



Decision on Insurance Policy Selection by weighted Correlation Approach on Neutrosophic Fuzziness through TOPSIS

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Abstract. Risk management recognizes and controls the hazards connected with our decisions and activities, whether they involve our assets or health. The fundamental goal of insurance is to mitigate the risks an insured person experiences. Term insurance is a kind of life insurance that is polarized among insurance policy buyers. The quantitative and qualitative attributes of term insurance are often articulated using language terminology. Consequently, the selection of a term insurance policy can be characterized as an uncertain multi-criteria decision-making (MCDM) problem. In this work, we substituted the distance measure in the technique for order preference by similarity to an ideal solution (TOPSIS) with a neutrosophic weighted correlation coefficient to address the ambiguity involved in the selection of distance measure in the TOPSIS approach. This work also offers weighted closeness measures and an index coefficient required in the neutrosophic TOPSIS approach. The suggested neutrosophic TOPSIS is used to identify the best term insurance policy for its clients. The outcome of the neutrosophic TOPSIS suggests that the tenth insurance firm exhibits the highest level of acceptance, whereas the first firm represents the least favorable option. The consistency and robustness of the proposed approach are established through comparison and sensitivity analysis.

Keywords: Single-valued Neutrosophic set; MCDM; Correlation measure; TOPSIS; Term insurance policy.

1. Introduction

Risk management is the rational creation and implementation of a loss mitigation strategy [1]. Risk management entails detecting possible risks in advance, analyzing them, and taking preventative measures to mitigate them. If we divide risk into pure and speculative categories, the pure risk is insurable while the speculative risk is not. Insurance is an essential component of risk management, but it is not the sole method. Protecting human life is the primary objective of loss prevention. Modern insurance may be divided into two categories: life and non-life. Life insurance is insurance in which the insurer agrees to pay a certain amount upon the insured's death or at the expiration of a specified time, with the insured paying a premium. An individual can take life insurance in two ways: term insurance with only death benefits and endowment insurance with a built-in savings component. Term insurance is a form of life insurance that offers limited financial protection to the policyholder [2]. The firm pays the beneficiary the death benefit if the insured person passes away within the period of the policy. Term insurance policy (TIP) is gaining popularity due to its unique characteristics, including cheap premium, full life coverage, payout of amount assured, critical illness coverage, accidental death benefits, terminal disease coverage, and tax advantage. Term insurance spreads the insured's risk and provides peace of mind by removing uncertainty and any anxieties associated with the risk. The problem with picking insurance is that linguistic terms describe the alternatives' criteria that a buyer cannot classify. In this process, ambiguity is introduced into the decision-making procedure. To reflect the ambiguity, we describe these linguistic terms in our study as a single-valued neutrosophic set (SVNS). In this article, the problem of selecting a TIP option based on its several uncertain criteria is constructed as an MCDM problem.

Human decision-making is a complex mental process that requires evaluating all relevant aspects to reach a goal. MCDM involves several alternatives and criteria. To solve the MCDM problem, one must analyze each alternative's criteria and select the best one. Due to the rapid expansion of MCDM, researchers have created different MCDM methodologies to address various realistic issues. The TOPSIS approach introduced by Hwang and Yoon [3] and extended by Hwang et al. [4] is a well-known technique to solve MCDM issues. There are several reasons to solve an MCDM problem by TOPSIS, such as sound logic, greater flexibility, a more straightforward calculation procedure, and a distance measure simultaneously for the best and worst possible outcomes. These advantages make TOPSIS more effective and practical than other existing approaches. Several qualitative and quantitative factors often discuss the criteria of an alternative to the MCDM problem. Linguistic terms are used to describe the criteria by human judgment. Collecting accurate assessment data can take much work because human judgments are subjective and frequently ambiguous or imprecise. Data and information on the

criteria of an option are frequently combined with hesitancy, indeterminacy, and uncertainty in the MCDM problem. In particular, MCDM techniques are vulnerable to the subjectivity of the experts when they use linguistic ideas for assessment. The extensions of the conventional fuzzy set (FS) such as intuitionistic fuzzy set (IFS) [5], pythagorean fuzzy set [6–8], fermatean fuzzy set [9, 10], neutrosophic set (NS) [11], hesitant fuzzy set [12], type II fuzzy set [13, 14], have been developed to address this subjectivity and ambiguity in the assessment process. FS has a membership function that represents the degree of acceptance. The IFS has two membership functions, acceptance and rejection, but they are not independent. An SVN is a generalization of interval, fuzzy, and IFSs. It has three independent membership functions, all of which lie in $[0, 1]$ and represent truth, indeterminacy, and falsity. As a result, SVN is a better choice for representing the language word when evaluating alternative criteria in MCDM.

In this study, we use SVN to determine the uncertainty associated with the linguistic term and describe the criteria for an alternative. Experts in the respective fields define the weights of the criteria. This study introduces a weighted correlation measure based on SVN to replace the distance measure in the TOPSIS approach. We establish three weighted correlation coefficient components for each membership function since the SVN offers three distinct membership functions: truth, hesitation, and falseness. The weighted correlation coefficient between two SVNs is derived by averaging the three components and assigning equal weight to each. The weighted closeness measures of types I and II are presented based on the suggested correlation measure. A closeness index parameter is given to rank the alternatives of an MCDM.

2. Literature Review

This section explores the neutrosophic MCDM approach's application across various significant sectors and identifies potential research gaps in the neutrosophic TOPSIS approach.

2.1. Neutrosophic MCDM approach

The neutrosophic logic [15] is a generalization of the two membership fuzzy sets. It is capable of managing inaccurate information through three values (truth, indeterminacy, and falsity). The significance of neutrosophic MCDM approaches is exhibited in the representation of hesitancy in decision-making. Jana et al. [16] applied SVN for constructing several Hamacher operators. Suresh et al. [17] used Euclidean measure to rank neutrosophic trapezoidal fuzzy numbers. Luo et al. [18] established a distance measure on SVN for pattern recognition. Saber et al. [19] developed the topological concept on a single-valued neutrosophic soft set.

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Hashmi et al. [20] introduced m-polar NS, a topological concept for medical diagnosis and clustering analysis. Martinez et al. [21] presented a distance-based similarity measure based on NS to evaluate IoT problems in the supply chain. Singh et al. [22] proposed an interval-valued neutrosophic fuzzy approach through principal component analysis.

2.2. Neutrosophic TOPSIS

Several articles include in-depth simulation-based comparisons and mathematical analysis of TOPSIS to clarify the ambiguity over which one should be used to solve MCDM problems [23]. TOPSIS is a popular and effective way to handle MCDM issues in an uncertain environment. The conventional TOPSIS method considers only distance measures, not similarity or probability. Biswas et al. [24] solved TOPSIS by using the SVN Euclidean distance measure. Abdel et al. [25] employed the neutrosophic TOPSIS technique for selecting medical devices. Gul et al. [26] utilized an interval-valued spherical fuzzy set for analyzing marble manufacturing facilities. Nafei et al. [27] defined a score function on fuzzy NSs to solve hotel site selection problems by the TOPSIS approach. Pouresmaeil et al. [28] developed a score function for solving interval neutrosophic MCDM problems by the TOPSIS approach. Ridvan et al. [29] developed a TOPSIS that optimizes the distance, similarity, and magnitude closeness coefficients in a neutrosophic environment. Mollaoglu et al. [30] identified alternate fuel sources for ship investment choices using the SVNS-based TOPSIS approach. Based on the Dice and Jaccard vector measures, Ozlu et al. [31] created some similarity measures in TOPSIS under SVNS type-2 information. Some recent applications of the neutrosophic TOPSIS approach with its application are summarized in table 1.

TABLE 1. Recent applications of neutrosophic TOPSIS approach

Contributor	Decision making	Alt.	Crt.	Application
Chen et al. [32]	MADM	4	4	Green supplier selection
Garg et al. [33]	MCDM	5	4	Software company selection
Zulqarnain et al. [34]	MCDM	5	5	Supplier selection
Ridvan [29]	MCDM	5	4	Mask selection in COVID-19
Aydin et al. [35]	MCDM	6	6	Logistic selection
Pouresmaeil et al. [28]	MCDM	5	4	Power station location
Lin et al. [36]	MCDM	13	4	Risk assessment
Li et al. [37]	MCDM	5	5	Doctor selection
Mahapatra et al. [38]	MCDM	12	12	Insurance provider
Biswas et al. [39]	MAGDM	4	6	Tablet selection
Canizares et al. [40]	MCGDM	5	4	Software project selection
Lan et al. [41]	MCGDM	5	20	Tourist destination
Nafe et al. [42]	MCGDM	8	3	Green supplier selection

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Implementing various neutrosophic TOPSIS techniques in the MCDM and MCGDM (Multi-criteria group decision-making) problems is convenient, as seen in table 1. However, there are few papers on insurance policy selection treated as an MCDM problem and solved by fuzzy TOPSIS: for example, Sehhat et al. [43] used crisp TOPSIS approach, Sekar et al. [44] and Chu et al. [45] used fuzzy TOPSIS. The neutrosophy philosophy [11] is advantageous for addressing ambiguous, conflicting, reticent, and incomplete data. Consequently, it is more logical to use neutrosophic measures to evaluate the qualitative criteria of the numerous alternatives. Neutrosophic TOPSIS has a wide range of applications, although TIP selection is not mentioned as an MCDM problem in any current literature.

2.3. Fuzzy correlation coefficient

A correlation between two things is the connection between them. Karl Pearson first introduced the correlation coefficient to deal with crisp numbers. There are several extensions of the correlation coefficient in the fuzzy field, such as FS [46], IFS [47–49], PFS [50], SVNS [51,52], Interval-valued fuzzy set [53], hesitant fuzzy sets [54–57], picture fuzzy sets [58]. All the researchers developed fuzzy correlation coefficients in $[0, 1]$ except Zeng et al. [52]; hence we cannot capture the negative correlation between two SVNSSs. There are two objectives of the proposed methodology: (i) the replacement of the ambiguity of distance measures in TOPSIS methodology and (ii) the development of a correlation measure that is within the range of $[0, 1]$. This study introduces a TOPSIS approach that is based on weighted neutrosophic correlation in order to mitigate the deficiencies of the existing correlation coefficients.

3. Neutrosophic TOPSIS Approach to solve MCDM Problem

In this segment, we first discuss the fundamental idea of SVNS before moving on to the neutrosophic TOPSIS strategy. In real-life scenarios, most parameters are imprecise, which means inexact, invalid, or inaccurate. The SVNS can explain the given parameters' truth, hesitation, and falsity to overcome this impreciseness.

Definition 3.1. Single valued neutrosophic set [59]: Suppose X is the set of the universe of discourse. An SVNS (\tilde{S}) of a single-valued independent variable (x) is defined by $\tilde{S} = \{ \langle x; [\pi_{\tilde{S}}(x), \theta_{\tilde{S}}(x), \eta_{\tilde{S}}(x)] \rangle : x \in X \}$, where $\pi_{\tilde{S}}(x)$, $\theta_{\tilde{S}}(x)$, $\eta_{\tilde{S}}(x)$ represents the concept of truth, hesitation, and falsity membership functions, respectively. Here $\pi_{\tilde{S}} : \mathbf{R} \rightarrow [0, 1]$ is the truth membership function, $\theta_{\tilde{S}} : \mathbf{R} \rightarrow [0, 1]$ is the hesitation membership function, and the falsity membership function is $\eta_{\tilde{S}} : \mathbf{R} \rightarrow [0, 1]$ with $0 \leq \pi_{\tilde{S}} + \theta_{\tilde{S}} + \eta_{\tilde{S}} \leq 3$. For convenience, we will express an SVNS as $\langle \pi_{\tilde{S}}, \theta_{\tilde{S}}, \eta_{\tilde{S}} \rangle$ which is called a single-valued neutrosophic element (SVNE).

A diagram of the SVNNS is given in Figure 1.

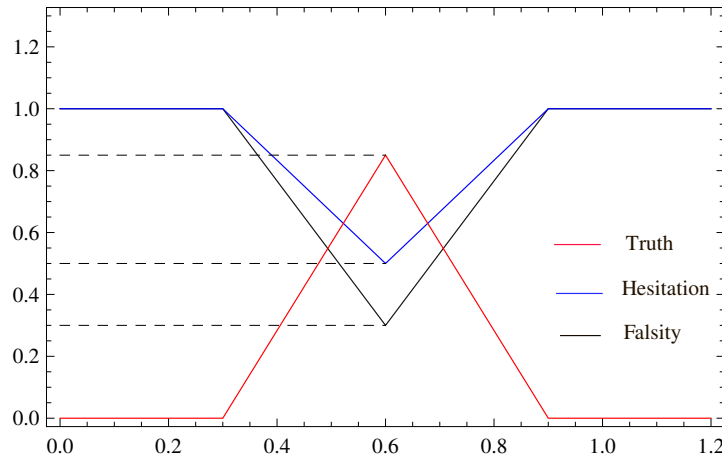


FIGURE 1. A diagram of a single-valued neutrosophic set

Definition 3.2. Arithmetic operations of SVNNS [60]: Suppose $\tilde{S} = \langle \pi_{\tilde{S}}, \theta_{\tilde{S}}, \eta_{\tilde{S}} \rangle$ and $\tilde{T} = \langle \pi_{\tilde{T}}, \theta_{\tilde{T}}, \eta_{\tilde{T}} \rangle$ be two SVNNS and $h \geq 0$ be real constant. The algebraic operations on SVNNS are described as follows:

$$\begin{aligned}
 &: \tilde{S} \oplus \tilde{T} = \langle \pi_{\tilde{S}} + \pi_{\tilde{T}} - \pi_{\tilde{S}}\pi_{\tilde{T}}, \theta_{\tilde{S}}\theta_{\tilde{T}}, \eta_{\tilde{S}}\eta_{\tilde{T}} \rangle \\
 &: \tilde{S} \otimes \tilde{T} = \langle \pi_{\tilde{S}}\pi_{\tilde{T}}, \theta_{\tilde{S}} + \theta_{\tilde{T}} - \theta_{\tilde{S}}\theta_{\tilde{T}}, \eta_{\tilde{S}} + \eta_{\tilde{T}} - \eta_{\tilde{S}}\eta_{\tilde{T}} \rangle \\
 &: h\tilde{S} = \langle 1 - (1 - \pi_{\tilde{S}})^h, \theta_{\tilde{S}}^h, \eta_{\tilde{S}}^h \rangle \\
 &: \tilde{S}^h = \langle \pi_{\tilde{S}}^h, 1 - (1 - \theta_{\tilde{S}})^h, 1 - (1 - \eta_{\tilde{S}})^h \rangle
 \end{aligned}$$

3.1. Neutrosophic TOPSIS Approach

The current section illustrates the neutrosophic TOPSIS methodology for solving the MCDM problem with uncertainty. Firstly, the expert gives an SVNNS rating for an alternative criterion. Then, a neutrosophic decision matrix is constructed using the SVNNS rating. The decision matrix defines the positive ideal solution (PIS) and negative ideal solution (NIS), type I and II correlation measures, the Index function, and their properties.

Let us consider m distinct alternatives defined as $\Lambda = \{\Lambda_1, \Lambda_2, \dots, \Lambda_m\}$ and n criteria $\Pi = \{\Pi_1, \Pi_2, \dots, \Pi_n\}$. Let $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$ be the respective weights for the n criteria $\Pi = \{\Pi_1, \Pi_2, \dots, \Pi_n\}$ where $\varsigma_j \in [0, 1] \forall j = 1, 2, \dots, n$. Let Π_B and Π_C represent the collection of benefit and cost criteria, respectively. Where $\Pi = \Pi_B \cup \Pi_C$ and $\Pi_B \cap \Pi_C = \emptyset$.

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Each entry in the decision matrix $\mathbb{M} = (\mathbb{M}_{ij})_{m \times n}$ are SVNSe as shown below:

$$\mathbb{M} = \begin{matrix} & \Pi_1 & \Pi_2 & \Pi_3 & \dots & \Pi_n \\ \begin{matrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_m \end{matrix} & \left(\begin{matrix} (\xi_{11}, \zeta_{11}, \kappa_{11}) & (\xi_{12}, \zeta_{12}, \kappa_{12}) & (\xi_{13}, \zeta_{13}, \kappa_{13}) & \dots & (\xi_{1n}, \zeta_{1n}, \kappa_{1n}) \\ (\xi_{21}, \zeta_{21}, \kappa_{21}) & (\xi_{22}, \zeta_{22}, \kappa_{22}) & (\xi_{23}, \zeta_{23}, \kappa_{23}) & \dots & (\xi_{2n}, \zeta_{2n}, \kappa_{2n}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (\xi_{m1}, \zeta_{m1}, \kappa_{m1}) & (\xi_{m2}, \zeta_{m2}, \kappa_{m2}) & (\xi_{m3}, \zeta_{m3}, \kappa_{m3}) & \dots & (\xi_{mn}, \zeta_{mn}, \kappa_{mn}) \end{matrix} \right) \end{matrix} \quad (1)$$

The element $\mathbb{M}_{ij} = (\xi_{ij}, \zeta_{ij}, \kappa_{ij})$ denote the ij^{th} rating of SVNSe (1) by the expert. The characteristic m_i corresponding to the alternative Λ_i in the i^{th} row is represented as

$$\begin{aligned} m_i &= \{(c_1, f_{i1}), (c_2, f_{i2}), \dots, (c_n, f_{in})\} \\ &= \{(c_1, \xi_{i1}, \zeta_{i1}, \kappa_{i1}), (c_2, \xi_{i2}, \zeta_{i2}, \kappa_{i2}), \dots, (c_n, \xi_{in}, \zeta_{in}, \kappa_{in})\}. \end{aligned}$$

Definition 3.3. Neutrosophic PIS and NIS [34]: Let \mathbb{M}_+ and \mathbb{M}_- denote the neutrosophic PIS and NIS represented as $\mathbb{M}_+ = \{(c_1, \mathbb{M}_{+1}), (c_2, \mathbb{M}_{+2}), \dots, (c_n, \mathbb{M}_{+n})\}$ and $\mathbb{M}_- = \{(c_1, \mathbb{M}_{-1}), (c_2, \mathbb{M}_{-2}), \dots, (c_n, \mathbb{M}_{-n})\}$, where \mathbb{M}_{+j} and \mathbb{M}_{-j} are defined as $\mathbb{M}_{+j} = (\xi_{+j}, \zeta_{+j}, \kappa_{+j})$ where

$$(\xi_{+j}, \zeta_{+j}, \kappa_{+j}) = \begin{cases} \left(\max_{i=1}^m \xi_{ij}, \min_{i=1}^m \zeta_{ij}, \min_{i=1}^m \kappa_{ij} \right) & \text{if } c_j \in \Pi_B \\ \left(\min_{i=1}^m \xi_{ij}, \max_{i=1}^m \zeta_{ij}, \max_{i=1}^m \kappa_{ij} \right) & \text{if } c_j \in \Pi_C \end{cases} \quad (2)$$

And $\mathbb{M}_{-j} = (\xi_{-j}, \zeta_{-j}, \kappa_{-j})$ where

$$(\xi_{-j}, \zeta_{-j}, \kappa_{-j}) = \begin{cases} \left(\min_{i=1}^m \xi_{ij}, \max_{i=1}^m \zeta_{ij}, \max_{i=1}^m \kappa_{ij} \right) & \text{if } c_j \in \Pi_B \\ \left(\max_{i=1}^m \xi_{ij}, \min_{i=1}^m \zeta_{ij}, \min_{i=1}^m \kappa_{ij} \right) & \text{if } c_j \in \Pi_C \end{cases} \quad (3)$$

4. Single-Valued weighted Neutrosophic Correlation Coefficient

To solve the MCDM problem using the TOPSIS method, we need to calculate the distance between PIS and NIS. We intend a novel neutrosophic correlation coefficient to replace the distance measure in TOPSIS. Some essential properties of the neutrosophic correlation coefficient are introduced to establish it. The neutrosophic correlation coefficient between two neutrosophic characteristics is defined as follows:

Definition 4.1. Neutrosophic correlation coefficient [38]: Let \mathbb{M}_s and \mathbb{M}_t represent PIS and NIS of an alternative in the neutrosophic decision matrix \mathbb{M} respectively, where

$$\begin{aligned} \mathbb{M}_s &= \{(c_1, \xi_{s1}, \zeta_{s1}, \kappa_{s1}), (c_2, \xi_{s2}, \zeta_{s2}, \kappa_{s2}), \dots, (c_n, \xi_{sn}, \zeta_{sn}, \kappa_{sn})\} \\ \mathbb{M}_t &= \{(c_1, \xi_{t1}, \zeta_{t1}, \kappa_{t1}), (c_2, \xi_{t2}, \zeta_{t2}, \kappa_{t2}), \dots, (c_n, \xi_{tn}, \zeta_{tn}, \kappa_{tn})\} \end{aligned}$$

Also consider $\bar{\xi}_j = \sum_{i=1}^m \xi_{ij}/m$, $\bar{\zeta}_j = \sum_{i=1}^m \zeta_{ij}/m$, and $\bar{\kappa}_j = \sum_{i=1}^m \kappa_{ij}/m$, then the neutrosophic correlation coefficient is

$$\vartheta(\mathbb{M}_s, \mathbb{M}_t) = \frac{1}{3} [\rho_{\xi}(\mathbb{M}_s, \mathbb{M}_t) + \rho_{\zeta}(\mathbb{M}_s, \mathbb{M}_t) + \rho_{\kappa}(\mathbb{M}_s, \mathbb{M}_t)]$$

where

$$\begin{aligned} \rho_{\xi}(\mathbb{M}_s, \mathbb{M}_t) &= \frac{\sum_{j=1}^n [\xi_{sj}^2 - \bar{\xi}_j^2] [\xi_{tj}^2 - \bar{\xi}_j^2]}{\sqrt{\sum_{j=1}^n [\xi_{sj}^2 - \bar{\xi}_j^2]^2} \sqrt{\sum_{j=1}^n [\xi_{tj}^2 - \bar{\xi}_j^2]^2}} \\ \rho_{\zeta}(\mathbb{M}_s, \mathbb{M}_t) &= \frac{\sum_{j=1}^n [\zeta_{sj}^2 - \bar{\zeta}_j^2] [\zeta_{tj}^2 - \bar{\zeta}_j^2]}{\sqrt{\sum_{j=1}^n [\zeta_{sj}^2 - \bar{\zeta}_j^2]^2} \sqrt{\sum_{j=1}^n [\zeta_{tj}^2 - \bar{\zeta}_j^2]^2}} \\ \rho_{\kappa}(\mathbb{M}_s, \mathbb{M}_t) &= \frac{\sum_{j=1}^n [\kappa_{sj}^2 - \bar{\kappa}_j^2] [\kappa_{tj}^2 - \bar{\kappa}_j^2]}{\sqrt{\sum_{j=1}^n [\kappa_{sj}^2 - \bar{\kappa}_j^2]^2} \sqrt{\sum_{j=1}^n [\kappa_{tj}^2 - \bar{\kappa}_j^2]^2}}. \end{aligned}$$

We assume that the denominators of $\rho_{\xi}(\mathbb{M}_s, \mathbb{M}_t)$, $\rho_{\zeta}(\mathbb{M}_s, \mathbb{M}_t)$, $\rho_{\kappa}(\mathbb{M}_s, \mathbb{M}_t)$ are not equal to zero.

The weighted neutrosophic correlation coefficient between two features is obtained by adding the weight vector, ς , to the correlation measure.

Definition 4.2. Weighted neutrosophic correlation coefficient [38]: Let $\mathbb{M}_s, \mathbb{M}_t$ be the two neutrosophic characteristics in the neutrosophic decision-making matrix \mathbb{M} and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\sum_{j=1}^n \varsigma_j = 1$ be the weight vector related with the criteria. Then, the weighted neutrosophic correlation coefficient between $\mathbb{M}_s, \mathbb{M}_t$ is defined as

$$\vartheta^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = \frac{1}{3} [\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) + \rho_{\zeta}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) + \rho_{\kappa}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t)] \quad (4)$$

where,

$$\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = \frac{\sum_{j=1}^n \varsigma_j [\xi_{sj}^2 - \bar{\xi}_j^2] \cdot [\xi_{tj}^2 - \bar{\xi}_j^2]}{\sqrt{\sum_{j=1}^n \varsigma_j [\xi_{sj}^2 - \bar{\xi}_j^2]^2} \cdot \sqrt{\sum_{j=1}^n \varsigma_j [\xi_{tj}^2 - \bar{\xi}_j^2]^2}} \quad (5)$$

$$\rho_{\zeta}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = \frac{\sum_{j=1}^n \varsigma_j [\zeta_{sj}^2 - \bar{\zeta}_j^2] \cdot [\zeta_{tj}^2 - \bar{\zeta}_j^2]}{\sqrt{\sum_{j=1}^n \varsigma_j [\zeta_{sj}^2 - \bar{\zeta}_j^2]^2} \cdot \sqrt{\sum_{j=1}^n \varsigma_j [\zeta_{tj}^2 - \bar{\zeta}_j^2]^2}} \quad (6)$$

$$\rho_{\kappa}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = \frac{\sum_{j=1}^n \varsigma_j [\kappa_{sj}^2 - \bar{\kappa}_j^2] \cdot [\kappa_{tj}^2 - \bar{\kappa}_j^2]}{\sqrt{\sum_{j=1}^n \varsigma_j [\kappa_{sj}^2 - \bar{\kappa}_j^2]^2} \cdot \sqrt{\sum_{j=1}^n \varsigma_j [\kappa_{tj}^2 - \bar{\kappa}_j^2]^2}}. \quad (7)$$

The denominator in the formulas above is expected to be non-zero, just like in the unweighted preceding instance.

Theorem 4.3. The membership component $\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t)$ in the weighted neutrosophic correlation coefficient $\vartheta^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t)$ for two neutrosophic characteristics \mathbb{M}_s and \mathbb{M}_t satisfy the following properties:

- (i) $\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = \rho_{\xi}^{\varsigma}(\mathbb{M}_t, \mathbb{M}_s)$
- (ii) $\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = 1$ if $\xi_{sj} = \xi_{tj} \forall c_j \in \Pi$
- (iii) $|\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t)| \leq 1$

Proof. The properties (i) and (ii) are trivial as

$$\rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) = \frac{\sum_{j=1}^n \varsigma_j \left[\xi_{sj}^2 - \bar{\xi}_j^2 \right]^2}{\left[\sqrt{\sum_{j=1}^n \varsigma_j \left[\xi_{sj}^2 - \bar{\xi}_j^2 \right]^2} \right]^2} = 1.$$

To prove of (iii) it is known that $-1 \leq \left((\xi_{sj})^2 - (\bar{\xi}_j)^2 \right) \cdot \left((\xi_{tj})^2 - (\bar{\xi}_j)^2 \right) \leq 1$.

Thus, $-1 \leq \sum_{j=1}^n \varsigma_j \left((\xi_{sj})^2 - (\bar{\xi}_j)^2 \right) \cdot \left((\xi_{tj})^2 - (\bar{\xi}_j)^2 \right) \leq 1$ since $\sum_{j=1}^n \varsigma_j = 1$.

Now let the denominator of (5) as, $0 \leq \sum_{j=1}^n \left((\xi_{sj})^2 - (\bar{\xi}_j)^2 \right)^2 \leq n$ and $0 \leq \sum_{j=1}^n \left((\xi_{tj})^2 - (\bar{\xi}_j)^2 \right)^2 \leq n$.

So it is obvious that $0 \leq \sum_{j=1}^n \varsigma_j \left((\xi_{sj})^2 - (\bar{\xi}_j)^2 \right)^2 \leq 1$ and $0 \leq \sum_{j=1}^n \varsigma_j \left((\xi_{tj})^2 - (\bar{\xi}_j)^2 \right)^2 \leq 1$ since $\sum_{j=1}^n \varsigma_j = 1$.

Hence, $\sqrt{\sum_{j=1}^n \varsigma_j \left((\xi_{sj})^2 - (\bar{\xi}_j)^2 \right)^2} \cdot \sqrt{\sum_{j=1}^n \varsigma_j \left((\xi_{tj})^2 - (\bar{\xi}_j)^2 \right)^2} \leq \sqrt{1} \cdot \sqrt{1} = 1$.

Similarly, it can be shown $-1 \leq \rho_{\xi}^{\varsigma}(\mathbb{M}_s, \mathbb{M}_t) \leq 1$, this establishes the theorem. \square

Definition 4.4. Weighted type I and type II closeness measures [61]: Let the neutrosophic PIS and NIS of the neutrosophic characteristic of an alternative Λ_i be \mathbb{M}_+ , \mathbb{M}_- . Also, let \mathbb{M} be any neutrosophic characteristic. Let ς_j be the weight of the criteria Π_i where $0 \leq \varsigma_j \leq 1$ and $\sum_{j=1}^n \varsigma_j = 1$. Let $\mathfrak{M}_I^{\varsigma}(\mathbb{M}_i)$ and $\mathfrak{M}_{II}^{\varsigma}(\mathbb{M}_i)$ denote the weighted closeness measure of type I and type II, then

$$\mathfrak{M}_I^{\varsigma}(\mathbb{M}_i) = \frac{1 - \mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_-)}{2 - \mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_+) - \mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_-)} \quad (8)$$

and

$$\mathfrak{M}_{II}^{\varsigma}(\mathbb{M}_i) = \frac{1 + \mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_+)}{2 + \mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_+) + \mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_-)} \quad (9)$$

It is assumed that the denominator of $\mathfrak{M}_I^{\varsigma}(\mathbb{M}_i)$ and $\mathfrak{M}_{II}^{\varsigma}(\mathbb{M}_i)$ are not zero.

Theorem 4.5. For each neutrosophic characteristic \mathbb{M}_i in the neutrosophic decision matrix \mathbb{M} , the weighted type I closeness measure $\mathfrak{M}_I^{\varsigma}(\mathbb{M}_i)$ follows the following criteria:

- (i) $0 \leq \mathfrak{M}_I^{\varsigma}(\mathbb{M}_i) \leq 1$
- (ii) $\mathfrak{M}_I^{\varsigma}(\mathbb{M}_i) = 1$ if $\mathfrak{C}^{\varsigma}(\mathbb{M}_i, \mathbb{M}_+) = 1$

- (iii) $\mathfrak{M}_I^\varsigma(\mathbb{M}_i) = 0$ if $\mathfrak{C}^\varsigma(\mathbb{M}_i, \mathbb{M}_-) = 1$
- (iv) $\mathfrak{M}_I^\varsigma(\mathbb{M}_-) = 0$ if $\mathfrak{M}_I^\varsigma(\mathbb{M}_+) = 1$
- (v) $\mathfrak{M}_I^\varsigma(\mathbb{M}_i) = \mathfrak{M}_I(\mathbb{M}_i)$ if $\varsigma = (1/n, 1/n, \dots, 1/n)$

Proof. From previous theorems we know that $-1 \leq \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_+) \leq 1$ and $-1 \leq \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_-) \leq 1$. So, $0 \leq 1 - \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_+) \leq 2$, $0 \leq 1 - \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_-) \leq 2$ and $0 \leq 2 - \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_+) - \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_-) \leq 4$. Therefore, $0 \leq \mathfrak{M}_I^\varsigma(\mathbb{M}_i) \leq 1$. Hence (i) is true.

The proofs of (ii), (iii), (iv), and (v) are obvious. \square

Theorem 4.6. *The weighted type II closeness measure $\mathfrak{M}_{II}^\varsigma(\mathbb{M}_i)$ meets the following conditions for every neutrosophic characteristic \mathbb{M}_i in the neutrosophic decision matrix \mathbb{M} .*

- (i) $0 \leq \mathfrak{M}_{II}^\varsigma(\mathbb{M}_i) \leq 1$
- (ii) $\mathfrak{M}_{II}^\varsigma(\mathbb{M}_i) = 0$ if $\mathfrak{C}^\varsigma(\mathbb{M}_i, \mathbb{M}_+) = -1$
- (iii) $\mathfrak{M}_{II}^\varsigma(\mathbb{M}_i) = 1$ if $\mathfrak{C}^\varsigma(\mathbb{M}_i, \mathbb{M}_-) = -1$
- (iv) $\mathfrak{M}_{II}^\varsigma(\mathbb{M}_-) \leq \mathfrak{M}_{II}^\varsigma(\mathbb{M}_+)$
- (v) $\mathfrak{M}_{II}^\varsigma(\mathbb{M}_i) = \mathfrak{M}_{II}(\mathbb{M}_i)$ if $\varsigma = (1/n, 1/n, \dots, 1/n)$

Proof. From previous theorems, we know that $-1 \leq \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_+) \leq 1$ and $-1 \leq \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_-) \leq 1$. So, $-2 \leq \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_+) + \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_-) \leq 2$ and $0 \leq 2 + \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_+) + \mathfrak{C}(\mathbb{M}_i, \mathbb{M}_-) \leq 4$. Therefore, $0 \leq \mathfrak{M}_{II}^\varsigma(\mathbb{M}_i) \leq 1$. Hence (i) is true.

The proof of (ii), (iii), (iv), and (v) are obvious. \square

Definition 4.7. Weighted neutrosophic index coefficient [61]: Let $\mathfrak{M}_I^\varsigma(\mathbb{M}_i)$, $\mathfrak{M}_{II}^\varsigma(\mathbb{M}_i)$ be the weighted type I and type II closeness measures for the alternative Λ_i , respectively. Let ε denote a closeness parameter, where $0 \leq \varepsilon \leq 1$. The weighted neutrosophic index coefficient $\mathcal{I}^\varsigma(\mathbb{M}_i)$ of the alternative Λ_i is defined as follows:

$$\mathcal{I}^\varsigma(\mathbb{M}_i) = \varepsilon \mathfrak{M}_I^\varsigma(\mathbb{M}_i) + (1 - \varepsilon) \mathfrak{M}_{II}^\varsigma(\mathbb{M}_i) \quad (10)$$

Theorem 4.8. *The weighted neutrosophic index coefficient $\mathcal{I}^\varsigma(\mathbb{M}_i)$ meets the following conditions for every neutrosophic characteristic \mathbb{M}_i in the neutrosophic decision matrix \mathbb{M} .*

- (i) $0 \leq \mathcal{I}^\varsigma(\mathbb{M}_i) \leq 1$
- (ii) $\mathcal{I}^\varsigma(\mathbb{M}_i) = \mathfrak{M}_I^\varsigma(\mathbb{M}_i)$ if $\varepsilon = 1$
- (iii) $\mathcal{I}^\varsigma(\mathbb{M}_i) = \mathfrak{M}_{II}^\varsigma(\mathbb{M}_i)$ if $\varepsilon = 0$
- (iv) $\mathcal{I}^\varsigma(\mathbb{M}_-) \leq \mathcal{I}^\varsigma(\mathbb{M}_+)$
- (v) $\mathcal{I}^\varsigma(\mathbb{M}_i) = \mathcal{I}(\mathbb{M}_i)$ if $\varsigma = (1/n, 1/n, \dots, 1/n)$

Proof. According to the definition 4.4, equation (9) since both $0 \leq \mathfrak{M}_I \leq 1$ and $0 \leq \mathfrak{M}_{II} \leq 1$, it is evident that $0 \leq \mathcal{I}^\varsigma(\mathbb{M}_i) \leq 1$. Hence (i) is proved. Proof (ii) and (iii) are obvious.

For (iv), it is already proved that $\mathfrak{M}_{II}(\mathbb{M}_-) \leq \mathfrak{M}_{II}(\mathbb{M}_+)$ hence $\mathcal{I}^s(\mathbb{M}_-) \leq \mathcal{I}^s(\mathbb{M}_+)$. Similarly, the proof of (v) is valid. \square

5. Proposed Neutrosophic TOPSIS Approach

This section elucidates the steps involved in the proposed neutrosophic TOPSIS methodology.

Step I: Construct an MCDM problem using m alternatives $\Lambda = \{\Lambda_1, \Lambda_2, \dots, \Lambda_m\}$ and n criteria $\Pi = \{\Pi_1, \Pi_2, \dots, \Pi_n\}$, partitioned into Π_B and Π_C .

Step II: Set the weight vector $\varsigma = \{\varsigma_1, \varsigma_2, \dots, \varsigma_n\}$ to the n criteria. A uniform weight will be assigned to an unweighted scenario in relation to the criteria.

Step III: Assign the neutrosophic rating \mathbb{M}_{ij} to each alternative Λ_i based on the criterion Π_j as determined by the experts.

Step IV: Formulate the weighted neutrosophic decision matrix $\mathbb{M} = (\mathbb{M}_{ij})_{m \times n}$ using equation (1). Define the neutrosophic characteristic \mathbb{M}_i for each $\Lambda_i \in \Lambda$.

Step V: Identify the neutrosophic PIS \mathbb{M}_+ and neutrosophic NIS \mathbb{M}_- using the equations (2) and (3).

Step VI: Compute the components of membership $\rho_\xi^s(\mathbb{M}_i, \mathbb{M}_+)$, $\rho_\xi^s(\mathbb{M}_i, \mathbb{M}_-)$, hesitancy $\rho_\zeta^s(\mathbb{M}_i, \mathbb{M}_+)$, $\rho_\zeta^s(\mathbb{M}_i, \mathbb{M}_-)$, and non-membership $\rho_\kappa^s(\mathbb{M}_i, \mathbb{M}_+)$, $\rho_\kappa^s(\mathbb{M}_i, \mathbb{M}_-)$ for each $\Lambda_i \in \Lambda$, using equations (5), (6) and (7), respectively.

Step VII: Apply equation (4) to calculate the weighted neutrosophic correlation coefficient $\vartheta^s(\mathbb{M}_i, \mathbb{M}_+)$ and $\vartheta^s(\mathbb{M}_i, \mathbb{M}_-)$ between \mathbb{M}_i , \mathbb{M}_+ , and \mathbb{M}_i , \mathbb{M}_- respectively for every $\Lambda_i \in \Lambda$.

Step VIII: Apply the equations (8) and (9) to determine the weighted Type I and Type II closeness measure $\mathfrak{M}_I^s(\mathbb{M}_i)$ and $\mathfrak{M}_{II}^s(\mathbb{M}_i)$, respectively for every $\Lambda_i \in \Lambda$.

Step IX: Define the closeness parameter $\varepsilon, 0 \leq \varepsilon \leq 1$ and determine $\mathcal{I}^s(\mathbb{M}_i)$ for each $\Lambda_i \in \Lambda$ using equation (10).

Step X: Determine the preference order of m alternatives according to descending order of $\mathcal{I}^s(\mathbb{M}_i)$ values.

The flowchart of the proposed approach is shown in figure 2.

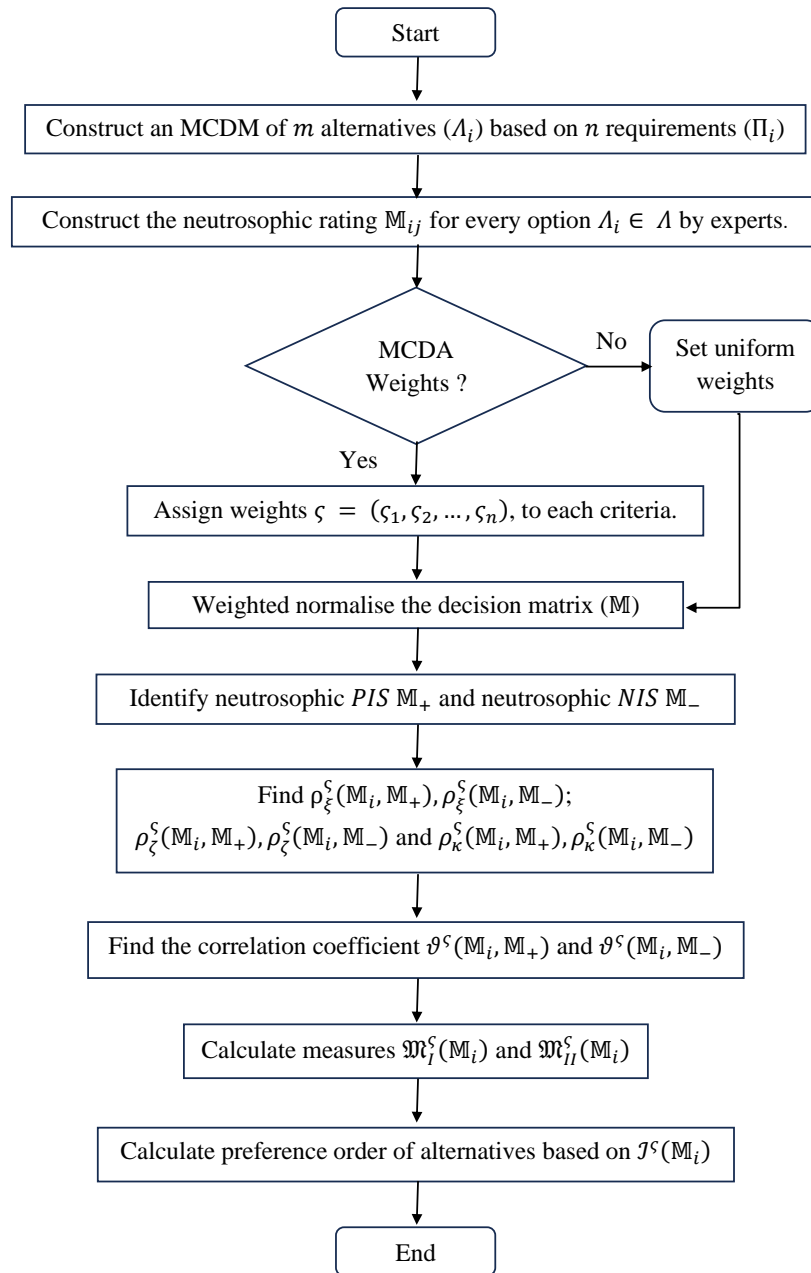


FIGURE 2. Proposed neutrosophic MCDA approach framework.

6. Numerical Problem on Term Policy Selection

Insurance is vital as it compensates for any unexpected loss of property and life. In this study, we will address the TIP selection, the significance of which still needs to be realized by most individuals.

6.1. Analyzing the criteria and alternatives

A variety of attributes, such as flexible premium payment options, a broad range of coverage options, a reputable insurance firm with a high claim settlement ratio, an affordable premium, and excellent customer feedback, are associated with an insurance policy. In this problem we choose five important criteria, such as insurance premium (Π_1), claim settlement ratio (Π_2), reputation of the insurance firm (Π_3), range of coverage (Π_4) and customer feedback (Π_5) and ten term insurance policy firms $\Lambda = \{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, \Lambda_7, \Lambda_8, \Lambda_9, \Lambda_{10}\}$.

6.2. Solution procedure by the proposed method

In Step I, we select ten alternatives and five criteria, with $\Pi_C = \{\Pi_1\}$ and $\Pi_B = \{\Pi_2, \Pi_3, \Pi_4, \Pi_5\}$. In Step II, the weights according to the importance of the criteria are predetermined as $\varsigma = \{0.30, 0.35, 0.15, 0.10, 0.10\}$.

In Step III, the expert evaluates each criterion Π_i associated with each alternative using the neutrosophic ratings outlined in Table 2.

TABLE 2. Linguistic terms and corresponding SVNS

Linguistic Variables	Membership functions
Very Low (VL)	(0.1,0.2,0.8)
Low (L)	(0.3,0.3,0.75)
Medium Low (ML)	(0.4,0.25,0.7)
Medium (M)	(0.6,0.2,0.5)
Medium High (MH)	(0.7,0.15,0.4)
High (H)	(0.8,0.15,0.3)
Very High (VH)	(0.9,0.2,0.1)

The neutrosophic decision matrix is formulated using the neutrosophic rating of SVNSs from table 2.

TABLE 3. Linguistic term based decision matrix

Criteria Weight	0.3	0.35	0.15	0.1	0.1
Criteria → Alternatives ↓	Π_1	Π_2	Π_3	Π_4	Π_5
Λ_1	ML	M	M	VL	VH
Λ_2	VL	ML	M	VH	VH
Λ_3	MH	VH	H	VH	ML
Λ_4	M	M	M	M	M
Λ_5	H	M	M	VH	ML
Λ_6	M	MH	MH	M	H
Λ_7	L	VL	M	VH	M
Λ_8	ML	H	ML	ML	ML
Λ_9	H	H	VH	ML	M
Λ_{10}	VH	L	VH	MH	ML

The neutrosophic rating is distributed among the alternatives by their credibility in table 3. The decision maker has assigned higher ratings to the alternatives Λ_3 , Λ_6 , Λ_9 , and Λ_{10} , while rating Λ_1 , Λ_2 , and Λ_8 relatively lower. Therefore, it is possible to predict that the alternatives Λ_3 , Λ_6 , Λ_9 , and Λ_{10} will acquire the initial positions, while the alternatives Λ_1 , Λ_2 , and Λ_8 will be at the bottom of the ranking sequence.

The weighted normalized neutrosophic decision matrix is determined by integrating criteria weights with the neutrosophic decision matrix in Step IV.

$$\mathbb{M} = \begin{bmatrix} (0.120, 0.075, 0.210) & (0.210, 0.070, 0.175) & (0.090, 0.030, 0.075) & (0.010, 0.020, 0.080) & (0.090, 0.020, 0.010) \\ (0.030, 0.060, 0.240) & (0.140, 0.087, 0.245) & (0.090, 0.030, 0.075) & (0.090, 0.020, 0.010) & (0.090, 0.020, 0.010) \\ (0.210, 0.045, 0.120) & (0.315, 0.070, 0.035) & (0.120, 0.023, 0.045) & (0.090, 0.020, 0.010) & (0.040, 0.025, 0.070) \\ (0.180, 0.060, 0.150) & (0.210, 0.070, 0.175) & (0.090, 0.030, 0.075) & (0.060, 0.020, 0.050) & (0.060, 0.020, 0.050) \\ (0.240, 0.045, 0.090) & (0.210, 0.070, 0.175) & (0.090, 0.030, 0.075) & (0.090, 0.020, 0.010) & (0.040, 0.025, 0.070) \\ (0.180, 0.060, 0.150) & (0.245, 0.053, 0.140) & (0.105, 0.023, 0.060) & (0.060, 0.020, 0.050) & (0.080, 0.015, 0.030) \\ (0.090, 0.090, 0.225) & (0.035, 0.070, 0.280) & (0.090, 0.030, 0.075) & (0.090, 0.020, 0.010) & (0.060, 0.020, 0.050) \\ (0.120, 0.075, 0.210) & (0.280, 0.053, 0.105) & (0.060, 0.038, 0.105) & (0.040, 0.025, 0.070) & (0.040, 0.025, 0.070) \\ (0.240, 0.045, 0.090) & (0.280, 0.053, 0.105) & (0.135, 0.030, 0.015) & (0.040, 0.025, 0.070) & (0.060, 0.020, 0.050) \\ (0.270, 0.060, 0.030) & (0.105, 0.105, 0.263) & (0.135, 0.030, 0.015) & (0.070, 0.015, 0.040) & (0.040, 0.025, 0.070) \end{bmatrix}$$

Table 4 displays the neutrosophic PIS \mathbb{M}_+ and NIS \mathbb{M}_- calculated using equations (2) and (3), respectively.

TABLE 4. Neutrosophic PIS \mathbb{M}_+ and Neutrosophic NIS \mathbb{M}_-

Neutrosophic ideal solutions	Components	Π_1	Π_2	Π_3	Π_4	Π_5
PIS	Membership	0.030	0.315	0.135	0.090	0.090
	Hesitant	0.090	0.053	0.023	0.015	0.015
	Non-membership	0.240	0.035	0.015	0.010	0.010
NIS	Membership	0.270	0.035	0.060	0.010	0.040
	Hesitant	0.045	0.105	0.038	0.025	0.025
	Non-membership	0.030	0.280	0.105	0.080	0.070

The components of weighted correlation coefficients are calculated from neutrosophic PIS and NIS using equation (4), as shown in table 5.

TABLE 5. Weighted correlation coefficient using neutrosophic PIS and NIS

Solution	Components	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8	Λ_9	Λ_{10}
PIS	$\rho_{\zeta}^{\zeta}(\mathbb{M}_i, \mathbb{M}_+)$	0.5627	-0.2869	0.7896	0.1958	-0.3005	0.8157	-0.6587	0.9742	0.5067	-0.8461
	$\rho_{\xi}^{\zeta}(\mathbb{M}_i, \mathbb{M}_+)$	0.8791	-0.5246	-0.8641	-0.8509	-0.8804	0.4053	0.8795	0.8995	-0.1570	-0.4959
	$\rho_{\kappa}^{\zeta}(\mathbb{M}_i, \mathbb{M}_+)$	0.6810	0.0880	0.4172	-0.6931	-0.8328	0.6148	-0.2317	0.9662	0.0558	-0.9195
NIS	$\rho_{\zeta}^{\zeta}(\mathbb{M}_i, \mathbb{M}_-)$	-0.8134	-0.0952	-0.4989	0.1745	0.6374	-0.5355	0.3250	-0.8723	-0.1426	0.9739
	$\rho_{\xi}^{\zeta}(\mathbb{M}_i, \mathbb{M}_-)$	-0.2556	0.9788	0.2461	0.2532	0.2575	-0.1058	-0.2560	-0.8969	-0.6118	0.9713
	$\rho_{\kappa}^{\zeta}(\mathbb{M}_i, \mathbb{M}_-)$	-0.2790	0.3705	-0.7814	0.8615	0.4747	-0.4425	0.6385	-0.8853	-0.4978	0.9856

In the initial stages of the proposed neutrosophic TOPSIS, weighted neutrosophic correlation measures from ideal solutions are helpful for predicting the final ranking of the alternatives. The highest membership function of the neutrosophic correlation from PIS signifies that the corresponding alternative is optimal, whereas the highest non-membership of neutrosophic correlation from PIS indicates that the corresponding alternative is the least favourable option. If the non-membership of the neutrosophic correlation from NIS is at its lowest, it signifies that this option is the least favourable. Conversely, if the membership is at its highest, it also indicates that this alternative is the least favourable. Table 5 is employed to generate the figures 3 and 4, which are used to identify the most appropriate alternatives through the proposed neutrosophic TOPSIS approach.

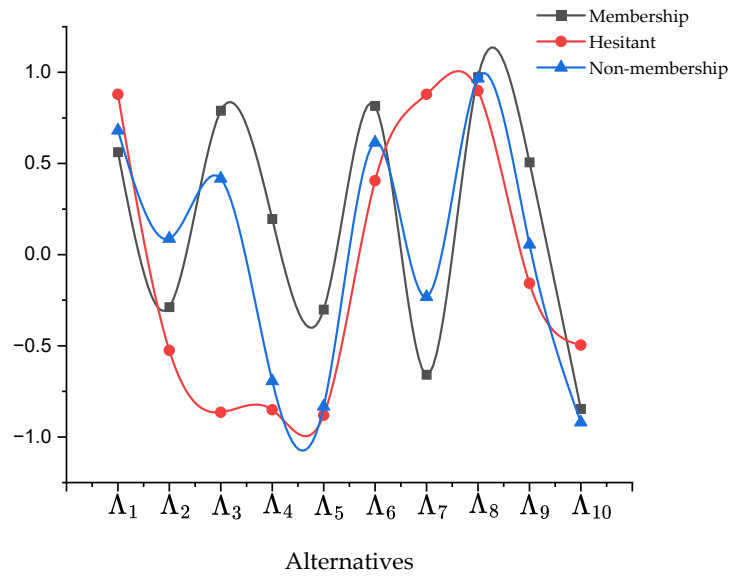


FIGURE 3. Neutrosophic component of the correlation coefficient from PIS

The membership components of the weighted neutrosophic correlation from PIS, as illustrated in figure 3, are comparatively higher for the alternatives Λ_1 , Λ_3 , Λ_6 , and Λ_8 . In contrast, the nonmembership components are significantly lower for the alternative Λ_{10} .

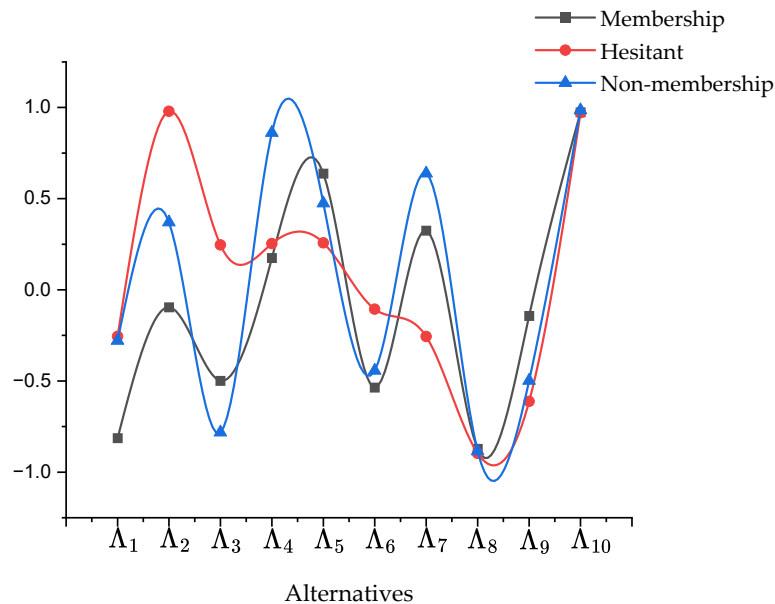


FIGURE 4. Neutrosophic component of the correlation coefficient from NIS

The non-membership components of the weighted neutrosophic correlation from NIS, as illustrated in figure 4, are notably higher for the alternatives Λ_4 and Λ_{10} . In contrast, the

membership components from NIS are lower for the alternatives Λ_1 and Λ_8 . Therefore, based on figures 3 and 4, it can be predicted that the top-ranking positions will be attained by the alternatives Λ_1 , Λ_8 , and Λ_{10} . However, the alternatives presented in figures 3 and 4 may attain the initial positions in the ranking sequence based on their average scores.

Table 6 provides the weighted neutrosophic correlation coefficients, type I and type II closeness measures according to Step VII and Step VIII.

TABLE 6. Weighted correlation coefficients, type I and type II closeness measures

Alt.	$\vartheta^c(\mathbb{M}_i, \mathbb{M}_+)$	$\vartheta^c(\mathbb{M}_i, \mathbb{M}_-)$	$\mathfrak{M}_I(\mathbb{M}_i)$	$\mathfrak{M}_{II}(\mathbb{M}_i)$
Λ_1	0.7076	-0.4493	0.1679	0.7562
Λ_2	-0.2412	0.4180	0.6808	0.3486
Λ_3	0.1142	-0.3447	0.3971	0.6297
Λ_4	-0.4494	0.4297	0.7176	0.2780
Λ_5	-0.6712	0.4565	0.7546	0.1842
Λ_6	0.6119	-0.3613	0.2218	0.7162
Λ_7	-0.0036	0.2359	0.5677	0.4464
Λ_8	0.9466	-0.8848	0.0275	0.9441
Λ_9	0.1352	-0.4174	0.3789	0.6608
Λ_{10}	-0.7538	0.9769	0.9870	0.1107

The index parameter is set 0.5 to provide same relevance to type I and type II closeness measurements. The weighted index values of the options are determined using equation (10). Table 7 presents the weighted index values and their corresponding rankings of the options.

TABLE 7. Index values and ranking of alternatives

Alternatives	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8	Λ_9	Λ_{10}
Index values	0.4620	0.5147	0.5134	0.4978	0.4694	0.4690	0.5070	0.4858	0.5199	0.5489
Ranking	10	3	4	6	8	9	5	7	2	1

The suggested neutrosophic TOPSIS technique identifies alternative Λ_{10} as the ideal choice and Λ_1 as the least beneficial option.

6.3. Comparison study

The proposed neutrosophic correlation-based TOPSIS is applied to the decision matrix of Zeng et al. [52] which considers five alternatives and four criteria for comparison of the obtained outcome. The weight vector for the four criteria is $\varsigma = \{0.2, 0.25, 0.3, 0.25\}$. The neutrosophic TOPSIS approach used divergence measure [33], Euclidean distance [24], OWA Golui et al., Decision on Insurance Policy Selection by weighted Correlation Approach on Neutrosophic Fuzziness through TOPSIS

distance [32]. The neutrosophic decision matrix \mathbb{M} of Zeng et al. [52] article is

$$\mathbb{M} = \begin{bmatrix} (0.5, 0.3, 0.4) & (0.3, 0.2, 0.1) & (0.2, 0.2, 0.6) & (0.5, 0.2, 0.5) \\ (0.7, 0.3, 0.6) & (0.5, 0.2, 0.2) & (0.4, 0.5, 0.2) & (0.7, 0.0, 0.2) \\ (0.5, 0.3, 0.4) & (0.5, 0.1, 0.3) & (0.6, 0.1, 0.1) & (0.6, 0.2, 0.4) \\ (0.7, 0.0, 0.3) & (0.6, 0.4, 0.2) & (0.6, 0.3, 0.2) & (0.7, 0.2, 0.3) \\ (0.4, 0.1, 0.3) & (0.4, 0.3, 0.6) & (0.4, 0.1, 0.5) & (0.5, 0.1, 0.2) \end{bmatrix}$$

The ranking orders derived from the aforementioned approaches are presented in Table 8.

TABLE 8. Comparison of the proposed and existing approaches

Approach	Results
Garg [33]	$\Lambda_4 \succ \Lambda_3 \succ \Lambda_1 \succ \Lambda_5 \succ \Lambda_2$
Biswas et al. [24]	$\Lambda_3 \succ \Lambda_1 \succ \Lambda_4 \succ \Lambda_2 \succ \Lambda_5$
Chen et al. [32]	$\Lambda_1 \succ \Lambda_3 \succ \Lambda_4 \succ \Lambda_5 \succ \Lambda_2$
Zeng et al. [52]	$\Lambda_3 \succ \Lambda_4 \succ \Lambda_2 \succ \Lambda_1 \succ \Lambda_5$
Proposed method	$\Lambda_3 \succ \Lambda_2 \succ \Lambda_4 \succ \Lambda_5 \succ \Lambda_1$

The ideal solution using distance measurements has potential variations, as shown in Table 8, which might mislead the decision-maker. The proposed technique differs from the [52], but Λ_3 is the best alternate in both approaches. In comparison to any distance measure, the suggested approach is relatively straightforward. Because the suggested neutrosophic TOPSIS technique is more flexible and effectively captures the uncertainty, it may improve existing approaches. As a result, the researcher might consider the presented method as an alternate solution to the uncertain MCDM issue. Table 9 displays correlation coefficients between findings from different methodologies in the comparative study.

TABLE 9. Correlation coefficient of the comparison study

	Garg [33]	Biswas et al. [24]	Chen et al. [32]	Zeng et al. [52]	Proposed method
Garg [33]	1				
Biswas et al. [24]	0.61	1			
Chen et al. [32]	0.49	0.8	1		
Zeng et al. [52]	0.73	0.7	0.2	1	
Proposed method	0.24	0.3	-0.3	0.8	1

The results of the proposed neutrosophic TOPSIS significantly correspond with the findings of Zeng et al. [52]. The correlation coefficients of the proposed approach and that of Zeng et al. [52] both lie within the interval $[-1, 1]$. The remaining methods utilised either divergence or distance measures in TOPSIS, which differ internally from the correlation-based TOPSIS approach. Thus, the proposed correlation-based neutrosophic TOPSIS presents a viable option for researchers addressing hesitancy in decision-making.

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6.4. Sensitivity analysis

In this section, we analyse the sensitivity for different values of ε to check the robustness and credibility of the proposed neutrosophic TOPSIS. The ranking of the ten alternatives for the values of ε is presented in figure 5.

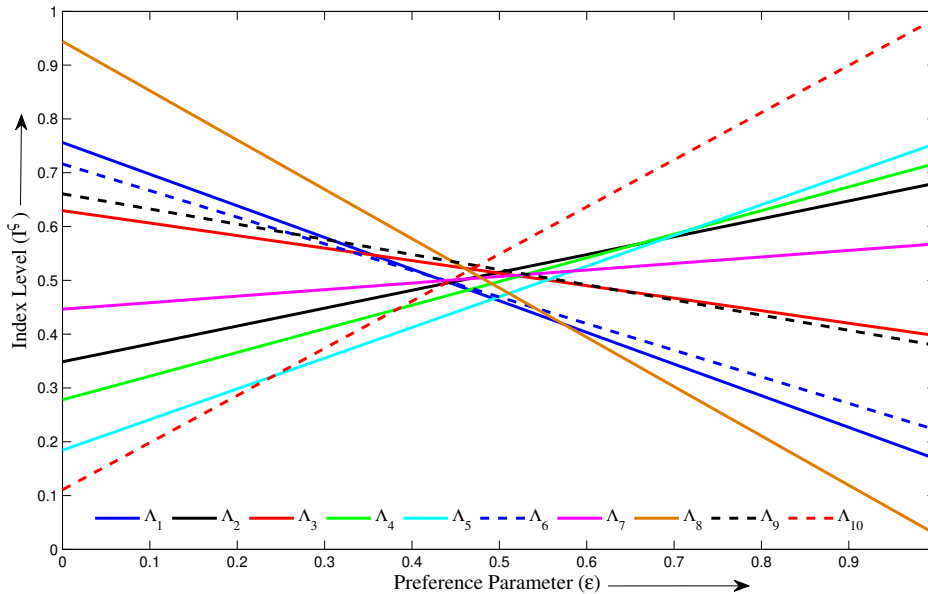


FIGURE 5. Grade of the alternatives for different ε

The preference parameter ε depends on the decision-maker. If the decision-maker considers type I weighted closeness measure (\mathfrak{M}_I^s) more than type II closeness measure (\mathfrak{M}_{II}^s), $\varepsilon \geq 0.5$, and if type II is more important, $\varepsilon \leq 0.5$. We provide the figure 5 to summarize the importance of type I and type II weighted closeness measures. The decision-making for each value of ε in the range $[0, 1]$ is shown in Figure 5. In particular, from the graph, we can conclude that when $\varepsilon \in [0, 0.3)$, the best choice is Λ_8 and the grading of the alternatives is $\Lambda_8, \Lambda_1, \Lambda_6, \Lambda_9, \Lambda_3, \Lambda_7, \Lambda_2, \Lambda_4, \Lambda_5, \Lambda_{10}$. The ranking order of the alternatives is $\Lambda_{10}, \Lambda_5, \Lambda_4, \Lambda_2, \Lambda_7, \Lambda_3, \Lambda_9, \Lambda_6, \Lambda_1, \Lambda_8$ when $\varepsilon \in (0.7, 1]$. The weighted closeness index is nearly close to each other when $\varepsilon \in [0.4, 0.6]$. Hence, the ranking order of the alternatives largely depends on the preference parameter ε . i.e., The ranking order of the alternatives varies with the preference parameter ε , as seen in 5.

7. Conclusions

The proposed approach effectively addresses the MCDM challenges associated with the selection of term insurance policies, particularly in ambiguous situations. The proposed weighted

neutrosophic correlation measure lies in $[-1, 1]$ aligns well with the classical correlation coefficient, ensuring the consistency and reliability of the decision-making process. The study reveals that customers place a higher emphasis on criteria such as the insurance provider's reputation and the policy's coverage area, showing a willingness to pay higher premiums for better service quality. In contrast, factors like claim settlement ratio and customer feedback are of lesser concern to most customers. The suggested strategy provides substantial advantages for both clients and insurance companies. Firms can prioritize the most significant criteria to successfully meet customer expectations, while customers can make more informed decisions by comprehending the most essential criteria. Furthermore, by recognizing and rectifying weaknesses, companies may refine their products and elevate consumer contentment, hence drawing a broader client. The limitations of the suggested technique include predetermined criterion weights and a single decision-maker. The forthcoming study may enhance outcomes through the incorporation of group decision-making processes, extension through type II fuzzy sets, complex spherical fuzzy sets, and Diophantine fuzzy sets. The suggested technique is capable of addressing any real-world MCDM issue, such as risk assessment, green energy selection, and supplier selection.

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