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Q-Neutrosophic Spherical-Cubic Soft Algebra for Enhancing Residential Space Art Design Courses Quality in IoT-Enabled Smart Factories

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Abstract: This paper introduces a new mathematical model called Q-Neutrosophic Spherical-Cubic Soft Evaluation Algebra (Q-NSCSEA) to support the improvement of educational practices in residential space art design courses. These courses are increasingly integrated with smart factory environments powered by the Internet of Things (IoT). The model combines ideas from Q-neutrosophic logic, spherical sets, cubic soft sets, and algebraic quasigroup structures to form a powerful mathematical system. It helps evaluate and compare student design projects based on artistic, functional, and industrial standards. Our approach builds a structured neutrosophic system using truth (T), indeterminacy (I), and falsity (F) values to describe the level of aesthetic quality, uncertainty in design structure, and failure to meet IoT-smart production requirements.

Several new mathematical definitions and operations are proposed, including neutrosophic union, intersection, and project composition under quasigroup behavior. Realistic numerical examples are provided to demonstrate how the model works in educational design assessments. The proposed model supports better decision-making for educators and designers by linking abstract creative ideas with real-world smart manufacturing constraints.

Keywords: Neutrosophic logic, art design, IoT, smart factory, Q-soft set, cubic sets, quasigroup.

1. Introduction

The rapid evolution of industrial systems has positioned smart factories as a cornerstone of modern manufacturing, leveraging technologies such as the Internet of Things (IoT), automation, and real-time data analytics to enhance efficiency and precision [1]. Concurrently, educational programs in design and architecture, particularly those focused on residential space art design, face the challenge of aligning creative expression with the demands of these technologically advanced production environments. Students in these programs are tasked with creating designs that are not only aesthetically compelling and functionally sound but also compatible with automated, IoT-driven manufacturing processes. Traditional evaluation methods in art and design education often prioritize artistic merit over practical manufacturability, resulting in a disconnect between creative outputs and industrial feasibility [2]. This misalignment can lead to designs that, while innovative, are impractical to produce in smart factory settings due to constraints such as material limitations, production scalability, or automation compatibility.

To bridge this gap, a novel evaluation framework is needed one that accommodates the subjective nature of artistic creativity while incorporating the objective constraints of industrial systems. Neutrosophic logic, introduced by Smarandache [3], provides a promising foundation for such a framework. Unlike traditional binary or fuzzy logic, neutrosophic logic independently models truth (T), indeterminacy (I), and falsity (F), offering a flexible approach to handle the ambiguity and multi-dimensionality inherent in design evaluation [3]. This logic is particularly suited for residential space art design, where subjective aesthetic judgments must coexist with objective technical requirements.

This study proposes a new algebraic structure, termed Q-NSCSEA, which integrates neutrosophic logic with spherical and cubic sets, Q-soft sets, and quasigroup theory to evaluate student design projects. The model balances artistic freedom with the practical needs of smart factories, supported by detailed mathematical operations and practical examples. By embedding IoT-driven constraints into the evaluation process, Q-NSCSEA aims to provide educators with a robust tool to assess design projects in a way that ensures both creativity and manufacturability are adequately addressed.

2. Literature Review

The application of neutrosophic logic to complex systems has gained traction in recent years, particularly in domains requiring multi-criteria decision-making under uncertainty. Neutrosophic sets, first conceptualized by Smarandache [3], offer a framework that captures truth, indeterminacy, and falsity as distinct components, enabling nuanced modeling of ambiguous or incomplete information [4]. This approach has proven effective in fields such as decision-making, medical diagnosis, and expert systems, where traditional logic systems fall short in addressing uncertainty [5].

Extensions of neutrosophic sets, such as Neutrosophic Spherical Sets and Neutrosophic Cubic Sets, have been developed to enhance modeling precision. Spherical sets incorporate geometric representations to better capture multi-dimensional relationships, while cubic sets use interval-based approaches to model uncertainty within defined bounds [6]. These structures have been applied to problems requiring precise yet flexible evaluation frameworks, such as quality control and system optimization [6]. Similarly, Q-Neutrosophic Soft Sets have emerged as a tool for handling soft decision environments, where parameters are partially defined or subjective [7]. By integrating Q-soft sets with algebraic structures like quasigroups and loops, researchers have uncovered valuable properties for organizing and evaluating complex, uncertain data [8].

Despite these advancements, the application of neutrosophic models to educational design systems, particularly in art and architecture, remains underexplored. Most existing studies focus on technical domains such as engineering or information systems, with limited attention to creative disciplines [9]. Moreover, there is a notable absence of frameworks that integrate neutrosophic evaluation with IoT-based industrial constraints, particularly in the context of smart factories [1]. Current evaluation models in design education often fail to connect creative outputs with measurable industrial feedback, leaving a gap in assessing designs for both artistic and technical merit [2].

The proposed Q-NSCSEA model addresses these shortcomings by synthesizing neutrosophic logic, spherical and cubic sets, Q-soft sets, and quasigroup theory into a unified algebraic framework tailored for residential space art design education. Unlike previous models, Q-NSCSEA explicitly incorporates IoT-driven smart factory constraints, enabling a holistic evaluation of student projects that balances creativity with manufacturability. This approach builds on prior work in neutrosophic systems [3, 6, 7] while addressing the unique challenges of design education in an increasingly automated industrial landscape [1].

3. Method

This section presents a complete and original algebraic methodology based on neutrosophic logic for evaluating residential space art design projects within IoT-based smart factory environments. The proposed structure (Q-NSCSEA) is constructed by integrating several key mathematical concepts: neutrosophic sets, spherical cubic representations, soft sets, and quasigroup operations.

3.1 Foundations of Neutrosophic Evaluation

A neutrosophic set models each element using three functions:
Truth $T(x) \in [0,1]$: represents the degree of correctness or success.
Indeterminacy $I(x) \in [0,1]$: represents uncertainty, ambiguity, or vagueness.
Falsity $F(x) \in [0,1]$: represents failure, incorrectness, or contradiction.
The three values are independent, and no constraint like $T + I + F = 1$ is required.

Soft Parameter Structure in Design Evaluation

Let:
$X = \{x_1, x_2, \ldots, x_n\}$: a finite set of student design projects.
$P = \{p_1, p_2, \ldots, p_m\}$: a finite set of evaluation parameters. Examples:
p_1 : Aesthetic quality
p_2 : Functional layout
p_3 : Material compatibility with loT smart factory
p_4 : Manufacturability via automation
Each parameter represents a soft condition. Different designs may satisfy different parameters to varying degrees.

Definition of Q-Neutrosophic Spherical-Cubic Soft Set

We now define the core structure:

$$Q - NSCSES = \{(x_i, p_j, T_{ij}, I_{ij}, F_{ij}) \mid x_i \in X, p_j \in P\}$$

Where:

T_{ij} : degree to which project x_i satisfies parameter p_j

I_{ij} : degree of uncertainty in evaluating x_i under p_j

F_{ij} : degree to which project x_i fails to meet parameter p_j

These form a Q-soft set with spherical cubic representation:

The spherical constraint holds:

$$T_{ij}^2 + I_{ij}^2 + F_{ij}^2 \leq 1$$

This allows geometric interpretation and control over feasibility.

Q-NSCSEA Evaluation Space

We define an evaluation algebra over student projects using binary operations.

Let:

$D = \{d_1, d_2, \dots, d_n\}$: the design space

*: a binary composition operation on D

Define the composition:

$$d_a * d_b = d_c \text{ where:}$$

$$T_c = \frac{T_a + T_b}{2}, I_c = \frac{I_a + I_b}{2}, F_c = \frac{F_a + F_b}{2}$$

This operation satisfies:

Closure: The result is still a neutrosophic spherical cubic value.

Quasigroup Property: For every pair d_a, d_b, there exists d_c such that $d_a * d_b = d_c$.

Reverse solutions exist due to the symmetric nature of the operation.

This structure provides a logical way to "combine" two student designs under shared parameters and evaluate the resulting fusion or similarity.

3.2 Neutrosophic Soft Operations

We define:

(i) Neutrosophic Union:

$$(x, p, T_1, I_1, F_1) \cup (x, p, T_2, I_2, F_2) = (x, p, \max(T_1, T_2), \min(I_1, I_2), \min(F_1, F_2))$$

(ii) Neutrosophic Intersection:

$$(x, p, T_1, I_1, F_1) \cap (x, p, T_2, I_2, F_2) = (x, p, \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2))$$

These operations are used to compare multiple evaluations (e.g., from two reviewers or two production scenarios) and produce consistent outputs.

Educational Evaluation Rule

We define a simple evaluation function $E(x_i)$ for project x_i :

$$E(x_i) = \frac{1}{m} \sum_{j=1}^{m} [\alpha T_{ij} - \beta I_{ij} - \gamma F_{ij}]$$

Where $\alpha, \beta, \gamma \in [0,1]$: are evaluator-defined weights (e.g., aesthetics are more important than manufacturability).
m : total number of parameters.
The result is used to rank and score student projects. Design of Evaluation presented in Table 1 where each student is evaluated with respect to each parameter. These values will be used later in the Examples and Analysis section.

Table 1 Design of Evaluation

Project x_i	Parameter p_j	T_{ij}	I_{ij}	F_{ij}
Design A	Aesthetic	0.9	0.1	0.0
Design A	Layout	0.7	0.2	0.1
Design A	IoT Compatibility	0.4	0.5	0.3
Design A	Manufacturability	0.5	0.4	0.4

4. Proposed Model and Mathematical Framework

This section presents the full mathematical model behind the Q-NSCSEA. It builds on the methodology to define all essential mathematical structures, operations, and properties. The goal is to evaluate and rank student residential space design projects in an IoT-enabled smart factory environment using rigorous neutrosophic algebra.

Definition: Neutrosophic Spherical-Cubic Soft Element

Let $x_i \in X$ be a student design project and $p_j \in P$ be an evaluation parameter.
A Q-Neutrosophic Spherical-Cubic Soft Element is a 5-tuple:

$$\theta_{ij} = \left(x_i, p_j, T_{ij}, I_{ij}, F_{ij}\right)$$

Where:
$T_{ij}, I_{ij}, F_{ij} \in [0,1]$
$T_{ij}^2 + I_{ij}^2 + F_{ij}^2 \leq 1$ (Spherical constraint)

Neutrosophic Evaluation Set (Q-NSCSES)

$$Q - NSCSES = \left\{\theta_{ij} \mid x_i \in X, p_j \in P\right\}$$

This set holds all evaluations across all students and parameters.

Operations on Q-NSCSES

A. Soft Neutrosophic Union

$$\theta_{ij} \cup \phi_{ij} = \left(x_i, p_j, \max\left(T_{ij}, T'_{ij}\right), \min\left(I_{ij}, I'_{ij}\right), \min\left(F_{ij}, F'_{ij}\right)\right)$$

B. Soft Neutrosophic Intersection

$$\theta_{ij} \cap \phi_{ij} = \left(x_i, p_j, \min\left(T_{ij}, T'_{ij}\right), \max\left(I_{ij}, I'_{ij}\right), \max\left(F_{ij}, F'_{ij}\right)\right)$$

C. Neutrosophic Complement

$$\theta_{ij}^c = \left(x_i, p_j, F_{ij}, I_{ij}, T_{ij}\right)$$

Quasigroup Composition of Designs

Let $d_1, d_2 \in D$ be two project evaluations:

$$d_1 = (T_1, I_1, F_1), d_2 = (T_2, I_2, F_2)$$

We define:

$$d_1 \circ d_2 = d_3 = \left(\frac{T_1 + T_2}{2}, \frac{I_1 + I_2}{2}, \frac{F_1 + F_2}{2}\right)$$

Proof of Closure:

Since all values are in [0,1], and since the mean of two values in [0,1] stays in [0,1], we have:

$$T_3^2 + I_3^2 + F_3^2 \leq \frac{T_1^2 + T_2^2 + I_1^2 + I_2^2 + F_1^2 + F_2^2}{2} \leq 1$$

Thus, $d_1 \circ d_2 \in D$, so the set is closed under $\circ$.

Neutrosophic Scoring Function

Let a project $x_i \in X$ have evaluations for m parameters:

$$\theta_i = \{(p_j, T_{ij}, I_{ij}, F_{ij}) \mid j = 1,2, \dots, m\}$$

We define the evaluation score:

$$E(x_i) = \frac{1}{m} \sum_{j=1}^{m} [\alpha T_{ij} - \beta I_{ij} - \gamma F_{ij}]$$

Where:

$\alpha, \beta, \gamma \in [0,1]$ are weight parameters.

$\alpha + \beta + \gamma = 1$

Theorem 1: Boundedness of Evaluation

For all projects $x_i \in X$, we have:

$$-\beta - \gamma \leq E(x_i) \leq \alpha$$

Proof:

Maximum: when $T_{ij} = 1, I_{ij} = F_{ij} = 0 \Rightarrow E(x_i) = \alpha$

Minimum: when $T_{ij} = 0, I_{ij} = 1, F_{ij} = 1 \Rightarrow E(x_i) = -(\beta + \gamma)$

Q.E.D.

Theorem 2: Neutral Design Property

If $T_{ij} = I_{ij} = F_{ij}$ for all j, then:

$$E(x_i) = T_{ij}(\alpha - \beta - \gamma)$$

If all components are equal, the score depends only on weight balance. If $\alpha > \beta + \gamma$, the design scores positively.

Distance Between Projects

Define:

$$\delta(d_1, d_2) = \sqrt{(T_1 - T_2)^2 + (I_1 - I_2)^2 + (F_1 - F_2)^2}$$

This distance function helps measure similarity between two student projects under neutrosophic evaluation.

Properties:

$\delta(d, d) = 0$

$\delta(d_1, d_2) = \delta(d_2, d_1)$

$\delta(d_1, d_3) \leq \delta(d_1, d_2) + \delta(d_2, d_3)$

Thus, it satisfies the conditions of a metric on design space.Example for Scoring illustrated in Table 2.

Table 2.Example for Scoring

Project	Parameter	T_{ij}	I_{ij}	F_{ij}
A	Aesthetic	0.8	0.1	0.1
A	Functional Layout	0.7	0.2	0.1
A	loT Material Fit	0.5	0.3	0.2
A	Manufacturability	0.4	0.4	0.3

Let $\alpha = 0.5, \beta = 0.3, \gamma = 0.2$
Then $E(A) = \frac{1}{4}\sum\ [0.5T - 0.3I - 0.2F]$
Calculating row by row:
Row 1: $0.5(0.8) - 0.3(0.1) - 0.2(0.1) = 0.4 - 0.03 - 0.02 = 0.35$
Row 2:0.5(0.7)-0.3(0.2)-0.2(0.1) $= 0.35 - 0.06 - 0.02 = 0.27$
Row 3: $0.5(0.5) - 0.3(0.3) - 0.2(0.2) = 0.25 - 0.09 - 0.04 = 0.12$
Row 4: $0.5(0.4) - 0.3(0.4) - 0.2(0.3) = 0.2 - 0.12 - 0.06 = 0.02$

$$E(A) = \frac{0.35 + 0.27 + 0.12 + 0.02}{4} = \frac{0.76}{4} = 0.19$$

5. Results & Analysis

This section presents and interprets the outcomes of the Q-NSCSEA applied to student design projects. The results demonstrate how the proposed model evaluates the quality and feasibility of residential space art designs in smart factory contexts.

We consider a hypothetical cohort of three student projects evaluated across four design parameters, as described in the table below. These parameters are chosen to reflect typical evaluation aspects in a smart, IoT-driven manufacturing environment.

<u>Parameters:</u>

p_1 : Aesthetic quality
p_2 : Functional layout
p_3 : IoT material compatibility
p_4 : Manufacturability in smart factory

<u>Weight values:</u>

$\alpha = 0.5$: truth weight
$\beta = 0.3$: indeterminacy weight
$\gamma = 0.2$: falsity weight

Table 3. Calculating Evaluation Scores

Project	Parameter	T_{ij}	I_{ij}	F_{ij}
A	Aesthetic	0.8	0.1	0.1
A	Layout	0.7	0.2	0.1
A	loT Compatibility	0.5	0.3	0.2
A	Manufacturability	0.4	0.4	0.3
B	Aesthetic	0.6	0.2	0.2

B	Layout	0.5	0.3	0.3
B	IoT Compatibility	0.4	0.4	0.3
B	Manufacturability	0.3	0.3	0.4
C	Aesthetic	0.9	0.1	0.0
c	Layout	0.8	0.1	0.1
C	loT Compatibility	0.7	0.2	0.2
C	Manufacturability	0.6	0.2	0.2

We apply the evaluation function:

$$E(x_i) = \frac{1}{4}\sum_{j=1}^{4}\left(\alpha T_{ij} - \beta I_{ij} - \gamma F_{ij}\right)$$

For Project A:

Row 1: $0.5(0.8) - 0.3(0.1) - 0.2(0.1) = 0.35$

Row 2: $0.5(0.7) - 0.3(0.2) - 0.2(0.1) = 0.27$

Row 3: $0.5(0.5) - 0.3(0.3) - 0.2(0.2) = 0.12$

Row 4: $0.5(0.4) - 0.3(0.4) - 0.2(0.3) = 0.02$

$$E(A) = \frac{0.35 + 0.27 + 0.12 + 0.02}{4} = \frac{0.76}{4} = 0.19$$

For Project B:

Row 1: $0.5(0.6) - 0.3(0.2) - 0.2(0.2) = 0.22$

Row 2: $0.5(0.5) - 0.3(0.3) - 0.2(0.3) = 0.14$

Row 3: $0.5(0.4) - 0.3(0.4) - 0.2(0.3) = 0.08$

Row 4: $0.5(0.3) - 0.3(0.3) - 0.2(0.4) = 0.01$

$$E(B) = \frac{0.22 + 0.14 + 0.08 + 0.01}{4} = \frac{0.45}{4} = 0.1125$$

For Project C:

Row 1: $0.5(0.9) - 0.3(0.1) - 0.2(0.0) = 0.42$

Row 2: $0.5(0.8) - 0.3(0.1) - 0.2(0.1) = 0.37$

Row 3: $0.5(0.7) - 0.3(0.2) - 0.2(0.2) = 0.29$

Row 4: $0.5(0.6) - 0.3(0.2) - 0.2(0.2) = 0.22$

$$E(C) = \frac{0.42 + 0.37 + 0.29 + 0.22}{4} = \frac{1.30}{4} = 0.325$$

Table 5. Results Summary

Project	Final Score $E(x_i)$
A	0.190
B	0.113
C	0.325

5.2 Evaluation and Robustness of the Model

Among the evaluated projects, Project C received the highest score, indicating a strong overall balance between artistic quality and compatibility with smart production systems. Its low levels of uncertainty and error suggest that it meets both the creative expectations of the course and the technical requirements of a smart factory environment. This makes it a strong candidate for real-world application.

Project A showed moderate results. Its design satisfies visual and structural expectations to a fair degree but lacks clarity in manufacturability. The presence of increased uncertainty and slight technical limitations lowered its score. While promising in concept, it may need refinement to better align with industrial standards.

Project B performed the weakest across the evaluation. Its score reflects a combination of low design impact and poor compatibility with the practical aspects of production. The higher levels of indeterminacy and failure indicate a project that might struggle in both artistic and industrial criteria.

The strength of the proposed model lies in its structured and customizable approach. By assigning weights to truth, indeterminacy, and falsity, the system allows instructors to emphasize certain evaluation priorities based on course goals. The model also accepts partial satisfaction of evaluation parameters, enabling more flexible and fair judgment in cases where design ideas are innovative but incomplete. Most importantly, the mathematical foundation of the model ensures that all projects can be assessed using a consistent and logically sound process.

6. Discussion

The proposed Q-Neutrosophic Spherical-Cubic Soft Evaluation Algebra offers a structured and logical method for assessing student design work in educational settings that are influenced by modern industrial technologies. This model allows educators to evaluate not only the creative aspects of a project but also its potential performance in environments supported by IoT, automation, and smart manufacturing.

One key advantage of the model is its ability to represent multiple dimensions of evaluation using independent values for truth, indeterminacy, and falsity. This separation allows for more flexible analysis. For example, a project can be highly creative (high truth) but still show uncertainty or risk in manufacturability (indeterminacy or falsity), which is often the case in student work. Traditional grading methods do not reflect this kind of nuance.

Incorporating this model into design education introduces a more objective and structured approach to decision-making. Since the evaluation is based on clearly defined mathematical functions and weights, educators can explain each score and justify their assessments more transparently. This improves student understanding and trust in the evaluation process.

Moreover, the model supports adaptability. It can be applied to a range of course objectives by adjusting the weights given to each component. A course that emphasizes industrial feasibility can increase the weight of falsity, while one focused on aesthetics can emphasize truth. This flexibility makes the model suitable for various teaching goals and project types.

Another important contribution is the way the model handles ambiguity in design performance. Many student projects include uncertain or incomplete ideas that are hard to evaluate using fixed rubrics. With the inclusion of an indeterminacy measure, the model captures this uncertainty and treats it as a valid part of the evaluation, rather than ignoring or penalizing it blindly.

Overall, this neutrosophic-based system brings academic evaluation closer to real-world conditions. It helps students learn how to balance creativity with constraints and prepares them for industries where designs must be both innovative and technically executable. It also offers a scalable framework that can grow with the integration of new technologies and design tools in education.

7. Conclusion

This research introduced a novel mathematical model called the Q-NSCSEA to evaluate student projects in residential space art design courses within IoT-enabled smart factory environments. The proposed model combines elements from neutrosophic logic, spherical sets, cubic soft sets, and quasigroup algebra to create a unified evaluation system that reflects both aesthetic quality and technical feasibility.

By using the triple components for each design parameter, the model enables educators to analyze student designs from multiple perspectives. The scoring function allows for customizable weights, making it adaptable to different teaching priorities.

Through full mathematical definitions, rigorous proofs, and detailed numerical examples, the paper demonstrates how this system can be used to:

— Compare and rank student projects
— Identify strengths and weaknesses
— Align creative ideas with the requirements of smart manufacturing systems

Importantly, the model supports fairness, transparency, and real-world relevance in design education—helping students prepare for environments where creative ideas must be grounded in technological constraints.

This work lays a foundation for future development of neutrosophic-based educational evaluation systems in other fields where uncertainty, aesthetics, and production must coexist.

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