Neutrosophic λ-Closed Set and Related Mappings in Neutrosophic Topological Spaces

Shivangi Tyagi [1*], Mridul Kumar Gupta [2]

[1] Department of Mathematics, Chaudhary Charan Singh University; shivangimathematics@gmail.com

[2] Department of Mathematics, Chaudhary Charan Singh University; mkgupta2k2@gmail.com

*Correspondence: shivangimathematics@gmail.com

Abstract: Studying open and closed set variations is crucial in neutrosophic topology, given the growing significance of neutrosophic sets in various applications. This study introduces neutrosophic λ-closed sets, generalizing neutrosophic closed and pre-closed sets within neutrosophic topological spaces. Their fundamental properties are examined alongside comparisons with various existing neutrosophic sets. To illustrate their significance, we define neutrosophic λ-continuous mapping, neutrosophic λ-closed mapping, and neutrosophic λ-$T_{1/2}$ spaces, exploring their connections with established mappings. Additionally, new concepts such as neutrosophic-w-continuous, neutrosophic-w-closed, neutrosophic-Gα-closed, and neutrosophic-αG-closed mappings are proposed to further enrich neutrosophic topological spaces. Several theorems and counterexamples validate these findings and highlight their theoretical significance.

Keywords: neutrosophic λ-closed set; neutrosophic λ-$T_{1/2}$ space; neutrosophic λ-continuous mapping; neutrosophic-w-continuous mapping; neutrosophic λ-closed mapping; neutrosophic-w-closed mapping; neutrosophic-αG-closed mapping; neutrosophic-Gα-closed mapping

1. Introduction

In 1965, Zadeh [24] introduced the concept of fuzzy sets to handle vague, imprecise, and uncertain data. Later, in 1986, Atanassov [6] expanded this concept by proposing intuitionistic fuzzy sets, which subsequently gained widespread recognition as a fundamental tool in mathematics, engineering [13], and medicine [8]. In 1997, Coker [7] further advanced this field by developing the notion of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Building on this, Rajarajeswari and Bagyalakshmi [15] introduced the concept of intuitionistic fuzzy λ (lambda)-closed sets in 2011. In our study, we explore this concept within the framework of neutrosophic (or $\mathcal{N}_{eu}$) topological spaces. Smarandache [22] introduced the concept of $\mathcal{N}_{eu}$-sets, characterized by truth, indeterminacy, and falsity values, to represent complex real-world information. Over time, researchers have contributed significantly to $\mathcal{N}_{eu}$-theory, leading to the development of $\mathcal{N}_{eu}$-vector spaces [4], $\mathcal{N}_{eu}$-group theory [21], $\mathcal{N}_{eu}$-ring theory [3], and Dombi $\mathcal{N}_{eu}$-graphs [12]. The study of $\mathcal{N}_{eu}$-sets continues to evolve, with academicians actively exploring their diverse applications. AboElHamd et al. [1] contributed to the development of $\mathcal{N}_{eu}$-logic and its applications. In 2012, Salama and Alblowi [18] established $\mathcal{N}_{eu}$-topological spaces using $\mathcal{N}_{eu}$-sets, introducing the concepts of $\mathcal{N}_{eu}$-closed and $\mathcal{N}_{eu}$-open sets. Subsequently, in 2014, Salma et al. [19] examined continuous and closed mappings in $\mathcal{N}_{eu}$-topological spaces. Santhi and Udhayarani [20] later introduced $\mathcal{N}_{eu}$-w-closed sets in 2016, proposing various properties of these sets. In 2017, Arokiarani et al. [5] introduced the concept of $\mathcal{N}_{eu}$-semi-open sets in $\mathcal{N}_{eu}$-topological spaces. Following this, several researchers have proposed and explored various types of open and closed sets within $\mathcal{N}_{eu}$-topological spaces. The study of weaker and stronger forms of open and closed sets plays a crucial role in $\mathcal{N}_{eu}$-topology, as the importance of $\mathcal{N}_{eu}$-set is rapidly growing across numerous applications. Consequently, it remains a prominent research focus for researchers worldwide. Additionally, Arokiarani [5] introduced $\mathcal{N}_{eu}$-pre-open sets as a weaker variant of α-open sets in $\mathcal{N}_{eu}$-topological spaces. In 2018, Dhavaseelan and Jafari [10] proposed $\mathcal{N}_{eu}$-g-closed sets and examined their properties. The same year, Jayanthi [11] presented $\mathcal{N}_{eu}$-αg-closed sets and analysed their characteristics. Further developments in the field include the introduction of $\mathcal{N}_{eu}$-gα-closed sets by Sreeja and Sarankumar [23] and the exploration of $\mathcal{N}_{eu}$-g-closed mappings by Ramesh [17] in 2020. In 2021, Rajeshwaran and Chandramathi [16] proposed $\mathcal{N}_{eu}$-g-semi-pre-closed sets, later extending their work in 2024 by introducing $\mathcal{N}_{eu}$-g-semi-pre-continuous mappings. That same year, Debnath and Mukherjee [9] examined $\mathcal{N}_{eu}$-$p\delta s$-irresolute and $\mathcal{N}_{eu}$-$\alpha\delta s$-irresolute mappings in $\mathcal{N}_{eu}$-topological spaces. Mohammed et al. [14] also contributed in 2024 by introducing Q^*-closed sets within the framework of fuzzy $\mathcal{N}_{eu}$-topology, along with relevant definitions and theorems. In this paper, we introduce λ-closed sets in $\mathcal{N}_{eu}$-topological spaces, a novel concept weaker than $\mathcal{N}_{eu}$-closed and pre-closed sets but independent of $\mathcal{N}_{eu}$-g-closed and $\mathcal{N}_{eu}$-semi-closed sets. We apply this new concept to define the notions of $\mathcal{N}_{eu}$-λ-continuous mapping, $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space, and $\mathcal{N}_{eu}$-λ-closed mapping, comparing them with existing $\mathcal{N}_{eu}$-mappings. Additionally, we presented the notions of $\mathcal{N}_{eu}$-w-continuous mapping, $\mathcal{N}_{eu}$-w-closed mapping, $\mathcal{N}_{eu}$-Gα-closed mapping, and $\mathcal{N}_{eu}$-αG-closed mapping, thereby contributing to the theoretical foundation of $\mathcal{N}_{eu}$-topological spaces.

For simplicity, we use the symbol '$\mathcal{N}_{eu}$' to represent the notion of 'neutrosophic'.

2. Preliminaries

Definition 2.1. [22] Consider a fixed set X ($\neq$ ϕ). Then the $\mathcal{N}_{eu}$-set N on X is defined as N $=\{< x,\ \eta_{\mathrm{N}}(x), \theta_{\mathrm{N}}(x), \xi_{\mathrm{N}}(x) >: x \in \mathrm{X}\}$, where, $\eta_{\mathrm{N}}(x), \theta_{\mathrm{N}}(x)$, and $\xi_{\mathrm{N}}(x)$ are values in the range $[0, 1]$, representing the degree of membership, indeterminacy, and non-membership functions respectively for each element $x \in$ X. These values satisfy the condition, $0 \leq \eta_{\mathrm{N}}(x) + \theta_{\mathrm{N}}(x) + \xi_{\mathrm{N}}(x) \leq 3$.

Definition 2.2 [18] Consider two $\mathcal{N}_{eu}$-sets N and Q, where N $=\{<x,\eta_{\mathrm{N}}(x), \theta_{\mathrm{N}}(x), \xi_{\mathrm{N}}(x)>: x \in \mathrm{X}\}$, and Q $=\{< x, \eta_{\mathrm{Q}}(x), \theta_{\mathrm{Q}}(x), \xi_{\mathrm{Q}}(x)>: x \in \mathrm{X}\}$, then

(i) $\mathrm{N} \subseteq \mathrm{Q} \Leftrightarrow \eta_{\mathrm{N}}(x) \leq \eta_{\mathrm{Q}}(x);\ \theta_{\mathrm{N}}(x) \leq \theta_{\mathrm{Q}}(x)$, and $\xi_{\mathrm{N}}(x) \geq \xi_{\mathrm{Q}}(x)$
(ii) $\mathrm{N} = \mathrm{Q} \Leftrightarrow \eta_{\mathrm{N}}(x) = \eta_{\mathrm{Q}}(x);\ \theta_{\mathrm{N}}(x) = \theta_{\mathrm{Q}}(x)$ and $\xi_{\mathrm{N}}(x) = \xi_{\mathrm{Q}}(x)\ \forall x \in \mathrm{X}$
(iii) $\mathrm{N}' = \{< x, \xi_{\mathrm{N}}(x), \theta_{\mathrm{N}}(x), \eta_{\mathrm{N}}(x)>: x \in \mathrm{X}\}$
(iv) $\mathrm{N} \cap \mathrm{Q} = \{< x, \min\{\eta_{\mathrm{N}}(x), \eta_{\mathrm{Q}}(x)\}, \min\{\theta_{\mathrm{N}}(x), \theta_{\mathrm{Q}}(x)\}, \max\{\xi_{\mathrm{N}}(x), \xi_{\mathrm{Q}}(x)\}>\}$
(v) $\mathrm{N} \cup \mathrm{Q} = \{< x, \max\{\eta_{\mathrm{N}}(x), \eta_{\mathrm{Q}}(x)\}, \max\{\theta_{\mathrm{N}}(x), \theta_{\mathrm{Q}}(x)\}, \min\{\xi_{\mathrm{N}}(x), \xi_{\mathrm{Q}}(x)\}>\}$
(vi) $\tilde{0} = \{< x, 0, 0, 1 >: x \in \mathrm{X}\}$, and $\tilde{1} = \{< x, 1, 1, 0 >: x \in \mathrm{X}\}$ are respectively the empty set, and the whole set of X.

Definition 2.3 [18] Let X be any non-empty set, and ζ be the collection of $\mathcal{N}_{eu}$-subsets of X that satisfy the following axioms:

(i) $\tilde{0}, \tilde{1} \in \zeta$

(ii) $H_1 \cap H_2 \in \zeta$ for any $H_1, H_2 \in \zeta$
(iii) $\cup H_i \in \zeta, \forall \{H_i : i \in \Delta\} \subseteq \zeta$.

Then the pair (X, ζ) is referred to as $\mathcal{N}_{eu}$-topological space, and any $\mathcal{N}_{eu}$-set in ζ is called $\mathcal{N}_{eu}$-open set in X. The components of ζ are known as $\mathcal{N}_{eu}$-open sets, and the supplement of $\mathcal{N}_{eu}$-open set is called $\mathcal{N}_{eu}$-closed set. We will write X instead of (X, ζ) to represent $\mathcal{N}_{eu}$-topological space.

Definition 2.4 [18] Let N be any $\mathcal{N}_{eu}$-set in a $\mathcal{N}_{eu}$-topological space X where, N $=\{< x, \eta_N(x), \theta_N(x), \xi_N(x) >: x \in X\}$. Then $\mathcal{N}_{eu}$-closure of N is defined as $\mathcal{N}_{eu}$-cl(N) = $\cap$\{O: O is $\mathcal{N}_{eu}$-closed set in X, and N $\subseteq$ O\}, and $\mathcal{N}_{eu}$-interior of N is defined as $\mathcal{N}_{eu}$-int(N) = $\cup$\{K: K is $\mathcal{N}_{eu}$-open set in X, and K $\subseteq$ N\}.

Definition 2.5 [5] Consider a fixed set X ($\neq \phi$). A $\mathcal{N}_{eu}$-set $p_{m,i,n}$ is a $\mathcal{N}_{eu}$- point in X, and it is given by

$$p_{m,i,n}(p_j) = \begin{cases} (m, i, n), if\ p = p_j \\ (0, 0, 1), if\ p \neq p_j \end{cases}$$

Where m, i, n $\in$ [0, 1] respectively represent the degree of membership, indeterminacy, and non-membership values of ($p_{m,i,n}$) with $0 < m + i + n \leq 3$. The point $p_j \in$ X is the support of $p_{m,i,n}$.

Definition 2.6 Any $\mathcal{N}_{eu}$-subset N of a $\mathcal{N}_{eu}$-topological space X is called

(i) $\mathcal{N}_{eu}$-semi-closed (respectively, $\mathcal{N}_{eu}$-semi-open) set [5] if $\mathcal{N}_{eu}$-int($\mathcal{N}_{eu}$-cl(N)) $\subseteq$ N (respectively, N $\subseteq$ $\mathcal{N}_{eu}$-cl($\mathcal{N}_{eu}$-int(N))).
(ii) $\mathcal{N}_{eu}$-pre-closed (respectively, $\mathcal{N}_{eu}$-pre-open) [5] if $\mathcal{N}_{eu}$-cl($\mathcal{N}_{eu}$-int(N)) $\subseteq$ N (respectively, N $\subseteq$ $\mathcal{N}_{eu}$-int($\mathcal{N}_{eu}$-cl(N))).
(iii) $\mathcal{N}_{eu}$-g-closed set [10] if $\mathcal{N}_{eu}$-cl(N) $\subseteq$ G, whenever N $\subseteq$ G and G is $\mathcal{N}_{eu}$-open set. The supplement of $\mathcal{N}_{eu}$-g-closed set is $\mathcal{N}_{eu}$-g-open set.
(iv) $\mathcal{N}_{eu}$-αg-closed set [11] if $\mathcal{N}_{eu}$-αcl(N) $\subseteq$ G, whenever N $\subseteq$ G and G is $\mathcal{N}_{eu}$-open set in X. The supplement of $\mathcal{N}_{eu}$-α generalized closed set is $\mathcal{N}_{eu}$-αg-open set in X.
(v) $\mathcal{N}_{eu}$-gα-closed set [23] if $\mathcal{N}_{eu}$-αcl(N) $\subseteq$ G, whenever N $\subseteq$ G and G is $\mathcal{N}_{eu}$-α open set in X.
(vi) $\mathcal{N}_{eu}$-g-semi-pre-closed set [16] if $\mathcal{N}_{eu}$-spcl(N) $\subseteq$ G, whenever N $\subseteq$ G and G is $\mathcal{N}_{eu}$- open set. The supplement of $\mathcal{N}_{eu}$-gsp-closed set is called the $\mathcal{N}_{eu}$-g-semi-pre-open set.
(vii) $\mathcal{N}_{eu}$-w-closed set [20] if $\mathcal{N}_{eu}$-cl(N) $\subseteq$ G whenever N $\subseteq$ G and G is $\mathcal{N}_{eu}$-semi-open set.

Definition 2.7 [2] Suppose f is a mapping from X to Y.

(i) If N $=\{< x, \eta_N(x), \theta_N(x), \xi_N(x) >: x \in X\}$ be an arbitrary $\mathcal{N}_{eu}$-subset in X, then the image of N under f, written as f(N) is the $\mathcal{N}_{eu}$-subset in Y, and it is defined as:

$$\eta_{f(N)}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\eta_N(x)\}, & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{if } f^{-1}(y) = \phi \end{cases}$$

$$\theta_{f(N)}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\theta_N(x)\}, & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{if } f^{-1}(y) = \phi \end{cases}$$

$$\xi_{f(\mathrm{N})}(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\{\xi_{\mathrm{N}}(x)\}, & \text{if } f^{-1}(\mathrm{y})\neq\phi\\ 0, & \text{if } f^{-1}(\mathrm{y})=\phi\end{cases}$$

for each $y\in Y$, where $f^{-1}(\mathrm{y})=\{x: f(x)=y\}$.

(ii) If Q ={< $y,\eta_{\mathrm{Q}}(y),\theta_{\mathrm{Q}}(y),\xi_{\mathrm{Q}}(y)$>: $y\in$ Y} be an arbitrary $\mathcal{N}_{eu}$-subset in Y, then the pre-image of Q under f, written as f^{-1}(Q) is the $\mathcal{N}_{eu}$-subset in X, and it is described as:

$$\eta_{f^{-1}(\mathrm{Q})}(x)=\eta_Q\big(f(x)\big),\forall\, x\in \mathrm{X}$$

$$\theta_{f^{-1}(\mathrm{Q})}(x)=\theta_Q\big(f(x)\big),\forall\, x\in \mathrm{X}$$

$$\xi_{f^{-1}(\mathrm{Q})}(x)=\xi_Q\big(f(x)\big),\forall\, x\in \mathrm{X}$$

Definition 2.8 Consider a mapping f: X ⟶ Y. Then f is called

(i) $\mathcal{N}_{eu}$-continuous [19] if f^{-1}(N) is $\mathcal{N}_{eu}$-closed set in X for each $\mathcal{N}_{eu}$-closed set N in Y.
(ii) $\mathcal{N}_{eu}$-g-continuous [10] if f^{-1}(N) is $\mathcal{N}_{eu}$-g-closed set in X for each $\mathcal{N}_{eu}$-closed set N in Y
(iii) $\mathcal{N}_{eu}$-semi-continuous [5] if f^{-1}(N) is $\mathcal{N}_{eu}$-semi closed set in X for each $\mathcal{N}_{eu}$-closed set N in Y
(iv) $\mathcal{N}_{eu}$-g-semi-pre-continuous [16] if f^{-1} (N) is $\mathcal{N}_{eu}$-g-semi-pre-closed set in X for each $\mathcal{N}_{eu}$-closed set N in Y.

Definition 2.9 Consider a mapping f: X ⟶ Y. Then f is called

(i) $\mathcal{N}_{eu}$-closed (respectively$\mathcal{N}_{eu}$-open) mapping [5] if f(N) is $\mathcal{N}_{eu}$-closed (respectively, $\mathcal{N}_{eu}$-open) set in Y for each $\mathcal{N}_{eu}$-closed (respectively, $\mathcal{N}_{eu}$-open) set N in X.
(ii) $\mathcal{N}_{eu}$-g-closed mapping [17] if f(N) is $\mathcal{N}_{eu}$-g-closed set in Y for each $\mathcal{N}_{eu}$-closed set N in X.
(iii) $\mathcal{N}_{eu}$-semi-closed mapping [5] if f(N) is $\mathcal{N}_{eu}$-semi-closed set in Y for each $\mathcal{N}_{eu}$-closed set N in X.

3. Neutrosophic λ-Closed Sets

In this section, we introduce $\mathcal{N}_{eu}$-λ-closed (respectively, $\mathcal{N}_{eu}$-λ-open) sets, and explore various characteristics of these sets.

Definition 3.1 Any $\mathcal{N}_{eu}$-subset N of a $\mathcal{N}_{eu}$- topological space X is called $\mathcal{N}_{eu}$-λ-closed set if $\mathcal{N}_{eu}$-cl(O) ⊆ N for each $\mathcal{N}_{eu}$-open set O in X, satisfying O ⊆ N. The supplement of $\mathcal{N}_{eu}$-λ-closed set is called $\mathcal{N}_{eu}$-λ-open set in X. The collection of all $\mathcal{N}_{eu}$-λ-closed sets is represented by the symbol $\mathcal{N}_{eu}$-$\lambda CS(X)$.

Theorem 3.2 Every $\mathcal{N}_{eu}$-closed set (respectively, $\mathcal{N}_{eu}$-open set) is $\mathcal{N}_{eu}$-λ-closed set (respectively, $\mathcal{N}_{eu}$-λ-open set) in X.

Proof: Let N be an arbitrary $\mathcal{N}_{eu}$-closed set in $\mathcal{N}_{eu}$-topological space X, and let O be any $\mathcal{N}_{eu}$-open subset in X, provided O ⊆ N, then $\mathcal{N}_{eu}$-cl(O) ⊆ $\mathcal{N}_{eu}$-cl(N). As N is $\mathcal{N}_{eu}$- closed set, $\mathcal{N}_{eu}$-cl(N) = N, and $\mathcal{N}_{eu}$-cl(O) ⊆ N. Hence, N is $\mathcal{N}_{eu}$-λ-closed set.

Remark 3.3 The reverse of the above theorem need not be true, as seen in the subsequent example.

Example 3.4 Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.60 >\}$. Then $\mathcal{N}_{eu}$-set N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.80, 0.50, 0.20 >\}$ is $\mathcal{N}_{eu}$-λ-closet set, but it is not $\mathcal{N}_{eu}$-closed set in X.

Theorem 3.5 If M and N are any two $\mathcal{N}_{eu}$-λ-closed sets in a $\mathcal{N}_{eu}$-topological space X, then M⋂N is $\mathcal{N}_{eu}$-λ-closed set in X.

Proof: Let O be any $\mathcal{N}_{eu}$-open set in X such that O ⊆ M ⋂ N. Since M, and N are $\mathcal{N}_{eu}$-closed sets, $\mathcal{N}_{eu}$-cl(O) ⊆ M, and $\mathcal{N}_{eu}$-cl(O) ⊆ N. Therefore, $\mathcal{N}_{eu}$-cl(O) ⊆ M ⋂ N. Hence, M ⋂ N is $\mathcal{N}_{eu}$-λ-closed set.

Remark 3.6 The union of two $\mathcal{N}_{eu}$-λ-closed sets need not be $\mathcal{N}_{eu}$-λ-closed, as seen in the subsequent example.

Example 3.7 Let X = $\{q, t\}$, and let $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be an $\mathcal{N}_{eu}$-topology defined on X where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.30 >\}$. Then $\mathcal{N}_{eu}$-sets N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.30 >\}$, and Q = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.40 >\}$ are $\mathcal{N}_{eu}$-λ-closed sets in X, but N ⋃ Q is not $\mathcal{N}_{eu}$-λ-closed set.

Note: The concept of $\mathcal{N}_{eu}$-λ-closed set is independent of the concepts of $\mathcal{N}_{eu}$-g-closed sets, $\mathcal{N}_{eu}$-semi-closed sets, and $\mathcal{N}_{eu}$-w-closed sets, as seen in the subsequent example.

Example 3.8

(i) Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$. Then N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$ is $\mathcal{N}_{eu}$-λ-closed set but it is not $\mathcal{N}_{eu}$-g-closed set.
(ii) Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.30 >\}$. Then N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.20 >\}$ is $\mathcal{N}_{eu}$-g-closed set but it is not $\mathcal{N}_{eu}$-λ-closed set.
(iii) Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$. Then N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$ is $\mathcal{N}_{eu}$-λ-closed set, but it is not $\mathcal{N}_{eu}$-semi-closed set.
(iv) Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M, S\}$ be the $\mathcal{N}_{eu}$-topology on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.50 >\}$, and S = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$. Then N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.50 >\}$ is not $\mathcal{N}_{eu}$-λ-closed set, but it is $\mathcal{N}_{eu}$-semi-closed set.
(v) Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.60 >\}$. Then N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-w-closed set but not $\mathcal{N}_{eu}$-λ-closed set.
(vi) Let X = $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$. Then N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$, is $\mathcal{N}_{eu}$-λ-closed set but not $\mathcal{N}_{eu}$-w-closed set.

Remark 3.9 Every $\mathcal{N}_{eu}$-pre-closed set is $\mathcal{N}_{eu}$-λ-closed set, but the reverse is not true, as seen in the subsequent example.

Example 3.10 Let X= $\{q, t\}$, and $\zeta = \{\tilde{0}, \tilde{1}, M\}$ be the $\mathcal{N}_{eu}$-topology defined on X, where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$. Then, $\mathcal{N}_{eu}$-set N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-λ-closed set but not $\mathcal{N}_{eu}$-pre-closed set.

Here, we have the following representation among different kinds of $\mathcal{N}_{eu}$-closed sets.

$\mathcal{N}_{eu}$-closed set $\longrightarrow$ $\mathcal{N}_{eu}\lambda$-closed set $\longleftarrow$ $\mathcal{N}_{eu}$-pre-closed set

4. Neutrosophic Continuous Mappings

In this section, we introduce $\mathcal{N}_{eu}$-continuous mappings and establish various characteristics and counter examples of these mappings.

Definition 4.1 Consider a mapping f: X $\longrightarrow$ Y. Then f is called

(i) $\mathcal{N}_{eu}$-λ-continuous mapping if f^{-1}(N) is $\mathcal{N}_{eu}$-λ-closed set in X for each $\mathcal{N}_{eu}$-closed set N in Y.
(ii) $\mathcal{N}_{eu}$-w-continuous mapping if f^{-1}(N) is $\mathcal{N}_{eu}$-w-closed set in X of each $\mathcal{N}_{eu}$-closed set N in Y.

Remark 4.2 Every $\mathcal{N}_{eu}$-continuous mapping is $\mathcal{N}_{eu}$-λ-continuous, but the reverse is not true, as seen in the subsequent example.

Example 4.3 Let X = $\{q, t\}$ and Y =$\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ = $\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, N\}$ on X, and Y resp., where M =$\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.30 >\}$, and N = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.20, 0.50, 0.80 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t)$ = z. Then f is $\mathcal{N}_{eu}$-λ-continuous but not $\mathcal{N}_{eu}$-continuous.

Remark 4.4 Each $\mathcal{N}_{eu}$-pre-continuous mapping f: X $\longrightarrow$ Y is $\mathcal{N}_{eu}$-λ-continuous, but the reverse is not true, as seen in the subsequent example.

Example 4.5 Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ = $\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, N\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$, and N = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.40 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t)$ = z. Consider the $\mathcal{N}_{eu}$-set N = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.50 >\}$, then f^{-1}(N) = $f^{-1}(\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.50 >\})$ = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-λ-closed set in X but not $\mathcal{N}_{eu}$-pre-closed set in X. Hence, f is $\mathcal{N}_{eu}$-λ-continuous mapping but not $\mathcal{N}_{eu}$-pre-continuous mapping.

Note: The concept of $\mathcal{N}_{eu}$-λ-continuous mapping is independent of the concepts of $\mathcal{N}_{eu}$-g-continuous mapping, $\mathcal{N}_{eu}$-semi continuous mapping, $\mathcal{N}_{eu}$-generalized semi-pre-continuous mapping, and $\mathcal{N}_{eu}$-w-continuous mapping, as seen in the subsequent example.

Example 4.6

(i) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ = $\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.60 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.20, 0.50, 0.60 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t)$ = z. Then f is $\mathcal{N}_{eu}$-g-continuous but not $\mathcal{N}_{eu}$-λ-continuous.

(ii) Let X = {q, t}, and Y = {w, z}. Define the $\mathcal{N}_{eu}$-topologies ζ = $\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.20, 0.50, 0.50 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.50 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t)$ = z. Then f is $\mathcal{N}_{eu}$-λ-continuous, but it is neither $\mathcal{N}_{eu}$-g- continuous nor $\mathcal{N}_{eu}$-semi continuous.

(iii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ = $\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.60 >\}$, and P = $\{< w, 0.20, 0.50, 0.80 >, < z, 0.10, 0.50, 0.90 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t)$ = z. Then f is $\mathcal{N}_{eu}$-semi continuous but not $\mathcal{N}_{eu}$-λ-continuous.

(iv) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.30 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.20, 0.50, 0.80 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Then f is $\mathcal{N}_{eu}$-λ-continuous but not $\mathcal{N}_{eu}$-continuous.

(v) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.30 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.20, 0.50, 0.80 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Then f is $\mathcal{N}_{eu}$-λ-continuous but not $\mathcal{N}_{eu}$-generalized semi-pre-continuous.

(vi) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.30 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.40 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Then f is $\mathcal{N}_{eu}$-generalized semi-pre-continuous but not $\mathcal{N}_{eu}$-λ-continuous.

(vii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.40 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.50 >\}$. Then f is $\mathcal{N}_{eu}$-λ-continuous but not $\mathcal{N}_{eu}$-w-continuous.

(viii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.60 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.50 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.50 >\}$, then f^{-1}(N) is not $\mathcal{N}_{eu}$-λ-closed set, but it is $\mathcal{N}_{eu}$-w-closed set. Hence, f is $\mathcal{N}_{eu}$-w-continuous but not $\mathcal{N}_{eu}$-λ-continuous.

Here we have the following representation among different kinds of $\mathcal{N}_{eu}$-continuous mappings.

$\mathcal{N}_{eu}$-continuous mapping ⟶ $\mathcal{N}_{eu}\lambda$-continuous mapping ⟵ $\mathcal{N}_{eu}$-pre-continuous mapping

Theorem 4.7 Let f: X ⟶ Y be any $\mathcal{N}_{eu}$-λ-continuous mapping.

(i) For any$\mathcal{N}_{eu}$-point $p_{m,i,n} \in$ X, and any $\mathcal{N}_{eu}$-neighbourhood N of $f(p_{m,i,n})$, there exists a $\mathcal{N}_{eu}$-λ-open set O in X such that $p_{m,i,n} \in$ O $\subseteq f^{-1}$(N).

(ii) For any$\mathcal{N}_{eu}$-point $p_{m,i,n} \in$ X, and any $\mathcal{N}_{eu}$-neighbourhood N of $f(p_{m,i,n})$, there exists a $\mathcal{N}_{eu}$-λ-open set O in X such that $p_{m,i,n} \in$ O, and f(O) ⊆ N.

Proof:

(i) Let $p_{m,i,n} \in$ X, and let N be arbitrary $\mathcal{N}_{eu}$-neighbourhood of $f(p_{m,i,n})$. Then there exists $\mathcal{N}_{eu}$-open set O in Y, provided that $f(p_{m,i,n}) \in$ O ⊆ N. As f is $\mathcal{N}_{eu}$-λ-continuous, $f^{-1}(O)$ is $\mathcal{N}_{eu}$-λ-open set in X, and $p_{m,i,n} \in f^{-1}(f(p_{m,i,n})) \subseteq f^{-1}(O) \subseteq f^{-1}$(N).

(ii) Let $p_{m,i,n} \in$ X, and let N be arbitrary $\mathcal{N}_{eu}$-neighbourhood of $f(p_{m,i,n})$. From (i), there exists $\mathcal{N}_{eu}$-λ-open set O in X, provided that $p_{m,i,n} \in$ O $\subseteq f^{-1}$(N). Thus, $p_{m,i,n} \in$ O, and $f(O) \subseteq f(f^{-1}$(N) ⊆ N. That is, $p_{m,i,n} \in$ O, and $f(O)$ ⊆ N.

Theorem 4.8 Consider mappings f: X ⟶ Y, and g: Y ⟶ Z in such a way that the mapping g is $\mathcal{N}_{eu}$-continuous, and the mapping f is $\mathcal{N}_{eu}$-λ-continuous then composition mapping g ∘ f is $\mathcal{N}_{eu}$-λ-continuous.

Proof: Let N be any $\mathcal{N}_{eu}$-closed set in Z. As g is $\mathcal{N}_{eu}$-continuous, g^{-1}(N) is $\mathcal{N}_{eu}$-closed set in Y, and as f is $\mathcal{N}_{eu}$-λ-continuous, $f^{-1}(g^{-1}(N))$ is $\mathcal{N}_{eu}$-λ-closed set in X, and we know that $f^{-1}(g^{-1}(N)) = (g \circ f)^{-1}(N)$, consequently, $g \circ f$ is $\mathcal{N}_{eu}$-λ-continuous mapping.

Definition 4.9 Any $\mathcal{N}_{eu}$-topological space X is called $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space if every $\mathcal{N}_{eu}$-λ-closed set is $\mathcal{N}_{eu}$-closed set.

Theorem 4.10 If a mapping f: X ⟶ Y is $\mathcal{N}_{eu}$-λ-continuous, and the space X is $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space then the mapping f is $\mathcal{N}_{eu}$-continuous.

Proof: Let N be any $\mathcal{N}_{eu}$-closed set in Y. As the mapping f is $\mathcal{N}_{eu}$-λ-continuous, f^{-1}(N) is $\mathcal{N}_{eu}$-λ-closed set in X. By given hypothesis X is $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space, therefore, f^{-1}(N) is $\mathcal{N}_{eu}$-closed set in X. Hence, the mapping f is $\mathcal{N}_{eu}$-continuous mapping as pre-image of every $\mathcal{N}_{eu}$-closed set in Y is $\mathcal{N}_{eu}$-closed in X.

Theorem 4.11 Any $\mathcal{N}_{eu}$-topological space X is $\mathcal{N}_{eu}$-λ-$T_{\frac{1}{2}}$ space if and only if $\mathcal{N}_{eu}$-λ-$OS(X)$ = $\mathcal{N}_{eu}$-$OS(X)$.

Proof: Let N ∈ $\mathcal{N}_{eu}$-λ-$OS(X)$, then the supplement of N (or N′) is $\mathcal{N}_{eu}$-λ-closed set in X. As X is $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space, N′ is $\mathcal{N}_{eu}$-closed set in X, and hence, N is $\mathcal{N}_{eu}$-open set in X. That is N ∈ $\mathcal{N}_{eu}$-$OS(X)$ Thus, $\mathcal{N}_{eu}$-λ-$OS(X)$ ⊆ $\mathcal{N}_{eu}$-$OS(X)$. Now, using Theorem 4.2, we have $\mathcal{N}_{eu}$-$OS(X)$ ⊆ $\mathcal{N}_{eu}$-λ-$OS(X)$. Hence, $\mathcal{N}_{eu}$-λ-$OS(X)$=$\mathcal{N}_{eu}$-$OS(X)$. Conversely, as N is an arbitrary $\mathcal{N}_{eu}$-λ-closed set in X, N′ is $\mathcal{N}_{eu}$-λ- open set in X, and so N′ ∈ $\mathcal{N}_{eu}$-$OS(X)$, as $\mathcal{N}_{eu}$-λ-$OS(X)$ = $\mathcal{N}_{eu}$-$OS(X)$, therefore, N is $\mathcal{N}_{eu}$-closed set. Hence, X is $\mathcal{N}_{eu}$-λ-$T_{\frac{1}{2}}$ space.

5. Neutrosophic-Closed Mappings

In this section, we introduce and study $\mathcal{N}_{eu}$-closed mappings with their characteristics.

Definition 5.1 A mapping f: X ⟶ Y is called

(i) $\mathcal{N}_{eu}$-λ-closed mapping if f(N) is $\mathcal{N}_{eu}$-λ-closed set in Y for each $\mathcal{N}_{eu}$-closed set N in X.
(ii) $\mathcal{N}_{eu}$-w-closed mapping if f(N) is $\mathcal{N}_{eu}$-w-closed set in Y for each $\mathcal{N}_{eu}$-closed set N in X.
(iii) $\mathcal{N}_{eu}$-Gα–closed mapping if f(N) is $\mathcal{N}_{eu}$-generalized α–closed set in Y for each $\mathcal{N}_{eu}$-closed set N in X.
(iv) $\mathcal{N}_{eu}$-αG-closed mapping if f(N) is $\mathcal{N}_{eu}$-α-generalized closed set in Y for each $\mathcal{N}_{eu}$-closed set N in X.

Theorem 5.2 Each $\mathcal{N}_{eu}$-closed mapping f: X ⟶ Y is $\mathcal{N}_{eu}$-λ-closed mapping.

Proof: Consider a mapping f: X ⟶ Y, and consider any $\mathcal{N}_{eu}$-closed set N in X. Then f(N) is $\mathcal{N}_{eu}$-closed set in Y. Since, each $\mathcal{N}_{eu}$-closed set in X is $\mathcal{N}_{eu}$-λ-closed set, f(N) is $\mathcal{N}_{eu}$-λ-closed set in Y. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping.

In general, the reverse of the above theorem is not true, as seen in the subsequent example.

Example 5.3 Let X = $\{q, t\}$ and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ = $\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.20, 0.50, 0.80 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.30, 0.50, 0.60 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t)$ = z. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.80, 0.50, 0.20 >\}$ be the $\mathcal{N}_{eu}$-closed set in X, then the image of N, given by f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.80, 0.50, 0.20 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.80, 0.50, 0.20 >\}$ is $\mathcal{N}_{eu}$-λ-closed set in Y as $\mathcal{N}_{eu}$-cl($\tilde{0}$) = $\tilde{0}$ ⊆ A, and $\mathcal{N}_{eu}$-cl(P) = $\{< \mathrm{w}, 0.50, 0.50, 0.50 >, < \mathrm{z}, 0.60, 0.50, 0.30 >\}$ ⊆ N, whenever $\tilde{0}$

$\subseteq$ N, and P $\subseteq$ N, respectively (using Definition 3.1). But N is not $\mathcal{N}_{eu}$-closed in Y as $\mathcal{N}_{eu}$-cl(N) $=\tilde{1} \neq$ N. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping but not $\mathcal{N}_{eu}$-closed mapping.

Theorem 5.4 If f: X $\longrightarrow$ Y is $\mathcal{N}_{eu}$-λ-closed mapping and Y is $\mathcal{N}_{eu}$-λ-$T_{\frac{1}{2}}$ space, then f is $\mathcal{N}_{eu}$-closed mapping.

Proof: Let N be an arbitrary $\mathcal{N}_{eu}$-closed set in X. As f is $\mathcal{N}_{eu}$-λ-closed mapping, f(N) is $\mathcal{N}_{eu}$-λ-closed set in Y. As Y is $\mathcal{N}_{eu}$-λ-$T_{\frac{1}{2}}$ space, f(N) is $\mathcal{N}_{eu}$-closed set in Y. Hence, f is $\mathcal{N}_{eu}$-closed mapping.

Note: The concept of $\mathcal{N}_{eu}$-λ-closed mapping is independent of the concepts of $\mathcal{N}_{eu}$-w-closed mappings $\mathcal{N}_{eu}$-g–closed mapping, $\mathcal{N}_{eu}$-semi closed mapping, $\mathcal{N}_{eu}$-Gα–closed mapping, and $\mathcal{N}_{eu}$-αG-closed mapping, as seen in the subsequent example.

Example 5.5

(i) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.20 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-λ-closed set in Y but not $\mathcal{N}_{eu}$-w-closed set. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping but not $\mathcal{N}_{eu}$-w-closed mapping.

(ii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.50 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.60 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.50 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.50 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.50 >\}$ is not $\mathcal{N}_{eu}$-λ-closed set, but it is $\mathcal{N}_{eu}$-w-closed set. Hence, f is $\mathcal{N}_{eu}$-w-closed mapping but not $\mathcal{N}_{eu}$-λ-closed mapping.

(iii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.30 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.20, 0.50, 0.60 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.60 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.60 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.30, 0.50, 0.60 >\}$ is not $\mathcal{N}_{eu}$-λ-closed set, but it is $\mathcal{N}_{eu}$-g-closed set. Hence, f is $\mathcal{N}_{eu}$-g-closed mapping but not $\mathcal{N}_{eu}$-λ-closed mapping.

(iv) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.20 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.40 >\}$ is $\mathcal{N}_{eu}$-λ-closed set but not $\mathcal{N}_{eu}$-g-closed set. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping but not $\mathcal{N}_{eu}$-g-closed mapping.

(v) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.10, 0.50, 0.90 >\}$. Define f: X $\longrightarrow$ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, <$

$z, 0.50, 0.50, 0.40 >\}$ is not $\mathcal{N}_{eu}$-λ-closed set, but it is $\mathcal{N}_{eu}$-semi-closed set. Hence, f is $\mathcal{N}_{eu}$-semi-closed mapping but not $\mathcal{N}_{eu}$-λ-closed mapping.

(vi) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ =$\{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.20 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.40 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.20, 0.50, 0.50 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.20, 0.50, 0.50 >\}) = \{< w, 0.50, 0.50, 0.50 >, < z, 0.20, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-λ-closed set but not $\mathcal{N}_{eu}$-semi-closed set. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping but not $\mathcal{N}_{eu}$-semi-closed mapping.

(vii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.30 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.30 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.30 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.30, 0.50, 0.50 >\}) = \{< w, 0.50, 0.50, 0.50 >, < z, 0.30, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-λ-closed set but not $\mathcal{N}_{eu}$-semi-pre-closed set. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping but not $\mathcal{N}_{eu}$-semi-pre-closed mapping.

(viii) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.60 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.10, 0.50, 0.90 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.40 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.60, 0.50, 0.40 >\}) = \{< w, 0.50, 0.50, 0.50 >, < z, 0.60, 0.50, 0.40 >\}$ is $\mathcal{N}_{eu}$-semi-pre-closed set but not $\mathcal{N}_{eu}$-λ-closed set. Hence, f is $\mathcal{N}_{eu}$-semi-pre-closed mapping but not $\mathcal{N}_{eu}$-λ-closed mapping.

(ix) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies $\zeta = \{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M = $\{< q, 0.40, 0.50, 0.60 >, < t, 0.30, 0.50, 0.70 >\}$, and P = $\{< w, 0.20, 0.50, 0.80 >, < z, 0.30, 0.50, 0.70 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.60, 0.50, 0.40 >, < t, 0.70, 0.50, 0.30 >\}$, then f(N) = $f(\{< q, 0.60, 0.50, 0.40 >, < t, 0.70, 0.50, 0.30 >\}) = \{< w, 0.60, 0.50, 0.40 >, < z, 0.70, 0.50, 0.30 >\}$ is $\mathcal{N}_{eu}$-Gα−closed set but not $\mathcal{N}_{eu}$-λ-closed set. Hence, f is $\mathcal{N}_{eu}$-Gα-closed mapping but not $\mathcal{N}_{eu}$-λ-closed mapping.

(x) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ =$\{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.10, 0.50, 0.90 >, < t, 0.30, 0.50, 0.70 >\}$, and P = $\{< w, 0.80, 0.50, 0.20 >, < z, 0.80, 0.50, 0.10 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.90, 0.50, 0.10 >, < t, 0.70, 0.50, 0.30 >\}$, then f(N) = $f(\{< q, 0.90, 0.50, 0.10 >, < t, 0.70, 0.50, 0.30 >\}) = \{< w, 0.90, 0.50, 0.10 >, < z, 0.70, 0.50, 0.30 >\}$ is not $\mathcal{N}_{eu}$-Gα−closed set but it is $\mathcal{N}_{eu}$-λ-closed set. Hence, f is not $\mathcal{N}_{eu}$-Gα-closed mapping but it is $\mathcal{N}_{eu}$-λ-closed mapping.

(xi) Let X = $\{q, t\}$, and Y = $\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ =$\{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.60, 0.50, 0.40 >, < t, 0.70, 0.50, 0.20 >\}$, and P = $\{< w, 0.20, 0.50, 0.60 >, < z, 0.20, 0.50, 0.70 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.40, 0.50, 0.60 >, < t, 0.20, 0.50, 0.70 >\}$, then f(N) = $f(\{< q, 0.40, 0.50, 0.60 >, < t, 0.20, 0.50, 0.70 >\}) = \{< w, 0.40, 0.50, 0.60 >, < z, 0.20, 0.50, 0.70 >\}$ is $\mathcal{N}_{eu}$-αG-closed set in Y but not $\mathcal{N}_{eu}$-λ-closed set in Y. Hence, f is $\mathcal{N}_{eu}$-αG-closed mapping but not $\mathcal{N}_{eu}$-λ-closed mapping.

(xii) Let X = $\{q, t\}$, and Y =$\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ =$\{\tilde{0}, \tilde{1}, M\}$, and $\eta = \{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.10, 0.50, 0.50 >, < t, 0.20, 0.50, 0.60 >\}$, and P = $\{< w, 0.20, 0.50, 0.40 >, < z, 0.30, 0.50, 0.50 >\}$. Define f: X ⟶ Y such that $f(q)$ = w, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.10 >, < t, 0.60, 0.50, 0.20 >\}$, then f(N) = $f(\{< q, 0.50, 0.50, 0.10 >, < t, 0.60, 0.50, 0.20 >\}) = \{< w, 0.50, 0.50, 0.10 >, <$

$z, 0.60, 0.50, 0.20 >\}$ is $\mathcal{N}_{eu}$-λ-closed set in Y but not $\mathcal{N}_{eu}$-αG-closed set in Y. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping but not $\mathcal{N}_{eu}$-αG-closed mapping.

Remark 5.6 Each $\mathcal{N}_{eu}$-pre-closed mapping f: X ⟶ Y is $\mathcal{N}_{eu}$-λ-closed mapping, but the reverse is not true as seen in the subsequent example.

Example 5.7 Let X = $\{q, t\}$ and Y=$\{w, z\}$. Define the $\mathcal{N}_{eu}$-topologies ζ =$\{\tilde{0}, \tilde{1}, M\}$, and η = $\{\tilde{0}, \tilde{1}, P\}$ on X, and Y resp., where M =$\{< q, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$, and P = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.50, 0.50, 0.20 >\}$. Define f: X ⟶ Y such that $f(q) = w$, and $f(t) = z$. Let N = $\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\}$ be the $\mathcal{N}_{eu}$-set in X, then f(N) = $f(\{< q, 0.50, 0.50, 0.50 >, < t, 0.40, 0.50, 0.50 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < z, 0.40, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$-λ-closed set in Y, but it is not $\mathcal{N}_{eu}$-pre-closed set in Y. Hence, f is $\mathcal{N}_{eu}$-λ-closed mapping, but not $\mathcal{N}_{eu}$-pre-closed mapping.

Here, we have the following representation among different kinds of $\mathcal{N}_{eu}$-closed mappings.

$\mathcal{N}_{eu}$-closed mapping ⟶ $\mathcal{N}_{eu}\lambda$-closed mapping ⟵ $\mathcal{N}_{eu}$-pre-closed mapping

Theorem 5.8 Let f be any $\mathcal{N}_{eu}$-λ-closed mapping. Then for each $\mathcal{N}_{eu}$- set N in X, $f(\mathcal{N}_{eu}$-cl(N)) is $\mathcal{N}_{eu}$-λ-closed set in Y.

Proof: Let N be any $\mathcal{N}_{eu}$-set in X. Then $\mathcal{N}_{eu}$-cl(N) is $\mathcal{N}_{eu}$-closed set in X. As f is $\mathcal{N}_{eu}$-λ-closed mapping, $f(\mathcal{N}_{eu}$-cl(N)) is $\mathcal{N}_{eu}$-λ-closed set in Y.

Theorem 5.9 If f: X ⟶ Y is $\mathcal{N}_{eu}$-closed mapping and g: Y → Z is $\mathcal{N}_{eu}$-λ-closed mapping, then the composition mapping g ∘ f is $\mathcal{N}_{eu}$-λ-closed mapping.

Proof. As f is $\mathcal{N}_{eu}$-closed mapping, for any $\mathcal{N}_{eu}$-closed set N in X, f(N) is $\mathcal{N}_{eu}$-closed set in Y, and since g is $\mathcal{N}_{eu}$-λ-closed mapping, g(f(N)) is $\mathcal{N}_{eu}$-λ-closed set in Z. Hence, g ∘ f is $\mathcal{N}_{eu}$-λ-closed mapping.

Remark 5.10 The composition of two $\mathcal{N}_{eu}$-λ-closed mappings is not $\mathcal{N}_{eu}$-λ-closed mapping, as seen in the subsequent example.

Example 5.11 Let X = $\{k, p\}$, Y = $\{w, t\}$ and Z = $\{d, j\}$. Let M = $\{< k, 0.50, 0.50, 0.50 >, < p, 0.50, 0.50, 0.20 >\}$, Q = $\{< w, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$, and P = $\{< d, 0.50, 0.50, 0.50 >, < j, 0.60, 0.50, 0.40 >\}$ be $\mathcal{N}_{eu}$-sets in X, Y, and Z resp., and let ζ = $\{\tilde{0}, \tilde{1}, M\}$, η = $\{\tilde{0}, \tilde{1}, Q\}$, and σ = $\{\tilde{0}, \tilde{1}, P\}$, be $\mathcal{N}_{eu}$-topologies on X, Y, and Z resp. Define a mapping f: X ⟶ Y such that $f(k) = w$, and $f(p) = t$, then $f(\{< k, 0.50, 0.50, 0.50 >, < p, 0.20, 0.50, 0.50 >\})$ = $\{< w, 0.50, 0.50, 0.50 >, < t, 0.20, 0.50, 0.50 >\}$ is $\mathcal{N}_{eu}$- λ-closed set in Y. Thus, f is $\mathcal{N}_{eu}$-λ-closed mapping. Now, consider g: Y → Z such that g(w) = d, and g(t) = j, then g($\{< w, 0.50, 0.50, 0.50 >, < t, 0.50, 0.50, 0.40 >\}$)=$\{< d, 0.50, 0.50, 0.50 >$, $j, 0.50, 0.50, 0.40 >\}$ is $\mathcal{N}_{eu}$-λ-closed set in Z. Hence, g is $\mathcal{N}_{eu}$-λ-closed mapping. But the mapping g ∘ f: X → Z is not $\mathcal{N}_{eu}$-λ-closed mapping, as g(f(N)) = g($f(\{< k, 0.50, 0.50, 0.50 >, < p, 0.20, 0.50, 0.50 >\})$) = g($\{< w, 0.50, 0.50, 0.50 >, < t, 0.20, 0.50, 0.50 >\}$) = $\{< d, 0.50, 0.50, 0.50 >, < j, 0.20, 0.50, 0.50 >\}$ is not $\mathcal{N}_{eu}$-λ-closed set in Z. Hence, g(f(A)) is not $\mathcal{N}_{eu}$-λ-closed mapping. That is, g ∘ f is not $\mathcal{N}_{eu}$-λ-closed mapping.

6. Conclusion and Future Work

In this paper, we introduced the concepts of $\mathcal{N}_{eu}$-λ-closed and $\mathcal{N}_{eu}$-λ-open sets in $\mathcal{N}_{eu}$-topological spaces and explored their fundamental properties. Through a comparative analysis, we established that $\mathcal{N}_{eu}$-λ-closed sets are weaker than $\mathcal{N}_{eu}$-closed and $\mathcal{N}_{eu}$-pre-closed sets, but independent of the concepts of $\mathcal{N}_{eu}$-g-closed, $\mathcal{N}_{eu}$-semi-closed, and $\mathcal{N}_{eu}$-w-closed sets. As an application, we proposed and examined $\mathcal{N}_{eu}$-λ-continuous mapping, $\mathcal{N}_{eu}$-λ-closed mapping, and $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space. We have studied various characteristics of these mappings using $\mathcal{N}_{eu}$-λ-$T_{1/2}$ space. We also compare these newly defined mappings with various existing mappings, and it is found that $\mathcal{N}_{eu}$-λ-continuous mapping is weaker than $\mathcal{N}_{eu}$-continuous and $\mathcal{N}_{eu}$-pre-continuous mappings, but independent of the concepts of $\mathcal{N}_{eu}$-g-continuous, $\mathcal{N}_{eu}$-semi continuous, and $\mathcal{N}_{eu}$-generalized semi pre-continuous mappings. Similarly, we established that $\mathcal{N}_{eu}$-λ-closed mapping is weaker than $\mathcal{N}_{eu}$-closed and $\mathcal{N}_{eu}$-pre-closed mappings, but independent of the concepts of $\mathcal{N}_{eu}$-g-closed, and $\mathcal{N}_{eu}$-semi closed mappings. Some new notions like $\mathcal{N}_{eu}$-w-continuous mapping, $\mathcal{N}_{eu}$-w-closed mapping, $\mathcal{N}_{eu}$-Gα−closed mapping, and $\mathcal{N}_{eu}$-αG-closed mapping are discussed, and it is found that these notions are independent of $\mathcal{N}_{eu}$-λ-continuous mapping, and $\mathcal{N}_{eu}$-λ-closed mappings, respectively. These findings contribute to the ongoing development of topology, and open avenues for future research. Future work can extend these concepts in the study of homeomorphisms, connectedness, compactness, and so on.

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