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Multi-Attribute Decision-Making for Road Slope Treatment Selection Based on Spherical Single-Valued Neutrosophic Value Triangular Aggregation Operators

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Abstract: Areas in the East China Sea are often affected by bad weather such as typhoons and rainy seasons, so geological disasters such as road slope landslides and avalanches often occur in these areas. To prevent the geological disasters, it is necessary to perform comprehensive treatments of road slopes to ensure their stability and safety. Due to the uncertainty and vagueness of decision makers' judgements and cognitions in the evaluation process of slope treatment schemes, there is a Single-Valued Neutrosophic Value (SvNV) uncertainty in a neutrosophic decision scenario. To effectively express the hybrid information of a crisp SvNV and its uncertain space (sphere with a radius), we need to develop a Spherical Single-Valued Neutrosophic Set/Value (S-SvNS/S-SvNV) and its Multi-Attribute Decision Making (MADM) technique. Therefore, this study requires the following new content to address the current gaps in neutrosophic research. First, we propose an S-SvNS and the basic relations, trigonometric operation laws, and score and accuracy formulae of S-SvNVs. Second, the S-SvNV trigonometric weighted averaging and geometric aggregation operators are established for the aggregation of S-SvNVs. Third, a MADM technique based on the established two aggregation operators and the score and accuracy formulae of S-SvNVs is developed for solving MADM problems with unknown attribute weights and periodicity in the scenario of S-SvNSs. Fourth, the developed technique is applied to an actual selection example of road slope treatment schemes and then its efficiency is verified by sensitivity analysis and comparison with the existing MADM techniques under the scenarios of SvNSs and S-SvNSs.

Keywords: Spherical single-valued neutrosophic value; Trigonometric weighted averaging aggregation operator; Trigonometric weighted geometric aggregation operator; Decision making; Road slope treatment schemes

1. Introduction

The evaluation and selection of Slope Treatment Schemes (STSs) is a systematic process that requires comprehensive consideration of technical, economic, environmental and other factors. Through scientific and reasonable evaluation and selection, the risk of landslides and slope failures can be effectively reduced, and the safety of the surrounding environment and people can be ensured. Therefore, the slope stability evaluation and STS election are critical steps in ensuring slope stability and safety [1–4]. Since there are uncertainties and inaccuracies in the evaluation/prediction of slope stability and the selection of STSs, fuzzy theory [5] is a very suitable tool in uncertain scenarios. Therefore, a fuzzy comprehensive evaluation approach [6] and a fuzzy multi-objective and Group Decision Making (GDM) approach [7] were proposed for landslide treatment scheme selection. Then, fuzzy or triangular fuzzy Analytic Hierarchy Processing (AHP) techniques [8, 9] were presented for landslide susceptibility assessment. In terms of Intuitionistic Fuzzy Sets (IFSs) including membership and nonmembership degrees [10], Liu et al. [11] introduced a large-scale GDM approach based on IFSs and three-way decision and applied it to the selection of landslide treatment schemes. In the scenario of simplified neutrosophic sets (SNSs) [12], including interval-valued and/or single-valued neutrosophic sets (IvNSs and/or SvNSs), they can be represented independently by true, false, and indeterminate membership degrees, extending the expressive capability of IFSs. Recently, Yong et al. [13] presented a Multi-Attribute Decision Making (MADM) technique using the Aczel-Alsina Weighted Aggregation Operators (WAOs) of SNSs and applied it to the selection of STSs. Under the single and interval-valued hybrid neutrosophic multivalued scenario, Ye et al. [14] developed a GDM model using correlation coefficients of credibility IvNSs for the choice of landslide treatment schemes. Ye et al. [15] also proposed a MADM model using the trigonometric WAOs of Single-Valued Neutrosophic Values (SvNVs) and used it for the choice of STSs. In the single-valued neutrosophic credibility value (SvNCV) scenario, Ye et al. [16] presented a MADM model using the trigonometric WAOs of SvNCVs for the selection of STSs.

Based on another real extension of IFS, a Circular IFS (C-IFS) was proposed by Atanassov [17] and Atanassov and Marinov [18], where a Circular Intuitionistic Fuzzy Value (C-IFV) is composed of an IFV and a circle with a radius around each element to effectively describe the hybrid information of the exact IFV and its uncertain circle composed of membership and nonmembership degrees. Then, they also defined relations, operations, and distances for C-IFSs. After that, the divergence measures of C-IFSs were used for multi-periodic medical diagnosis [19]. The C-IFS AHP and VIKOR (VlseKriterijumska Optimizacija Kompromisno Resenje) approach [20] and the C-IFV TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) approach [21] were utilized for the multi-expert supplier evaluation and the choice of pandemic hospital sites, respectively. Some researchers developed the MADM methods based on the distances of C-IFSs [22, 23], the multiple criteria optimization and compromise solutions of C-IFSs [24], the score and accuracy formulae of C-IFVs [25], the interval-valued C-IFS AHP [26], the assignment model of C-IFVs with a parameterized scoring rule [27], the similarity and entropy measures of C-IFSs [28], and the compromise decision support and median ranking models using the scoring formulae of C-IFVs [29, 30].

In neutrosophic decision theories and methods, SvNSs [31] have been wildly applied to MADM/GDM problems because they can dependently depict true, false and indeterminate information in uncertain and inconsistent environments. Especially SvNV WAOs show a critical mathematical tool in MADM/GDM applications. For example, many researchers have developed Single-Valued Neutrosophic Value (SvNV) Einstein WAOs [32], Dombi WAOs [33], power Muirhead WAOs [34], Dombi power WAOs [35], Einstein interactive WAOs [36], trigonometric WAOs [15], and trigonometric Dombi WAOs [37] to address MADM/GDM problems in SvNS scenarios. However, SvNV cannot represent its uncertain evaluation value for each attribute, i.e., it cannot contain the hybrid information of both an exact SvNV and an uncertain spherical space composed of true, false, and indeterminate membership degrees. Therefore, existing SvNV MADM methods [15, 32–37] cannot deal with SvNV MADM/GDM problems including the uncertain SvNV information, which shows their research gap. On the other hand, since C-IFS cannot independently depict true, false, and indeterminate information and their uncertain spherical space with a radius, the C-IFV MADM techniques imply their limitations/insufficiencies. To effectively express the hybrid information of both an exact SvNV and an uncertain space (a sphere with a radius) for any attribute evaluation in uncertainty, we need to extend C-IFS/C-IFV to a spherical SvNS/SvNV (S-SvNS/S-SvNV) and develop its MADM technique in the S-SvNS scenario. Motivated by both the emerging research needs and the current research gap of SvNSs, the purposes of this study are: (1) to propose S-SvNS and the basic relations and score and accuracy formulae of S-SvNVs in terms of the hybrid information of an SvNV and its uncertain sphere with a radius, (2) to present the Trigonometric Operation Laws (TOLs) of S-SvNVs based on the sine t-norm and cosine t-conorm, (3) to establish the S-SvNV Trigonometric Weighted Averaging (S-SvNVTWA) and S-SvNV Trigonometric Weighted Geometric (S-SvNVTWG) operators for the aggregation of S-SvNVs, (4) to develop a MADM technique utilizing the S-SvNVTWA and S-SvNVTWG operators and score and accuracy formulae of S-SvNVs for solving MADM problems with unknown attribute weights and periodicity in the scenario of S-SvNSs, and (5) to apply the developed MADM technique to an actual selection example of Road Slope Treatment Schemes (RSTSs) and then to verify the efficiency of the developed technique by sensitivity analysis and comparison with the existing SvNV MADM techniques under the scenarios of SvNSs and S-SvNSs.

In order to fill the current research gap of SvNSs, this article mainly addresses the following problems:

(1) The S-SVNS is proposed to adequately represent the hybrid information of an SvNV and a sphere with a radius under the SvNV uncertainty. Its advantage is able to contain the more comprehensive information of the exact SvNV and its uncertain spherical space to each attribute evaluation in complex MADM problems.

(2) The basic relations and score and accuracy formulae of S-SvNVs are presented to provide a reasonable ranking approach for S-SvNVs.

(3) The TOLs and S-SvNVTWA and S-SvNVTWG operators are established to address the periodic aggregation operation issues of S-SvNVs for the modelling of MADM in the scenario of S-SvNSs.

(4) The MADM technique is developed to solve MADM problems with unknown attribute weights and periodicity in the scenario of S-SvNSs, where the scoring entropy weight approach of S-SvNVs can effectively derive the attribute weights to avoid the given situation of subjective weights in the MADM process.

(5) The developed MADM technique is applied to the selection problem of RSTSs in Ningbo City, China, and then the importance of uncertain sphere information to the ranking of RSTSs in S-SvNV decision information was verified by sensitivity analysis and comparison with the existing SvNV MADM techniques in SvNS and S-SvNS scenarios.

The remaining parts of this article are presented below. The section 2 introduces the concepts of C-IFS and SvNS and the WAOs and ranking rules of SvNVs from the literature to clearly understand the new concepts and contributions of this study for the readers. The section 3 proposes the S-SvNS and the relations, TOLs, and score and accuracy formulae of S-SvNVs. The section 4 presents the S-SvNVTWA and S-SvNVTWG operators of S-SvNVs and their properties. In the section 5, a MADM technique is developed based on the proposed trigonometric WAOs and score and accuracy formulae of S-SvNVs to address MADM problems with unknown attribute weights and periodicity in the S-SvNS scenario. In the section 6, the developed MADM technique is applied to the actual selection sample of RSTSs in Ningbo City, China, and then its efficiency is verified by sensitivity analysis and comparison with the existing SvNV MADM techniques in the SvNS and S-SvNS scenarios. The section 7 presents the conclusions and future research.

2. Preliminaries of C-IFS and SvNS

This section describes S-IFS and SvNS and their related operations as preliminaries to this article.

First, some concepts of C-IFSs and C-IFVs are introduced.

Based on an extension of the IFS, a concept of C-IFS was first introduced by Atanassov [17] and Atanassov and Marinov [18].

A C-IFS C_S in a fixed universe set G is represented by $C_S = \{<g, s_{td}(g), s_{fd}(g); c_r>|g \in G\}$, in which $s_{fd}(g)$, $s_{td}(g)$: $G \to [0, 1]$ are the nonmembership and membership degrees and $c_r \in [0, \sqrt{2}]$ is a circular radius around g for $g \in G$ and $0 \le s_{td}(g) + s_{fd}(g) \le 1$. A single element $<g, s_{td}(g), s_{fd}(g); c_r>$ in C_S is denoted as the C-IFV $g_c = <s_{td}, s_{fd}; c_r>$ for simplicity. However, the S-IFS degenerates to the IFS $I_S = \{<g, s_{td}(g), s_{fd}(g)>|g \in G\}$ if $c_r = 0$.

Then, as another extension of IFS, the notion of SvNS was introduced by Wang et al. [31].

A SvNS S_N in a fixed universe set G is defined as $S_N = \{<g, s_{td}(g), s_{ud}(g), s_{fd}(g)> | g \in G\}$, where $s_{td}(g)$, $s_{ud}(g)$, $s_{fd}(g)$: $E \to [0, 1]$ are the membership degrees of the truth, indeterminacy and falsehood with $0 \le s_{td}(g) + s_{ud}(g) + s_{fd}(g) \le 3$. A single element $<g, s_{td}(g), s_{fd}(g), s_{fd}(g)>$ in S_N is denoted as the SvNV $s_N = <s_{td}, s_{ud}, s_{fd}>$ for simplicity.

Suppose that there are two SvNVs $s_{N1} = <s_{td1}, s_{ud1}, s_{fd1}>$, $s_{N2} = <s_{td2}, s_{ud2}, s_{fd2}>$, and $q > 0$. Then, their operational relations are defined below [12, 31, 32]:

(1) $s_{N1} \subseteq s_{N2} \Leftrightarrow s_{td1} \le s_{td2}, s_{ud1} \ge s_{ud2}, s_{fd1} \ge s_{fd2}$;

(2) $s_{N1} = s_{N2} \Leftrightarrow s_{N1} \subseteq s_{N2}, s_{N1} \supseteq s_{N2}$;

(3) $s_{N1} \cap s_{N2} = \left\langle s_{td1} \wedge s_{td2}, s_{ud1} \vee s_{ud2}, s_{fd1} \vee s_{fd2} \right\rangle$;

(4) $s_{N1} \cup s_{N2} = \left\langle s_{td1} \vee s_{td2}, s_{ud1} \wedge s_{ud2}, s_{fd1} \wedge s_{fd2} \right\rangle$;

(5) $s_{N1} \oplus s_{N2} = \left\langle s_{td1} + s_{td2} - s_{td1}s_{td2}, s_{ud1}s_{ud2}, s_{fd1}s_{fd2} \right\rangle$;

(6) $s_{N1} \otimes s_{N2} = \left\langle s_{td1}s_{td2}, s_{ud1} + s_{ud2} - s_{ud1}s_{ud2}, s_{fd1} + s_{fd2} - s_{fd1}s_{fd2} \right\rangle$;

(7) $qs_{N1} = \left\langle 1-(1-s_{td1})^q, s_{ud1}^q, s_{fd1}^q \right\rangle$;

(8) $s_{N1}^q = \left\langle s_{td1}^q, 1-(1-s_{ud1})^q, 1-(1-s_{fd1})^q \right\rangle$;

(9) $s_{N1}^c = \left\langle s_{fd1}, 1-s_{ud1}, s_{td1} \right\rangle$ (Complement of s_{N1}).

Set $s_{Nj} = <s_{tdj}, s_{udj}, s_{fdj}>$ ($j = 1, 2, \ldots, h$) as a collection of SvNVs with their weight vector $\boldsymbol{q} = (q_1, q_2, \ldots, q_h)$ subject to $0 \le q_j \le 1$ and $\sum_{j=1}^{h} q_j = 1$. Then, the SvNV weighted averaging (SvNVWA) and SvNV weighted geometric (SvNVWG) operators are introduced below [32]:

$$SvNVWA(s_{N1}, s_{N2}, \ldots, s_{Nh}) = \sum_{j=1}^{h} q_j s_{Nj} = \left\langle 1-\prod_{j=1}^{h}\left(1-s_{tdj}\right)^{q_j}, \prod_{j=1}^{h} s_{udj}^{q_j}, \prod_{k=1}^{h} s_{fdj}^{q_j} \right\rangle, \tag{1}$$

$$SvNVWG(s_{N1}, s_{N2}, \ldots, s_{Nh}) = \prod_{j=1}^{h} s_{Nj}^{q_j} = \left\langle \prod_{k=1}^{h} s_{tdj}^{q_j}, 1-\prod_{j=1}^{h}\left(1-s_{udj}\right)^{q_j}, 1-\prod_{j=1}^{h}\left(1-s_{fdj}\right)^{q_j} \right\rangle. \tag{2}$$

Based on the cotangent t-norm and tangent t-conorm, Ye et al. [15] proposed the SvNV trigonometric weighted averaging (SvNVTWA) and SvNV trigonometric weighted geometric (SvNVTWG) operators:

$$SvNVTWA(s_{N1}, s_{N2}, \ldots, s_{Nh}) = \sum_{j=1}^{h}{}_{T}\, q_j s_{Nj} = \left\langle \begin{array}{c} 2/\pi \tan^{-1}\left(\sum_{j=1}^{h} q_j \tan(0.5\pi s_{tdj})\right), \\ 2/\pi \cot^{-1}\left(\sum_{j=1}^{h} q_j \cot(0.5\pi s_{udj})\right), \\ 2/\pi \cot^{-1}\left(\sum_{j=1}^{h} q_j \cot(0.5\pi s_{fdj})\right) \end{array} \right\rangle, \tag{3}$$

$$SvNVTWG(s_{N1}, s_{N2}, ..., s_{Nh}) = \prod_{j=1}^{h}{}_{T} s_{Nj}^{q_j} = \left\langle \begin{array}{l} 2/\pi \cot^{-1}\left(\sum_{j=1}^{h} q_j \cot(0.5\pi s_{tdj})\right), \\ 2/\pi \tan^{-1}\left(\sum_{j=1}^{h} q_j \tan(0.5\pi s_{udj})\right), \\ 2/\pi \tan^{-1}\left(\sum_{j=1}^{h} q_j \tan(0.5\pi s_{fdj})\right) \end{array} \right\rangle. \quad (4)$$

Then, the ranking of two SvNVs s_{Nj} = <s_{tdj}, s_{udj}, s_{fdj}> (j = 1, 2) can be derived by their score and accuracy formulae [15, 32]:

$$C(s_{Nj}) = \frac{2 + s_{tdj} - s_{udj} - s_{fdj}}{3} \quad \text{for} \quad C(s_{Nj}) \in [0,1], \quad (5)$$

$$D(s_{Nj}) = s_{tdj} - s_{fdj} \quad \text{for} \quad D(s_{Nj}) \in [-1,1]. \quad (6)$$

Consequently, the ranking rules of two SvNVs are introduced below [15, 32]:

(a) When $C(s_{N1}) > C(s_{N2})$, $s_{N1} > s_{N2}$;

(b) When $C(s_{N1}) = C(s_{N2})$ and $D(s_{N1}) > D(s_{N2})$, $s_{N1} > s_{N2}$;

(c) When $C(s_{N1}) = C(s_{N2})$ and $D(s_{N1}) = D(s_{N2})$, $s_{N1} \cong s_{N2}$.

3. S-SvNSs

This section extends the S-IFS concept [17, 18] to propose an S-SvNS that expresses the hybrid information of an SvNV and a sphere with a radius around each element.

Definition 1. Set G as a finite set. An S-SvNS R_{SN} in G is represented by the following form:

$$R_{SN} = \left\{ \left\langle g, s_{td}(g), s_{ud}(g), s_{fd}(g); s_r \right\rangle \mid g \in G \right\},$$

where $s_{td}(g)$, $s_{ug}(g)$, $s_{fd}(g)$: $G \rightarrow [0, 1]$ are the true, indeterminate, and false membership degrees, subject to the condition $0 \leq s_{td}(g) + s_{ud}(g) + s_{fd}(g) \leq 3$, and $s_r \in [0, \sqrt{3}\,]$ is a spherical radius around an element $g \in G$.

For simplicity, the single element <g, $s_{td}(g)$, $s_{ud}(g)$, $s_{fd}(g)$; s_r> in R_{SN} is simply denoted as r_{SN} = <s_{td}, s_{ud}, s_{fd}; s_r>, which is named S-SvNV. Its geometric representation in a single-valued neutrosophic cube is shown in Figure 1.

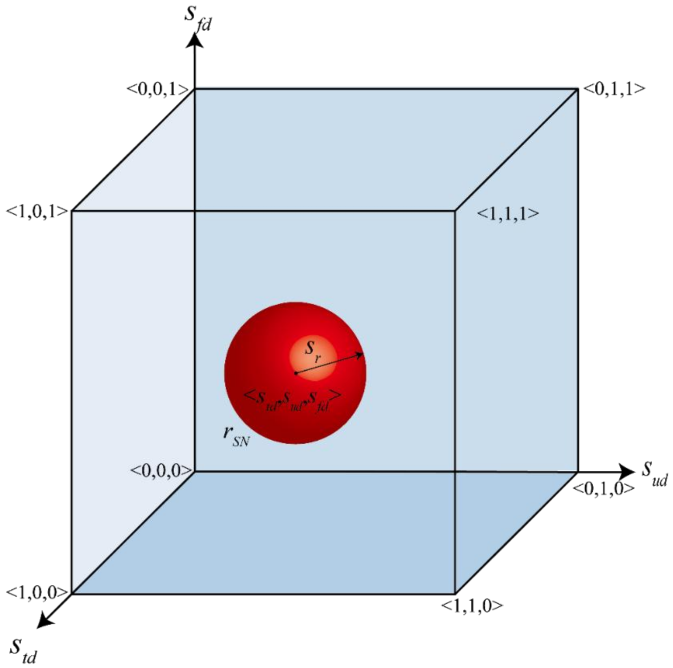

Figure 1. Geometric representation of the S-SvNV r_{SN} in a single-valued neutrosophic cube.

Definition 2. Let two S-SvNVs be r_{SN1} = <s_{td1}, s_{ud1}, s_{fd1}; s_{r1}>, r_{SN2} = <s_{td2}, s_{ud2}, s_{fd2}; s_{r2}>, and q > 0. Then, their relations are defined below:

(1) $r_{SN1} \subseteq r_{SN2} \Leftrightarrow s_{td1} \le s_{td2}, s_{ud1} \ge s_{ud2}, s_{fd1} \ge s_{fd2}, s_{r1} \le s_{r2}$;

(2) $r_{SN1} = r_{SN2} \Leftrightarrow r_{SN1} \subseteq r_{SN2}, r_{SN1} \supseteq r_{SN2}$;

(3) $r_{SN1} \cap r_{SN2} = \left\langle s_{td1} \wedge s_{td2}, s_{ud1} \vee s_{ud2}, s_{fd1} \vee s_{fd2}; s_{r1} \wedge s_{r2} \right\rangle$;

(4) $r_{SN1} \cup r_{SN2} = \left\langle s_{td1} \vee s_{td2}, s_{ud1} \wedge s_{ud2}, s_{fd1} \wedge s_{fd2}; s_{r1} \vee s_{r2} \right\rangle$;

(5) $r_{SN1}^{c} = \left\langle s_{fd1}, (1 - s_{ud1}), s_{td1}; \sqrt{3} - s_{r1} \right\rangle$ (Complement of r_{SN1}).

Based on the sine t-norm $H_S(t, v)$: $[0, 1]^2 \to [0, 1]$ and the cosine t-conorm $H_C(t, v)$: $[0, 1]^2 \to [0,1]$ for $t, v \in [0, 1]$, we introduce the following trigonometric operations [16]:

$$H_S(t, v) = 2/\pi \sin^{-1}(\sin(0.5\pi t)\sin(0.5\pi v)), \tag{7}$$

$$H_C(t, v) = 2/\pi \cos^{-1}\left(\cos(0.5\pi t)\cos(0.5\pi v)\right). \tag{8}$$

In terms of Eqs. (7) and (8), we can propose TOLs of S-SvNVs below.

Definition 3. Let r_{SN1} = <s_{td1}, s_{ud1}, s_{fd1}; s_{r1}> and r_{SN2} = <s_{td2}, s_{ud2}, s_{fd2}; s_{r2}> be two S-SvNVs and q > 0. Then, they are defined as the following TOLs:

(1) $$r_{SN1} \oplus_T r_{SN2} = \left\langle \begin{array}{l} 2/\pi \cos^{-1}\left(\cos(0.5\pi s_{td1})\cos(0.5\pi s_{td2})\right), \\ 2/\pi \sin^{-1}\left(\sin(0.5\pi s_{ud1})\sin(0.5\pi s_{ud2})\right), \\ 2/\pi \sin^{-1}\left(\sin(0.5\pi s_{fd1})\sin(0.5\pi s_{fd2})\right); \\ 6/(\sqrt{3}\pi)\cos^{-1}\left(\cos(\sqrt{3}\pi s_{r1}/6)\cos(\sqrt{3}\pi s_{r2}/6)\right) \end{array} \right\rangle;$$

(2) $$r_{SN1} \otimes_T r_{SN2} = \left\langle \begin{array}{l} 2/\pi \sin^{-1}\left(\sin(0.5\pi s_{td1})\sin(0.5\pi s_{td2})\right), \\ 2/\pi \cos^{-1}\left(\cos(0.5\pi s_{ud1})\cos(0.5\pi s_{ud2})\right), \\ 2/\pi \cos^{-1}\left(\cos(0.5\pi s_{fd1})\cos(0.5\pi s_{fd2})\right); \\ 6/(\sqrt{3}\pi)\sin^{-1}\left(\sin(\sqrt{3}\pi s_{r1}/6)\sin(\sqrt{3}\pi s_{r2}/6)\right) \end{array} \right\rangle;$$

(3) $$q r_{SN1} = \left\langle \begin{array}{l} 2/\pi \cos^{-1}\left(\cos(0.5\pi s_{td1})\right)^q, 2/\pi \sin^{-1}\left(\sin(0.5\pi s_{ud1})\right)^q, \\ 2/\pi \sin^{-1}\left(\sin(0.5\pi s_{fd1})\right)^q; 6/(\sqrt{3}\pi)\cos^{-1}\left(\cos(\sqrt{3}\pi s_{r1}/6)\right)^q \end{array} \right\rangle;$$

(4) $$(r_{SN1})^q = \left\langle \begin{array}{l} 2/\pi \sin^{-1}\left(\sin(0.5\pi s_{td1})\right)^q, 2/\pi \cos^{-1}\left(\cos(0.5\pi s_{ud1})\right)^q \\ 2/\pi \cos^{-1}\left(\cos(0.5\pi s_{fd1})\right)^q; 6/(\sqrt{3}\pi)\sin^{-1}\left(\sin(\sqrt{3}\pi s_{r1}/6)\right)^q \end{array} \right\rangle.$$

Example 1. Set two S-SvNVs as r_{SN1} = <(0.6, 0.2, 0.4; 0.3)> and r_{SN2} = <0.7, 0.2, 0.3; 0.2)> with q = 0.7. Using the TOLs (1) – (4) in Definition 3, we derive the results:

(1) $$r_{SN1} \oplus_T r_{SN2} = \left\langle \begin{array}{l} 2/\pi \cos^{-1}\left(\cos(0.5\times\pi\times0.6)\cos(0.5\times\pi\times0.7)\right), \\ 2/\pi \sin^{-1}\left(\sin(0.5\times\pi\times0.2)\sin(0.5\times\pi\times0.2)\right), \\ 2/\pi \sin^{-1}\left(\sin(0.5\times\pi\times0.4)\sin(0.5\times\pi\times0.3)\right); \\ 6/(\sqrt{3}\pi)\cos^{-1}\left(\cos(\sqrt{3}\times\pi\times0.3/6)\cos(\sqrt{3}\times\pi\times0.2/6)\right) \end{array} \right\rangle$$

$$= \left\langle 0.8280, 0.0609, 0.1720; 0.3592 \right\rangle;$$

(2) $$r_{SN1}\otimes_T r_{SN2}=\left\langle\begin{array}{l}2/\pi\sin^{-1}\left(\sin(0.5\times\pi\times0.6)\sin(0.5\times\pi\times0.7)\right),\\ 2/\pi\cos^{-1}\left(\cos(0.5\times\pi\times0.2)\cos(0.5\times\pi\times0.2)\right),\\ 2/\pi\cos^{-1}\left(\cos(0.5\times\pi\times0.4)\cos(0.5\times\pi\times0.3)\right);\\ 6/(\sqrt{3}\pi)\sin^{-1}\left(\sin(\sqrt{3}\times\pi\times0.3/6)\sin(\sqrt{3}\times\pi\times0.2/6)\right)\end{array}\right\rangle$$

$$=\langle 0.5125,\ 0.2805,\ 0.4875;\ 0.0535\rangle;$$

(3) $$qr_{SN1}=\left\langle\begin{array}{l}2/\pi\cos^{-1}\left(\cos(0.5\pi0.6)\right)^{0.7},2/\pi\sin^{-1}\left(\sin(0.5\pi0.2)\right)^{0.7},\\ 2/\pi\sin^{-1}\left(\sin(0.5\pi0.4)\right)^{0.7};6/(\sqrt{3}\pi)\cos^{-1}\left(\cos(\sqrt{3}\pi0.3/6)\right)^{0.7}\end{array}\right\rangle$$

$$=\langle 0.5158,\ 0.2897,\ 0.4842;\ 0.2515\rangle;$$

(4) $$(r_{SN1})^q=\left\langle\begin{array}{l}2/\pi\sin^{-1}\left(\sin(0.5\times\pi\times0.6)\right)^{0.7},2/\pi\cos^{-1}\left(\cos(0.5\times\pi\times0.2)\right)^{0.7}\\ 2/\pi\cos^{-1}\left(\cos(0.5\times\pi\times0.4)\right)^{0.7};6/(\sqrt{3}\pi)\sin^{-1}\left(\sin(\sqrt{3}\times\pi\times0.3/6)\right)^{0.7}\end{array}\right\rangle$$

$$=\langle 0.6617,\ 0.1678,\ 0.3383;\ 0.4521\rangle.$$

It is obvious that the above trigonometric operational results are still S-SvNVs.

To compare the S-SvNVs r_{SNj} = <s_{tdj}, s_{udj}, s_{fdj}; s_{rj}> for j =1, 2, their score and accuracy formulae are defined below:

$$T(r_{SNj})=\frac{2+s_{tdj}-s_{udj}-s_{fdj}+s_{rj}/\sqrt{3}}{4}\quad\text{for}\quad T(r_{SNj})\in[0,1],\tag{9}$$

$$Z(r_{SNj})=s_{tdj}-s_{fdj}\quad\text{for}\quad Z(r_{SNj})\in[-1,1].\tag{10}$$

Thus, there are the following ranking rules:

(a) When $T(r_{SN1}) > T(r_{SN2})$, $r_{SN1} > r_{SN2}$;

(b) When $T(r_{SN1}) = T(r_{SN2})$ and $Z(r_{SN1}) > Z(r_{SN2})$, $r_{SN1} > r_{SN2}$;

(c) When $T(r_{SN1}) = T(r_{SN2})$ and $Z(r_{SN1}) = Z(r_{SN2})$, $r_{SN1} \cong r_{SN2}$.

Example 2. Set two S-SvNVs as r_{SN1} = <0.7, 0.1, 0.2; 0.3> and r_{SN2} = <0.8, 0.2, 0.4; 0.2)>. Then, their ranking is given below.

Using Eq. (9), the score values of r_{SN1} and r_{SN2} are $T(r_{SN1}) = (2 + 0.7 - 0.1 - 0.2 + 0.3/\sqrt{3})/4 = 0.6433$ and $T(r_{SN2}) = (2 + 0.8 - 0.2 - 0.4 + 0.2/\sqrt{3})/4 = 0.5789$. Since $T(r_{SN1}) > T(r_{SN2})$, the ranking of both is $r_{SN1} > r_{SN2}$.

4. Trigonometric WAOs of S-SvNVs

According to the TOLs in Definition 3, this section establishes the S-SvNVTWA and S-SvNVTWG operators of S-SvNVs in the scenario of S-SvNSs.

First, we define the S-SvNVTWA and S-SvNVTWG operators of S-SvNVs.

Definition 4. Let r_{SNj} = <s_{tdj}, s_{udj}, s_{fdj}; s_{rj}> (j = 1, 2, …, h) be a group of S-SvNVs and S-SvNVTWA, S-SvNVTWG: $\Omega^h \rightarrow \Omega$. Then, the S-SvNVTWA and S-SvNVTWG operators are defined respectively below:

$$S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNh})=q_1r_{SN1}\oplus_T q_2r_{SN2}\oplus_T...\oplus_T q_hr_{SNh}=\sum_{j=1}^{h}{}_T\, q_jr_{SNj}\,,\tag{11}$$

$$S-SvNVTWG(r_{SN1},r_{SN2},...,r_{SNh})=r_{SN1}^{q_1}\otimes_T r_{SN2}^{q_2}\otimes_T...\otimes_T r_{SNh}^{q_h}=\prod_{j=1}^{h}{}_T\, r_{SNj}^{q_j}\,.\tag{12}$$

where q_j (j = 1, 2, …, h) is the weight of r_{SNj} with $0 \le q_j \le 1$ and $\sum_{j=1}^{h} q_j = 1$.

Theorem 1. Let r_{SNj} = <s_{tdj}, s_{udj}, s_{fdj}; s_{rj}> (j = 1, 2, …, h) be a group of S-SvNVs and let q_j be the weight of r_{SNj} (j = 1, 2, …, h) with $0 \le q_j \le 1$ and $\sum_{j=1}^{h} q_j = 1$. Then, the aggregated value of the S-SvNVTWA operator is still S-SvNV, which is derived by

$$S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNh})=\sum_{j=1}^{h}{}_{T}\,q_j r_{SNj}$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{tdj})\right)^{q_j}\right),2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{fdj})\right)^{q_j}\right);6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right) \end{array}\right\rangle. \quad (13)$$

Proof:

Mathematical induction is introduced to the proof of Eq. (13) below.

(1) When $h = 2$, the TOLs (1) and (3) in Definition 3 can obtain the result:

$$S-SvNVTWA(r_{SN1},r_{SN2})=q_1 r_{SN1}\oplus_T q_2 r_{SN2}=\sum_{j=1}^{2}{}_{T}\,q_j r_{SNj}$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\cos\left((2/\pi)\times(\pi/2)\cos^{-1}\left(\cos(0.5\pi s_{td1})\right)^{q_1}\right)\cos\left((2/\pi)\times(\pi/2)\cos^{-1}\left(\cos(0.5\pi s_{td2})\right)^{q_2}\right)\right),\\ 2/\pi\sin^{-1}\left(\sin\left((2/\pi)\times(\pi/2)\sin^{-1}\left(\sin(0.5\pi s_{ud1})\right)^{q_1}\right)\sin\left((2/\pi)\times(\pi/2)\sin^{-1}\left(\sin(0.5\pi s_{ud2})\right)^{q_2}\right)\right),\\ 2/\pi\sin^{-1}\left(\sin\left((2/\pi)\times(\pi/2)\sin^{-1}\left(\sin(0.5\pi s_{fd1})\right)^{q_1}\right)\sin\left((2/\pi)\times(\pi/2)\sin^{-1}\left(\sin(0.5\pi s_{fd2})\right)^{q_2}\right)\right);\\ 6/\sqrt{3}\pi\cos^{-1}\left(\begin{array}{l}\cos\left((6/\sqrt{3}\pi)\times(\sqrt{3}\pi/6)\cos^{-1}\left(\cos(\sqrt{3}\pi s_{r1}/6)\right)^{q_1}\right)\\ \times\cos\left((6/\sqrt{3}\pi)\times(\sqrt{3}\pi/6)\cos^{-1}\left(\cos(\sqrt{3}\pi s_{r2}/6)\right)^{q_2}\right)\end{array}\right) \end{array}\right\rangle$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\left(\cos(0.5\pi s_{td1})\right)^{q_1}\left(\cos(0.5\pi s_{td2})\right)^{q_2}\right),\\ 2/\pi\sin^{-1}\left(\left(\sin(0.5\pi s_{ud1})\right)^{q_1}\left(\sin(0.5\pi s_{ud2})\right)^{q_2}\right),\\ 2/\pi\sin^{-1}\left(\left(\sin(0.5\pi s_{fd1})\right)^{q_1}\left(\sin(0.5\pi s_{fd2})\right)^{q_2}\right);\\ 6/(\sqrt{3}\pi)\cos^{-1}\left(\left(\cos(\sqrt{3}\pi s_{r1}/6)\right)^{q_1}\left(\cos(\sqrt{3}\pi s_{r2}/6)\right)^{q_2}\right) \end{array}\right\rangle \quad (14)$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\prod_{j=1}^{2}\left(\cos(0.5\pi s_{tdj})\right)^{q_j}\right),2/\pi\sin^{-1}\left(\prod_{j=1}^{2}\left(\sin(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{2}\left(\sin(0.5\pi s_{fdj})\right)^{q_j}\right);6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{2}\left(\cos(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right) \end{array}\right\rangle.$$

(2) When $h = m$, Eq. (13) can keep the formula:

$$S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNm})=\sum_{j=1}^{m}{}_{T}\,q_j r_{SNj}$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\prod_{j=1}^{m}\left(\cos(0.5\pi s_{tdj})\right)^{q_j}\right),2/\pi\sin^{-1}\left(\prod_{j=1}^{m}\left(\sin(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{m}\left(\sin(0.5\pi s_{fdj})\right)^{q_j}\right);6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{m}\left(\cos(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right) \end{array}\right\rangle, \quad (15)$$

(3) When $h = m+1$, based on the TOLs (1) and (3) in Definition 3 and Eqs. (14) and (15), we have the result:

$$S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNm},r_{SNm+1})=\sum_{j=1}^{m}{}_T\, q_j r_{SNj}\oplus_T q_{m+1}r_{SNm+1}$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\prod_{j=1}^{m}\left(\cos(0.5\pi s_{tdj})\right)^{q_j}\right),2/\pi\sin^{-1}\left(\prod_{j=1}^{m}\left(\sin(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{m}\left(\sin(0.5\pi s_{fdj})\right)^{q_j}\right);6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{m}\left(\cos(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right)\end{array}\right\rangle\oplus_T q_{m+1}r_{SNm+1}$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\prod_{j=1}^{m+1}\left(\cos(0.5\pi s_{tdj})\right)^{q_j}\right),2/\pi\sin^{-1}\left(\prod_{j=1}^{m+1}\left(\sin(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{m+1}\left(\sin(0.5\pi s_{fdj})\right)^{q_j}\right);6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{m+1}\left(\cos(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right)\end{array}\right\rangle$$

Regarding the above results, Eq. (13) can exist for all h.

Therefore, the proof of Theorem 1 is completed.

Theorem 2. Let r_{SNj} = < s_{tdj}, s_{udj}, s_{fdj}; s_{rj} > (j = 1, 2, …, h) be a group of S-SvNVs and let q_j be the weight of r_{SNj} (j = 1, 2, …, h) with $0 \le q_j \le 1$ and $\sum_{j=1}^{h} q_j = 1$. Then, the aggregated value of the S-SvNVTWG operator is still S-SvNV, which is derived by

$$S-SvNVTWG(r_{SN1},r_{SN2},...,r_{SNh})=\prod_{j=1}^{h}{}_T\, r_{SNj}^{q_j}$$
$$=\left\langle \begin{array}{l} 2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{tdj})\right)^{q_j}\right),2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{fdj})\right)^{q_j}\right);6/(\sqrt{3}\pi)\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right)\end{array}\right\rangle, \quad (16)$$

A similar proof way of Theorem 1 can be used for the proof of Eq. (16), which is omitted here.

Theorem 3. The S-SvNVTWA operator of Eq. (13) and the S-SvNVTWG operator of Eq. (16) include the following properties:

(1) Idempotency: Let r_{SNj} = < s_{tdj}, s_{udj}, s_{fdj}; s_{rj} > (j = 1, 2, …, h) be a group of S-SvNVs. When $r_{SNJ} = r_{SN}$ (j = 1, 2, ..., h), there are $S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNh})=r_{SN}$ and $S-SvNVTWG(r_{SN1},r_{SN2},...,r_{SNh})=r_{SN}$.

(2) Boundedness: Let r_{SNj} = < s_{tdj}, s_{udj}, s_{fdj}; s_{rj} > (j = 1, 2, …, h) be a group of S-SvNVs and let the minimum and maximum S-SvNVs:

$$r_{SN\min}=\left\langle \min_j(s_{tdj}),\max_j(s_{udj}),\max_j(s_{fdj});\min_j(s_{rj})\right\rangle,$$
$$r_{SN\max}=\left\langle \max_j(s_{tdj}),\min_j(s_{udj}),\min_j(s_{fdj});\max_j(s_{rj})\right\rangle.$$

Subsequently, there are $r_{SN\min}\le S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNh})\le r_{SN\max}$ and $r_{SN\min}\le S-SvNVTWG(r_{SN1},r_{SN2},...,r_{SNh})\le r_{SN\max}$.

(3) Monotonicity: Let r_{SNj} = < s_{tdj}, s_{udj}, s_{fdj}; s_{rj} > and $r_{SNj}^{*}=\left\langle s_{tdj}^{*},s_{udj}^{*},s_{fdj}^{*};s_{rj}^{*}\right\rangle$ (k = 1, 2, …, q) be two groups of S-SvNVs. When $r_{SNj}\le r_{SNj}^{*}$, there are $S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNh})\le S-SvNVTWA(r_{SN1}^{*},r_{SN2}^{*},...,r_{SNh}^{*})$ and $S-SvNVTWG(r_{SN1},r_{SN2},...,r_{SNh})\le S-SvNVTWG(r_{SN1}^{*},r_{SN2}^{*},...,r_{SNh}^{*})$.

Proof:

(1) When $r_{SNj} = r_{SN}$ (j = 1, 2, ..., h), Eqs. (13) and (16) have the following results:

$$S-SvNVTWA(r_{SN1},r_{SN2},...,r_{SNh})=\sum_{j=1}^{h}{}_{T}\,q_j r_{SNj}$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{tdj})\right)^{q_j}\right), 2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{fdj})\right)^{q_j}\right); 6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right)\end{array}\right\rangle,$$
$$=\left\langle \begin{array}{l} 2/\pi\cos^{-1}\left(\cos(0.5\pi s_{td})\right)^{\sum_{j=1}^{h}q_j}, 2/\pi\sin^{-1}\left(\sin(0.5\pi s_{ud})\right)^{\sum_{j=1}^{h}q_j},\\ 2/\pi\sin^{-1}\left(\sin(0.5\pi s_{fd})\right)^{\sum_{j=1}^{h}q_j}; 6/(\sqrt{3}\pi)\cos^{-1}\left(\cos(\sqrt{3}\pi s_r/6)\right)^{\sum_{j=1}^{h}q_j}\end{array}\right\rangle$$
$$=\left\langle s_{td},s_{ud},s_{fd};s_r\right\rangle = r_{SN},$$
$$S-SvNVTWG(r_{SN1},r_{SN2},...,r_{SNh})=\prod_{j=1}^{h}{}_{T}\,r_{SNj}^{q_j}$$
$$=\left\langle \begin{array}{l} 2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{tdj})\right)^{q_j}\right), 2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{udj})\right)^{q_j}\right),\\ 2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{fdj})\right)^{q_j}\right); 6/(\sqrt{3}\pi)\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(\sqrt{3}\pi s_{rj}/6)\right)^{q_j}\right)\end{array}\right\rangle.$$
$$=\left\langle \begin{array}{l} 2/\pi\sin^{-1}\left(\sin(0.5\pi s_{td})\right)^{\sum_{j=1}^{h}q_j}, 2/\pi\cos^{-1}\left(\cos(0.5\pi s_{ud})\right)^{\sum_{j=1}^{h}q_j},\\ 2/\pi\cos^{-1}\left(\cos(0.5\pi s_{fd})\right)^{\sum_{j=1}^{h}q_j}; 6/(\sqrt{3}\pi)\sin^{-1}\left(\sin(\sqrt{3}\pi s_r/6)\right)^{\sum_{j=1}^{h}q_j}\end{array}\right\rangle$$
$$=\left\langle s_{td},s_{ud},s_{fd};s_r\right\rangle = r_{SN}.$$

(2) Since r_{SNmin} and r_{SNmax} are the minimum and maximum S-SvNVs, there is the inequality $r_{SNmin} \le r_{SNj} \le r_{SNmax}$. Therefore, $\sum_{j=1}^{h}{}_{T}\,q_j r_{SN\min} \le \sum_{j=1}^{h}{}_{T}\,q_j r_{SNj} \le \sum_{j=1}^{h}{}_{T}\,q_j r_{SN\max}$ and $\prod_{j=1}^{h}{}_{T}\,r_{SN\min}^{q_j} \le \prod_{j=1}^{h}{}_{T}\,r_{SNj}^{q_j} \le \prod_{j=1}^{h}{}_{T}\,r_{SN\max}^{q_j}$ also exist. Based on the property (1) and the trigonometric properties, there are $r_{SN\min} \le \sum_{j=1}^{h}{}_{T}\,q_j r_{SNj} \le r_{SN\max}$ and $r_{SN\min} \le \prod_{j=1}^{h}{}_{T}\,r_{SNj}^{q_j} \le r_{SN\max}$, i.e. $r_{SN\min} \le S-SvNVTWA\left(r_{SN1},r_{SN2},...,r_{SNh}\right) \le r_{SN\max}$ and $r_{SN\min} \le S-SvNVTWG\left(r_{SN1},r_{SN2},...,r_{SNh}\right) \le r_{SN\max}$.

(3) When $r_{SNj} \le r_{SNj}^*$, there exist $\sum_{j=1}^{h}{}_{T}\,q_j r_{SNj} \le \sum_{j=1}^{h}{}_{T}\,q_j r_{SNj}^*$ and $\prod_{j=1}^{h}{}_{T}\,r_{SNj}^{q_j} \le \prod_{j=1}^{h}{}_{T}\,(r_{SNj}^*)^{q_j}$, i.e., $S-SvNVTWA\left(r_{SN1},r_{SN2},...,r_{SNh}\right) \le S-SvNVTWA\left(r_{SN1}^*,r_{SN2}^*,...,r_{SNh}^*\right)$ and $S-SvNVTWG\left(r_{SN1},r_{SN2},...,r_{SNh}\right) \le S-SvNVTWG\left(r_{SN1}^*,r_{SN2}^*,...,r_{SNh}^*\right)$.

Consequently, all the above properties are true.

Example 3. Set three S-SvNVs as r_{SN1} = <0.7, 0.3, 0.2; 0.4>, r_{SN2} = <0.6, 0.2, 0.2; 0.3>, and r_{SN3} = <0.8, 0.4, 0.5; 0.2> subject to the weight vector $\boldsymbol{q}$ = (0.4, 0.3, 0.3). Using Eqs. (13) and (16), there are the aggregation results of the S-SvNVTWA and S-SvNVTWG operators:

$$S-SvNVTWA(r_{SN1},r_{SN2},r_{SN3})=\sum_{j=1}^{3}{}_{T}q_j r_{SNj}$$

$$=\left\langle\begin{array}{l}2/\pi\cos^{-1}\left((\cos(0.5\times\pi\times0.7))^{0.4}(\cos(0.5\times\pi\times0.6))^{0.3}(\cos(0.5\times\pi\times0.8))^{0.3}\right),\\ 2/\pi\sin^{-1}\left((\sin(0.5\times\pi\times0.3))^{0.4}(\sin(0.5\times\pi\times0.2))^{0.3}(\sin(0.5\times\pi\times0.4))^{0.3}\right),\\ 2/\pi\sin^{-1}\left((\sin(0.5\times\pi\times0.2))^{0.4}(\sin(0.5\times\pi\times0.2))^{0.3}(\sin(0.5\times\pi\times0.5))^{0.3}\right);\\ 6/(\sqrt{3}\times\pi)\cos^{-1}\left(\left(\cos(\sqrt{3}\times\pi\times0.4/6)\right)^{0.4}\left(\cos(\sqrt{3}\times\pi\times0.3/6)\right)^{0.3}\left(\cos(\sqrt{3}\times\pi\times0.2/6)\right)^{0.3}\right)\end{array}\right\rangle$$

$$=\langle 0.7120,\ 0.2880,\ 0.2593;\ 0.3215\rangle,$$

$$S-SvNVTWG(r_{SN1},r_{SN2},r_{SN3})=\prod_{j=1}^{3}r_{SNj}^{q_j}$$

$$=\left\langle\begin{array}{l}2/\pi\sin^{-1}\left((\sin(0.5\times\pi\times0.7))^{0.4}(\sin(0.5\times\pi\times0.6))^{0.3}(\sin(0.5\times\pi\times0.8))^{0.3}\right),\\ 2/\pi\cos^{-1}\left((\cos(0.5\times\pi\times0.3))^{0.4}(\cos(0.5\times\pi\times0.2))^{0.3}(\cos(0.5\times\pi\times0.4))^{0.3}\right),\\ 2/\pi\cos^{-1}\left((\cos(0.5\times\pi\times0.2))^{0.4}(\cos(0.5\times\pi\times0.2))^{0.3}(\cos(0.5\times\pi\times0.5))^{0.3}\right);\\ 6/(\sqrt{3}\times\pi)\sin^{-1}\left(\left(\sin(\sqrt{3}\times\pi\times0.4/6)\right)^{0.4}\left(\sin(\sqrt{3}\times\pi\times0.3/6)\right)^{0.3}\left(\sin(\sqrt{3}\times\pi\times0.2/6)\right)^{0.3}\right)\end{array}\right\rangle$$

$$=\langle 0.6885,\ 0.3115,\ 0.3281;\ 0.2974\rangle.$$

5. MADM Technique Using the S-SvNVTWA and S-SvNVTWG Operators

This section develops a MADM technique utilizing the S-SvNVTWA and S-SvNVTWG operators and the score and accuracy formulae of S-SvNVs in the scenario of S-SvNSs.

In general, a MADM problem usually needs to select the optimal alternative from a set of alternatives $S_A = \{S_{A1}, S_{A2}, \ldots, S_{Ap}\}$ among those that satisfy the requirements of the multiple attributes (denoted as a set $R_S = \{R_{E1}, R_{E2}, \ldots, R_{Eh}\}$). During the evaluation process, the evaluation results of the alternatives over the attributes can be represented by the S-SvNVs $r_{SNij} = <s_{tdij}, s_{udij}, s_{fdij}; s_{rij}>$ ($j = 1, 2, \ldots, h$; $i = 1, 2, \ldots, p$), which are composed of SvNVs and spherical radii, and constructed as their decision matrix $R_M = (r_{SNij})_{p\times h}$. Then, a MADM technique in the scenario of S-SvNSs is established and described by the following decision process.

Step 1. In terms of the scoring entropy values of the evaluated S-SvNVs of each attribute across the alternatives provided by the decision makers, the attribute weights q_j ($j = 1, 2, \ldots, h$) are derived by the formulae:

$$E_{Sj}=-\frac{1}{\ln p}\sum_{i=1}^{p}\left(\frac{T(r_{SNij})}{\sum_{i=1}^{p}T(r_{SNij})}\ln\frac{T(r_{SNij})}{\sum_{i=1}^{p}T(r_{SNij})}\right), \tag{17}$$

$$q_j=\frac{1-E_{Sj}}{\sum_{j=1}^{h}(1-E_{Sj})},\ j=1,2,\ldots,h. \tag{18}$$

Step 2. The aggregated values r_{SNi} for S_{Ai} ($i = 1, 2, \ldots, p$) are obtained by one of the S-SvNVTWA and S-SvNVTWG operators:

$$r_{SNi}=S-SvNVTWA(r_{SNi1},r_{SNi2},\ldots,r_{SNih})=\sum_{j=1}^{h}{}_{T}q_j r_{SNij}$$

$$=\left\langle\begin{array}{l}2/\pi\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{tdij})\right)^{q_j}\right),2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{udij})\right)^{q_j}\right),\\ 2/\pi\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{fdij})\right)^{q_j}\right);6/(\sqrt{3}\pi)\cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(\sqrt{3}\pi s_{rij}/6)\right)^{q_j}\right)\end{array}\right\rangle, \tag{19}$$

$$r_{SNi} = S-SvNVTWG(r_{SNi1}, r_{SNi2}, ..., r_{SNih}) = \prod_{j=1}^{h}{}_{T}\, r_{SNij}^{q_j}$$
$$= \left\langle \begin{array}{l} 2/\pi \sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(0.5\pi s_{tdij})\right)^{q_j}\right), 2/\pi \cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{udij})\right)^{q_j}\right), \\ 2/\pi \cos^{-1}\left(\prod_{j=1}^{h}\left(\cos(0.5\pi s_{fdij})\right)^{q_j}\right); 6/(\sqrt{3}\pi)\sin^{-1}\left(\prod_{j=1}^{h}\left(\sin(\sqrt{3}\pi s_{rij}/6)\right)^{q_j}\right) \end{array} \right\rangle. \quad (20)$$

Step 3. The score values of $T(r_{SNi})$ (accuracy values of $T(r_{SNi})$ for necessity) (i = 1, 2, …, p) are derived by Eqs. (9) and (10).

Step 4. Alternatives are ranked subject to a descending order of the score values and the optimal one is determined.

Step 5. End.

6. MADM Application of RSTS Selection

In China, the city of Ningbo is located on the East China Sea, and it is often affected by bad weather such as typhoons and rainy seasons, so geological disasters such as road slope landslides and avalanches often occur in Ningbo. In order to prevent the geological disasters, it is necessary to carry out the comprehensive treatment of road slopes to ensure their stability and safety. In order to verify the effectiveness of the developed MADM technique, this section applies the developed MADM technique to an actual example of RSTS selection, and then compares it with the existing SvNV MADM techniques to show the appropriateness and advantages of the developed technique in the scenario of S-SvNSs.

6.1 Selection Example of RSTSs

To comprehensively treat the risk areas of road slopes in Ningbo City, the engineering department provided four RSTSs based on the previous treatment experience and actual requirements as follows:

(a) The RSTS S_{A1}: Drainage ditches, concrete retaining walls, slope protection stones, and protective nets;

(b) The RSTS S_{A2}: Masonry slope protection, retaining walls, slope trimming, and drainage ditches;

(c) The RSTS S_{A3}: Drainage ditches, retaining walls, slope cutting, and gutters;

(d) The RSTS S_{A4}: Drainage ditches, retaining walls, and shotgun technology (surface shotcrete and fixed anchors).

Then they have to satisfy four critical factors/attributes: geological conditions (S_{A1}), technical feasibility (S_{A2}), economics (S_{A3}), and climatic status (S_{A4}). Regarding this MADM problem, the satisfactory degrees of each RSTS with respect to the four factors are assessed by decision-makers/experts to select the optimal one among the four RSTSs.

In the assessment process, the decision makers/experts provide the assessment values of each RSTS over the four factors by the S-SvNVs $r_{SNij} = \langle s_{tdij}, s_{udij}, s_{fdij}; s_{rij}\rangle$ (j, i = 1, 2, 3, 4), which consist of the true, false and indeterminate values and spherical radii. Then, all S-SvNVs form their decision matrix $R_M = (r_{SNij})_{4\times4}$:

$$R_M = \begin{bmatrix} \langle 0.8,0.2,0.2;0.4\rangle & \langle 0.9,0.2,0.1;0.2\rangle & \langle 0.8,0.1,0.1;0.2\rangle & \langle 0.8,0.2,0.2;0.3\rangle \\ \langle 0.8,0.1,0.1;0.3\rangle & \langle 0.8,0.1,0.1;0.3\rangle & \langle 0.7,0.2,0.2;0.3\rangle & \langle 0.8,0.1,0.2;0.4\rangle \\ \langle 0.7,0.3,0.2;0.2\rangle & \langle 0.7,0.2,0.2;0.3\rangle & \langle 0.8,0.1,0.1;0.2\rangle & \langle 0.7,0.2,0.4;0.2\rangle \\ \langle 0.6,0.2,0.1;0.3\rangle & \langle 0.8,0.2,0.3;0.2\rangle & \langle 0.7,0.3,0.3;0.3\rangle & \langle 0.6,0.1,0.1;0.3\rangle \end{bmatrix}.$$

In this actual selection problem of RSTSs, the developed MADM technique is applied and represented by the following decision procedures.

Step 1. Using Eqs. (17) and (18), the attribute weights q_j (j = 1, 2, 3, 4) are derived below:

q_1 = 0.2221, q_2 = 0.2343, q_3 = 0.2628, and q_4 = 0.2809.

Step 2. Applying Eq. (19) or (20), the aggregated values r_{SNi} for S_{Ai} (i = 1, 2, 3, 4) are obtained below:

r_{SN1} = <0.8303, 0.1664, 0.1414; 0.2846>, r_{SN2} = <0.7779, 0.1198, 0.1455; 0.3314>, r_{SN3} = <0.7308, 0.1816, 0.2004; 0.2275>, and r_{SN4} = <0.6868, 0.1822, 0.1712; 0.2799>.

Or r_{SN1} = <0.8182, 0.1794, 0.1587; 0.2610>, r_{SN2} = <0.7690, 0.1340, 0.1625; 0.3251>, r_{SN3} = <0.7224, 0.2087, 0.2598; 0.2198>, and r_{SN4} = <0.6614, 0.2124, 0.2246; 0.2727>.

Step 3. By Eq. (9), the score values of $T(r_{SNi})$ for S_{Ai} (i = 1, 2, 3, 4) are yielded below:

$T(r_{SN1})$ = 0.6717, $T(r_{SN2})$ = 0.6760, $T(r_{SN3})$ = 0.6201, and $T(r_{SN4})$ = 0.6237.

Or $T(r_{SN1})$ = 0.6577, $T(r_{SN2})$ = 0.6650, $T(r_{SN3})$ = 0.5952, and $T(r_{SN4})$ = 0.5955.

Step 4. The ranking of the four RSTSs is $S_{A2} > S_{A1} > S_{A4} > S_{A3}$ and the optimal one is S_{A2}.

6.2 Sensitivity Analysis

To analyze the sensitivity of the spherical radii in the S-SvNV matrix R_M to the ranking of the four RSTSs, it is assumed that s_{rij} = 0 is set in all r_{SNij} = <s_{tdij}, s_{udij}, s_{fdij}; s_{rij}> (j, i = 1, 2, 3, 4) in the decision matrix R_M. In this case, the decision procedures are indicated as follows:

Step 1. Using Eqs. (17) and (18), the attribute weights q_j (j = 1, 2, 3, 4) are derived below:

q_1 = 0.1957, q_2 = 0.1908, q_3 = 0.4019, and q_4 = 0.2115.

Step 2. Applying Eq. (19) or (20), the aggregated values r_{SNi} for S_{Ai} (i = 1, 2, 3, 4) are obtained below:

r_{SN1} = <0.8250, 0.1511, 0.1324; 0>, r_{SN2} = <0.7651, 0.1319, 0.1527; 0>, r_{SN3} = <0.7457, 0.1631, 0.1736; 0>, and r_{SN4} = <0.6895, 0.2022, 0.1901; 0>.

Or r_{SN1} = <0.8147, 0.1674, 0.1494; 0>, r_{SN2} = <0.7543, 0.1488, 0.1688; 0>, r_{SN3} = <0.7351, 0.1952, 0.2341; 0>, and r_{SN4} = <0.6684, 0.2328, 0.2410; 0>.

Step 3. By Eq. (9), the score values of $T(r_{SNi})$ for S_{Ai} (i = 1, 2, 3, 4) are yielded below:

$T(r_{SN1})$ = 0.6354, $T(r_{SN2})$ = 0.6201, $T(r_{SN3})$ = 0.6022, and $T(r_{SN4})$ = 0.5743.

Or $T(r_{SN1})$ = 0.6245, $T(r_{SN2})$ = 0.6092, $T(r_{SN3})$ = 0.5764, and $T(r_{SN4})$ = 0.5486.

Step 4. The ranking of the four RSTSs is $S_{A1} > S_{A2} > S_{A3} > S_{A4}$ and the optimal one is S_{A1}.

Since there is the ranking difference when the values of the spherical radii are zero and not zero in the S-SvNV scenario, we see that the values of the spherical radii are sensitive to the ranking of the four RSTSs. Therefore, the values of the spherical radii can influence the ranking of the four RSTSs, which implies the importance of the spherical radius information in the S-SvNS scenario.

6.3 Related Comparison

To show the superiority of the proposed MADM technique in the setting of S-SvNSs, a comparison with the existing SvNV MADM techniques is provided by the actual example of RSTS selection.

To apply the existing SvNV MADM techniques [15, 32] to the actual example, we have to ignore all the values of the spherical radii in the S-SvNV decision matrix R_M. Based on this special case, the S-SvNV decision matrix R_M is degenerated to the SvNV decision matrix:

$$R_M^* = \begin{bmatrix} \langle 0.8,0.2,0.2\rangle & \langle 0.9,0.2,0.1\rangle & \langle 0.8,0.1,0.1\rangle & \langle 0.8,0.2,0.2\rangle \\ \langle 0.8,0.1,0.1\rangle & \langle 0.8,0.1,0.1\rangle & \langle 0.7,0.2,0.2\rangle & \langle 0.8,0.1,0.2\rangle \\ \langle 0.7,0.3,0.2\rangle & \langle 0.7,0.2,0.2\rangle & \langle 0.8,0.1,0.1\rangle & \langle 0.7,0.2,0.4\rangle \\ \langle 0.6,0.2,0.1\rangle & \langle 0.8,0.2,0.3\rangle & \langle 0.7,0.3,0.3\rangle & \langle 0.6,0.1,0.1\rangle \end{bmatrix}.$$

Then, the weight vector of the four attributes is set as $\boldsymbol{q}$ = (0.1957, 0.1908, 0.4019, 0.2115). In this case, the existing SvNV MADM techniques [15, 32] based on the aggregation operators of Eqs. (1) – (4) and the score and accuracy formulae of Eqs. (5) and (6) can be applied to the actual example of RSTS selection, and then the decision results of the existing MADM techniques [15, 32] can be obtained in the SvNS scenario. All the decision results of different MADM techniques in the SvNS and S-SvNS scenarios are given in Table 1 for convenient comparison.

Table 1. All decision results based on different MADM techniques in the SvNS and S-SvNS scenarios.

MADM technique	Score value	Ranking	The optimal RSTS
MADM technique using the SvNVWA operator [32]	0.8405, 0.8373, 0.7818, 0.7763	$S_{A1} > S_{A2} > S_{A3} > S_{A4}$	S_{A1}
MADM technique using the SvNVWG operator [32]	0.8319, 0.8297, 0.7624, 0.7537	$S_{A1} > S_{A2} > S_{A3} > S_{A4}$	S_{A1}
MADM technique using the SvNVTWA operator [15]	0.8543, 0.8329, 0.8147, 0.7811	$S_{A1} > S_{A2} > S_{A3} > S_{A4}$	S_{A1}
MADM technique using the SvNVTWG operator [15]	0.8387, 0.8183, 0.7821, 0.7426	$S_{A1} > S_{A2} > S_{A3} > S_{A4}$	S_{A1}
Developed MADM technique using the S-SvNVTWA operator	0.6717, 0.6760, 0.6201, 0.6237	$S_{A2} > S_{A1} > S_{A4} > S_{A3}$	S_{A2}
Developed MADM technique using the S-SvNVTWG operator	0.6577, 0.6650, 0.5952, 0.5955	$S_{A2} > S_{A1} > S_{A4} > S_{A3}$	S_{A2}

In the decision results in Table 1, the ranking results and the optimal RSTS based on the existing MADM techniques [15, 32] are completely consistent in the SvNS scenario. Then, the ranking results and the optimal RSTS of the existing MADM techniques and the developed MADM technique imply their difference between the SvNS scenario and the S-SvNS scenario. Therefore, the developed MADM technique not only reveals the information importance of the spherical radii for the ranking of the four RSTSs in the S-SvNS scenario, but also extends the existing MADM capability and scope since S-SvNSs are the generalization of SvNSs in decision information. Due to the fuzziness and uncertainty of human cognitions/judgments under the MADM complication and uncertainty, it is obvious that the S-SvNV evaluation values in the decision process can capture the hybrid information of both the SvNV and the spherical space with a radius to effectively express the crisp SvNV and its uncertain spherical information in a single-valued neutrosophic cube. However, the existing MADM techniques [15, 32] are unable to perform MADM problems with S-SvNV information and unknown attribute weights, then they may also lead to unreasonable/distorted decision results for RSTS selection due to a lack of useful spherical information in MADM applications. In general, the developed MADM technique not only reveals its superiority over the existing techniques, but also effectively improves the decision capability and scope in the S-SvNS scenario.

7. Conclusion

Based on an extension of C-IFS, the proposed S-SvNS can effectively express the hybrid information of both SvNVs and spheres with radii within a single-valued neutrosophic cube. Then, the presented TOLs, S-SvNVTWA and S-SvNVTWG operators, and score and accuracy formulae of S-SvNVs solved the operations and ranking problems of S-SvNVs. In terms of the S-SvNVTWA and S-SvNVTWG operators and score and accuracy formulae of S-SvNVs, the developed MADM technique in the S-SvNS setting can extend the MADM scope and capability and solve MADM problems with periodicity and unknown attribute weights in the S-SvNS scenario. Furthermore, the developed MADM technique was applied to the actual selection example of RSTSs, and then the optimal selection of RSTSs was provided in the S-SvNS scenario. Through sensitivity analysis and comparison with existing SvNV MADM techniques, the efficiency and superiority of the developed technique were verified in the S-SvNS scenario.

Generally, this study includes the following advantages/achievements:

(1) The proposed S-SvNV can effectively represent the hybrid information of both an SvNV and a spherical radius based on an exact SvNV and its uncertain spherical space.

(2) The proposed S-SvNVTWA and S-SvNVTWG operators can perform the S-SvNV aggregation operations with periodicity/multitemporal stages as an extension of the existing SvNVWA, SvNVWG, SvNVTWA, and SvNVTWG operators.

(3) The developed MADM technique using the S-SvNVTWA and S-SvNVTWG operators can solve MADM problems with the requirement of periodicity/multitemporal stages and unknown attribute weights in the S-SvNS scenario based on the generalization of existing MADM techniques in SvNS scenarios.

Although this study has achieved the above, in the future, more S-SvNV trigonometric aggregation operators will be developed based on Enistein, Aczel-Alsina, and Dombi operations and their decision/evaluation models will be established to tackle GDM, slope stability classification and evaluation, environmental risk and mine safety evaluation, as well as image processing and medical diagnosis in S-SvNS scenarios.

Conflicts of Interest: The authors declare that they have no conflicts of interest.

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