Application of Information Theoretic Measures in Neutrosophic Soft Environment

Surender Singh[1,3],* and Sonam Sharma[2]

[1] School of Mathematics, Shri Mata Vaishno Devi University, Katra-182320, Jammu and Kashmir, India
surender.singh@smvdu.ac.in
[2] School of Mathematics, Shri Mata Vaishno Devi University, Katra-182320, Jammu and Kashmir, India
20dmt001@smvdu.ac.in
[3] Department of Mathematics, Kurukshetra University, Kurukshetra-136119, Haryana, India

* Correspondence: surender.singh@smvdu.ac.in; surender1976@gmail.com

Abstract: Soft set theory is a valuable mathematical tool for modeling and analyzing uncertain systems. A neutrosophic soft set is a hybrid entity of a neutrosophic set, and a soft set that enables a more comprehensive analysis of uncertainty in a system. In this article, we introduce some novel information theoretic measures in a single-valued neutrosophic soft environment. Additionally, we study the data-dimensionality reduction using two-pronged approach, leveraging the score matrix and neutrosophic soft entropy measure. The complexity of decision-making problems involving numerous factors can be alleviated using the dimensionality reduction technique. Finally, the comparative analysis is presented with the help of an illustrative example utilizing a measure of performance. The comparative study highlighted the advantage of the proposed methods.

Keywords: single-valued neutrosophic set, neutrosophic soft set, dimensionality reduction, score matrix, entropy measure, similarity measure.

1. Introduction

Uncertainty is an inherent part of real-world systems, and several methods are available in the literature for the representation of uncertain data or information. The most prevalent methods include probability theory, rough set theory, fuzzy set theory, and intuitionistic fuzzy set theory, and are utilized for modeling human-centric and expert-based systems. The real-life applications concerned with decision-making, clustering analysis, pattern recognition, anomaly detection, image analysis, and many more have been investigated using these methods. However, all these theories have their own limitations in varied situations. This article aims to put forth a neutrosophic soft information measure, and its implication. In [1], Molodtsov introduced the concept of a soft set by integrating the parametrization tool with the classical set. Soft set theory has been widely applied in a variety of fields such as decision-making [2-8], data analysis [9], forecasting [10], simulation [11], optimization [12], texture classification [13], etc. Afterward, by combining a Soft set [1] and a fuzzy set (FS) [14], Maji et al.[8] suggested the concept of Fuzzy soft sets. Similarly, Maji et al. [15] introduced the concept of intuitionistic fuzzy soft sets (IFSSs) due to the fusion of soft sets and intuitionistic fuzzy sets (IFS) [16]. Thereafter, various researchers studied diverse mathematical hybrid structures such as generalized intuitionistic fuzzy soft sets [17], generalized fuzzy soft sets [18] possibility intuitionistic fuzzy soft sets [19], vague soft sets [20], interval-valued fuzzy soft sets [21], interval-valued intuitionistic fuzzy soft sets [22], etc. Furthermore, Maji [23] introduced the notion of the neutrosophic soft set by fusion of a neutrosophic set (NS) [24] and a soft set [1]. Maji [25] suggested some operations and propositions related to a weighted neutrosophic soft set. Wang

[26] suggested a subclass of neutrosophic soft sets. Marei [27] developed a rough set approach in single-valued neutrosophic soft settings. Neutrosophic soft sets (NSSs), due to their ability to deal with intermediate, inconsistent, and neutrosophic parameters found extensive implications in a variety of fields.

The essential information theoretic measures like entropy and Knowledge measure quantify the information content of a fuzzy/non-fuzzy set. Furthermore, similarity and distance measures evaluate the extent of closeness and discrimination between two fuzzy/non-fuzzy sets. In the problems related to pattern recognition, the concept of similarity/distance measure is commonly employed to verify the authenticity of an object/document. In recent years, several studies [28-41] suggested different information-theoretic measures concerned with fuzzy sets, fuzzy soft sets, intuitionistic fuzzy sets, intuitionistic fuzzy soft sets, single-valued neutrosophic sets, and single-valued neutrosophic soft sets. Broumi [29] studied similarity measures for neutrosophic soft sets. Dey et al. [37] presented a neutrosophic soft similarity measure for selecting the suitable alternative based on a grey relational analysis involving multiple decision-makers. Karaaslan [38] suggested a decision-making method and a group decision-making method in the neutrosophic soft environment. Sahin and Kucuk [39] suggested various distance measures between neutrosophic soft sets and introduced an axiomatic definition of entropy for a neutrosophic soft set. The proposed research work is aimed to introduce neutrosophic soft information measures, and their application to decision-making and data dimensionality reduction.

The dimensionality reduction technique is instrumental in mitigating the curse of dimensionality, enabling the effective management of high-dimensional data sets and the elimination of irrelevant features. This approach offers a threefold benefit: Simplifying data, visualizing complex relationships, and managing multicollinearity. As a result, this method has become a vital area of study in diverse computational disciplines, especially those characterized by extreme data modality. In the fuzzy soft set, the dimension reduction technique of big data was utilized to convert soft tables into fuzzy soft set tables [42]. The concept of Pythagorean fuzzy soft matrix, introduced by Bajaj [43] has paved the way for the development of a new generation of dimensionality reduction approaches, tailored to address the complexities of MCDM problems. Picture fuzzy soft matrix was suggested in Devi *et al.* [44] to solve decision-making problems and the dimensionality reduction can be dealt with in a better and broader sense of human opinion. However, there has been rather little work completed for entropy, and similarity measure in the context of single-valued neutrosophic soft sets and their applications.

1.1. *Contribution*

The main contribution of this paper is as follows:

- We propose novel entropy, and similarity measures for the SVNSSs to overcome the shortcomings of existing measures.
- The newly proposed measures are applied for solving the MCDM problem and for data reduction technique.
- Finally, a comparative analysis has been done to check the effectiveness of the proposed measure based on performance measures.

1.2. *Importance of neutroshopy in the present work*

Neutrosophy provides a more rational extension to traditional fuzzy logic by explicitly modelling indeterminacy. It enhances data dimensionality reduction by guiding the selection of relevant, lownoise features and improves decision-making processes by providing a structured framework to deal with incomplete, contradictory, and uncertain information. The independent choice of trueness, falsity, and indeterminacy in a single valued neutrosophic set make the neutrosophic information measures more robust and indispensable tools in the real-life problems concerned with of classification, pattern-recognition and decision-support.

The content of this article is organised as follows.

Section 2 presents basic concepts relevant to this study. Section 3 introduces the similarity, and entropy measure in a single-valued neutrosophic soft environment. Section 4 demonstrates the application of the proposed measure in MCDM problem as well as in data dimensionality reduction. In section 5. We contrast the performance of the proposed methods with existing methods. Section 6 concludes the article.

2. Preliminaries

In this section, we present some essential fundamental concepts concerned with this article.

Definition 2.1 ([1]). Let U be a generic universe and P be a set of parameters in U. Consider $E \subseteq P$. A pair (F, E) is said to be a soft set over the universe set in which F is a mapping from E to 2^U, where 2^U is a power set of U. In short, a soft set over U is a parameterized family of a subset of U.

Definition 2.2 ([8]). Suppose U is a generic universe of objects. Consider P be a set of parameters in U and $E \subseteq P$. A pair (F, E) is said to be a fuzzy soft set over the universe set U, in which F is a mapping from E to FS^U, where FS^U is a set of all fuzzy subsets of the universe set U.

Definition 2.3 ([15]). Let U be a generic universe of objects and P be a set of parameters in U. Consider $E \subseteq P$ and let IFS^U denote the collection of all intuitionistic fuzzy sets of U. A pair (F, E) is said to be intuitionistic fuzzy soft set over the universe set U, where F is a mapping F: $E \to IFS^U$.

Definition 2.2 ([24]**).** Let U be a universe of discourse with generic element y in U. A single-valued neutrosophic set B in U is characterized by truth-membership degree $T_B(y_i)$, indeterminacy degree $I_B(y_i)$ and falsity-membership degree $F_B(y_i)$. For each $y_i \in Y$, $T_B(y_i)$, $I_B(y_i)$, $F_B(y_i) \in [0,1]$. A single-valued neutrosophic set B can be denoted by a triplet i.e., $B = \{\langle T_B(y_i), I_B(y_i), F_B(y_i)\rangle | \, y_i \in Y\}$ with $T_B(y_i) + I_B(y_i) + F_B(y_i) \in [0,3]$.

Definition 2.4 ([23]). Let U be a universe of objects and P be a set of parameters in U. Consider $E \subseteq P$ and NS^U denote the collection of all neutrosophic sets of U. A pair (F, E) is said to be neutrosophic soft set over the universe set U, where F is a mapping F: $E \to NS^U$.

Operations on Single-Valued Neutrosophic Soft Sets: Let (F, E), (G, E), and (H, E) be three single-valued neutrosophic soft sets, then we have the following operations.

Union: $(F, E) \cup (G, E) = \left(max.\left(T_{F(e)}(y), T_{G(e)}(y)\right), \frac{I_{F(e)}(y) + I_{G(e)}(y)}{2}, min.\left(T_{F(e)}(y), T_{G(e)}(y)\right) \right)$ if $e \in A \cap B$.

Intersection: $(F,E)\cap(G,E)=\left(min.\left(T_{F(e)}(y),T_{G(e)}(y)\right),\frac{I_{F(e)}(y)+I_{G(e)}(y)}{2},max.\left(T_{F(e)}(y),T_{G(e)}(y)\right)\right)$ if $e\in A\cap B$.

Complement: $(F,E)^c=\left(T_{F^c(e)(y)}=F_{F(e)(y)},I_{F^c(e)(y)}=I_{F(e)(y)},\ F_{F^c(e)(y)}=T_{F(e)(y)}\right)$.

Subset: If $(F,E)\subseteq(G,\ E)$ then $T_{F(e)}(y)\leq T_{G(e)}(y),I_{F(e)}(y)\leq I_{G(e)}(y),F_{F(e)}(y)\geq F_{G(e)}(y)$.

Definition 2.5 ([27]). Let U be the universal set and P be a set of parameters. Consider $E\subseteq P$ and, let$SVNS^U$ denote the set of all single–valued neutrosophic sets of U. The collection (F,E) is said to be a single-valued neutrosophic soft set over the universe set U, where F is a mapping $F\colon E\to SVNS^U$.

Example 2.1 ([23]). Let $U=\{y_1,y_2,y_3,y_4,y_5\}$be a set of five models of car out of which one car is to be purchased and $E=\{e_1=\text{elegant},e_2=\text{trustworthy},e_3=\text{sporty},e_4=\text{comfortable},e_5=\text{modern}\}$be the set of parameters to select the car. The SVNSS (F,E) in this example can be presented in the following Table 1.

Table 1. Tabular representation of (F,E) of Example 1.

U	e_1	e_2	e_3	e_4
y_1	(0.6,0.3, 0.4)	(0.4,0.3, 0.3)	(0.1,0.1, 0.1)	(0.7,0.5, 0.6)
y_2	(0.4,0.3, 0.4)	(0.5,0.6, 0.4)	(0,0, 0.4)	(0.8,0.3, 0.3)
y_3	(0.6,0.5, 0.4)	(0.3,0.3, 0.4)	(0.2,0.3, 0.7)	(0.3,0.3, 0.3)
y_4	(0, 0.3, 0.4)	(0.1,0.2, 0.4)	(0.2,0.2, 0.4)	(0.2,0.9, 0.1)
y_5	(0.2,0.3, 0.1)	(0.7,0.3, 0.4)	(0.9,0.3, 0.3)	(0.8,0.5, 0.2)

Sahin and Kucuk [39] and I.Arockiarani [45] proposed entropy measures for neutrosophic soft environments satisfying some axiomatic requirements. An entropy measure of a SVNSS should satisfy axiomatic requirements given in the following definition of entropy measure.

Definition 2.6. Let $U=\{y_1,y_2,y_3,\ldots,y_m\}$ be a generic universe and $E=\{e_1,e_2,e_3,\ldots,e_n\}$ be the set of parameters. A function $EM\colon NSS(Y)\to[0,1]$ is said to be an entropy measure on $NSS(Y)$ if EM satisfies the following axioms:

$\boldsymbol{NSEM1}.EM(F,E)=0\iff\forall e\in E;\ (F,E)$ is a soft set;

$\boldsymbol{NSEM2}.EM(F,E)=EM(F,E)^c$;

$\boldsymbol{NSEM3}.EM(F,E)=1$ iff $T_{F(e_i)}(y_j)=I_{F(e_i)}(y_j)=F_{F(e_i)}(y_j)\ \forall\ e\in E,y\in Y$;

NSEM4. If $(G,E)\subseteq(F,E)$then $EM(F,E)\leq EM(G,E)$.

Definition 2.7 ([35])Let $U=\{y_1,y_2,y_3,\ldots,y_m\}$ be a generic universe and $E=\{e_1,e_2,e_3,\ldots,e_n\}$ be the set of parameters. Let (F,E), and (G,E) be two neutrosophic soft sets over U, where F,G are mappings given by $F,G\colon E\to NS(U)$. Then $SM((F,E),(G,E))$ is said to be a similarity measure between (F,E) and (G,E) if it satisfies the following axioms:

$\boldsymbol{NSSM1}.\, 0 \leq SM\big((F,E),(G,E)\big) \leq 1;$

$\boldsymbol{NSSM2}.\, SM\big((F,E),(G,E)\big) = SM\big((G,E),(F,E)\big);$

$\boldsymbol{NSSM3}.\, SM\big((F,E),(G,E)\big) = 1$ iff $(F,E) = (G,E);$

NSSM4. If $(F,E) \subseteq (G,E) \subseteq (H,E)$ then $SM((F,E),(H,E)) \leq SM((G,E),(H,E))$.

Definition 2.9 ([36]). The performance measure of a similarity method (say M) that satisfies the optimality criteria to solve an IFSS-based decision-making problem is defined as

$$P_M = SM_{rt} + \frac{1}{\sum_{i=1\,(r,t)\neq(i,j)}^{n-1}\sum_{j=i+1}(1-SM_{ij})}\ ;\ SM_{rt} > SM_{ij},$$

where SM_{rt} denotes the highest similarity value of an object and SM_{ij} is the similarity value of the remaining object.

Performance measure represents the sum of the highest similarity value of an object and an inverse of the summation of the non-similarity values of the remaining objects.

3. The proposed Information Measure for SVNSSs

In this section, we suggest a similarity measure, and an entropy measure in a single-valued neutrosophic soft environment.

3.1 Similarity Measure

Consider $U = \{y_1, y_2, y_3, \dots, y_m\}$ be the universe of discourse and $E = \{e_1, e_2, e_3, \dots, e_n\}$ be the set of parameters. Then similarity measure between two single-valued neutrosophic soft sets (F,E), and (G,E) is defined as

$$SM_{SVNSSs}(F,G) = \frac{1}{m}\sum_{i=1}^{n}\Big(1 - \frac{1}{4n}\sum_{j=1}^{m}[\big|S_{F(e_i)}(y_j) - S_{G(e_i)}(y_j)\big| + \big|T_{F(e_i)}(y_j) - T_{G(e_i)}(y_j)\big| + \big|I_{F(e_i)}(y_j) - I_{G(e_i)}(y_j)\big| + |F_{F(e_i)}(y_j) - F_{G(e_i)}(y_j)|]\Big) \quad (1)$$

Where, $S_{F(e_i)}(y_j) = \big(T_{F(e_i)}(y_j) - I_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)\big)$, and

$S_{G(e_i)}(y_j) = \big(T_{G(e_i)}(y_j) - I_{G(e_i)}(y_j) - F_{G(e_i)}(y_j)\big)$.

Theorem 3.1. $SM_{SVNSSs}(F,G)$ is a valid similarity measure between SVNSSs (F,E) and (G,E).

Proof. To check the validity of $SM_{SVNSSs}(F,G)$, we verify the axiomatic requirements given in definition 2.7.

NSSM1. Since $S_{F(e_i)}(y_j) \in [-1,1]$ and $S_{G(e_i)}(y_j) \in [-1,1]$. Then $\big|S_{F(e_i)}(y_j) - S_{G(e_i)}(y_j)\big| \leq 1$. Also, $\big|T_{F(e_i)}(y_j) - T_{G(e_i)}(y_j)\big| \leq 1$, $\big|I_{F(e_i)}(y_j) - I_{G(e_i)}(y_j)\big| \leq 1$, $\big|F_{F(e_i)}(y_j) - F_{G(e_i)}(y_j)\big| \leq 1$, which implies $SM_{SVNSSs}(F,G) \in [0,1]$.

NSSM2. $SM_{SVNSSs}(F,G) = \frac{1}{m}\sum_{i=1}^{n}\Big(1 - \frac{1}{4n}\sum_{j=1}^{m}[\big|S_{F(e_i)}(y_j) - S_{G(e_i)}(y_j)\big| + \big|T_{F(e_i)}(y_j) - T_{G(e_i)}(y_j)\big| +$

$\big|I_{F(e_i)}(y_j) - I_{G(e_i)}(y_j)\big| + |F_{F(e_i)}(y_j) - F_{G(e_i)}(y_j)|]\Big) = \frac{1}{m}\sum_{i=1}^{n}\Big(1 - \frac{1}{4n}\sum_{j=1}^{m}([\big|S_{G(e_i)}(y_j) - S_{F(e_i)}(y_j)\big| +$

$\big|T_{G(e_i)}(y_j) - T_{F(e_i)}(y_j)\big| + \big|I_{G(e_i)}(y_j) - I_{F(e_i)}(y_j)\big| + |F_{G(e_i)}(y_j) - F_{F(e_i)}(y_j)|])\Big) = SM_{SVNSSs}(G,F)$.

NSSM3. $SM_{SVNSSs}(F,G)=1 \Leftrightarrow \frac{1}{m}\sum_{i=1}^{n}\left(1-\frac{1}{4n}\sum_{j=1}^{m}[|S_{F(e_i)}(y_j)-S_{G(e_i)}(y_j)|+|T_{F(e_i)}(y_j)-T_{G(e_i)}(y_j)|+|I_{F(e_i)}(y_j)-I_{G(e_i)}(y_j)|+|F_{F(e_i)}(y_j)-F_{G(e_i)}(y_j)|]\right)=0 \Leftrightarrow |S_{F(e_i)}(y_j)-S_{G(e_i)}(y_j)|=0,\ |T_{F(e_i)}(y_j)-T_{G(e_i)}(y_j)|=0, |I_{F(e_i)}(y_j)-I_{G(e_i)}(y_j)|=0, |F_{F(e_i)}(y_j)-F_{G(e_i)}(y_j)|=0 \Leftrightarrow T_{F(e_i)}(y_j)=T_{G(e_i)}(y_j), I_{F(e_i)}(y_j)=I_{G(e_i)}(y_j), F_{F(e_i)}(y_j)=F_{G(e_i)}(y_j) \Leftrightarrow (F,E)=(G,E)$.

NSSM4. Consider $(F,E)\subseteq(G,E)\subseteq(H,E)$. Then
$T_{F(e_i)}(y_j)\le T_{G(e_i)}(y_j)\le T_{H(e_i)}(y_j)$,
$I_{F(e_i)}(y_j)\le I_{G(e_i)}(y_j)\le I_{H(e_i)}(y_j)$, and
$F_{F(e_i)}(y_j)\ge F_{G(e_i)}(y_j)\ge F_{H(e_i)}(y_j)$.
Therefore,

$$|T_{F(e_i)}(y_j)-T_{H(e_i)}(y_j)|\ge|T_{G(e_i)}(y_j)-T_{H(e_i)}(y_j)|. \tag{2}$$

$$|I_{F(e_i)}(y_j)-I_{H(e_i)}(y_j)|\ge|I_{G(e_i)}(y_j)-I_{H(e_i)}(y_j)|. \tag{3}$$

$$|F_{F(e_i)}(y_j)-F_{H(e_i)}(y_j)|\ge|F_{G(e_i)}(y_j)-F_{H(e_i)}(y_j)|. \tag{4}$$

Therefore, $S_{F(e_i)}(y_j)-S_{H(e_i)}(y_j)=\left(T_{F(e_i)}(y_j)-I_{F(e_i)}(y_j)-F_{F(e_i)}(y_j)\right)-\left(T_{H(e_i)}(y_j)-I_{H(e_i)}(y_j)-F_{H(e_i)}(y_j)\right)$

$$=\left(T_{F(e_i)}(y_j)-T_{H(e_i)}(y_j)\right)+\left(I_{H(e_i)}(y_j)-I_{F(e_i)}(y_j)\right)+\left(F_{H(e_i)}(y_j)-F_{F(e_i)}(y_j)\right)$$

Similarly, $S_{F(e_i)}(y_j)-S_{G(e_i)}(y_j)=\left(T_{F(e_i)}(y_j)-I_{F(e_i)}(y_j)-F_{F(e_i)}(y_j)\right)-\left(T_{G(e_i)}(y_j)-I_{G(e_i)}(y_j)-F_{G(e_i)}(y_j)\right)$

$$=\left(T_{F(e_i)}(y_j)-T_{G(e_i)}(y_j)\right)+\left(I_{G(e_i)}(y_j)-I_{F(e_i)}(y_j)\right)+\left(F_{G(e_i)}(y_j)-F_{F(e_i)}(y_j)\right)$$

This implies, $|S_{F(e_i)}(y_j)-S_{H(e_i)}(y_j)|\ge|S_{F(e_i)}(y_j)-S_{G(e_i)}(y_j)|$. (5)

By combining (2), (3), (4), and (5), we get
$SM_{SVNSSs}(F,H)\le SM_{SVNSSs}(G,H)$.
This completes the proof.

3.2. Entropy Measure

Suppose $U=\{y_1,y_2,y_3,\dots,y_m\}$ be the universe of discourse and $E=\{e_1,e_2,e_3,\dots,e_n\}$be the set of parameters. Then entropy measure of a single valued neutrosophic soft set (F,E) is denoted and defined as

$$EM_{SVNSSs}(F,E)=\frac{1}{\sqrt{2}-1}\frac{1}{m}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j)-F_{F(e_i)}(y_j)}{4}\right)\pi-1\right) \tag{6}$$

Theorem 3.3. $EM_{SVNSSs}(F,E)$ is a valid entropy measure of SVNSSs (F,E).

Proof. To check the validity of $EM_{SVNSSs}(F,E)$, we verify the axiomatic requirements given in definition 2.6.

NSEM1. $EM_{SVNSS}(F,E)=0$ if and only if

$$\frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j)-F_{F(e_i)}(y_j)}{4}\right)\pi-1\right)=0$$

if and only if $\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j)-F_{F(e_i)}(y_j)}{4}\right)\pi-1\right)=0.$

If and only if $T_{F(e_i)}(y_j)=0\ or F_{F(e_i)}(y_j)=1$ or $F_{F(e_i)}(y_j)=0\ or\ T_{F(e_i)}(y_j)=1, \forall\ e_i\in E, y\in U$.

NSEM2. $EM_{SVNSS}(F,E)=1$

If and only if $\frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)}{4}\right)\pi - 1\right) = 1.$

If and only if $\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)}{4}\right)\pi - 1\right) = \sqrt{2}-1$.

If and only if $Cos\left(\frac{T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)}{4}\right)\pi = 1.$

If and only if $T_{F(e_i)}(y_j) = F_{F(e_i)}(y_j)$.

NSEM3. $EM_{SVNSS}(F,E) = \frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)}{4}\right)\pi - 1\right)$

$= \frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{F_{F^c(e_i)}(y_j) - T_{F^c(e_i)}(y_j)}{4}\right)\pi - 1\right) = EM_{SVNSS}(F,E)^c.$

NSEM4. $\forall\, e_i \in E, y \in U$, when $(G,E) \subseteq (F,E)$ then $T_{F(e_i)}(y_j) \leq T_{G(e_i)}(y_j)$; $I_{F(e_i)}(y_j) \leq I_{G(e_i)}(y_j)$; $F_{F(e_i)}(y_j) \geq F_{G(e_i)}(y_j)$ which implies $T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j) \leq T_{G(e_i)}(y_j) - F_{G(e_i)}(y_j)$ or $\frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)}{4}\right)\pi - 1\right) \leq \frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{G(e_i)}(y_j) - F_{G(e_i)}(y_j)}{4}\right)\pi - 1\right)$

and hence we get, $EM_{SVNSS}(F,E) \subseteq EM_{SVNSS}(G,E)$.

In the next section, we present some applications of our suggested measures i.e., similarity, and entropy measure.

4. Applications

In this section, we apply the suggested similarity measure to a decision-making problem.

4.1. Application to Suitable Location of Industrial Unit

In this subsection, we introduce a method for solving a decision-making problem based on the proposed similarity measure. The concept of an ideal point has been utilized to identify the most suitable alternative in decision-making processes. Although an ideal alternative may not exist in real-world scenarios, it provides a valuable theoretical framework for evaluating alternatives. We define the ideal alternative y^*as the SVNS ${y^*}_j = (T^*, I^*, F^*) = (1,0,0)\ \forall\, j$. To utilize our proposed measure for the selection of a suitable location of an industrial unit, following algorithm is suggested.

Algorithm 1.

Step 1. Identify the alternatives and parameters, and obtain the single-valued neutrosophic soft set (F,E) as shown in the Table 1.

Table 1. Tabular representation of (F,E)

Alternative/Parameter → ↓	e_1	e_2	…	e_n
y_1	$F_{(e_1)}(y_1)$	$F_{(e_2)}(y_1)$	…	$F_{(e_n)}(y_1)$
y_1	$F_{(e_1)}(y_2)$	$F_{(e_2)}(y_2)$	…	$F_{(e_n)}(y_2)$
.			….	
.				
.				

y_m	$F_{(e_1)}(y_m)$	$F_{(e_2)}(y_m)$	$\dots$	$F_{(e_n)}(y_m)$

Where $F_{(e_i)}(y_j) = \left(T_{F(e_i)}(y_i), I_{F(e_i)}(y_j), F_{F(e_i)}(y_j)\right)$.

Step 2. Normalize the SVN soft set (F, E) into (F', E) using the following framework.

$$\begin{Bmatrix} \left(T_{F(e_i)}(y_j), I_{F(e_i)}(y_j), F_{F(e_i)}(y_j)\right), e_i \in B \\ \left(F_{F(e_i)}(y_j), 1 - I_{F(e_i)}(y_j), T_{F(e_i)}(y_j)\right), e_i \in C \end{Bmatrix}$$

Where B is the benefit parameter set and C is the cost parameter set.
Step 3. Compute the similarity measure $SM(y_j, y^*), j = 1, 2, 3, \dots m$ by using Eq. (1).
Step 4. Evaluate the performance measure corresponding to the similarity value of the alternatives.
Step 5. Choose or select the suitable measure that offers a higher accuracy rate/performance measure.
Next, we consider the following numerical example to illustrate the procedure.
Example 4.1 Industrial site selection is a complex decision-making process that involves evaluating multiple factors, including technical, economic, social, environmental, and political considerations, highlighting the need for a robust tool and knowledge base to support data collection, analysis, and site management. Let us suppose a company X wants to select a suitable location for setting up an industry. Assume that there are four locations: location A, location B, location C, and location D. The company selects six parameters to evaluate the four locations. Let $U = \{y_1 = location\ A, y_2 = location\ B,\ y_3 = location\ C, y_4 = location\ D\}$ be the set of locations and $E = \{e_1, e_2, e_3, e_4,\ e_5, e_6\}$ is a set of parameter, where $e_1 =$ raw material, $e_2 =$ power, $e_3 =$ labour, $e_4 =$ Transport, $e_5 =$ vulnerability to nature, e_6 = investment climate. The decision data given by the expert is shown in Table 2, decision scenario is visualized in Figure 1.
Now, we utilize the above algorithm 1 to select the suitable industry under single-valued neutrosophic soft information.

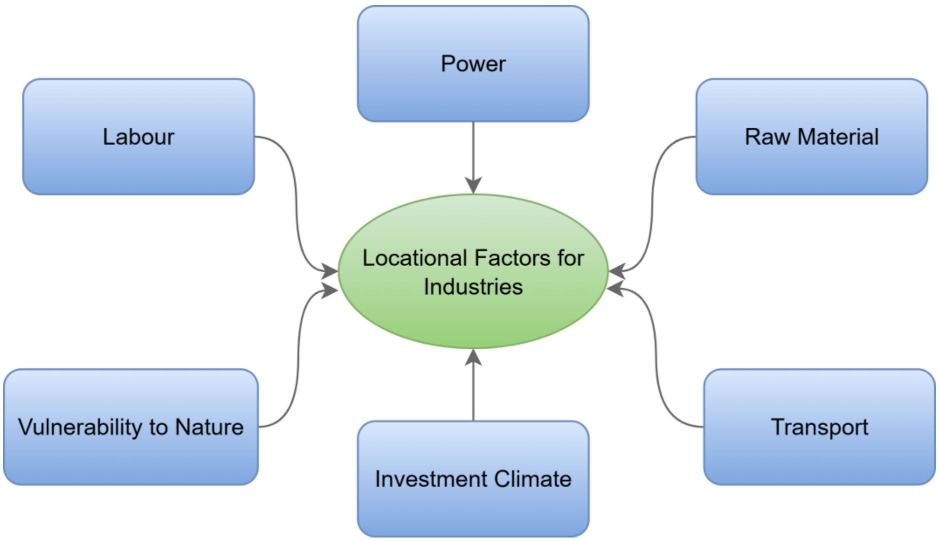

Figure 1. Decision scenario for location of industrial unit

Implementation of Algorithm 1.

Step 1. Select the alternatives and parameters, and obtain the single-valued neutrosophic soft set (F, E) as shown in Table 2.

Table 2. Tabular representation of (F, E)

Alternative	e_1	e_2	e_3	e_4	e_5	e_6
A	(0.6, 0.2, 0.3)	(0.7, 0.3, 0.4)	(0.4, 0.3. 0.6)	(0.8, 0.2, 0.3)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)
B	(0.4, 0.1, 0.2)	(0.3, 0.1, 0.2)	(0.2, 0.2, 0.4)	(0.3, 0.1, 0.4)	(0.2, 0.1, 0.3)	(0.2, 0.3, 0.5)
C	(0.7, 0.3, 0.4)	(0.8, 0.2, 0.5)	(0.4, 0.2, 0.5)	(0.8, 0.1, 0.2)	(0.5, 0.3, 0.2)	(0.3, 0.2, 0.5)
D	(0.3, 0.2, 0.3)	(0.2, 0.2, 0.3)	(0.3, 0.1, 0.3)	(0.3, 0.2, 0.3)	(0.1 0.2, 0.2)	(0.3, 0.2, 0.5)

Step 2. As all the parameters are benefit parameters, so there is no need to normalize. Therefore, the normalized SVNS (F', E) is similar to Table 2.

Step 3. Compute the similarity measure $SM(y_j, y^*), j = 1, 2, 3, \dots m$ by using equation (1) and (6), shown as follows:

$SM(y_1, y^*) = 0.8604$, $SM(y_2, y^*) = 0.8479, SM(y_3, y^*) = 0.8729, SM(y_4, y^*) = 0.8437$

Step 4. Evaluate the performance measure corresponding to the similarity values given above. Firstly, we consider some existing measures which is given as below.

$SM_1 = \frac{1}{1+L\,(F,G\,)}$, where

$$L\,(F,G\,) = \frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|T_{F(e_i)}(y_j) - T_{G(e_i)}(y_j)\right| + \left|I_{F(e_i)}(y_j) - I_{G(e_i)}(y_j)\right| + \left|F_{F(e_i)}(y_j) - F_{G(e_i)}(y_j)\right|$$

Mukherjee and Sarkar [34]

$$SM_2 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(T_{F(e_i)}(y_j)\wedge T_{G(e_i)}(y_j)\right) + \left(I_{F(e_i)}(y_j)\wedge I_{G(e_i)}(y_j)\right) + \left(F_{F(e_i)}(y_j)\wedge F_{G(e_i)}(y_j)\right)\right\}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(T_{F(e_i)}(y_j)\vee T_{G(e_i)}(y_j)\right) + \left(I_{F(e_i)}(y_j)\vee I_{G(e_i)}(y_j)\right) + \left(F_{F(e_i)}(y_j)\vee F_{G(e_i)}(y_j)\right)\right\}}$$

Mukherjee and Sarkar [35]

$$SM_3 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(T_{F(e_i)}(y_j)T_{G(e_i)}(y_j)\right) + \left(I_{F(e_i)}(y_j)I_{G(e_i)}(y_j)\right) + \left(F_{F(e_i)}(y_j)F_{G(e_i)}(y_j)\right)\right\}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(T_{F(e_i)^2}(y_j)\vee T_{G(e_i)^2}(y_j)\right) + \left(I_{F(e_i)^2}(y_j)\vee I_{G(e_i)^2}(y_j)\right) + \left(F_{F(e_i)^2}(y_j)\vee F_{G(e_i)^2}(y_j)\right)\right\}}$$

Sinha and Majumdar [46]

$$SM_4 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\sqrt{\left(T_{FG(e_i)}(y_j)\right)^2 + \left(I_{FG(e_i)}(y_j)\right)^2 + \left(F_{FG(e_i)}(y_j)\right)^2}\right\}}{max(|\alpha(e_i)|, |\beta(e_i)|)}$$

Binu and Paul [33]

Where $T_{FG(e_i)}(y_j) = T_{F(e_i)}(y_j)I_{G(e_i)}(y_j) - I_{F(e_i)}(y_j)T_{G(e_i)}(y_j)$,

$I_{FG(e_i)}(y_j) = T_{F(e_i)}(y_j)F_{G(e_i)}(y_j) - F_{F(e_i)}(y_j)I_{G(e_i)}(y_j)$,
$F_{FG(e_i)}(y_j) = F_{F(e_i)}(y_j)T_{G(e_i)}(y_j) - T_{F(e_i)}(y_j)F_{G(e_i)}(y_j)$,

$\alpha(e_i) = \sqrt{\left(T_{F(e_i)}(y_j)\right)^2 + \left(I_{F(e_i)}(y_j)\right)^2 + \left(F_{F(e_i)}(y_j)\right)^2}$, and

$\beta(e_i) = \sqrt{\left(T_{G(e_i)}(y_j)\right)^2 + \left(I_{G(e_i)}(y_j)\right)^2 + \left(F_{G(e_i)}(y_j)\right)^2}$.

$SM_5 = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} max.\{|T_{F(e_i)}(y_j) - T_{G(e_i)}(y_j)|, |I_{F(e_i)}(y_j) - I_{G(e_i)}(y_j)|, |F_{F(e_i)}(y_j) - F_{G(e_i)}(y_j)|\}$.

Sarkar and Ghosh [47]

The computed values of the existing and proposed measures are shown in Table 3 along with their performance measure (given in definition 2.9 and definition 2.10).

Table 3. Similarity measure values along with performance measure

Measures	A	B	C	D	**PM1**
SM_1	0.8453	0.8328	0.8570	0.8283	2.882
SM_2	0.333	0.33	0.3738	0.1659	0.8057
SM_3	0.4444	0.743	0.4878	0.2176	0.9636
SM_4	0.3833	0.562	0.3833	0.3166	0.8917
SM_5	0.0205	0.55	0.0198	0.0307	0.3720
Proposed SM	0.8604	0.8479	0.8729	0.8437	3.1050

Step 5. From the performance measure of the proposed measures and existing measures shown in Table 3, we conclude that our suggested measures have a high degree of accuracy while comparing with the existing measures.

4.2. Dimensionality Reduction Technique for SVN Soft Matrix in Decision- Making

In the present subsection, we investigate two dimensionality-reduction techniques i.e., score-based dimensionality reduction technique and entropy-based dimensionality-reduction technique. Firstly, we present some relevant definition of object-oriented SVN soft matrix, parameter-oriented SVN soft matrix, score matrix, and threshold value of SVN soft matrix. In the following, we present some essential definitions to understand the techniques of data dimensionality reduction.

Definition 4.1 ([44]). Let $E = \{e_1, e_2, e_3, \dots, e_n\}$ be parametrs and $U = \{y_1, y_2, y_3, \dots, y_m\}$ be the universe of discourse, then for SVN soft set (F, E),

$$O_i = \left[\sum_j \frac{T_{ij}}{|E|}, \sum_j \frac{I_{ij}}{|E|}, \sum_j \frac{F_{ij}}{|E|}\right]$$

is known as oriented-object grade with respect to parameters.

Also,

$$E_j = \left[\sum_j \frac{T_{ij}}{|U|}, \sum_j \frac{I_{ij}}{|U|}, \sum_j \frac{F_{ij}}{|U|}\right]$$

is known as oriented parameter grade with respect to objects. Where $|U|$ and $|E|$ denotes the cardinality of universal set and parameter set.

Definition 4.2. The threshold value of the SVN soft matrix using Entropy measure is computed as follows.

$$E_{TH} = \frac{1}{\sqrt{2}-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sqrt{2}\cos\left(\frac{T_{F(e_i)}(y_j) - F_{F(e_i)}(y_j)}{4}\right)\pi - 1\right)$$

where $TH = (T_{TH}, I_{TH}, F_{TH}) = \left[\sum_{i,j}\frac{T_{ij}}{|U\times P|}, \sum_{i,j}\frac{I_{ij}}{|U\times P|}, \sum_{i,j}\frac{F_{ij}}{|U\times P|}\right]$.

Definition 4.3.([48]**)** The threshold value of the SVN soft matrix from the matrix itself is computed as follows.

$\overline{SM}_{TH} = [s_{ij}] = [T_{ij} - I_{ij}F_{ij}] \quad \forall\, i, j$

where $TH = (T_{TH}, I_{TH}, F_{TH}) = \left[\sum_{i,j}\frac{T_{ij}}{|U\times P|}, \sum_{i,j}\frac{I_{ij}}{|U\times P|}, \sum_{i,j}\frac{F_{ij}}{|U\times P|}\right]$.

The two algorithms of dimensionality reduction are as follows.

Algorithm 2. (Score-based dimensionality reduction technique)

Step 1. We construct the SVN neutrosophic soft matrix.

Step 2. Using definition 4.1, compute the object-oriented matrix for the object O_i and the parameter-oriented matrix E_j for the parameters.

Step 3. Next, compute their score matrix using definition 4.3.

Step 4. Find the threshold element and threshold value of the neutrosophic soft matrix as presented in definition 4.3.

Step 5. Remove those objects and parameters for which $\overline{SM}(O_i) < \overline{SM}(TH)$ and $\overline{SM}(E_j) > \overline{SM}(TH)$, respectively.

Step 6. The new neutrosophic soft matrix is the desired dimensionality-reduced matrix.

Algorithm 3. (Entropy-based dimensionality reduction technique)

Step 1. Construct the SVN soft matrix.

Step 2. Compute the object-oriented and parameter-oriented SVN soft matrix by using definition 4.1.

Step 3. Evaluate the entropy measure of the object-oriented and parameter-oriented SVN soft matrix by using definition 4.2

Step 4. Evaluate the threshold element $TH$of the SVN soft matrix and compute its entropy measure given in definition 4.2.

Step 5. Remove those objects for which $EM(O_i) > EM(TH)$ and those parameters for which $EM(E_j) < EM(TH)$.

Step 6. The remaining SVN soft matrix is the desired dimensionality-reduced matrix and the object corresponding to the lowest entropy value is the best one.

Fig. 2 presents the flowchart of the proposed dimensionality reduction technique.

Now, we consider an illustrative example to present the applicability of the proposed measure in the light of algorithm 2 and algorithm 3.

Example 4.2. Let Mr. Y wants to select the most suitable house from five number of houses concerning five parameters. Our problem is to select the most suitable house i.e., the object which dominates each of the house of the spectrum of the parameters. To solve this decision-making problem, we consider a numerical example, which is adapted from the reference [23] and [25].

Suppose there are five houses $H = \{h_1, h_2, h_3, h_4, h_5\}$ and $E = \{e_1$ = beautiful, e_2= cheap, e_3 = in good repairing, e_4 = moderate, e_5= wooden} be the set of parameters.

Firstly, we solve this problem with the help of existing score-based data reduction, to check the consistency.

Implementation of Algorithm 2

Step 1. Consider the SVN soft matrix $(SVNSM)$.

	e_1	e_2	e_3	e_4	e_5
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)

Step 2. Construct the object-oriented O_i and the parameter-oriented SVN soft matrix E_j; $i, j = 1, 2, 3, 4, 5$.

	e_1	e_2	e_3	e_4	e_5	O_i
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)	(0.64, 0.4, 0.56)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)	(0.68, 0.38, 0.6)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)	(0.66, 0.34, 0.36)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)	(0.72, 0.5, 0.7)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)	(0.7, 0.58, 0.6)
E_j	(0.72, 0.38, 0.62)	(0.6, 0.48, 0.58)	(0.64, 0.52, 0.6)	(0.72, 0.52, 0.38)	(0.72, 0.3, 0.6)	

Now, evaluate the score matrix of parameter and object-oriented SVN soft matrix $\overline{SM}(E_j)$ and $\overline{SM}(O_i)$, as given in Guleria and Bajaj [48].

	e_1	e_2	e_3	e_4	e_5	O_i	$SM(O_i)$
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)	(0.64, 0.4, 0.56)	(0.416)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)	(0.68, 0.38, 0.6)	(0.452)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)	(0.66, 0.34, 0.36)	(0.5376)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)	(0.72, 0.5, 0.7)	(0.37)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)	(0.7, 0.58, 0.6)	(0.352)
E_j	(0.72, 0.38, 0.62)	(0.6, 0.48, 0.58)	(0.64, 0.52, 0.6)	(0.72, 0.52, 0.38)	(0.72, 0.3, 0.6)		
$SM(E_j)$	0.4844	0.3216	0.328	0.5244	0.54		

Step 3. Compute the threshold element of the SVN soft matrix and determine its threshold value by using the score matrix. We have

$$TH = (0.68, 0.432, 0.556) \text{ and } \quad \overline{SM}(TH) = 0.4398$$

Step 4. Using the values obtained in step 3, we remove those alternatives for which condition $\overline{SM}(O_j) < \overline{SM}(TH)$ and those parameters for which condition $\overline{SM}(E_j) > \overline{SM}(TH)$ holds. Thus, the desired matrix is given as:

$$\begin{matrix} & e_2 & e_3 & O_i & SM(O_i) \\ \begin{matrix} h_2 \\ h_3 \\ E_j \\ SM(E_j) \end{matrix} & \begin{pmatrix} (0.6, 0.3, 0.7) & (0.7, 0.5, 0.6) & (0.68, 0.38, 0.6) & (0.452) \\ (0.8, 0.5, 0.1) & (0.3, 0.5, 0.6) & (0.66, 0.34, 0.36) & (\mathbf{0.5376}) \\ (0.6, 0.48, 0.58) & (0.64, 0.52, 0.6) & & \\ 0.3216 & 0.328 & & \end{pmatrix} \end{matrix}$$

From the above matrix, it can be seen that the data size has been reduced by approximately 50%. It can be concluded that the same decision partition stated in [23] and [25], that Mr. Y selected the house h_3.

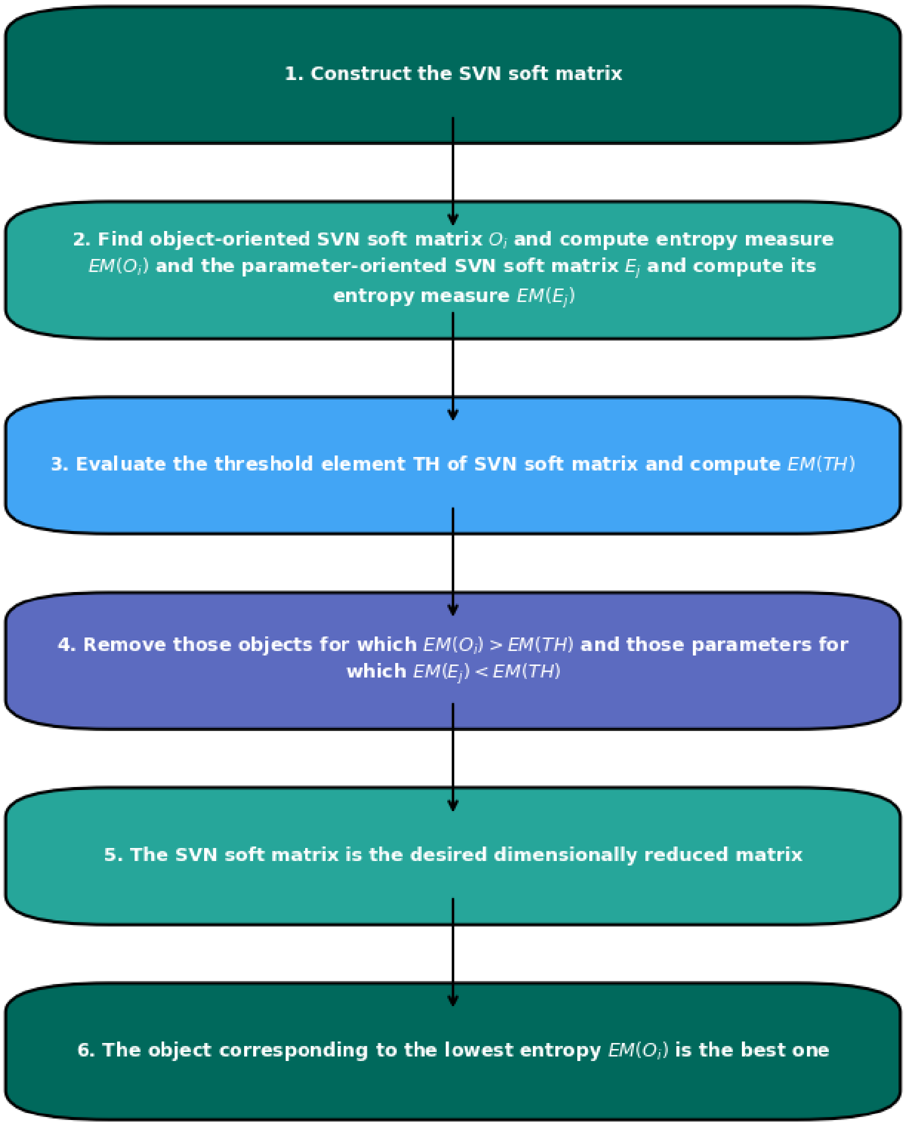

Figure 2. Flowchart of algorithm 3 for dimensionality reduction technique for SVN soft environment

Implementation of Algorithm 2 (Entropy-based data reduction technique)

Step 1. Consider the SVN soft matrix $(SVNSM)$.

	e_1	e_2	e_3	e_4	e_5
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)

Step 2. Construct the object-oriented O_i and the parameter-oriented SVN soft matrix E_j; $i, j = 1, 2, 3, 4, 5$.

	e_1	e_2	e_3	e_4	e_5	O_i
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)	(0.64, 0.4, 0.56)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)	(0.68, 0.38, 0.6)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)	(0.66, 0.34, 0.36)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)	(0.72, 0.5, 0.7)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)	(0.7, 0.58, 0.6)
E_j	(0.72, 0.38, 0.62)	(0.6, 0.48, 0.58)	(0.64, 0.52, 0.6)	(0.72, 0.52, 0.38)	(0.72, 0.3, 0.6)	

Now, evaluate the entropy measure of parameter and object-oriented SVN soft matrix $EM(E_j)$ and $EM(O_i)$ by using Definition 4.2 which is given below.

	e_1	e_2	e_3	e_4	e_5	O_i	$EM(O_i)$
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)	(0.64, 0.4, 0.56)	(0.2583)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)	(0.68, 0.38, 0.6)	(0.2583)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)	(0.66, 0.34, 0.36)	(0.1472)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)	(0.72, 0.5, 0.7)	(0.2664)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)	(0.7, 0.58, 0.6)	(0.2535)
E_j	(0.72, 0.38, 0.62)	(0.6, 0.48, 0.58)	(0.64, 0.52, 0.6)	(0.72, 0.52, 0.38)	(0.72, 0.3, 0.6)		
$EM(E_j)$	0.2535	0.2664	0.2648	0.1134	0.2476		

Step 3. Compute the threshold element of the SVN soft matrix and determine its threshold value using Definition 4.2, we have

$$TH = (0.68, 0.432, 0.556) \text{ and } EM(TH) = 0.2463$$

Step 4. Next, by using the values obtained in step 3, we remove those alternatives for which condition $EM(O_i) > EM(TH)$ and those parameters for which condition $EM(E_j) < EM(TH)$ holds. Thus, the desired matrix is as follows.

	e_1	e_2	e_3	e_5	O_i	$EM(O_i)$
h_3	(0.8,0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.6)	(0.66, 0.34, 0.36)	(0.0272)
E_j	(0.72,0.38, 0.62)	(0.6, 0.48, 0.58)	(0.64, 0.52, 0.6)	(0.72, 0.3, 0.6)		
$EM(E_j)$	0.2535	0.2664	0.2648	0.2476		

From the above matrix, it can be seen that the data size has been reduced by approximately 50%. Mr. Y selected house h_3. So, our proposed measure is consistent with the existing method.

5. Comparative Study

To show the effectiveness of our proposed measure over the existing measures, we consider the following illustrative example.

Example 5.1. [23]. Consider $U = \{y_1, y_2\}$ be the universe of discourse where $y_1 =$ severe, $y_2 =$ mild. Here the set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of certain visible symptoms, where e_1 =headache, $e_2 =$ fatigue,$e_3 =$ nausea and vomiting, $e_4 =$ skin changes, $e_5 =$ weakness. In this example, our proposed method is applied to determine whether an ill person having some visible symptoms is suffering from cancer or not suffering from cancer. To illustrate and compare our proposed measures, we consider some existing measures which are given in section 4. The results obtained from the evaluation of proposed measures and existing measures are given in Table 4.

Table 4. Similarity measure between the proposed and existing measures

Measures	(F, G)	(G, H)	PM2
SM_1	0.69	0.31	2.139
SM_2	0.75	0.33	2.242
SM_3	0.335	0.743	2.248
SM_4	0.624	0.562	2.909
SM_5	0.09	0.55	1.64
Proposed SM	0.95	0.76	5.116

Now, we consider another example to show the effectiveness of the proposed measure.

Example 5.2. Let $(F,E),(G,E)$, and (H,E) be three SVNSSs, whose SVN soft matrices are given as below.

$$(F,E) = \begin{pmatrix} (0.6, 0.2, 0.1) & (0.4, 0.5, 0.2) & (0.8, 0.1, 0.2) \\ (0.5, 0.3, 0) & (0.7, 0.1, 0.2) & (0.6, 0.3, 0.2) \\ (0.8, 0.2, 0.1) & (0.6, 0, 0) & (0.9, 0, 0.1) \end{pmatrix},$$

$$(G,E) = \begin{pmatrix} (0.5, 0.3\ 0.2) & (0.7, 0, 0.2) & (0.6, 0.3, 0.1) \\ (0.6, 0.2, 0.1) & (0.4, 0, 0.1) & (0.5, 0.1, 0.2) \\ (0.9, 0, 0.1) & (0.5, 0.1, 0.2) & (0.8, 0, 0.2) \end{pmatrix}, \text{ and}$$

$$(H,E) = \begin{pmatrix} (0.4, 0.4, 0.2) & (0.6, 0.2, 0.1) & (0.5, 0.1, 0.2) \\ (0.3, 0.2, 0.1) & (0.7, 0.1, 0.2) & (0.5, 0.4, 0.1) \\ (0.2, 0, 0.2) & (0.5, 0, 0.1) & (0.1, 0.8, 0) \end{pmatrix}.$$

Now, compute the similarity measure $SM((F,E),(H,E))$, $SM((F,E),(G,E))$ and $SM((G,E),(H,E))$ which is shown as

Table 5. Similarity values between SVNSSs due to proposed and existing measures

Measures	(F,H)	(G,H)	(F,G)	PM3
SM_1	0.6476	0.6476	0.6211	2.0150
SM_2	0.5749	0.6176	0.5489	1.7588
SM_3	0.7295	0.6564	0.8510	2.4793
SM_4	0.5989	0.5710	0.5885	1.788
SM_5	0.2883	0.5824	0.21	1.222

Proposed SM	0.9428	0.9335	0.9428	9.0268

Now, we represent the graphical representation of the performance measures given in Table 3, Table 4, and Table 5 of the existing and proposed measures. From Fig 3., it can be concluded that the proposed similarity measure boasts a significantly higher accuracy rate than existing measures.

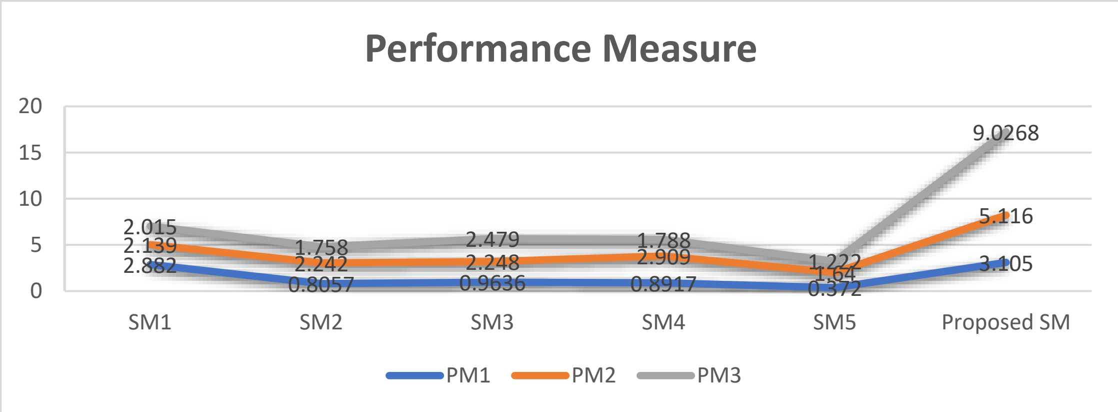

Figure 3. Graphical representation of performance measure of proposed measure and existing measure.

6. Conclusion

This article introduced some information theoretic measures in the SVNS framework. Our approach is grounded in the conviction that entropy, and similarity measures as indispensable tools to investigate the uncertain information with soft representation. Based on the score matrix and entropy measure, a new technique of dimensionality reduction has been investigated in the SVNS soft environment. By using two techniques of data reduction, we observed that data size has been substantially reduced to 50% and despite reduction techniques, the data still supports the same decision partition suggested in Maji ([23] [25]). Furthermore, the effectiveness of the proposed measures has been buttressed by illustrative examples. The evaluation of performance measure elucidated the higher accuracy of the proposed measures. The present study deals with applications of proposed methods using artificial dataset. In future, the relevant real-data can be explored to investigate more interdisciplinary applications.

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