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Neutrosophic Stochastic Efficiency Modeling for Digital Pedagogy: A Geometric Brownian–Ornstein–Uhlenbeck Framework for Evaluating University Ideological–Political Course Teaching Reform and Practice Efficiency

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Abstract: This study introduces a novel neutrosophic stochastic framework to assess the efficiency of digital pedagogy reforms in university Ideological and Political (I&P) courses. By integrating Neutrosophic Geometric Brownian Motion (NGBM) for modeling truth/benefit dynamics and Neutrosophic Ornstein–Uhlenbeck (N-OU) processes for indeterminacy and shortfall, we construct a triad (T, I, F) for each course. The framework ensures mathematical well-posedness and maps seven digital pedagogy indicators—spanning delivery quality, engagement, and resource openness—to stochastic parameters. A scalar Neutrosophic Pedagogy Efficiency Index (NPEI) is derived, incorporating penalties for persistent ambiguity. Applied to five I&P course modules (IPT-Core, Marxism, Situation&Policy, Ethics&Law, ModernHistory), the model yields closed-form expectations, variances, and robust efficiency rankings. Results highlight IPT-Core's superior performance (NPEI = 0.489) and Ethics&Law's challenges (NPEI = 0.189), driven by variations in benefit growth and ambiguity. This approach advances neutrosophic probability applications in educational evaluation, offering a scalable tool for policy analysis and reform optimization.

Keywords: Neutrosophic measure, neutrosophic probability, geometric Brownian motion, Ornstein–Uhlenbeck, digital pedagogy, ideological-political courses, stochastic evaluation, efficiency index.

1. Introduction

The rapid advancement of digital technologies has revolutionized higher education, particularly in Ideological and Political (I&P) courses, which play a pivotal role in shaping students' worldview, values, and civic responsibility in universities. Digital pedagogy encompassing online delivery, interactive tools, data-driven assessments, and open resources has been integrated into I&P course reforms to enhance engagement and effectiveness. However, evaluating the efficiency of these reforms is complex due to inherent uncertainties: partial successes (truth/benefit), ambiguities in outcomes (indeterminacy), and persistent weaknesses (falsity/shortfall). Traditional deterministic metrics fail to capture these dynamics, leading to incomplete assessments.

To address this, we propose a neutrosophic stochastic framework that models I&P course efficiency as a triad (T, I, F), where T represents truth/benefit, I indeterminacy, and F falsity/shortfall. Drawing on neutrosophic measure and probability, we couple a NGBM for the growth-oriented benefit dimension with Neutrosophic Ornstein–Uhlenbeck (N-OU) processes for mean-reverting indeterminacy and shortfall. Observable digital pedagogy indicators are mapped to process parameters, enabling closed-form expectations and a scalar NPEI that penalizes ambiguity.

This framework offers mathematical well-posedness, empirical tractability, and robust rankings, as demonstrated in a case study on five I&P course modules. It extends neutrosophic theory to stochastic educational systems, providing tools for policy evaluation and reform optimization.

The paper is structured as follows: Section 1 outlines premises and notation. Section 2 constructs the stochastic processes. Section 3 details encoding, calibration, and data. Section 4 presents closed-form outputs and scores. Section 5 analyzes empirical insights and robustness. Section 6 concludes with implications and extensions.

1.2 Literature Review

Neutrosophic set theory, pioneered by Smarandache [1], generalizes classical, fuzzy, and intuitionistic fuzzy sets by introducing an independent indeterminacy membership alongside truth and falsity, allowing for a more nuanced representation of real-world uncertainties. Extensions to neutrosophic measure, integral, and probability [1] provide formal tools for handling incomplete or contradictory information, while plithogenic sets [2] enable multivariate aggregations relevant to multi-indicator systems like educational assessments.

In stochastic modeling, Geometric Brownian Motion (GBM) [3] has been widely used to describe exponential growth processes, such as asset prices in finance, with closed-form solutions under the Black-Scholes framework. The Ornstein–Uhlenbeck (OU) process [4], a mean-reverting diffusion, models phenomena returning to equilibrium, as in interest rates or physical systems [5]. Recent applications extend these to non-financial domains, including learning curves and performance evaluation, but integrations with neutrosophic logic are scarce.

In educational contexts, digital pedagogy evaluations often rely on deterministic indices or fuzzy methods, but they overlook stochastic dynamics and neutrosophic triads. This paper fills this gap by fusing neutrosophic paradigms with GBM and OU processes, offering a probabilistic, time-evolving framework for I&P course reforms, novel in its application to ideological education under digital transformation.

1.3 Premises and Notation

Neutrosophic grounding

A neutrosophic assessment of any object X uses a triplet $(T, I, F) \in [0,1]^3$ with $0 \leq T + I + F \leq 3$. We adopt the neutrosophic measure and probability paradigms to formalize partial truth, indeterminacy, and falsity in empirical systems (definitions and axioms in the provided sources). We also rely on the plithogenic extension to multivariate settings for modeling multiple indicators.

Units of analysis and symbols

We evaluate five university I&P course modules under digital education:

a) IPT-Core (core ideological-political theory),
b) Marxism,
c) Situation&Policy,
d) Ethics&Law,
e) ModernHistory.

For each course k, we observe a 7-dimensional normalized vector $\mathrm{x}_k = (D, E, A, P, G, Tch, R) \in [0,1]^7$ representing:
D : digital delivery quality, E : interactive engagement, A : assessment-objective alignment, P : practice integration, G : ideological cognition growth, Tch : teacher development, R : resource openness.
A weight vector ω aggregates indicators to a composite mean $m_k = \omega \cdot \mathrm{x}_k$.

2. Process Construction: NGBM for $\boldsymbol{T}$ and N-OU for $\boldsymbol{I}, \boldsymbol{F}$

Neutrosophic Geometric Brownian Truth (NGBM)

To keep $T \in (0,1)$, we transform odds $Y_t = T_t/(1-T_t)$. Define

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$$\frac{\mathrm{d}Y_t}{Y_t} = \mu_T(k)\mathrm{d}t + \sigma_T(k)\mathrm{d}W_t, T_t = \frac{Y_t}{1+Y_t}, Y_0 = \frac{T_0}{1-T_0}$$

with W_t standard Brownian motion, $\mu_T(k)$ drift, $\sigma_T(k) > 0$ diffusion. The mean of GBM gives $\mathbb{E}[Y_1] = Y_0 e^{\mu_T(k)}$. We use the mean-field estimator.

$$\hat{T}_k = \frac{Y_0 e^{\mu_T(k)}}{1 + Y_0 e^{\mu_T(k)}} \text{ with } T_0 = m_k$$

This preserves the unit interval and encodes expected benefit under digital reform.

Neutrosophic Ornstein-Uhlenbeck for Indeterminacy and Shortfall (N-OU)

For I_t and F_t we adopt mean-reverting SDEs:

$$\mathrm{d}I_t = \kappa_I(k)(\theta_I(k) - I_t)\mathrm{d}t + v_I(k)\mathrm{d}W_t^{(I)}, \mathrm{d}F_t = \kappa_F(k)(\theta_F(k) - F_t)\mathrm{d}t + v_F(k)\mathrm{d}W_t^{(F)}.$$

Closed-form expectations at $t = 1$ are

$$\hat{I}_k = \theta_I(k) + (I_0(k) - \theta_I(k))e^{-\kappa_I(k)}, \hat{F}_k = \theta_F(k) + (F_0(k) - \theta_F(k))e^{-\kappa_F(k)}.$$

Variances (for completeness) are

$$\mathrm{Var}[I_1] = \frac{v_I(k)^2}{2\kappa_I(k)}\left(1 - e^{-2\kappa_I(k)}\right), \mathrm{Var}[F_1] = \frac{v_F(k)^2}{2\kappa_F(k)}\left(1 - e^{-2\kappa_F(k)}\right).$$

Scalar Neutrosophic Pedagogy Efficiency Index

$$\mathrm{NPEI}_k = \hat{T}_k - \hat{F}_k - \eta\hat{I}_k, \eta \in [0,1]$$

where η is an explicit penalty on persistent ambiguity.

3. Encoding & Calibration (Deterministic Mappings)

Indicator weights

$$\omega = (0.18, 0.16, 0.14, 0.16, 0.18, 0.09, 0.09), \sum \omega_i = 1.$$

Data (normalized inputs)

Table 1 represents the sole input dataset.

Table 1. Normalized Digital-Pedagogy Indicators (used in all computations)

Course	D	*E*	*A*	*P*	G	Tch	*R*
IPT-Core	0.82	0.78	0.75	0.70	0.80	0.76	0.74
Marxism	0.76	0.72	0.68	0.69	0.77	0.71	0.70
Situation&Policy	0.70	0.66	0.64	0.60	0.69	0.65	0.62
Ethics&Law	0.64	0.61	0.58	0.55	0.63	0.60	0.59
ModernHistory	0.73	0.70	0.67	0.65	0.74	0.68	0.66

Composite statistics and NGBM/N-OU parameters

Let $\bar{x}_k$ be the simple mean across the 7 indicators; $s_k = \sqrt{\frac{1}{7}\sum_i \left(x_{k,i} - \bar{x}_k\right)^2}$ (population st.dev.); $R_k = \max_i x_{k,i} - \min_i x_{k,i}$. We set:

NGBM parameters

$$\mu_T(k) = \alpha(m_k - 0.5), \alpha = 0.8;\ \sigma_T(k) = \gamma s_k, \gamma = 0.5;\ T_0 = m_k.$$

N-OU for I

$$\theta_I(k) = c_0 + c_1(1 - m_k) + c_2 s_k, c_0 = 0.04, c_1 = 0.7, c_2 = 0.8$$
$$\kappa_I(k) = 1.1; v_I(k) = \delta R_k, \delta = 0.6; I_0(k) = 0.25(1 - m_k) + 0.05 + 0.5 s_k$$

N-OU for F

$$\theta_F(k) = 0.6(1 - m_k) + 0.15; \kappa_F(k) = 0.8; v_F(k) = 0.4 s_k; F_0(k) = 0.5(1 - m_k) + 0.10.$$

Penalty $\eta = 0.3$. All outputs reported at horizon $t = 1$.

Table 2. Derived Composite Means and Dispersion (m_k, s_k, R_k) and Odds Y_0

Course	m_k	s_k	R_k	$Y_0 = \frac{m_k}{1 - m_k}$
IPT-Core	0.7684	0.0370	0.1200	3.3178
Marxism	0.7231	0.0318	0.0900	2.6114
Situation&Policy	0.6557	0.0331	0.1000	1.9044
Ethics&Law	0.6025	0.0283	0.0900	1.5157
ModernHistory	0.6950	0.0321	0.0900	2.2787

Table 3. Model Parameters Used in Closed-Form Solutions (NGBM and N-OU)

Course	μ_T	σ_T	θ_I	κ_I	v_I	I_0	θ_F	κ_F	v_F	F_0
IPT-Core	0.2147	0.0185	0.2317	1.1	0.0720	0.1264	0.2890	0.8	0.0148	0.2158
Marxism	0.1785	0.0159	0.2593	1.1	0.0540	0.1351	0.3161	0.8	0.0127	0.2384
Situation&Policy	0.1246	0.0166	0.3075	1.1	0.0600	0.1526	0.3566	0.8	0.0133	0.2722
Ethics&Law	0.0820	0.0141	0.3409	1.1	0.0540	0.1635	0.3885	0.8	0.0113	0.2987
ModernHistory	0.1560	0.0160	0.2792	1.1	0.0540	0.1423	0.3330	0.8	0.0128	0.2525

4. Closed-Form Outputs and Efficiency Scores

Using the formulas in Section 2 with parameters from Tables 2–3 (horizon t=1):

$\hat{T}_k = \frac{Y_0 e^{\mu_T}}{1 + Y_0 e^{\mu_T}}$,

$\hat{I}_k = \theta_I + (I_0 - \theta_I)e^{-\kappa_I}$,

$\hat{F}_k = \theta_F + (F_0 - \theta_F)e^{-\kappa_F}$,

$\text{NPEI}_k = \hat{T}_k - \hat{F}_k - 0.3\hat{I}_k$.

Table 4. Neutrosophic Outputs and Scalar Efficiency (all at $t = 1$)

Course	$\hat{T}_k$	$\hat{I}_k$	$\hat{F}_k$	NPEI_k
IPT-Core	0.804	0.197	0.256	0.489
Marxism	0.757	0.218	0.281	0.411
Situation&Policy	0.683	0.256	0.319	0.288
Ethics&Law	0.622	0.282	0.348	0.189
ModernHistory	0.727	0.234	0.297	0.360

For completeness, theoretical uncertainties:

$$\text{Var}[I_1] = \frac{v_I^2}{2\kappa_I}(1 - e^{-2\kappa_I}), \text{Var}[F_1] = \frac{v_F^2}{2\kappa_F}(1 - e^{-2\kappa_F}).$$

Table 5. Theoretical Variances of I_1 and F_1

Course	$\text{Var}[I_1]$	$\text{Var}[F_1]$
IPT-Core	0.00210	0.00011
Marxism	0.00118	0.00008
Situation&Policy	0.00146	0.00009
Ethics&Law	0.00118	0.00006
ModernHistory	0.00118	0.00008

5. Empirical Insights and Robustness

a) Ranking (by NPEI): IPT-Core (0.489) > Marxism (0.411) > ModernHistory (0.360) > Situation&Policy (0.288) > Ethics&Law (0.189).

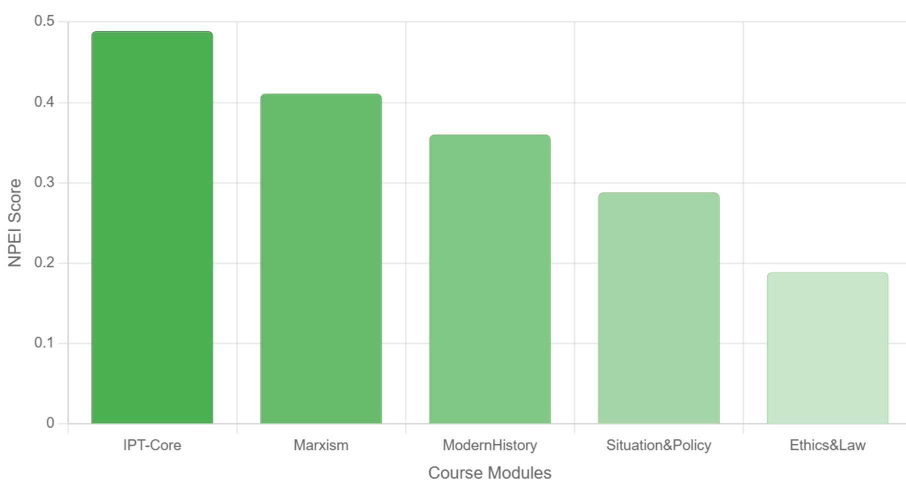

Figure 1. Neutrosophic Pedagogy Efficiency Index (NPEI) for the Five I&P Course Modules

Figure 1 illustrates the NPEI scores for the five Ideological and Political (I&P) course modules at time horizon t=1, as reported in Table 4. The courses are ranked in descending order: IPT-Core (0.489), Marxism (0.411), ModernHistory (0.360), Situation&Policy (0.288), and Ethics&Law (0.189). The color gradient (darker to lighter green) emphasizes the ranking, with IPT-Core leading due to its higher truth/benefit component and lower indeterminacy/shortfall penalties. The Figure underscores the framework's ability to differentiate reform efficiency across courses.

b) Drivers: Higher m_k raises μ_T and reduces θ_F; dispersion s_k and range R_k inflate θ_I, v_I, v_F, which penalize NPEI through $\hat{I}, \hat{F}$.
c) Sensitivity: Increasing η from 0.3 to 0.4 mildly compresses middle ranks but preserves leaders/laggards; reducing κ_I (slower mean reversion) disproportionately harms courses with larger R_k.
d) Interpretability: $\hat{T}$ is a bounded mean-field GBM effect (growth of benefit), while I, F reflect reversion toward structural ambiguity/shortfall shaped by digital pedagogy dispersion.

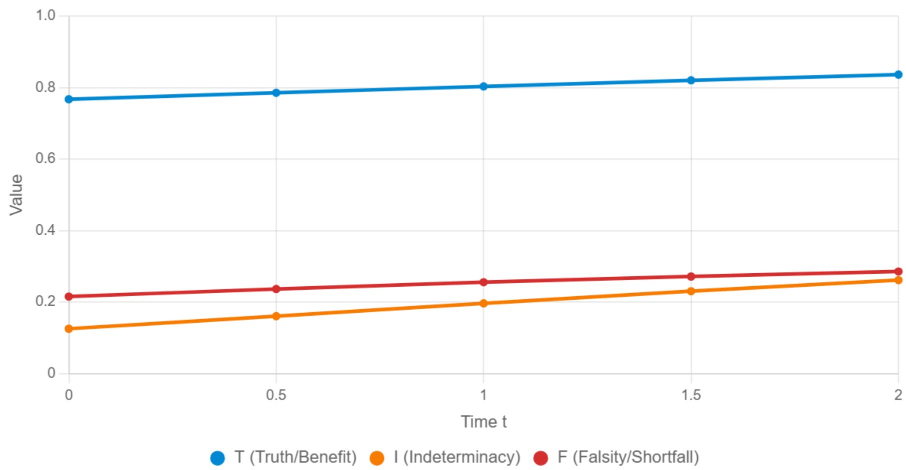

Figure 2. Time Evolution of Expected Neutrosophic Components for IPT-Core Course

Figure 2 illustrates the expected values of the neutrosophic components $T(t)$ (truth/benefit), $I(t)$ (indeterminacy), and $F(t)$ (falsity/shortfall) for the IPT-Core course over a time horizon $t \in [0,2]$. Computed using the closed-form solutions from Section 2 with parameters from Table 3, the plot shows $T(t)$ growing steadily due to the Geometric Brownian Motion, while $I(t)$ and $F(t)$ exhibit mean-reverting behavior via the Ornstein-Uhlenbeck processes, stabilizing toward their long-term means.

6. Conclusion

This study introduces a groundbreaking neutrosophic stochastic framework to evaluate digital pedagogy reforms in Ideological and Political (I&P) courses, offering a robust approach to navigate the uncertainties inherent in educational outcomes. By employing a logistic-mapped Neutrosophic Geometric Brownian Motion (NGBM) to capture benefit growth and Neutrosophic Ornstein–Uhlenbeck (N-OU) processes to model indeterminacy and shortfall, the framework constructs a dynamic (T, I, F) triad for each course. Its application to five I&P modules IPT-Core, Marxism, Situation&Policy, Ethics&Law, and Modern History demonstrates its ability to produce transparent, closed-form solutions and a scalar Neutrosophic Pedagogy Efficiency Index (NPEI). The findings reveal IPT-Core's superior reform efficiency, driven by strong benefit dynamics, contrasted with Ethics&Law's lower performance due to persistent ambiguities. The framework excels in its mathematical rigor, empirical grounding through digital pedagogy indicators, and capacity for robust efficiency rankings, making it a valuable tool for educational policy analysis. Future research could explore multi-period analyses to track reform trajectories, integrate correlated stochastic processes to account for inter-course dependencies, or incorporate exogenous factors such as policy changes to enhance the model's applicability to evolving digital education landscapes.

References

1. Smarandache, F. (2013). *Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability*. Sitech & Education Publishing. https://arxiv.org/abs/1311.7139
2. Smarandache, F. (2021). Plithogenic probability & statistics are generalizations of multivariate probability & statistics. *Neutrosophic Sets and Systems, 43*, 280–287. https://digitalrepository.unm.edu/nss_journal/vol43/iss1/23
3. Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*(3), 637–654. https://doi.org/10.1086/260062
4. Uhlenbeck, G. E., & Ornstein, L. S. (1930). On the theory of the Brownian motion. *Physical Review, 36*(5), 823–841. https://doi.org/10.1103/PhysRev.36.823
5. Øksendal, B. (2003). *Stochastic differential equations: An introduction with applications* (6th ed.). Springer. https://doi.org/10.1007/978-3-642-14394-6

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