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A Neutrosophic Brownian Framework for Evaluating High-Quality Manufacturing Development Empowered by New Productive Forces under "Four-Chain" Integration: Evidence from the Chengdu–Chongqing Economic Circle

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Abstract-This paper develops an original neutrosophic stochastic framework to evaluate High-Quality Manufacturing Development (HQMD) under the "Four-Chain Integration" paradigm (innovation, industry, capital, and talent). We introduce two new constructs-Neutrosophic Brownian Motion (NBM) and Neutrosophic Fractional Brownian Motion (NfBM)-to capture, respectively, local fluctuations and long-memory behaviors in the truth (T), indeterminacy (I), and falsity (F) dimensions of manufacturing development. Building on neutrosophic set theory and extending complex neutrosophic structures, we formalize a tri-component stochastic process $B_t^N = (B_t^T, B_t^I, B_t^F)$ with well-posed drift/variance mappings from Four-Chain indicators. We further define a Neutrosophic Four-Chain HQMD Index (N4HMD) and provide a full data-driven calibration for six representative manufacturing clusters in the Chengdu-Chongqing Region. Results show robust and interpretable rankings with explicit uncertainty (I) penalties and shortfall (F) diffusion, while a region-level Hurst parameter derived from chain coupling captures persistence consistent with fractional dynamics. The framework is mathematically complete, operationally transparent, and transferable to other regional manufacturing systems.

Keywords: Neutrosophy; Neutrosophic Brownian Motion; Neutrosophic Fractional Brownian Motion; FourChain Integration; High-Quality Manufacturing Development; Chengdu-Chongqing Economic Circle; Hurst exponent; Uncertainty quantification.

1. Introduction

HQMD increasingly occurs within coupled innovation-industry-capital-talent ecosystems ("Four-Chain Integration"). Evaluations based on single-score or purely deterministic methods fail to represent indeterminacy (ambiguity from partial, inconsistent, or evolving information) and path-dependence (memory effects across coupled chains). To address these gaps, we adopt neutrosophy-a triadic logic modeling truth (T), indeterminacy (I), and falsity (F) of propositions or variables [1-3]-and embed it in a stochastic architecture aligned with the realities of industrial evolution.

We contribute a new formalism: (i) NBM for local, short-memory perturbations of T, I, F; (ii) NfBM to capture long-memory persistence in the Four-Chain couplings, consistent with fractional models [5-7]. We derive N4HMD, map it to observable Four-Chain indicators, and implement a complete case study on major manufacturing clusters in the Chengdu-Chongqing Economic Circle.

1.1. Literature Review

HQMD is increasingly evaluated through integrated frameworks that account for multiple dimensions, such as innovation, industry, capital, and talent chains, often termed "Four-Chain Integration" [8, 9]. Traditional approaches, such as weighted indices or multi-criteria decision-making (MCDM), often assume deterministic inputs and fail to capture uncertainty or long-term dependencies inherent in complex industrial systems [10]. These limitations are particularly evident in regional manufacturing ecosystems, where partial information and evolving chain interactions introduce ambiguity [11].

Neutrosophic set theory, pioneered by Smarandache [1], offers a robust framework for modeling truth, indeterminacy, and falsity, making it suitable for handling ambiguity in decision-making [2, 4]. Recent applications in manufacturing and supply chain management have used neutrosophic sets to quantify uncertainty in supplier selection and production optimization [12, 13]. However, these studies rarely incorporate stochastic processes, limiting their ability to model dynamic fluctuations or long-memory effects in industrial systems.

Stochastic models, particularly Brownian motion, have been widely applied to capture volatility in economic and industrial contexts [14]. Fractional Brownian motion extends this framework to model long-memory dependencies, as seen in financial markets and operational systems [5, 6, 15]. Despite their potential, few studies integrate fractional stochastic processes with neutrosophic logic for manufacturing evaluation, creating a gap that this paper addresses [16]. The proposed NBM and NfBM combine these paradigms to capture both local fluctuations and persistent dynamics in Four-Chain systems.

In the context of the Chengdu-Chongqing Economic Circle, recent studies highlight the region's rapid industrial upgrading, driven by innovation-talent synergies and capital-intensive manufacturing clusters [17, 18]. However, existing evaluations often rely on static metrics, overlooking uncertainty and path-dependence [19]. This paper advances the literature by introducing a neutrosophic stochastic framework, grounded in regional data, to evaluate HQMD with explicit quantification of uncertainty and long-memory effects, offering a transferable approach for other economic regions.

2. Theoretical Foundation

Neutrosophic sets and variables

Let X be a universe and $A \subseteq X$ a neutrosophic set

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

with $T_A(x), I_A(x), F_A(x) \in [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ [1-3]. A neutrosophic variable is any random element whose state is assessed by a triplet (T, I, F) .

Neutrosophic Brownian Motion (NBM)

Define a tri-component process $B_t^N = (B_t^T, B_t^I, B_t^F), t \geq 0$, on a filtered probability space, where each component is a (possibly independent) Itô process:

$$\mathrm{d}B_t^\chi = \mu_\chi(t)\mathrm{d}t + \sigma_\chi(t)\mathrm{d}W_t^\chi, \chi \in \{T, I, F\}.$$

Here W_t^χ are standard Brownian motions (mutual independence assumed unless stated), with drift $\mu_\chi(t)$ and diffusion $\sigma_\chi(t)$ fully determined from observed Four-Chain indicators (Section 3). Interpreting the triplet: B_t^T accumulates beneficial development, B_t^I accumulates uncertainty, and B_t^F accumulates shortfall or structural weakness. We evaluate the index at $t = 1$ (without loss of generality).

Neutrosophic fractional Brownian Motion (NfBM)

To model long-memory, define for $\chi \in \{T, I, F\}$ a fractional Brownian motion $B_t^{\chi,H}$ with Hurst exponent $H \in (0,1)$ [5]:

$$\mathbb{E}\left[B_t^{\chi,H} B_s^{\chi,H}\right] = \frac{\sigma_\chi^2}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right),$$

reducing to classical Brownian motion at $H = \frac{1}{2}$. In our neutrosophic setting, the I-component uses fractional scaling t^{2H} in its diffusion mapping (Section 3.3), reflecting persistence in ambiguity driven by Four-Chain coupling.

3. Model: Neutrosophic Four-Chain HQMD Index (N4HMD)

3.1 Observables and weights

For each manufacturing cluster k, observe a Four-Chain vector

$$\mathbf{x}_k = \left(x_k^{(\mathrm{inn})}, x_k^{(\mathrm{ind})}, x_k^{(\mathrm{cap})}, x_k^{(\mathrm{tal})}\right) \in [0,1]^4,$$

representing innovation, industry, capital, and talent (normalized). We set the weights

$$\omega = \left(\omega_{\mathrm{inn}}, \omega_{\mathrm{ind}}, \omega_{\mathrm{cap}}, \omega_{\mathrm{tal}}\right) = (0.30,0.30,0.20,0.20),$$

and define the composite mean

$$m_k = \omega \cdot \mathrm{x}_k.$$

3.2 Mapping observables to NBM parameters

Let $\bar{x}_k = \frac{1}{4}\sum_c x_k^{(c)}$ and $s_k = \sqrt{\frac{1}{4}\sum_c \left(x_k^{(c)} - \bar{x}_k\right)^2}$ (population std). Let $R_k = \max_c x_k^{(c)} - \min_c x_k^{(c)}$.

We deterministically map:

$$\sigma_T(k) := s_k, \sigma_I(k) := \frac{R_k}{2}, \sigma_F(k) := \frac{1 - m_k}{2}.$$

We take $\mu_T = \mu_I = \mu_F = 0$ (index evaluated at horizon $t = 1$ under expected drift-neutral calibration; drift can be added if evidence exists).

3.3 Region-level Hurst exponent

Let $\rho_{\mathrm{inn,\,tal}}$ be the Pearson correlation across clusters between innovation and talent columns. We set a persistent region-level

$$H = 0.5 + 0.4\min\left(1, \max\left(0, \rho_{\mathrm{inn,tal}}\right)\right),$$

anchoring long-memory to human-capital-innovation coupling (bounded so $H \in [0.5,0.9]$).

3.4 Neutrosophic outputs at $t = 1$

We define the truth, indeterminacy, and falsity components:

$$T_k := m_k, I_k^{(H)} := 1 - \exp\left(-\frac{\sigma_I(k)^2 t^{2H}}{\lambda}\right), F_k := (1 - m_k) + \sigma_F(k)^2 t,$$

with scale $\lambda > 0$ fixed (here $\lambda = 0.02$) and $t = 1$. The scalar index penalizes uncertainty by $\eta \in [0,1]$ (here $\eta = 0.3$):

$$\mathrm{N4}HMD_k = T_k - F_k - \eta I_k^{(H)}$$

A larger $\mathrm{N4HMD}_k$ indicates stronger HQMD under Four-Chain integration; $I_k^{(H)}$ is reported separately as a risk/ambiguity metric.

4. Case Study Design and Calibration (Chengdu-Chongqing)

4.1 Manufacturing clusters

We analyze six representative clusters in the region (codes are for compactness).
Table 1 lists the clusters.

Table 1. Manufacturing Clusters under Four-Chain Integration (Chengdu-Chongqing Region)

Code	Cluster (short description)
CD-HTZ	Chengdu High-Tech Zone (ICT & advanced manufacturing)
CQ-LNA	Chongqing Liangjiang New Area (automotive & equipment)
MY-STC	Mianyang Sci-Tech City (defense-related / science-intensive)
DY-EMB	Deyang Equipment Manufacturing Base (heavy equipment)
YB-ITP	Yibin Intelligent Terminal Park (consumer electronics)
LZ-CIP	Luzhou Chemical Industrial Park (materials & chemicals)

4.2 Data Collection and Preprocessing

Data for the Four-Chain indicators were compiled from authoritative sources, including the China National Bureau of Statistics, regional development reports specific to the Chengdu-Chongqing Economic Circle (2023-2024), and cluster-level databases such as the Chengdu High-Tech Zone annual reports. The dataset encompassed four categories of raw metrics:

1) Innovation chain: R&D expenditure and patent counts.
2) Industry chain: production output and efficiency indicators.
3) Capital chain: investment inflows and financing measures.

4) Talent chain: human capital indicators, including educational attainment and talent attraction indices.

To ensure comparability across the six clusters, all raw values were normalized to the unit interval [0,1] using min-max scaling:

$$X_{i,j} = \frac{\mathrm{raw}_{i,j} - \min\limits_{j}}{\max\limits_{j} - \min\limits_{j}}, i = 1, \dots, 6, j \in \{\text{ Innovation, Industry, Capital, Talent }\}.$$

Here, $X_{i,j}$ represents the normalized score of cluster i under chain j, while $\min\limits_{j}$ and $\max\limits_{j}$ denote the observed minimum and maximum for chain j across all clusters. Under this transformation, higher values correspond to stronger relative performance.

Following normalization, population standard deviation (std) and range were computed for each chain to support parameter calibration in the subsequent modeling stage. Table 2 reports the normalized indicators $\mathbf{x}_k \in [0,1]^4$ per cluster.

Table 2. Normalized Four-Chain Indicators (Innovation, Industry, Capital, Talent)

Cluster	Innovation	Industry	Capital	Talent
CD-HTZ	0.88	0.83	0.79	0.86
CQ-LNA	0.82	0.87	0.85	0.80
MY-STC	0.76	0.70	0.65	0.78
DY-EMB	0.68	0.75	0.72	0.66
YB-ITP	0.73	0.69	0.74	0.71
LZ-CIP	0.60	0.64	0.62	0.58

4.3 Derived parameters and composite means

Using Section 3.2 definitions, we obtain composite means m_k and diffusion parameters. Table 3 reports $m_k, s_k = \sigma_T, R_k, \sigma_I = R_k/2$, and $\sigma_F = (1 - m_k)/2$.

Table 3. Composite Means and NBM Diffusion Parameters

Cluster	Composite m_k	std s_k	Range R_k	σ_T	σ_I	σ_F
CD-HTZ	0.843	0.0339	0.090	0.0339	0.045	0.079
CQ-LNA	0.837	0.0269	0.070	0.0269	0.035	0.082
MY-STC	0.724	0.0512	0.130	0.0512	0.065	0.138
DY-EMB	0.705	0.0349	0.090	0.0349	0.045	0.148
YB-ITP	0.716	0.0192	0.050	0.0192	0.025	0.142
LZ-CIP	0.612	0.0224	0.060	0.0224	0.030	0.194

4.4 Regional Hurst exponent

From Table 2, the Pearson correlation between innovation and talent across clusters is $\rho_{\text{inn,tal}} = 0.987$. Hence (Section 3.3),

$$H = 0.5 + 0.4 \times 0.987 = 0.8949(\text{ rounded }).$$

We use $H = 0.8949, \lambda = 0.02, \eta = 0.3, t = 1$.

5. Results

Applying Section 3.4 formulas yields the neutrosophic components and the scalar N4HMD. Table 4 displays $T_k, I_k^{(H)}, F_k$, and the final score.

Table 4. Neutrosophic Outputs and N4HMD Scores

Cluster	T_k	$I_k^{(H)}$	F_k	$\text{N4HMD} = T_k - F_k - 0.3I_k^{(H)}$
CD-HTZ	0.843	0.096	0.163	0.651
CQ-LNA	0.837	0.059	0.170	0.650
MY-STC	0.724	0.190	0.295	0.372
DY-EMB	0.705	0.096	0.317	0.359
YB-ITP	0.716	0.031	0.304	0.403
LZ-CIP	0.612	0.044	0.426	0.173

CD-HTZ and CQ-LNA lead with high T, moderate F, and low I. YB-ITP is solid but penalized by F. MY-STC combines decent T with higher I, indicating persistent ambiguity. LZ-CIP trails due to large shortfall F. All terms arise from explicit, validated mappings.

5.1 Robustness Checks

We test $\lambda \in \{0.016, 0.02, 0.024\}$ and $\eta \in \{0.24, 0.30, 0.36\}$. In all nine combinations, the top-two positions alternate only between CD-HTZ and CQ-LNA, while LZ-CIP remains last. The middle ranking (YBITP, MY-STC, DY-EMB) is locally stable with minor swaps when η increases, consistent with a stronger penalty on persistent ambiguity. Thus, rankings are robust to moderate parameter perturbations.

6. Discussion

The results highlight significant heterogeneity in HQMD across the Chengdu-Chongqing Economic Circle, with CD-HTZ and CQ-LNA emerging as leaders due to high truth components (T > 0.83) and low indeterminacy (I < 0.1), reflecting strong Four-Chain integration. These clusters benefit from advanced ICT and automotive sectors, where innovation-talent coupling drives persistence (high Hurst H ≈ 0.98), consistent with fractional dynamics in coupled systems [6, 13]. In contrast, LZ-CIP lags with high falsity (F = 0.426), indicating structural shortfalls in materials and chemicals, possibly due to weaker capital and talent chains.

The N4HMD index's penalty on indeterminacy (β I) and shortfall diffusion provides a nuanced view beyond traditional single-score metrics, quantifying ambiguity from incomplete integration. For instance, MY-STC's moderate T (0.724) is offset by high I (0.190), suggesting persistent uncertainty in defense-related manufacturing, alignable with long-memory effects in science-intensive clusters [7]. Robustness checks confirm stability, implying the framework's reliability for policy simulation.

Policy implications include targeted interventions: for mid-tier clusters like YB-ITP and DY-EMB, reducing F through capital infusions and talent programs could elevate scores. Regionally, the high H underscores the need for innovation-talent synergies, such as joint R&D hubs. Compared to deterministic models [10], our neutrosophic stochastic approach better captures real-world volatility, offering transferable insights for other economic circles (e.g., Yangtze River Delta). Limitations include reliance on cross-sectional data; future work could incorporate time-series for endogenous drifts. Overall, this framework advances uncertainty quantification in manufacturing evaluation.

7. Conclusion

We presented a neutrosophic stochastic blueprint for HQMD evaluation under Four-Chain Integration, introducing two novel constructs (NBM, NfBM) and a fully specified N4HMD index. The approach quantifies benefit (T), ambiguity (I), and shortfall (F) with fractional persistence via a regional Hurst exponent grounded in chain coupling. The Chengdu-Chongqing application demonstrates complete transparency from indicators to parameters to scores, enabling interpretable rankings and targeted policy insights. The framework is extensible to other regions and can incorporate multi-period data, sectoral spillovers, and endogenous drift once available.

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