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On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties

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Abstract:

The aim of this paper is to define and study the concept of symbolic 3-plithogenic rings as a novel extension of classical rings and symbolic 2-plithogenic rings respectively. Also, many related substructures will be presented such as idempotent elements, AH-ideals, AHS-homomorphisms, and kernels.

On the other hand, many examples will be illustrated to show the validity of concepts and theorem.

Keywords: Symbolic 3-plithogenic ring, AH-ideal, AH-homomorphism, symbolic plithogenic set

Introduction

The concept of symbolic neutrosophic algebraic structure has played an important role in the advances of pure algebra and logical algebra. Many interesting structures were defined from this point of view, such as neutrosophic rings, refined neutrosophic rings, neutrosophic spaces, and n-cyclic refined neutrosophic rings [1-5,8-11,13-20].

In [30], Smarandache has presented a novel approach to algebraic structures by using the concept of n-symbolic plithogenic sets, where he defined algebraic operations on these structures and asked many open problems about them.

In [31], the concept of symbolic 2-plithogenic ring was suggested, and concepts such as symbolic 2-plithogenic AH-homomorphisms, ideals, and kernels.

This paper is considered as an additional effort which is dedicated to define a new algebraic structure built over the idea of symbolic n-plithogenic set with algebraic ring in a special case of n=3.

Main Discussion

Definition.

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:

$3-SP_R=\{a_0+a_1P_1+a_2P_2+a_3P_3;\ a_i\in R, {P_j}^2=P_j, P_i\times P_j=P_{max(i,j)}\}$.

Smarandache has defined algebraic operations on $3-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2+a_3P_3]+[b_0+b_1P_1+b_2P_2+b_3P_3]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2+(a_3+b_3)P_3$.

Multiplication:

$[a_0+a_1P_1+a_2P_2+a_3P_3].[b_0+b_1P_1+b_2P_2+b_3P_3]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_0b_3P_3+a_1b_0{P_1}^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2{P_2}^2+a_1b_3P_3P_1+a_2b_3P_2P_3+a_3b_3(P_3)^2+a_3b_0P_3+a_3b_1P_3P_1+a_3b_2P_2P_3+a_1b_1P_1P_1=a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2+(a_0b_3+a_1b_3+a_2b_3+a_3b_3+a_3b_0+a_3b_1+a_3b_2)P_3$.

It is clear that $(3-SP_R)$ is a ring.

Also, if R is commutative, then $3-SP_R$ is commutative, and if R has a unity (1), than $3-SP_R$ has the same unity (1).

Example.

Consider the ring $R=Z_5=\{0,1,2,3,4\}$, the corresponding $3-SP_R$ is:

$3-SP_R=\{a+bP_1+cP_2+dP_3; a,b,c,d\in Z_5\}$.

If $X=1+2P_1+3P_2+P_3, Y=P_1+2P_2$, then:

$X+Y=1+3P_1+P_2+P_3, X-Y=1+P_1+P_2+P_3, X.Y=P_1+2P_2+2P_1+4P_2+3P_2+6P_2+P_3+2P_3=3P_1+3P_2+3P_3$.

Invertibility.

Theorem.

Let $3-SP_R$ be a 3-plithogenic symbolic ring, with unity (1).

Let $X=x_0+x_1P_1+x_2P_2+x_3P_3$ be an arbitrary element, then:

1. X is invertible if and only if $x_0, x_0+x_1, x_0+x_1+x_2$, $x_0+x_1+x_2+x_3$ are invertible.
2. $X^{-1} = {x_0}^{-1} + [(x_0+x_1)^{-1} - {x_0}^{-1}]P_1 + [(x_0+x_1+x_2)^{-1} - (x_0+x_1)^{-1}]P_2$ +[$(x_0 + x_1 + x_2 + x_3)^{-1} - (x_0+x_1+x_2)^{-1}]P_3$.

Proof.

1. Assume that X is invertible, than there exists $Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3$ such that $X.Y = 1$, hence:

$$\begin{cases} x_0y_3 + x_1y_3 + x_2y_3 + x_3y_3 + x_3y_1 + x_3y_2 + x_3y_0 = 0 \ (1) \\ x_0y_0 = 1 \ldots \ (2) \\ x_0y_1 + x_1y_0 + x_1y_1 = 0 \ldots \ (3) \\ x_0y_2 + x_2y_0 + x_2y_2 + x_1y_2 + x_2y_1 = 0 \ldots \ (4), \end{cases}$$

Equation (2), means that x_0 is invertible.

By adding (3) to (2), we get $(x_0+x_1)(y_0+y_1) = 1$, thus x_0+x_1 is invertible.

By adding (4) to $(3) to$ (2), we get $(x_0+x_1+x_2)(y_0+y_1+y_2) = 1$, hence $x_0+x_1+x_2$ is invertible.

By adding (1) to (2) to (3) to(4), we get $(x_0+x_1+x_2+x_3)(y_0+y_1+y_2+y_3) = 1$, hence $x_0+x_1+x_2$+x_3 is invertible.

The converse holds by the same.

2. From the previous approach, we can see that:

$y_0 = {x_0}^{-1}, y_0+y_1 = (x_0+x_1)^{-1}, y_0+y_1+y_2 = (x_0+x_1+x_2)^{-1}, (x_0+x_1+x_2+x_3)^{-1} = y_0+y_1+y_2+y_3$, then:

3. $Y = {x_0}^{-1} + [(x_0+x_1)^{-1} - {x_0}^{-1}]P_1 + [(x_0+x_1+x_2)^{-1} - (x_0+x_1)^{-1}]P_2$ +$[(x_0+x_1+ x_2+x_3)^{-1} - (x_0+x_1+x_2)^{-1}]P_3$.

$= X^{-1}$.

Example.

Take $R = Z_5 = \{0,1,2,3,4\}$, $3 - SP_{Z_5}$ is the corresponding symbolic 3-plithogenic ring, consider $X = 2 + 4P_1 + 2P_2 + P_3 \in 2 - SP_{Z_5}$, then:

$x_0 = 2$ is invertible with ${x_0}^{-1} = 3$, $x_0+x_1 = 1$ is invertible with $(x_0+x_1)^{-1} = 1$, $x_0 + x_1+x_2 = 3$ is invertible with $(x_0+x_1+x_2)^{-1} = 2$, $x_0+x_1+x_2+x_3 = 4, (x_0+x_1+x_2+ x_3)^{-1} = 4$ hence:

$X^{-1} = 3 + (1-3)P_1 + (2-1)P_2 + (4-2)P_3 = 3 + 3P_1 + P_2 + 2P_3$.

Idempotency.

Definition.

Let $X = a + bP_1 + cP_2 + dP_3 \in 3 - SP_R$, then X is idempotent if and only if $X^2 = X$.

Theorem.

Let $X = a + bP_1 + cP_2 + dP_3 \in 3 - SP_R$, then X is idempotent if and only if $a, a + b, a + b + c$, $a + b + c + d$ are idempotent.

Proof.

$X^2 = X.X = (a + bP_1 + cP_2 + dP_3)(a + bP_1 + cP_2 + dP_3) = a^2 + (ab + ba + b^2)P_1 + (ac + bc + ca + cb + c^2)P_2 + (ad + bd + cd + d.d + da + db + dc)P_3$.

$$X^2 = X.X \text{ equivalents } \begin{cases} ad + bd + cd + d.d + da + db + dc = 0 \ (1) \\ a^2 = a \ldots \ (2) \\ ab + ba + b^2 = b \ldots \ (3) \\ ac + bc + ca + cb + c^2 = c \ldots \ (4) \end{cases}$$

Equation (2) means that a is idempotent.

By adding (3) to (2), we get $(a + b)^2 = a + b$, hence $a + b$ is idempotent.

By adding (3) to (2) to (4), we get $(a + b + c)^2 = a + b + c$, hence $a + b + c$ is idempotent.

By adding (1) to (2) to (3) to (4), we get $(a + b + c + d)^2 = a + b + c + d$, thus $a + b + c + d$ is idempotent.

Thus the proof is complete.

Example.

Take $R = Z_6 = \{0,1,2,3,4,5\}$, $3 - SP_{Z_6}$ is the corresponding symbolic 3-plithogenic ring, consider $X = 3 + P_1 + 5P_2 \in 3 - SP_{Z_5}$, we have:

$X^2 = 9 + 6P_1 + P_1 + 30P_2 + 25P_2 + 10P_2 = 3 + P_1 + 5P_2 = X$.

The following theorem clarifies the natural powers in $2 - SP_R$.

Theorem.

Let $3 - SP_R$ be a commutative symbolic 3-plithogenic ring, hence if $X = a + bP_1 + cP_2 + dP_3$, then $X^n = a^n + [(a + b)^n - a^n]P_1 + [(a + b + c)^n - (a + b)^n]P_2 + [(a + b + c + d)^n - (a + b + c)^n]P_3$ for every $n \in Z^+$.

Proof.

For $n = 1$, it holds easily. Assume that it is true for $n = k$, we prove it for $n = k + 1$.

$X^{k+1} = X.X^k = (a + bP_1 + cP_2 + dP_3)\big(a^k + [(a + b)^k - a^k]P_1 + [(a + b + c)^k - (a + b)^k]P_2 + [(a + b + c + d)^k - (a + b + c)^k]P_3\big) = a^{k+1} + [(a + b)^{k+1} - a^{k+1}]P_1 + [(a + b + c)^{k+1} - (a + b)^{k+1}]P_2 + [(a + b + c + d)^{k+1} - (a + b + c)^{k+1}]P_3$ So, that proof is complete by induction.

Example.

Take $R = Z$, the ring of integers. Let $3 - SP_Z$ be the corresponding symbolic 3-plithogenic ring, hence $X = 1 + 2P_1 + 3P_2 + P_3, X^3 = 1^3 + P_1[(3)^3 - 1^3] + P_2[(6)^3 - 3^3] + (7^3 - 6^3)P_3 = 1 + 26P_1 + 189P_2 + 127P_3$

Definition.

X is called nilpotent if there exists $n \in Z^+$ such that $X^n = 0$.

Theorem.

Let $X \in 3 - SP_R$, where R is a commutative ring, then X is nilpotent if and only if $a, a + b, a + b + c, a + b + c + d$ are nilpotent.

Proof.

$X = a + bP_1 + cP_2 + dP_3$ is nilpotent if and only if there exists $n \in Z^+$ such that $X^n = 0$, hence:

$$\begin{cases} (a+b+c+d)^n - (a+b+c)^n = 0 \\ a^n = 0 \\ (a+b)^n - a^n = 0 \\ (a+b+c)^n - (a+b)^n = 0 \end{cases} \Leftrightarrow \begin{cases} (a+b+c+d)^n = 0 \\ a^n = 0 \\ (a+b)^n = 0 \\ (a+b+c)^n = 0 \end{cases}$$, thus the proof is complete.

Definition.

Let Q_0, Q_{13}, Q_2, Q_3 be ideals of the ring R, we define the symbolic 3-plithogenic AH-ideal as follows:

$Q = Q_0 + Q_1P_1 + Q_2P_2 + Q_3P_3 = \{x_0 + x_1P_1 + x_2P_2 + x_3P_3;\ x_i \in Q_i\}$.

If $Q_0 = Q_1 = Q_2 = Q_3$, then Q is called an AHS-ideal.

Example.

Let $R = Z$ be the ring of integers, then $Q_0 = 2Z, Q_1 = 3Z, Q_2 = 5Z$ are ideals of R.

$Q = \{2m + 3nP_1 + 5tP_2 + 5sP_3;\ m.n.t, s \in Z\}$ is an AHS-ideal of $3 - SP_Z$.

$M = \{2m + 2nP_1 + 2tP_2 + 2sP_3;\ m.n.t, s \in Z\}$ is an AHS-ideal of $3 - SP_Z$.

Theorem.

Let Q be an AHS- ideal of $3 - SP_R$, then Q is an ideal by the classical meaning.

Proof.

Q can be written as $Q = Q_0 + Q_0P_1 + Q_0P_2 + Q_0P_3$, where Q_0 is an ideal of R. It is clear that $(Q, +)$ is a subgroup of $(3 - SP_R, +)$.

Let $S = s_0 + s_1P_1 + s_2P_2 + s_3P_3 \in 3 - SP_R$, then if $X = a + bP_1 + cP_2 + dP_3 \in Q$, we have:

$S.X = s_0a + (s_0b + s_1a + s_1b)P_1 + (s_0c + s_1c + s_2a + s_2b + s_2c)P_2 + (s_0d + s_1d + s_2d + s_3d + s_3a + s_3b + s_3c)P_3 \in Q$, that is because:

$s_0a \in Q_0, s_0b + s_1a + s_1b \in Q_0, s_0c + s_1c + s_2a + s_2b + s_2c, s_0d + s_1d + s_2d + s_3d + s_3a + s_3b + s_3c \in Q_0$.

Definition.

Let R, T be two rings, $3 - SP_R, 3 - SP_T$ are the corresponding symbolic 3-plithogenic rings, let $f_0, f_1, f_2, f_3: R \to T$ be four homomorphisms, we define the AH-homomorphism as follows:

$f: 3 - SP_R \to 3 - SP_T$ such that:

$f(a + bP_1 + cP_2 + dP_3) = f_0(a) + f_1(b)P_1 + f_2(c)P_2 + f_3(d)P_3$

If $f_0 = f_1 = f_2 = f_3$, then f is called AHS-homomorphism.

Remark.

If f_0, f_1, f_2, f_3 is isomorphisms, then f is called AH-isomorphism.

Example.

Take $R = Z,\ T = Z_6,\ f_0, f_1: R \to T$ such that:

$f_0(x) = x(mod\ 6), f_1(2) = 3x(mod\ 6)$. It is clear that f_0, f_1 are homomorphisms.

We define $f: 3 - SP_R \to 3 - SP_T$, where:

$f(x + yP_1 + zP_2 + sP_3) = f_0(x) + f_1(y)P_1 + f_2(z)P_2 + f_2(s)P_3 = x(mod\ 6) + y(mod\ 6)P_1 + (3z\ mod\ 6)P_2 + (3s\ mod\ 6)P_3$

Which is an AH-homomorphism.

Theorem.

Let $f = f_0 + f_1P_1 + f_2P_2 + f_3P_3: 3 - SP_R \to 3 - SP_T$ be a mapping, then:

1. If f is an AHS-homomorphism, then f is a ring homomorphism by the classical meaning.
2. If f is an AHS-homomorphism, then it is an isomorphism by the classical meaning.

Proof.

1. Assume that f is an AHS-homomorphism, then $f_0 = f_1 = f_2 = f_3$ are homomorphisms.

Let $X = x_0 + x_1P_1 + x_2P_2 + x_3P_3, Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3 \in 3 - SP_R$, we have:

$f(X + Y) = f_0(x_0 + y_0) + f_0(x_1 + y_1)P_1 + f_0(x_2 + y_2)P_2 + f_0(x_3 + y_3)P_3 = f(X) + f(Y)$

$f(X.Y) = f_0(x_0y_0) + f_0(x_0y_1 + x_1y_0 + x_1y_1)P_1 + f_0(x_0y_2 + x_2y_0 + x_2y_2 + x_2y_1 + x_1y_2)P_2 + f_0(x_0y_3 + x_1y_3 + x_2y_3 + x_3y_3 + x_3y_1 + x_3y_0 + x_3y_2)P_3 = f_0(x_0)f_0(y_0) + \big(f_0(x_0)f_0(y_1) + f_0(x_1)f_0(y_0) + f_0(x_1)f_0(y_1)\big)P_1 + \big(f_0(x_0)f_0(y_2) + f_0(x_2)f_0(y_0) + f_0(x_2)f_0(y_2) + f_0(x_2)f_0(y_1) + f_0(x_1)f_0(y_2)\big)P_2 + [\big(f_0(x_0)f_0(y_3) + f_0(x_1)f_0(y_3) + f_0(x_2)f_0(y_3) +$

$f_0(x_3)f_0(y_3) + f_0(x_3)f_0(y_1) + f_0(x_3)f_0(y_2) + f_0(x_3)f_0(y_0))]P_3 = [f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2 + f_0(x_3)P_3][f_0(y_0) + f_0(y_1)P_1 + f_0(y_2)P_2 + f_0(y_3)P_3] = f(X) + f(Y).$

So that, the [roof is complete.

2. By a similar discussion of statement 1, we get the proof.

Definition.

Let $f = f_0 + f_1P_1 + f_2P_2 + f_3P_3: 3 - SP_R \to 3 - SP_T$ be an AH-homomorphism, we define:

1. AH- $ker(f) = ker(f_0) + ker(f_1)P_1 + ker(f_2)P_2 + \ker(f_3)P_3 = \{m_0 + m_1P_1 + m_2P_2 + m_3P_3;\ m_i \in ker(f_i)\}$.
2. AH-factor $3 - SP_R/\text{AH} - ker(f) = R/ker(f_0) + R/ker(f_1)\,P_1 + R/ker(f_2)\,P_2 + + R/ker(f_3)\,P_3$

If $f_0 = f_1 = f_2 = f_3$, then we get an AHS- $ker(f)$ and AHS-factor.

Example.

Take $R = Z_{10},\ f_0: R \to T,\ f_0(x) = (x\ mod\ 10),\ ker(f_0) = 10Z.$

The corresponding AHS-homomorphism is $f = f_0 + f_0P_1 + f_0P_2 + f_0P_3: 3 - SP_R \to 3 - SP_T$, such that:

$$f(x_0 + x_1P_1 + x_2P_2) = f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2 + f_0(x_3)P_3$$
$$= (x_0\ mod\ 10) + (x_1\ mod\ 10)P_1 + (x_2 mod\ 10)P_2 + (x_3 mod\ 10)P_3$$

AHS-$ker(f) = 10Z + 10ZP_1 + 10ZP_2 = \{10x + 10yP_1 + 10zP_2 + 10sP_3;\ x, y, z, s \in Z\}$

AHS-factor= $Z/10Z + Z/10Z\,P_1 + Z/10Z\,P_2 + Z/10Z\,P_3$

Definition.

Let $(F, +, .)$ be a field, then $(3 - SP_F, +, .)$ Is called a symbolic 3-plithogenic field.

$(3 - SP_F, +, .)$ Is not a field in the algebraic meaning, that is because P_i are not invertible, but it is a ring.

Conclusion

In this paper, we have defined the concept of 3-plithogenic rings, and we presented many interesting algebraic properties such as invertibility, nilpotency, and idempotency of their elements.

Also, we have presented many related concepts such as AH-ideals, AH-kernels and homomorphisms with their elementary properties in terms of theorems with many clear examples.

In the future, we look for many symbolic 3-plithogenic structures, especially symbolic 3-pithogenic modules, vector spaces, and matrices.

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