0.00036, 0.04735])
I_6	([0.00102, 0.08418] [0.00105, 0.08733] [0.00061, 0.06734] [0.00012, 0.01578])

Table 8. Evaluation of Seeds in accordance with the criteria

	C_3
I_1	([0.00073, 0.06030] [0.00063, 0.05527] [0.00029, 0.03015] [0.00036, 0.04522])
I_2	([0.00003, 0.00804] [0.00003, 0.00925] [0.00087, 0.07035] [0.00106, 0.09145])
I_3	([0.00087, 0.07035] [0.00065, 0.06532] [0.00079, 0.06532] [0.00031, 0.03517])
I_4	([0.00065, 0.06532] [0.00073, 0.07035] [0.00051, 0.04522] [0.00029, 0.03015])
I_5	([0.00001, 0.00301] [0.00001, 0.00255] [0.00036, 0.03517] [0.00099, 0.08542])
I_6	([0.00097, 0.08743] [0.00105, 0.08542] [0.00061, 0.05427] [0.00012, 0.01507])

The values in each square of the tables above indicate the growth factor of a seed over a specific period under a given criterion. The similarity measures between the ideal alternative and each individual alternative are computed as follows:

$\mathfrak{S}_1(b*, s_1) = 0.9878$, $\mathfrak{S}_1(b*, s_2) = 0.9849$, $\mathfrak{S}_1(b*, s_3) = 0.9891$, $\mathfrak{S}_1(b*, s_4) = 0.9870$, $\mathfrak{S}_1(b*, s_5) = 0.9830$, $\mathfrak{S}_1(b*, s_6) = 0.9928$.

$\mathfrak{S}_2(b*, s_1) = 0.9819$, $\mathfrak{S}_2(b*, s_2) = 0.9758$, $\mathfrak{S}_2(b*, s_3) = 0.9833$, $\mathfrak{S}_2(b*, s_4) = 0.9795$, $\mathfrak{S}_2(b*, s_5) = 0.9732$, $\mathfrak{S}_2(b*, s_6) = 0.9870$.

TABLE 9. Evaluation of Seeds in accordance with the criteria

	C_4
I_1	([0.02219, 0.04873] [0.01387, 0.04385] [0.01332, 0.03411] [0.02774, 0.05847])
I_2	([0.01443, 0.03801] [0.01165, 0.03801] [0.02330, 0.05262] [0.02497, 0.05360])
I_3	([0.03329, 0.06822] [0.03606, 0.07309] [0.02219, 0.04873] [0.00832, 0.02436])
I_4	([0.02330, 0.05457] [0.02219, 0.04873] [0.02497, 0.06334] [0.01942, 0.04385])
I_5	([0.01110, 0.02924] [0.01387, 0.03411] [0.01387, 0.04385] [0.02774, 0.05847])
I_6	([0.03995, 0.08381] [0.04161, 0.08283] [0.02219, 0.06822] [0.00555, 0.01949])

TABLE 10. Evaluation of Seeds in accordance with the criteria

	C_5
I_1	([0.04042, 0.07365] [0.04379, 0.08944] [0.02358, 0.04735] [0.00606, 0.01578])
I_2	([0.04379, 0.07891] [0.03705, 0.07891] [0.01684, 0.03683] [0.00674, 0.02630])
I_3	([0.04716,, 0.08418] [0.04716, 0.09470] [0.01011, 0.02630] [0.00337, 0.01052])
I_4	([0.01347, 0.03157] [0.01011, 0.02630] [0.03705, 0.06839] [0.03908, 0.07891])
I_5	([0.02358, 0.04735] [0.02358, 0.04735] [0.04042, 0.07365] [0.01347, 0.03157])
I_6	([0.03369, 0.06313] [0.03705, 0.06839] [0.03032, 0.05787] [0.01684, 0.03683])

TABLE 11. Evaluation of Seeds in accordance with the criteria

	C_6
I_1	([0.03063, 0.05465] [0.03063, 0.05465] [0.02858, 0.04708] [0.01701, 0.02943])
I_2	([0.02878, 0.04204] [0.02518, 0.03687] [0.02230, 0.03850] [0.02518, 0.04506])
I_3	([0.02929, 0.05375] [0.03083, 0.04633] [0.03468, 0.06023] [0.01927, 0.03243])
I_4	([0.05970, 0.08399] [0.05970, 0.08206] [0.02388, 0.03862] [0.00796, 0.01931])
I_5	([0.05713, 0.06950] [0.06153, 0.07413] [0.01758, 0.03243] [0.01758, 0.03243])
I_6	([0.05164, 0.06758] [0.05594, 0.07241] [0.01033, 0.02414] [0.02582, 0.03862])

Pumpkin Seed is selected for crop cultivation based on the results obtained from both similarity measures. It stands out as the most profitable crop choice, delivering the highest potential profit. Figures (3) and (4) illustrate the results of Example (6.2).

A farmer may make money from a crop only during a particular season. The aim here is to find a crop that will help farmers make money in a continuous way. The similarity index has been calculated to achieve this and the example above is an illustration.

Table 12. Evaluation of Seeds in accordance with the criteria

	C_7
I_1	([0.00001, 0.00316] [0.00002, 0.00263] [0.00005, 0.00473] [0.00012, 0.10311])
I_2	([0.00001, 0.00210] [0.00001, 0.00158] [0.00001, 0.00316] [0.00109, 0.08944])
I_3	([0.00003, 0.00526] [0.00003, 0.00421] [0.00003, 0.00421] [0.00093, 0.08102])
I_4	([0.00029, 0.03157] [0.00022, 0.02630] [0.00006, 0.00737] [0.00076, 0.06629])
I_5	([0.00000, 0.00053] [0.00000, 0.00026] [0.00004, 0.00631] [0.00090, 0.07681])
I_6	([0.00006, 0.00842] [0.00003, 0.00526] [0.00003, 0.00316] [0.00051, 0.04735])

Table 13. Evaluation of Seeds in accordance with the criteria

	C_8
I_1	([0.00105 0.09259] [0.00109, 0.08944] [0.00051, 0.04735] [0.00003, 0.00526])
I_2	([0.00073, 0.09470] [0.00121, 0.09785] [0.00022, 0.02630] [0.00004, 0.00737])
I_3	([0.00093, 0.08207] [0.00095, 0.07891] [0.00055, 0.05051] [0.00036, 0.03683])
I_4	([0.00029, 0.03157] [0.00022, 0.02630] [0.00006, 0.00737] [0.00076, 0.06629])
I_5	([0.00095, 0.07891] [0.00093, 0.08207] [0.00058, 0.08418] [0.00016, 0.02315])
I_6	([0.00109, 0.08944] [0.00103, 0.08838] [0.00051, 0.04735] [0.00013, 0.01684])

Table 14. Evaluation of Seeds in accordance with the criteria

	C_9
s_1	([0.04177, 0.08207], [0.04379, 0.07891], [0.01684, 0.03683], [0.00674, 0.02630])
s_1	([0.04716, 0.08628], [0.04783, 0.08733], [0.02358, 0.04735], [0.00674, 0.01789])
s_1	([0.04783, 0.07681], [0.04110, 0.07681], [0.03099, 0.06839], [0.00808, 0.02841])
s_1	([0.04042, 0.08418], [0.03773, 0.08207], [0.03032, 0.05787], [0.00876, 0.02315])
s_1	([0.04716, 0.08944], [0.04649, 0.08733], [0.02358, 0.04735], [0.00606, 0.01789])
s_1	([0.05390, 0.09470], [0.05322, 0.09785], [0.01684, 0.03683], [0.00067, 0.00737])

6.1. *Results and Discussion*

In Example (6.1) of the pharmaceutical industry, the output indicates that the Interval-Valued Complex Neutrosophic Fuzzy Set appears to be the preferred investment choice within specific periods of the year. In Example (6.2), pumpkin seeds are chosen for crop farming throughout the year, emerging as the most profitable option with the highest potential for profit.

7. Merits and limitations of the proposed approach

The proposed approach using IVCNFS offers significant advantages in terms of representing complex uncertainty and improving the precision of decision making. This enhanced representation of uncertainty can lead to more informed and accurate decision-making in complex scenarios. Furthermore, the use of IVCNFSs has the potential to improve the accuracy of solutions compared to existing methods. The versatility of this structure allows for its potential application across various domains. However, it also presents certain limitations, such as increased computational complexity, challenges in parameter selection and interpretation, and potential data requirements. The successful application of the proposed approach may require sufficient and reliable data to accurately estimate the parameters and membership functions of IVCNFSs.

8. Conclusion

In our present work, we have extended complex neutrosophic fuzzy sets (CNFSs) into interval-valued complex neutrosophic fuzzy sets (IVCNFSs). To assess the practical applications in real-time scenario, we apply IVCNFS and multivariable decision-making. The results, illustrated through relevant examples, serve to validate the effectiveness of our approach and also we have defined the Hamming and Euclidean distances, along with the proposal of similarity measures tailored for IVCNFS. These measures have been developed by establishing a connection between similarity measures and distances. Additionally, we have applied these similarity measures in the context of multicriteria decision-making within the Interval-Value Complex Neutrosophic Fuzzy structure. This approach involves an evaluation of the similarity between each alternative and the ideal alternative. By utilizing these similarity measures, we can effectively determine the ranking order of all available alternatives and identify the most suitable one. To illustrate the practical utility of our approach, we have provided two examples. The proposed similarity measures for IVCNFS demonstrate their suitability for real scientific and engineering applications. Moreover, these techniques extend the existing decision-making methods and offer a valuable tool for decision-makers. In future, we will continue to explore the application of these similarity measures for IVCNFS to various other domains.

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