Exploring the Structural Aspects of Interval-Valued Intuitionistic Neutrosophic Fuzzy Ẑ-Subalgebra

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Abstract: The article develops an updated methodology of interval-valued intuitionistic neutrosophic sets (IVINS) based on Ẑ-algebra theoretical structures. The new method presents an advanced solution to real-world uncertainties alongside inconsistent and indeterminate states found during decision-making and problem-solving activities. The investigation follows up by proving fundamental principles for Ẑ-algebra under this neutrosophic extension. Researchers have deeply studied homomorphism properties and Cartesian product operations of interval-valued intuitionistic neutrosophic sets contained in Ẑ-algebras. Research applications of fuzzy set theory span multiple domains that include real-world, theoretical, and imaginary contexts. Multiple algebraic structures such as BCK/BCI-algebras, UP-algebras, and B-algebras, alongside others have validated fuzzy sets mathematically, thus proving their extensive applicability in abstract mathematical systems.
Keywords: Fuzzy set, Ẑ-subalgebra, intuitionistic fuzzy Ẑ-subalgebra, interval-valued fuzzy Ẑ-subalgebra, neutrosophic fuzzy Ẑ-subalgebra.

1. Introduction

In decision-making difficulties, uncertainty and imprecision can be represented mathematically through fuzzy sets. A fuzzy set represents a collection of objects that have varying degrees of membership to a particular set. Overall, fuzzy sets and their extensions have proven to be valuable tools for dealing with uncertainty and have found appllications in various fields, including engineering, economics, and computer science. Zadeh [1] 1965, created the fuzzy set, which offered a mathematical foundation for managing with imprecision and uncertainty. This concept was a significant departure from traditional set theory, which only allowed for objects to belong to a set or not. Fuzzy sets allowed for partial membership, where objects could belong to a set of varying degrees. Following Zadeh's essential work, scholars have created several extensions of fuzzy set theory, including type-2 fuzzy sets, intuitionistic fuzzy sets, and hesitant fuzzy sets. These models improve the ability to describe and comprehend more complex types of uncertainty. For example, type-2 fuzzy sets can accommodate uncertainty in membership functions, making them very helpful in contexts with highly changeable data. As a result, fuzzy set theory continues to advance, giving powerful tools for real-world problem resolution in situations where ambiguity is inherent.

Atanassov [2] developed the intuitionistic fuzzy set as an extension of the classical fuzzy set, adding an extra dimension to reflect the degree of non-membership as well as the degree of membership. Later, Zadeh [3] improved on the concept of fuzzy sets, proposing interval-valued fuzzy sets and adding ideas from intuitionistic fuzzy sets. This process resulted in the invention of interval-valued intuitionistic fuzzy set theory, which combines the characteristics of interval-valued and intuitionistic fuzzy sets to better reflect data uncertainty. These advanced fuzzy set models have increased the flexibility and expressiveness of decision-making systems, particularly in situations when exact numerical values are difficult to attain. By representing membership and non-membership degrees as intervals rather than exact integers, interval-valued intuitionistic fuzzy sets provide a more comprehensive framework for modelling ambiguous or contradictory information. This is especially useful in applications like medical diagnosis, financial forecasting, and multi-criteria decision analysis, where expert opinions or data sources frequently contain ambiguity or discrepancies.

Smarandache [4] developed neutrosophic fuzzy set theory as a generalisation of intuitionistic fuzzy sets. This theory established an even more extensive framework for dealing with indeterminacy, allowing for the separate representation of truth, indeterminacy, and falsehood. This theory studied in mathematicians [5-16]. Following this, many mathematicians proceeded to investigate fuzzy sets further [17-26], particularly in the subject of algebra, where these ideas have proven to be extremely significant [28-38]. Chandramouleeswaran [39] introduced Ẑ-algebra, an important algebraic notion that offers new perspectives into investigating fuzzy structures. Sowmiya [40] developed fuzzy set structures and expansions using the Ẑ-algebra framework. This technique was eventually expanded to include intuitionistic fuzzy sets within the Ẑ-algebraic structure. Furthermore, fuzzy structures on Ẑ-algebra were created, providing more subtle approaches to handle imprecision in algebraic systems. The study of fuzzy sets, interval-valued fuzzy sets, and intuitionistic fuzzy sets is based on β-algebra. The combination of fuzzy set theory with algebraic structures has opened up new avenues for theoretical research and practical application. Integrating algebraic frameworks like Ẑ-algebra and β-algebra with fuzzy logic concepts allows researchers to better model complex systems. This fusion has resulted in advances in abstract algebra, soft computing, and modelling of uncertain systems, providing the framework for future multidisciplinary research in mathematics, computer science, and engineering.

In [41], the author explores the Cartesian product features of fuzzy Ẑ-ideals and defines a fuzzy Ẑ-ideal of a Ẑ-algebra under Ẑ-homomorphisms. Hemavathi et al. [42] introduced an interval-valued intuitionistic fuzzy β-subalgebra, broadening the definition of fuzzy algebraic structures. The neutrosophic fuzzy set is an advanced modification of the fuzzy set that gives a degree of truth, indeterminacy, and falsity to each member, improving the ability to model ambiguous and contradictory information. The neutrosophic set, described as a large and powerful mathematical framework [43], enables more precise interpretation of ambiguous data. Nagiah [44] pioneered the use of neutrosophic theory in algebra, opening up new study avenues. The intuitionistic neutrosophic set (INS) was discussed in [45], which combined the advantages of intuitionistic fuzzy sets and neutrosophic logic. The use of neutrosophic concepts in UP-algebras was investigated in [46], with a particular emphasis on the features and behaviour of homomorphic inverse pictures. In addition, MBJ-neutrosophic structures were introduced for β-subalgebras [47], leading to a more specialised and flexible framework in algebraic research. Furthermore, in the view of neutrosophic generalized semi generalized closed Sets, weak separation axioms, and generalized sg-closed sets with their continuity are deliberated by Imran et al. [51-53]. Abdulkadhim et al. [54,55] introduced the concepts of generalized alpha generalized closed sets spaces and neutrosophic generalized alpha generalized separation axioms. The ongoing evolution of fuzzy and neutrosophic set theories in algebra has laid the groundwork for more in-depth mathematical research and applications. These advancements enable the creation of more flexible models, which is especially valuable in systems with incomplete, inaccurate, or contradicting data. Whether in computational intelligence, formal logic systems, or abstract algebra, these hybrid approaches enable more robust analysis and decision-making processes in difficult situations.

The research develops a clear conceptual model of neutrosophic sets in Ẑ-subalgebras of Ẑ-algebras which further develops earlier exploration of intuitionistic neutrosophic sets within Ẑ-algebraic contexts. This research article introduces interval-valued intuitionistic neutrosophic sets (IVINS) into Ẑ-algebra structures to enhance the management of uncertain and inconsistent information in algebraic systems. The paper fully examines key algebraic concepts including homomorphisms and Cartesian products by diversifying them within the neutrosophic context.

2. Preliminaries

In the following preliminaries, discussed about the necessary and sufficient conditions for the research article. The particular notations are used in this research as follows X specified as $\mathfrak{V}$, x denoted as ϖ, y specified as $\mathfrak{y}$ and Y represented as $\mathfrak{F}$.

Definition 2.1. [1] If $\mathfrak{V}$ be a non-empty set and the fuzzy set ξ in $\mathfrak{V}$ is characterized by $\xi = \{ \varpi, \mu_\xi(\varpi) / \varpi \in \mathfrak{V} \}$, where the function μ_ξ: $\mathfrak{V} \to [0,1]$ specified a degree of membership element $\varpi \in \mathfrak{V}$.

Definition 2.2.[41] If $\mathfrak{V}$ be a non-empty set, an intuitionistic fuzzy set ξ of $\mathfrak{V}$ is having the form $\xi = \{ \varpi, \mu_\xi(\varpi), \vartheta_\xi(\varpi) / \varpi \in \mathfrak{V} \}$, then $\mu_\xi(\varpi)$:$\mathfrak{V} \to [0,1]$ be degree of membership and $\vartheta_\xi(\varpi)$: $\mathfrak{V} \to [0,1]$ be degree of non-membership function respectively, for every $\varpi \in \mathfrak{V}$, $0 \le \mu_\xi(\varpi) \le \vartheta_\xi(\varpi) \le 1$.

Definition 2.3. [48] An interval-valued fuzzy set ξ defined on the set $\mathfrak{V}$ is given by $\xi = \{ \varpi, \mu_\xi^L, \mu_\xi^U / \varpi \in \mathfrak{V} \}$ and it is denoted by $\xi = [\mu_\xi^L, \mu_\xi^U]$, (i.e) μ_ξ^L & μ_ξ^U are the two fuzzy sets in $\mathfrak{V}$ such that

$$\mu_\xi^L(\varpi) \le \mu_\xi^U(\varpi),\ \forall\ \varpi \in \mathfrak{V},\ \bar{\mu}_\xi(\varpi) = [\mu_\xi^L(\varpi),\ \mu_\xi^U(\varpi)],\ \forall\ \varpi \in \mathfrak{V}$$

Let $\mathfrak{D}$ [0,1] denotes the closed sub-intervals of [0,1] . If $\mu_\xi^L(\varpi) = \mu_\xi^U(\varpi) = c$, where $0 \le c \le 1$, So $\bar{\mu}_\xi(\varpi) = [c,c]$. Assume, $\bar{\mu}_\xi(\varpi) \in \mathfrak{D}\ [0,1]\ \forall\ \varpi \in \mathfrak{V}$.

$\therefore$ the interval-valued fuzzy set ξ is represented by $\xi = \{ \varpi, \bar{\mu}_\xi(\varpi)\}, \forall\ \varpi \in \mathfrak{V}$, then $\bar{\mu}_\xi(\varpi) : \mathfrak{V} \to \mathfrak{D}\ [0,1]$.

Consider two different elements from $\mathfrak{D}_1$ and $\mathfrak{D}_2$ and then $\mathfrak{D}_1 := [\gamma_1, \delta_1]$ & $\mathfrak{D}_2 := [\gamma_2, \delta_2] \in \mathfrak{D}\ [0,1]$.

$\therefore$ rmin $(\mathfrak{D}_1, \mathfrak{D}_2) = [\min\{\gamma_1, \gamma_2\}, \min\{\delta_1, \delta_2\}]$; $\mathfrak{D}_1 \ge \mathfrak{D}_2$, iff $\gamma_1 \ge \gamma_2$, $\delta_1 \ge \delta_2$.

Similarly, $\mathfrak{D}_1 \le \mathfrak{D}_2$, $\mathfrak{D}_1 = \mathfrak{D}_2$, $\mathfrak{D}_1 \ge \mathfrak{D}_2$.

Definition 2.4. [48] An interval-valued intuitionistic fuzzy set ξ over the universal set $\mathfrak{V}$ is in the form $\xi = \{ \varpi, \bar{\mu}_\xi(\varpi), \bar{\vartheta}_\xi(\varpi) / \varpi \in \mathfrak{V} \}$. $\bar{\mu}_\xi(\varpi) : \mathfrak{V} \to \mathfrak{D}\ [0,1]$ denotes the degree of membership function and $\bar{\vartheta}_\xi(\varpi)$: $\mathfrak{V} \to \mathfrak{D}\ [0,1]$ specifies the degree of non-membership function of the element ϖ from the set $\mathfrak{V}$, $\bar{\mu}_\xi(\varpi) = [\mu_\xi^L(\varpi), \mu_\xi^U(\varpi)]$ and $\bar{\vartheta}_\xi(\varpi) = [\vartheta_\xi^L(\varpi), \vartheta_\xi^U(\varpi)]$ for every $\varpi \in \mathfrak{V}$, with $0 \le \mu_\xi^L(\varpi) + \vartheta_\xi^L(\varpi) \le 1$ and $0 \le \mu_\xi^U(\varpi) + \vartheta_\xi^U(\varpi) \le 1$.

Also, $\bar{\mu}_{\xi}(\varpi)$ = [1-$\mu_{\xi}^{U}(\varpi)$,1-$\mu_{\xi}^{L}(\varpi)$] and $\bar{\bar{\vartheta}}_{\xi}$ = [1-$\vartheta_{\xi}^{U}(\varpi)$,$1-\vartheta_{\xi}^{L}(\varpi)$], where [$\bar{\mu}_{\xi}(\varpi)$, $\bar{\bar{\vartheta}}_{\xi}(\varpi)$] represents the complement of ϖ in ξ.

Definition 2.5.[49] Let the neutrosophic set ξ= { ϖ: $T_{\xi}(\varpi)$, $I_{\xi}(\varpi)$, $F_{\xi}(\varpi)$/ $\varpi \in \mathfrak{V}$ }, $T_{\xi}(\varpi)$: $\mathfrak{V} \rightarrow [0,1]$ as true membership function, $I_{\xi}(\varpi)$: $\mathfrak{V} \rightarrow [0,1]$ as indeterminacy membership function, $F_{\xi}(\varpi)$: $\mathfrak{V} \rightarrow [0,1]$ considered as False membership function.

Definition 2.6.[39] Let ($\mathfrak{V}$, ∗, 0) be non-empty set with the constant element 0 and a binary operation ∗ satisfying the following conditions then it is said to be Ẑ-algebra

i) $\varpi * 0 = 0$

ii) $0 * \varpi = \varpi$

iii) $\varpi * \varpi = \varpi$

iv) $\varpi * \mathfrak{y} = \mathfrak{y} * \varpi$, when $\varpi \neq 0$ & $\mathfrak{y} \neq 0$, $\forall \varpi, \mathfrak{y} \in \mathfrak{V}$.

Definition 2.7. If ($\mathfrak{V}$,∗,0) is Ẑ-algebra, then ξ in $\mathfrak{V}$ is the fuzzy set having membership function μ_{ξ} is defined as fuzzy Ẑ-subalgebra of Ẑ-algebra $\forall \varpi, \mathfrak{y} \in \mathfrak{V}$,

$$\mu_{\xi}(\varpi * \mathfrak{y}) \geq \min\{\mu_{\xi}(\varpi), \mu_{\xi}(\mathfrak{y})\}.$$

Definition 2.8.[41] The intuitionistic fuzzy set ξ = { ϖ: $\mu_{\xi}(\varpi)$, $\vartheta_{\xi}(\varpi)$/ $\varpi \in \mathfrak{V}$ } in $\mathfrak{V}$ is known as intuitionistic fuzzy Ẑ-subalgebra of $\mathfrak{V}$, if it holds

i) $\mu_{\xi}(\varpi * \mathfrak{y}) \geq \min\{\mu_{\xi}(\varpi), \mu_{\xi}(\mathfrak{y})\}$,

ii) $\vartheta_{\xi}(\varpi * \mathfrak{y}) \leq \max\{\vartheta_{\xi}(\varpi), \vartheta_{\xi}(\mathfrak{y})\}$.

Definition 2.9.[47] Let ($\mathfrak{V}$, ∗,0) be Ẑ-algebra then the interval-valued fuzzy $\bar{\mu}_{\xi}$ in $\mathfrak{V}$ is represented as an interval-valued fuzzy Ẑ-algebra of $\mathfrak{V}$, $\forall \varpi, \mathfrak{y} \in \mathfrak{V}$, if it satisfies

$$\bar{\mu}_{\xi}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{\mu}_{\xi}(\varpi), \bar{\mu}_{\xi}(\mathfrak{y})\}.$$

Definition 2.10.[50] Let the neutrosophic set ξ= { ϖ: $T_{\xi}(\varpi)$, $I_{\xi}(\varpi)$, $F_{\xi}(\varpi)$/ $\varpi \in \mathfrak{V}$ } if it holds the following conditions, then it is known to be neutrosophic fuzzy in Ẑ-algebra.

i) $T_{\xi}(\varpi * \mathfrak{y}) \geq \min\{T_{\xi}(\varpi), T_{\xi}(\mathfrak{y})\}$,

ii) $I_{\xi}(\varpi * \mathfrak{y}) \geq \min\{I_{\xi}(\varpi), I_{\xi}(\mathfrak{y})\}$,

iii) $F_{\xi}(\varpi * \mathfrak{y}) \leq \max\{F_{\xi}(\varpi), F_{\xi}(\mathfrak{y})\}$.

Definition 2.11.[49] If ($\mathfrak{V}$, ∗,0) is Ẑ-algebra with constant 0 & the binary operation ∗, then the interval-valued neutrosophic set is $\bar{\xi}$ = { ϖ: $\bar{T}_{\bar{\xi}}(\varpi)$, $\bar{I}_{\bar{\xi}}(\varpi)$, $\bar{F}_{\bar{\xi}}(\varpi)$/ $\varpi \in \mathfrak{V}$ }, if it satisfies the following conditions,

i) $\bar{T}_{\bar{\xi}}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{T}_{\bar{\xi}}(\varpi), \bar{T}_{\bar{\xi}}(\mathfrak{y})\}$,

ii) $\bar{I}_{\bar{\xi}}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{I}_{\bar{\xi}}(\varpi), \bar{I}_{\bar{\xi}}(\mathfrak{y})\}$,

iii) $\bar{F}_{\bar{\xi}}(\varpi * \mathfrak{y}) \leq \text{rmax}\{\bar{F}_{\bar{\xi}}(\varpi), \bar{F}_{\bar{\xi}}(\mathfrak{y})\}$.

Definition 2.12. [45] If ($\mathfrak{V}$, ∗,0) is Ẑ-algebra, ∗ is a binary operation & 0 is the constant, then the intuitionistic neutrosophic set ξ = { ϖ : $\mu_{T_{\xi}}(\varpi)$, $\mu_{I_{\xi}}(\varpi)$, $\mu_{F_{\xi}}(\varpi)$, $\vartheta_{T_{\xi}}(\varpi)$, $\vartheta_{I_{\xi}}(\varpi)$, $\vartheta_{F_{\xi}}(\varpi)$/ $\varpi \in \mathfrak{V}$ } if it holds the following properties

i) $\mu_{T_{\xi}}(\varpi * \mathfrak{y}) \geq \min\{\mu_{T_{\xi}}(\varpi), \mu_{T_{\xi}}(\mathfrak{y})\}$ & $\vartheta_{T_{\xi}}(\varpi * \mathfrak{y}) \leq \max\{\vartheta_{T_{\xi}}(\varpi), \vartheta_{T_{\xi}}(\mathfrak{y})\}$

ii) $\mu_{I_{\xi}}(\varpi * \mathfrak{y}) \geq \min\{\mu_{I_{\xi}}(\varpi), \mu_{I_{\xi}}(\mathfrak{y})\}$ & $\vartheta_{I_{\xi}}(\varpi * \mathfrak{y}) \leq \max\{\vartheta_{I_{\xi}}(\varpi), \vartheta_{I_{\xi}}(\mathfrak{y})\}$

iii) $\mu_{F_{\xi}}(\varpi * \mathfrak{y}) \leq \max\{\mu_{F_{\xi}}(\varpi), \mu_{F_{\xi}}(\mathfrak{y})\}$ & $\vartheta_{F_{\xi}}(\varpi * \mathfrak{y}) \geq \min\{\vartheta_{F_{\xi}}(\varpi), \vartheta_{F_{\xi}}(\mathfrak{y})\}$.

3. Interval-valued Intuitionistic neutrosophic in Ẑ-algebra

The following part discusses the fundamental properties and methodologies used in the research article.

Definition 3.1. An interval-valued intuitionistic neutrosophic set $\bar{\xi}$ = { ϖ, $\bar{\mu}_{T_{\bar{\xi}}}(\varpi)$, $\bar{\mu}_{I_{\bar{\xi}}}(\varpi)$, $\bar{\mu}_{F_{\bar{\xi}}}(\varpi)$, $\bar{\vartheta}_{T_{\bar{\xi}}}(\varpi)$, $\bar{\vartheta}_{I_{\bar{\xi}}}(\varpi)$, $\bar{\vartheta}_{F_{\bar{\xi}}}(\varpi)$ / $\varpi \in \mathfrak{V}$ } is defined as interval-valued intuitionistic fuzzy Ẑ-subalgebra of ($\mathfrak{V}$,⁎,0) by the operation ⁎ and constant 0, if it holds the following properties,

i) $\bar{\mu}_{T_{\bar{\xi}}}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{\mu}_{T_{\bar{\xi}}}(\varpi), \bar{\mu}_{T_{\bar{\xi}}}(\mathfrak{y})\}$ & $\bar{\vartheta}_{T_{\bar{\xi}}}(\varpi * \mathfrak{y}) \leq \text{rmax}\{\bar{\vartheta}_{T_{\bar{\xi}}}(\varpi), \bar{\vartheta}_{T_{\bar{\xi}}}(\mathfrak{y})\}$

ii) $\bar{\mu}_{I_{\bar{\xi}}}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{\mu}_{I_{\bar{\xi}}}(\varpi), \bar{\mu}_{I_{\bar{\xi}}}(\mathfrak{y})\}$ & $\bar{\vartheta}_{I_{\bar{\xi}}}(\varpi * \mathfrak{y}) \leq \text{rmax}\{\bar{\vartheta}_{I_{\bar{\xi}}}(\varpi), \bar{\vartheta}_{I_{\bar{\xi}}}(\mathfrak{y})\}$

iii) $\bar{\mu}_{F_{\bar{\xi}}}(\varpi * \mathfrak{y}) \leq \text{rmax}\{\bar{\mu}_{F_{\bar{\xi}}}(\varpi), \bar{\mu}_{F_{\bar{\xi}}}(\mathfrak{y})\}$ & $\bar{\vartheta}_{F_{\bar{\xi}}}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{\vartheta}_{F_{\bar{\xi}}}(\varpi), \bar{\vartheta}_{F_{\bar{\xi}}}(\mathfrak{y})\}$

Example: 3.2. Let ($\mathfrak{V}$, ⁎,0) be Ẑ-algebra, where $*: \mathfrak{V} \times \mathfrak{V} \rightarrow \mathfrak{V}$ is defined by table (1).

Table (1): ($\mathfrak{V}$, ⁎,0) be Ẑ-algebra

⁎	0	ϱ_1	ϱ_2	ϱ_3	ϱ_4
0	0	ϱ_1	ϱ_2	ϱ_3	ϱ_4
ϱ_1	0	ϱ_1	ϱ_4	ϱ_1	ϱ_3
ϱ_2	0	ϱ_4	ϱ_2	ϱ_1	ϱ_4
ϱ_3	0	ϱ_1	ϱ_1	ϱ_3	ϱ_2
ϱ_4	0	ϱ_3	ϱ_4	ϱ_2	ϱ_4

$$\bar{\mu}_{\bar{\xi}_{T,I,F}} = \begin{cases} [0.6,0.8] & \varpi = \text{otherwise} \\ [0.4,0.5] & \varpi = \varrho_1, \varrho_2 \\ [0.2,0.3] & \varpi = 0 \end{cases} \qquad \bar{\vartheta}_{\bar{\xi}_{T,I,F}} = \begin{cases} [0.2,0.4] & \varpi = \varrho_1, \varrho_3 \\ [0.4,0.5] & \varpi = 0 \\ [0.2,0.3] & \varpi = \text{otherwise} \end{cases}$$

Theorem 3.3. If ξ = { ϖ, $\bar{\mu}_{T_\xi}(\varpi)$, $\bar{\mu}_{I_\xi}(\varpi)$, $\bar{\mu}_{F_\xi}(\varpi)$, $\bar{\vartheta}_{T_\xi}(\varpi)$, $\bar{\vartheta}_{I_\xi}(\varpi)$, $\bar{\vartheta}_{F_\xi}(\varpi)$: $\varpi \in \mathfrak{V}$ } is an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{V}$, then the sets

$\mathfrak{V}_{\bar{\mu}_{T_\xi}}$ = { $\varpi \in \mathfrak{V}$ | $\bar{\mu}_{T_\xi}(\varpi) \geq \bar{\mu}_{T_\xi}(1)$}, $\mathfrak{V}_{\bar{\vartheta}_{T_\xi}}$ = { $\varpi \in \mathfrak{V}$ | $\bar{\vartheta}_{T_\xi}(\varpi) \leq \bar{\vartheta}_{T_\xi}(0)$},

$\mathfrak{V}_{\bar{\mu}_{I_\xi}}$ = { $\varpi \in \mathfrak{V}$ | $\bar{\mu}_{I_\xi}(\varpi) \geq \bar{\mu}_{I_\xi}(1)$}, $\mathfrak{V}_{\bar{\vartheta}_{I_\xi}}$ = { $\varpi \in \mathfrak{V}$ | $\bar{\vartheta}_{I_\xi}(\varpi) \leq \bar{\vartheta}_{I_\xi}(0)$},

$\mathfrak{V}_{\bar{\mu}_{F_\xi}}$ = { $\varpi \in \mathfrak{V}$ | $\bar{\mu}_{F_\xi}(\varpi) \leq \bar{\mu}_{F_\xi}(0)$}, and $\mathfrak{V}_{\bar{\vartheta}_{F_\xi}}$ = { $\varpi \in \mathfrak{V}$ | $\bar{\vartheta}_{F_\xi}(\varpi) \geq \bar{\vartheta}_{F_\xi}(1)$} are Ẑ-subalgebra of $\mathfrak{V}$.

Proof: Given ϖ, $\mathfrak{y} \in \mathfrak{V}_{\bar{\mu}_{T_\xi}}$, then we get $\bar{\mu}_{T_\xi}(\varpi) \geq \bar{\mu}_{T_\xi}(1)$, $\bar{\mu}_{T_\xi}(\mathfrak{y}) \geq \bar{\mu}_{T_\xi}(1)$.

Let us proceed, $\bar{\mu}_{T_\xi}(\varpi * \mathfrak{y}) \geq \text{rmin}\{\bar{\mu}_{T_\xi}(\varpi), \bar{\mu}_{T_\xi}(\mathfrak{y})\} \geq \text{rmin}\{\bar{\mu}_{T_\xi}(1), \bar{\mu}_{T_\xi}(1)\} = \bar{\mu}_{T_\xi}(1)$, then $\bar{\mu}_{T_\xi}(\varpi * \mathfrak{y}) \geq \bar{\mu}_{T_\xi}(1)$ and hence $\varpi * \mathfrak{y} \in \mathfrak{V}_{\bar{\mu}_{T_\xi}}$.

Similarly, $\bar{\mu}_{I_\xi}(\varpi * \mathfrak{y}) \geq \bar{\mu}_{I_\xi}(1)$, for any ϖ, $\mathfrak{y} \in \mathfrak{V}_{\bar{\mu}_{I_\xi}}$ and hence $\varpi * \mathfrak{y} \in \mathfrak{V}_{\bar{\mu}_{T_\xi}}$.

Also, let ϖ, $\mathfrak{y} \in \mathfrak{V}_{\bar{\mu}_{F_\xi}}$, then we obtain that $\mathfrak{V}_{\bar{\mu}_{F_\xi}}(\varpi) \leq \mathfrak{V}_{\bar{\mu}_{F_\xi}}(0)$, $\mathfrak{V}_{\bar{\mu}_{F_\xi}}(\mathfrak{y}) \leq \mathfrak{V}_{\bar{\mu}_{F_\xi}}(0)$.

$\bar{\mu}_{\bar{F}_\xi}$ $(\varpi * \mathfrak{y}) \le \text{rmax}\{\bar{\mu}_{\bar{F}_\xi}(\varpi), \bar{\mu}_{\bar{F}_\xi}(\mathfrak{y})\} \le \text{rmax}\{\bar{\mu}_{\bar{F}_\xi}(0), \bar{\mu}_{\bar{F}_\xi}(0)\} = \bar{\mu}_{\bar{F}_\xi}(0)$. Then, we have

$\bar{\mu}_{\bar{F}_\xi}$ $(\varpi * \mathfrak{y}) \le \bar{\mu}_{\bar{F}_\xi}(0)$ and hence $\varpi * \mathfrak{y} \in \mathfrak{V}_{\bar{\mu}_{\bar{F}_\xi}}$.

$\therefore$ $\mathfrak{V}_{\bar{\mu}_{\bar{T}_\xi}}$, $\mathfrak{V}_{\bar{\mu}_{I_\xi}}$ and $\mathfrak{V}_{\bar{\mu}_{T_\xi}}$ are $\hat{Z}$-subalgebras of $\mathfrak{V}$.

Let $\varpi, \mathfrak{y} \in \mathfrak{V}_{\bar{\vartheta}_{\bar{T}_\xi}}$, then $\bar{\vartheta}_{\bar{T}_\xi}(\varpi) \le \bar{\vartheta}_{\bar{T}_\xi}(0)$, $\bar{\vartheta}_{\bar{T}_\xi}(\mathfrak{y}) \le \bar{\vartheta}_{\bar{T}_\xi}(0)$.

$\bar{\vartheta}_{\bar{T}_\xi}(\varpi * \mathfrak{y}) \le \text{rmax}\{\bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\mathfrak{y})\} \le \text{rmax}\{\bar{\vartheta}_{\bar{T}_\xi}(0), \bar{\vartheta}_{\bar{T}_\xi}(0)\} = \bar{\vartheta}_{\bar{T}_\xi}(0)$. Then $\bar{\vartheta}_{\bar{T}_\xi}(\varpi * \mathfrak{y}) \le \bar{\vartheta}_{\bar{T}_\xi}(0)$ and hence $\varpi * \mathfrak{y} \in \mathfrak{V}_{\bar{\vartheta}_{\bar{T}_\xi}}$.

Similarly, $\bar{\vartheta}_{\bar{I}_\xi}(\varpi * \mathfrak{y}) \le \bar{\vartheta}_{\bar{I}_\xi}(0)$, for any $\varpi, \mathfrak{y} \in \mathfrak{V}_{\bar{\vartheta}_{I_\xi}}$ and hence $\varpi * \mathfrak{y} \in \mathfrak{V}_{\bar{\vartheta}_{I_\xi}}$.

Let $\varpi, \mathfrak{y} \in \mathfrak{V}_{\bar{\vartheta}_{\bar{F}_\xi}}$, then $\bar{\vartheta}_{\bar{F}_\xi}(\varpi) \ge \bar{\vartheta}_{\bar{F}_\xi}(1)$, $\bar{\vartheta}_{\bar{F}_\xi}(\mathfrak{y}) \ge \bar{\vartheta}_{\bar{F}_\xi}(1)$

$\bar{\vartheta}_{\bar{F}_\xi}(\varpi * \mathfrak{y}) \ge \text{rmin}\{\bar{\vartheta}_{\bar{F}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\mathfrak{y})\} \ge \text{rmin}\{\bar{\vartheta}_{\bar{F}_\xi}(1), \bar{\vartheta}_{\bar{F}_\xi}(1)\} = \bar{\vartheta}_{\bar{F}_\xi}(1)$, then $\bar{\vartheta}_{\bar{F}_\xi}(\varpi * \mathfrak{y}) \ge \bar{\vartheta}_{\bar{F}_\xi}(1)$ and hence $\varpi * \mathfrak{y} \in \mathfrak{V}_{\bar{\vartheta}_{\bar{F}_\xi}}$.

$\therefore \mathfrak{V}_{\bar{\vartheta}_{\bar{T}_\xi}}$, $\mathfrak{V}_{\bar{\vartheta}_{\bar{I}_\xi}}$ and $\mathfrak{V}_{\bar{\vartheta}_{\bar{F}_\xi}}$ *are* $\hat{Z}$-subalgebras of $\mathfrak{V}$.

Theorem 3.4. Let $\xi = \{ \varpi, \bar{\mu}_{\bar{T}_\xi}(\varpi), \bar{\mu}_{\bar{I}_\xi}(\varpi), \bar{\mu}_{\bar{F}_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{I}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\varpi) : \varpi \in \mathfrak{V} \}$ be interval-valued intuitionistic neutrosophic fuzzy set of $\mathfrak{V}$, Then $\xi = \{ \varpi, \bar{\mu}_{\bar{T}_\xi}(\varpi), \bar{\mu}_{\bar{I}_\xi}(\varpi), \bar{\mu}_{\bar{F}_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{I}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\varpi) : \varpi \in \mathfrak{V} \}$ is an interval-valued intuitionistic neutrosophic $\hat{Z}$-subalgebra of $\mathfrak{V}$ iff $\boxdot\, \xi = (\varpi, \bar{\vartheta}^c_{\bar{T}_\xi}, \bar{\vartheta}^c_{\bar{I}_\xi}, \bar{\vartheta}^c_{\bar{F}_\xi})$ is an interval-valued intuitionistic neutrosophic $\hat{Z}$-subalgebra of $\mathfrak{V}$.

Proof: If $\xi = \{ \varpi, \bar{\mu}_{\bar{T}_\xi}(\varpi), \bar{\mu}_{\bar{I}_\xi}(\varpi), \bar{\mu}_{F_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{I}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\varpi) : \varpi \in \mathfrak{V} \}$ is an interval-valued intuitionistic neutrosophic fuzzy set of $\mathfrak{V}$.

To Prove: $\boxdot\, \xi$ is an interval-valued intuitionistic neutrosophic fuzzy $\hat{Z}$-subalgebra of $\mathfrak{V}$.

$\bar{\vartheta}^c_{\bar{T}_\xi}(\varpi * \mathfrak{y}) \le \text{rmax}\{\bar{\vartheta}^c_{\bar{T}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{T}_\xi}(\mathfrak{y})\}$

For, $\bar{\vartheta}^c_{\bar{T}_\xi}(\varpi * \mathfrak{y}) \ge \text{rmin}\{\bar{\vartheta}^c_{\bar{T}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{T}_\xi}(\mathfrak{y})\}$

$\Leftrightarrow [0,0] - (\bar{\vartheta}_{\bar{T}_\xi})(\varpi * \mathfrak{y}) \ge [0,0] - \text{rmin}\{\bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\mathfrak{y})\}$

$\Leftrightarrow (\bar{\vartheta}^c_{\bar{T}_\xi})(\varpi * \mathfrak{y}) \le \text{rmax}\{([0,0] - \bar{\vartheta}_{\bar{T}_\xi}(\varpi)), ([0,0] - \bar{\vartheta}_{\bar{T}_\xi}(\mathfrak{y})\}$

$\Leftrightarrow (\bar{\vartheta}^c_{\bar{T}_\xi})(\varpi * \mathfrak{y}) \le \text{rmax}\{\bar{\vartheta}^c_{\bar{T}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{T}_\xi}(\mathfrak{y})\}$

Similarly, $(\bar{\vartheta}^c_{\bar{I}_\xi})(\varpi * \mathfrak{y}) \le \text{rmax}\{\bar{\vartheta}^c_{\bar{I}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{I}_\xi}(\mathfrak{y})\}$

$\bar{\vartheta}^c_{\bar{F}_\xi}(\varpi * \mathfrak{y}) \ge \text{rmin}\{\bar{\vartheta}^c_{\bar{F}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{F}_\xi}(\mathfrak{y})\}$

$\bar{\vartheta}^c_{\bar{F}_\xi}(\varpi * \mathfrak{y}) \le \text{rmax}\{\bar{\vartheta}^c_{\bar{F}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{F}_\xi}(\mathfrak{y})\}$

$\Leftrightarrow [1,1] - (\bar{\vartheta}_{\bar{F}_\xi})(\varpi * \mathfrak{y}) \ge [1,1] - \text{rmax}\{\bar{\vartheta}_{\bar{F}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\mathfrak{y})\}$

$\Leftrightarrow (\bar{\vartheta}^c_{\bar{F}_\xi})(\varpi * \mathfrak{y}) \le \text{rmax}\{([1,1] - \bar{\vartheta}_{\bar{F}_\xi}(\varpi)), ([1,1] - \bar{\vartheta}_{\bar{F}_\xi}(\mathfrak{y})\}$

$\Leftrightarrow (\bar{\vartheta}^c_{\bar{F}_\xi})(\varpi * \mathfrak{y}) \ge \text{rmin}\{\bar{\vartheta}^c_{\bar{F}_\xi}(\varpi), \bar{\vartheta}^c_{\bar{F}_\xi}(\mathfrak{y})\}$

Corollary 3.5. Let $\xi = \{\varpi, \bar{\mu}_{\bar{T}_\xi}(\varpi), \bar{\mu}_{\bar{I}_\xi}(\varpi), \bar{\mu}_{\bar{F}_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{I}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\varpi) / \varpi \in \mathfrak{V} \}$ be an interval-valued intuitionistic neutrosophic set of $\mathfrak{V}$, Then $\xi = \{\varpi, \bar{\mu}_{\bar{T}_\xi}(\varpi), \bar{\mu}_{\bar{I}_\xi}(\varpi), \bar{\mu}_{\bar{F}_\xi}(\varpi), \bar{\vartheta}_{\bar{T}_\xi}(\varpi), \bar{\vartheta}_{\bar{I}_\xi}(\varpi), \bar{\vartheta}_{\bar{F}_\xi}(\varpi) / \varpi \in \mathfrak{V} \}$ is an interval-valued intuitionistic neutrosophic fuzzy $\hat{Z}$-subalgebra of $\mathfrak{V}$ iff $\boxdot\, \xi = (\varpi, \bar{\vartheta}^c_\xi, \bar{\vartheta}_\xi)$ is an interval-valued intuitionistic neutrosophic fuzzy $\hat{Z}$-subalgebra of $\mathfrak{V}$ $\Leftrightarrow$ $\boxdot\, \xi^L = (\varpi, \left(\vartheta^L_{T_\xi}\right)^c, \vartheta^L_{T_\xi}, \left(\vartheta^L_{I_\xi}\right)^c,$

$\vartheta^L_{I_\xi}$, $\left(\vartheta^L_{F_\xi}\right)^c$, $\vartheta^L_{F_\xi}$) and $\boxdot\, \xi^U$ = (ϖ,$\left(\vartheta^U_{T_\xi}\right)^c$, $\vartheta^U_{T_\xi}$, $\left(\vartheta^U_{I_\xi}\right)^c$, $\vartheta^U_{I_\xi}$, $\left(\vartheta^U_{F_\xi}\right)^c$, $\vartheta^U_{F_\xi}$) are interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebra of 𝔙.

Theorem 3.6. Let ξ = {ϖ, $\bar{\mu}_{\bar{T}_\xi}$(ϖ), $\bar{\mu}_{\bar{I}_\xi}$(ϖ), $\bar{\mu}_{\bar{F}_\xi}$(ϖ), $\bar{\vartheta}_{\bar{T}_\xi}$(ϖ), $\bar{\vartheta}_{\bar{I}_\xi}$(ϖ), $\bar{\vartheta}_{\bar{F}_\xi}$(ϖ) / ϖ ∈ 𝔙 } is an interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebra of 𝔙 iff $\boxdot\, \xi$ = (ϖ, $\overline{(\mu}_{\bar{T}_\xi}$(ϖ), $\left(\bar{\mu}_{\bar{T}_\xi}\right)^c$), ($\bar{\mu}_{\bar{I}_\xi}$(ϖ), $\left(\bar{\mu}_{\bar{I}_\xi}\right)^c$), ($\bar{\mu}_{F_\xi}$(ϖ), $\left(\bar{\mu}_{F_\xi}\right)^c$) be an interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebra of 𝔙 ⇔ $\boxdot\, \xi^L$ = (ϖ,($\mu^L_{T_\xi}$,$\left(\mu^L_{T_\xi}\right)^c$),($\mu^L_{I_\xi}$, $\left(\mu^L_{I_\xi}\right)^c$), ($\mu^L_{F_\xi}$ $\left(\mu^L_{F_\xi}\right)^c$)) and $\boxdot\, \xi^U$ = (ϖ,($\mu^U_{T_\xi}$,$\left(\mu^U_{T_\xi}\right)^c$), ($\mu^U_{I_\xi}$, $\left(\mu^U_{I_\xi}\right)^c$),($\mu^U_{F_\xi}$, $\left(\mu^U_{F_\xi}\right)^c$)) are interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebra of 𝔙.

To Prove: $\boxdot\, \xi$ is interval-valued intuitionistic neutrosophic Ẑ-subalgebra

$\bar{\mu}^c_{\bar{T}_\xi}$(ϖ ∗ ŋ) ≤ rmax {$\bar{\mu}^c_{\bar{T}_\xi}$(ϖ), $\bar{\mu}^c_{\bar{T}_\xi}$(ŋ)}

For, $\bar{\mu}_{\bar{T}_\xi}$(ϖ ∗ ŋ) ≥ rmin{ $\bar{\mu}_{\bar{T}_\xi}$(ϖ), $\bar{\mu}_{\bar{T}_\xi}$(ŋ)}

⇔ [1,1] – ($\bar{\mu}_{\bar{T}_\xi}$ (ϖ ∗ ŋ) ≥ [1,1] – rmin{$\bar{\mu}_{\bar{T}_\xi}$(ϖ), $\bar{\mu}_{\bar{T}_\xi}$(ŋ)}

⇔ $\bar{\mu}^c_{\bar{T}_\xi}$(ϖ ∗ ŋ) ≤ rmax{([1,1]- $\bar{\mu}_{\bar{T}_\xi}$(ϖ)),([1,1]- $\bar{\mu}_{\bar{T}_\xi}$(ŋ)}

⇔ $\bar{\mu}^c_{\bar{T}_\xi}$(ϖ ∗ ŋ) ≤ rmax{ $\bar{\mu}^c_{\bar{T}_\xi}$(ϖ), $\bar{\mu}^c_{\bar{T}_\xi}$(ŋ)}

Similarly, $\bar{\mu}^c_{\bar{I}_\xi}$(ϖ ∗ ŋ) ≤ rmax{ $\bar{\mu}^c_{\bar{I}_\xi}$(ϖ), $\bar{\mu}^c_{\bar{I}_\xi}$(ŋ)}

$\bar{\mu}^c_{\bar{F}_\xi}$(ϖ ∗ ŋ) ≤ rmax {$\bar{\mu}^c_{\bar{F}_\xi}$(ϖ), $\bar{\mu}^c_{\bar{F}_\xi}$(ŋ)}

For, $\bar{\mu}_{\bar{F}_\xi}$(ϖ ∗ ŋ) ≥ rmin{ $\bar{\mu}_{\bar{F}_\xi}$(ϖ), $\bar{\mu}_{\bar{F}_\xi}$(ŋ)}

⇔ [1,1] – ($\bar{\mu}_{\bar{F}_\xi}$ (ϖ ∗ ŋ) ≤ [1,1] – rmax{$\bar{\mu}_{\bar{F}_\xi}$(ϖ), $\bar{\mu}_{\bar{F}_\xi}$(ŋ)}

⇔ $\bar{\mu}^c_{\bar{F}_\xi}$(ϖ ∗ ŋ) ≥ rmin{([1,1]- $\bar{\mu}_{\bar{F}_\xi}$(ϖ)),([1,1]- $\bar{\mu}_{\bar{F}_\xi}$(ŋ)}

⇔ $\bar{\mu}^c_{\bar{F}_\xi}$(ϖ ∗ ŋ) ≥ rmin{ $\bar{\mu}^c_{\bar{F}_\xi}$(ϖ), $\bar{\mu}^c_{\bar{F}_\xi}$(ŋ)}.

4. Homomorphism of interval-valued intuitionistic neutrosophic in Ẑ-subalgebra:

Definition 4.1. Let ξ = {ϖ, $\bar{\mu}_{\bar{T}_\xi}$(ϖ), $\bar{\mu}_{\bar{I}_\xi}$(ϖ), $\bar{\mu}_{\bar{F}_\xi}$(ϖ), $\bar{\vartheta}_{\bar{T}_\xi}$(ϖ), $\bar{\vartheta}_{\bar{I}_\xi}$(ϖ), $\bar{\vartheta}_{\bar{F}_\xi}$(ϖ) / ϖ ∈ 𝔙 } be an interval-valued intuitionistic neutrosophic fuzzy set in 𝔙 and f be a mapping from a set 𝔙 into a set 𝔉 then its image of ξ under f, f(ξ) is defined as

f(ξ) = {ϖ, $f_{rsup}\bar{\mu}_{\bar{T}_\xi}$(ϖ), $f_{rsup}\bar{\mu}_{\bar{I}_\xi}$(ϖ), $f_{rinf}\bar{\mu}_{\bar{F}_\xi}$(ϖ), $f_{rinf}\bar{\vartheta}_{\bar{T}_\xi}$(ϖ) $f_{rinf}\bar{\vartheta}_{\bar{I}_\xi}$(ϖ), $f_{rsup}\bar{\vartheta}_{F_\xi}$(ϖ)/ ϖ ∈ 𝔉 }

Where,

$$f_{rsup}(\bar{\mu}_{\bar{T}_\xi})(\mathfrak{y}) = \begin{cases} \underset{\varpi \in f^{-1}(\mathfrak{y})}{rsup}\ \bar{\mu}_{\bar{T}_\xi}(\varpi) & if\ f^{-1}(\mathfrak{y}) \neq \phi \\ [1,1] & otherwise \end{cases}$$

$$f_{rsup}(\bar{\mu}_{\bar{I}_\xi})(\mathfrak{y}) = \begin{cases} \underset{\varpi \in f^{-1}(\mathfrak{y})}{rsup}\ \bar{\mu}_{\bar{I}_\xi}(\varpi) & if\ f^{-1}(\mathfrak{y}) \neq \phi \\ [1,1] & otherwise \end{cases}$$

$$f_{rinf}(\bar{\mu}_{\bar{F}_\xi})(\mathfrak{y}) = \begin{cases} \underset{\varpi \in f^{-1}(\mathfrak{y})}{rinf}\ \bar{\mu}_{\bar{F}_\xi}(\varpi) & if\ f^{-1}(\mathfrak{y}) \neq \phi \\ [0,0] & otherwise \end{cases}$$

$$f_{rinf}(\bar{\vartheta}_{\bar{T}_\xi})(\mathfrak{y}) = \begin{cases} \underset{\varpi \in f^{-1}(\mathfrak{y})}{rinf}\ \bar{\vartheta}_{\bar{T}_\xi}(\varpi) & if\ f^{-1}(\mathfrak{y}) \neq \phi \\ [1,1] & otherwise \end{cases}$$

$$f_{rinf}(\bar{\vartheta}_{\bar{I}_\xi})(\mathfrak{y}) = \begin{cases} \underset{\varpi \in f^{-1}(\mathfrak{y})}{rinf}\ \bar{\vartheta}_{\bar{I}_\xi}(\varpi) & if\ f^{-1}(\mathfrak{y}) \neq \phi \\ [1,1] & otherwise \end{cases}$$

$$f_{rsup}(\bar{\vartheta}_{\bar{F}_\xi})(\mathfrak{y}) = \begin{cases} \underset{\varpi \in f^{-1}(\mathfrak{y})}{rinf}\ \bar{\vartheta}_{\bar{F}_\xi}(\varpi) & if\ f^{-1}(\mathfrak{y}) \neq \phi \\ [0,0] & otherwise \end{cases}$$

Definition 4.2. The interval-valued intuitionistic neutrosophic set ξ in any set $\mathfrak{B}$ is referred to be rsup-rsup-rinf property if for subset $\mathcal{T}$ of $\mathfrak{B}$ then, there exist $t_0 \in \mathcal{T}$ such that $\bar{\mu}_{\bar{T}_\xi}(t_0) = rsup_{t_0 \in \mathcal{T}}$, $\bar{\mu}_{\bar{I}_\xi}(t_0) = rsup_{t_0 \in \mathcal{T}}$, $\bar{\mu}_{\bar{F}_\xi}(t_0) = rinf_{t_0 \in \mathcal{T}}$, $\bar{\vartheta}_{\bar{T}_\xi}(t_0) = rinf_{t_0 \in \mathcal{T}}$, $\bar{\vartheta}_{\bar{I}_\xi}(t_0) = rinf_{t_0 \in \mathcal{T}}$, $\bar{\vartheta}_{\bar{F}_\xi}(t_0) = rsup_{t_0 \in \mathcal{T}}$ respectively.

Theorem 4.3. If f : $\mathfrak{B} \to \mathfrak{F}$ be the homomorphism of a Ẑ-subalgebra. If $\tau = \{ \varpi, \bar{\mu}_{\bar{T}_\tau}(\varpi), \bar{\mu}_{\bar{I}_\tau}(\varpi), \bar{\mu}_{\bar{F}_\tau}(\varpi), \bar{\vartheta}_{\bar{T}_\tau}(\varpi), \bar{\vartheta}_{\bar{I}_\tau}(\varpi), \bar{\vartheta}_{\bar{F}_\tau}(\varpi) / \varpi \in \mathfrak{B} \}$ is an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{F}$, then its inverse image $f^{-1}(\tau)$=1$\{\varpi, f^{-1}(\bar{\mu}_{\bar{T}_\tau}), f^{-1}(\bar{\mu}_{\bar{I}_\tau}), f^{-1}(\bar{\mu}_{\bar{F}_\tau}), f^{-1}(\bar{\vartheta}_{\bar{T}_\tau}), f^{-1}(\bar{\vartheta}_{\bar{I}_\tau}), f^{-1}(\bar{\vartheta}_{\bar{F}_\tau}) / \varpi \in \mathfrak{B} \}$ of τ under f is an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{B}$.

Proof: $f^{-1}(\bar{\mu}_{\bar{T}_\tau})(\varpi * \mathfrak{y}) = \bar{\mu}_{\bar{T}_\tau}(f(\varpi * \mathfrak{y}))$

$= \bar{\mu}_{\bar{T}_\tau}(f(\varpi) * f(\mathfrak{y}))$

$\geq rmin\{\bar{\mu}_{\bar{T}_\tau}(f(\varpi), \bar{\mu}_{\bar{T}_\tau}(f(\mathfrak{y}))\}$

$= rmin\{f^{-1}(\bar{\mu}_{\bar{T}_\tau})(\varpi), f^{-1}(\bar{\mu}_{\bar{T}_\tau})(\mathfrak{y})\}$

Similarly, $f^{-1}(\bar{\mu}_{\bar{I}_\tau})(\varpi * \mathfrak{y}) \geq rmin\{f^{-1}(\bar{\mu}_{\bar{I}_\tau})(\varpi), f^{-1}(\bar{\mu}_{\bar{I}_\tau})(\mathfrak{y})\}$

Then $f^{-1}(\bar{\mu}_{\bar{F}_\tau})(\varpi * \mathfrak{y}) = \bar{\mu}_{\bar{F}_\tau}(f(\varpi * \mathfrak{y}))$

$= \bar{\mu}_{\bar{F}_\tau}(f(\varpi) * f(\mathfrak{y}))$

$\leq rmax\{\bar{\mu}_{\bar{F}_\tau}(f(\varpi), \bar{\mu}_{\bar{F}_\tau}(f(\mathfrak{y}))\}$

$= rmax\{f^{-1}(\bar{\mu}_{\bar{F}_\tau})(\varpi), f^{-1}(\bar{\mu}_{\bar{F}_\tau})(\mathfrak{y})\}$

Also for, $f^{-1}(\bar{\vartheta}_{\bar{T}_\tau})(\varpi * \mathfrak{y}) = \bar{\vartheta}_{\bar{T}_\tau}(f(\varpi * \mathfrak{y}))$

$= \bar{\vartheta}_{\bar{T}_\tau}(f(\varpi) * f(\mathfrak{y}))$

$\leq rmax\{\bar{\vartheta}_{\bar{T}_\tau}(f(\varpi), \bar{\vartheta}_{\bar{T}_\tau}(f(\mathfrak{y}))\}$

$= rmax\{f^{-1}(\bar{\vartheta}_{\bar{T}_\tau})(\varpi), f^{-1}(\bar{\vartheta}_{\bar{T}_\tau})(\mathfrak{y})\}$

Similarly, $f^{-1}(\bar{\vartheta}_{\bar{I}_\tau})(\varpi * \mathfrak{y}) \leq rmax\{f^{-1}(\bar{\vartheta}_{\bar{I}_\tau})(\varpi), f^{-1}(\bar{\vartheta}_{\bar{I}_\tau})(\mathfrak{y})\}$

Then, $f^{-1}(\bar{\vartheta}_{\bar{F}_\tau})(\varpi * \mathfrak{y}) = \bar{\vartheta}_{\bar{F}_\tau}(f(\varpi * \mathfrak{y})$

$= \bar{\vartheta}_{\bar{F}_\tau}(f(\varpi) * f(\mathfrak{y}))$

$\geq rmin\{\bar{\vartheta}_{\bar{F}_\tau}(f(\varpi), \bar{\vartheta}_{\bar{F}_\tau}(f(\mathfrak{y}))\}$

$= rmin\{f^{-1}(\bar{\vartheta}_{\bar{F}_\tau})(\varpi), f^{-1}(\bar{\vartheta}_{\bar{F}_\tau})(\mathfrak{y})\}$

Theorem 4.4. If $(\mathfrak{B}, *, 0)$ & $(\mathfrak{F}, *, 0)$ be two Ẑ-algebras. Let f : $\mathfrak{B} \to \mathfrak{F}$ is an endomorphism. If ξ is the interval-valued intuitionistic neutrosophic Ẑ-subalgebras of $\mathfrak{B}$, defines to be

$f(\xi) = \{\varpi, \bar{\mu}_{\bar{T}_f}(\varpi) = \bar{\mu}_{\bar{T}}(f(\varpi)), \bar{\mu}_{\bar{I}_f}(\varpi) = \bar{\mu}_{\bar{I}}(f(\varpi)), \bar{\mu}_{\bar{F}_f}(\varpi) = \bar{\mu}_{\bar{F}}(f(\varpi)), \bar{\vartheta}_{\bar{T}_f}(\varpi) = \bar{\vartheta}_{\bar{T}}(f(\varpi)), \bar{\vartheta}_{\bar{I}_f}(\varpi) = \bar{\vartheta}_{\bar{I}}(f(\varpi)), \bar{\vartheta}_{\bar{F}_f}(\varpi) = \bar{\vartheta}_{\bar{F}}(f(\varpi)) / \varpi \in \mathfrak{B}\}$ then $f(\xi)$ is an interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebras of $\mathfrak{F}$.

Proof: Let $\varpi, \mathfrak{y} \in \mathfrak{B}$,

$\bar{\mu}_{\bar{T}_f}(\varpi * \mathfrak{y}) = \bar{\mu}_{\bar{T}}(f(\varpi * \mathfrak{y}))$

$= \bar{\mu}_{\bar{T}}(f(\varpi) * f(\mathfrak{y}))$

$\geq rmin\{\bar{\mu}_{\bar{T}}(f(\varpi), \bar{\mu}_{\bar{T}}(f(\mathfrak{y}))\}$

$= rmin\{\bar{\mu}_{\bar{T}_f}(\varpi), \bar{\mu}_{\bar{T}_f}(\mathfrak{y})\}$

Similarly, $\bar{\mu}_{\bar{I}_f}(\varpi * \mathfrak{y}) \geq rmin\{\bar{\mu}_{\bar{I}_f}(\varpi), \bar{\mu}_{\bar{I}_f}(\mathfrak{y})\}$

$\bar{\mu}_{\bar{F}_f}(\varpi * \mathfrak{y}) = \bar{\mu}_{\bar{F}}(f(\varpi * \mathfrak{y}))$

$= \bar{\mu}_{\bar{F}}(\mathrm{f}(\varpi) * \mathrm{f}(\mathfrak{y}))$

$\leq \mathrm{rmax}\{\bar{\mu}_{\bar{F}}(\mathrm{f}(\varpi),\ \bar{\mu}_{\bar{F}}(\mathrm{f}(\mathfrak{y}))\}$

$= \mathrm{rmax}\{\bar{\mu}_{\bar{F}_{\mathrm{f}}}(\varpi),\ \bar{\mu}_{\bar{F}_{\mathrm{f}}}(\mathfrak{y})\}$

$\bar{\vartheta}_{\bar{T}_{\mathrm{f}}}(\varpi * \mathfrak{y}) = \bar{\vartheta}_{\bar{T}}(\mathrm{f}(\varpi * \mathfrak{y}))$

$= \bar{\vartheta}_{\bar{T}}(\mathrm{f}(\varpi) * \mathrm{f}(\mathfrak{y}))$

$\leq \mathrm{rmax}\{\bar{\vartheta}_{\bar{T}}(\mathrm{f}(\varpi),\ \bar{\vartheta}_{\bar{T}}(\mathrm{f}(\mathfrak{y}))\}$

$= \mathrm{rmax}\{\bar{\vartheta}_{\bar{T}_{\mathrm{f}}}(\varpi),\ \bar{\vartheta}_{\bar{T}_{\mathrm{f}}}(\mathfrak{y})\}$

Similarly, $\bar{\vartheta}_{\bar{I}_{\mathrm{f}}}(\varpi * \mathfrak{y}) \leq \mathrm{rmax}\{\bar{\vartheta}_{\bar{I}_{\mathrm{f}}}(\varpi),\ \bar{\vartheta}_{\bar{I}_{\mathrm{f}}}(\mathfrak{y})\}$

$\bar{\vartheta}_{\bar{F}_{\mathrm{f}}}(\varpi * \mathfrak{y}) = \bar{\vartheta}_{\bar{F}}(\mathrm{f}(\varpi * \mathfrak{y}))$

$= \bar{\vartheta}_{\bar{F}}(\mathrm{f}(\varpi) * \mathrm{f}(\mathfrak{y}))$

$\geq \mathrm{rmin}\{\bar{\vartheta}_{\bar{F}}(\mathrm{f}(\varpi),\ \bar{\vartheta}_{\bar{F}}(\mathrm{f}(\mathfrak{y}))\}$

$= \mathrm{rmin}\{\bar{\vartheta}_{\bar{F}_{\mathrm{f}}}(\varpi),\ \bar{\vartheta}_{\bar{F}_{\mathrm{f}}}(\mathfrak{y})\}$

∴ f(ξ) is an interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebras of $\mathfrak{F}$.

5. Interval-valued intuitionistic neutrosophic fuzzy Ẑ-subalgebra In Cartesian Product

The following section discussed about the cartesian product and their definitions which are necessary for this research work.

Definition 5.1. Let ($\mathfrak{B}$, ∗,0) & ($\mathfrak{F}$, ∗,0) be two sets of Ẑ-subalgebra. Let $\xi_1 = \{\varpi, \bar{\mu}_{\bar{T}_{\xi_1}}(\varpi), \bar{\mu}_{\bar{I}_{\xi_1}}(\varpi), \bar{\mu}_{\bar{F}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{T}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{I}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{F}_{\xi_1}}(\varpi) / \varpi \in \mathfrak{B}\}$ and $\xi_2 = \{\mathfrak{y}, \bar{\mu}_{\bar{T}_{\xi_2}}(\mathfrak{y}), \bar{\mu}_{\bar{I}_{\xi_2}}(\mathfrak{y}), \bar{\mu}_{\bar{F}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{T}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{I}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{F}_{\xi_2}}(\mathfrak{y}) / \mathfrak{y} \in \mathfrak{F}\}$ be the interval-valued intuitionistic neutrosophic subsets in $\mathfrak{B}$ & $\mathfrak{F}$ respectively. The cartesian product ξ_1 and ξ_2 if the form $\xi_1 \times \xi_2 = \{(\varpi, \mathfrak{y}), \bar{\mu}_{\bar{T},\bar{I},\bar{F}_{\xi_1\times\xi_2}}(\varpi, \mathfrak{y}), \bar{\vartheta}_{\bar{T},\bar{I},\bar{F}_{\xi_1\times\xi_2}}(\varpi, \mathfrak{y}) / \varpi, \mathfrak{y} \in \mathfrak{B} \times \mathfrak{F}$, where $\bar{\mu}_{\bar{T},\bar{I},\bar{F}_{\xi_1\times\xi_2}} : \mathfrak{B} \times \mathfrak{F} \to \mathfrak{D}$ [0,1] and $\bar{\vartheta}_{\bar{T},\bar{I},\bar{F}_{\xi_1\times\xi_2}} : \mathfrak{B} \times \mathfrak{F} \to \mathfrak{D}$ [0,1] is given by

$\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}} \geq \mathrm{rmin}\{\bar{\mu}_{\bar{T}_{\xi_1}}(\varpi),\ \bar{\mu}_{\bar{T}_{\xi_2}}(\mathfrak{y})\}$, $\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}} \leq \mathrm{rmax}\{\bar{\vartheta}_{\bar{T}_{\xi_1}}(\varpi),\ \bar{\vartheta}_{\bar{T}_{\xi_2}}(\mathfrak{y})\}$

$\bar{\mu}_{\bar{I}_{\xi_1\times\xi_2}} \geq \mathrm{rmin}\{\bar{\mu}_{\bar{I}_{\xi_1}}(\varpi),\ \bar{\mu}_{\bar{I}_{\xi_2}}(\mathfrak{y})\}$, $\bar{\vartheta}_{\bar{I}_{\xi_1\times\xi_2}} \leq \mathrm{rmax}\{\bar{\vartheta}_{\bar{I}_{\xi_1}}(\varpi),\ \bar{\vartheta}_{\bar{I}_{\xi_2}}(\mathfrak{y})\}$

$\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}} \leq \mathrm{rmax}\{\bar{\mu}_{\bar{F}_{\xi_1}}(\varpi),\ \bar{\mu}_{\bar{F}_{\xi_2}}(\mathfrak{y})\}$, $\bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}} \geq \mathrm{rmin}\{\bar{\vartheta}_{\bar{F}_{\xi_1}}(\varpi),\ \bar{\vartheta}_{\bar{F}_{\xi_2}}(\mathfrak{y})\}$.

Theorem 5.2. Let $\xi_1 = \{\varpi, \bar{\mu}_{\bar{T}_{\xi_1}}(\varpi), \bar{\mu}_{\bar{I}_{\xi_1}}(\varpi), \bar{\mu}_{\bar{F}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{T}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{I}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{F}_{\xi_1}}(\varpi) / \varpi \in \mathfrak{B}\}$ and $\xi_2 = \{\mathfrak{y}, \bar{\mu}_{\bar{T}_{\xi_2}}(\mathfrak{y}), \bar{\mu}_{\bar{I}_{\xi_2}}(\mathfrak{y}), \bar{\mu}_{\bar{F}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{T}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{I}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{F}_{\xi_2}}(\mathfrak{y}) / \mathfrak{y} \in \mathfrak{F}\}$ be any two interval-valued intuitionistic neutrosophic Ẑ-subalgebras of $\mathfrak{B}$ and $\mathfrak{F}$ respectively and then $\xi_1 \times \xi_2$ is also interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{B} \times \mathfrak{F}$.

Proof: Let $\xi_1 = \{\varpi, \bar{\mu}_{\bar{T},\bar{I},\bar{F}_{\xi_1}}(\varpi), \bar{\vartheta}_{\bar{T},\bar{I},\bar{F}_{\xi_1}}(\varpi) / \varpi \in \mathfrak{B}\}$ and $\xi_2 = \{\mathfrak{y}, \bar{\mu}_{\bar{T},\bar{I},\bar{F}_{\xi_2}}(\mathfrak{y}), \bar{\vartheta}_{\bar{T},\bar{I},\bar{F}_{\xi_2}}(\mathfrak{y}) / \mathfrak{y} \in \mathfrak{F}\}$be the two interval-valued intuitionistic Ẑ-subalgebra of $\mathfrak{B}$ and $\mathfrak{F}$. Take (a,b) $\in \mathfrak{B} \times \mathfrak{F}$, where a = $(\varpi_1, \mathfrak{y}_1)$ and b= $(\varpi_2, \mathfrak{y}_2)$.

$\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi * \mathfrak{y}) = \bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}((\varpi_1, \mathfrak{y}_1) * (\varpi_2, \mathfrak{y}_2))$

$= [\mu^L_{T_{\xi_1\times\xi_2}}((\varpi_1, \mathfrak{y}_1) * (\varpi_2, \mathfrak{y}_2)),\ \mu^U_{T_{\xi_1\times\xi_2}}((\varpi_1, \mathfrak{y}_1) * (\varpi_2, \mathfrak{y}_2))]$

$\geq \min\{\mu^L_{T_{\xi_1\times\xi_2}}(\varpi_1, \mathfrak{y}_1),\ \mu^L_{T_{\xi_1\times\xi_2}}(\varpi_2, \mathfrak{y}_2)\}, \min\{\mu^U_{T_{\xi_1\times\xi_2}}(\varpi_1, \mathfrak{y}_1),\ \mu^U_{T_{\xi_1\times\xi_2}}(\varpi_2, \mathfrak{y}_2)\}]$

$= \mathrm{rmin}\{[\mu^L_{T_{\xi_1\times\xi_2}}(\varpi_1, \mathfrak{y}_1),\ \mu^U_{T_{\xi_1\times\xi_2}}(\varpi_1, \mathfrak{y}_1)],[\ \mu^L_{T_{\xi_1\times\xi_2}}(\varpi_2, \mathfrak{y}_2)\ \ \mu^U_{T_{\xi_1\times\xi_2}}(\varpi_2, \mathfrak{y}_2)]\}$

$= \mathrm{rmin}[\{\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_1, \mathfrak{y}_1),\ \bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_2, \mathfrak{y}_2)\}]$

$= \mathrm{rmin}\{\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\mathrm{a}),\ \bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\mathrm{b})\}$

Similarly, $\bar{\mu}_{\bar{I}_{\xi_1\times\xi_2}}(\varpi * \mathfrak{y}) \geq \mathrm{rmin}\{\bar{\mu}_{\bar{I}_{\xi_1\times\xi_2}}(\mathrm{a}),\ \bar{\mu}_{\bar{I}_{\xi_1\times\xi_2}}(\mathrm{b})\}$

Then, $\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi * \mathfrak{y}) = \bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}((\varpi_1, \mathfrak{y}_1) * (\varpi_2, \mathfrak{y}_2))$

$= [\mu^L_{F_{\xi_1\times\xi_2}}((\varpi_1, \mathfrak{y}_1) * (\varpi_2, \mathfrak{y}_2)),\ \mu^U_{F_{\xi_1\times\xi_2}}((\varpi_1, \mathfrak{y}_1) * (\varpi_2, \mathfrak{y}_2))]$

$\leq \max\{\mu^L_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \mu^L_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}, \max\{\mu^U_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \mu^U_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}]$

$= \text{rmax}\{[\mu^L_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \mu^U_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1)],[\ \mu^L_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\ \ \mu^U_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)]\}$

$= \text{rmax}[\{\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}]$

$= \text{rmax}\{\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\text{a}),\ \bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\text{b})\}$

Additionally,

$\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi *\ \mathfrak{y}) = \ \bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))$

$= [\vartheta^L_{T_{\xi_1\times\xi_2}}((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2)),\ \vartheta^U_{T_{\xi_1\times\xi_2}}((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))]$

$\leq \max\{\vartheta^L_{T_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \vartheta^L_{T_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}, \min\{\vartheta^U_{T_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \vartheta^U_{T_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}]$

$= \text{rmax}\{[\vartheta^L_{T_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \vartheta^U_{T_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1)],[\ \vartheta^L_{T_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\ \ \vartheta^U_{T_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)]\}$

$= \text{rmax}[\{\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}]$

$= \text{rmax}\{\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\text{a}),\ \bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\text{b})\}$

Similarly, $\bar{\vartheta}_{\bar{I}_{\xi_1\times\xi_2}}(\varpi *\ \mathfrak{y})\ \leq\ \text{rmax}\{\bar{\vartheta}_{\bar{I}_{\xi_1\times\xi_2}}(\text{a}),\ \bar{\vartheta}_{\bar{I}_{\xi_1\times\xi_2}}(\text{b})\}$

Then, $\bar{\vartheta}_{F_{\xi_1\times\xi_2}}(\varpi *\ \mathfrak{y}) = \ \bar{\vartheta}_{F_{\xi_1\times\xi_2}}((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))$

$= [\vartheta^L_{F_{\xi_1\times\xi_2}}((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2)),\ \vartheta^U_{F_{\xi_1\times\xi_2}}((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))]$

$\geq \min\{\vartheta^L_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \vartheta^L_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}, \min\{\vartheta^U_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \vartheta^U_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}]$

$= \text{rmin}\{[\vartheta^L_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \vartheta^U_{F_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1)],[\ \vartheta^L_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\ \ \vartheta^U_{F_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)]\}$

$= \text{rmin}[\{\bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}]$

$= \text{rmin}\{\bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}(\text{a}),\ \bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}(\text{b})\}$

Theorem 5.3. If $\xi_1\ \times\ \xi_2$ be an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{V}\ \times\ \mathfrak{F}$, then either ξ is an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{V}$ or ζ is an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{F}$.

Proof: Let $\xi_1\ \times\ \xi_2$ be an interval-valued intuitionistic neutrosophic Ẑ-subalgebra of $\mathfrak{V}\ \times\ \mathfrak{F}$

Take $(\varpi_1,\ \mathfrak{y}_1)$ and $(\varpi_2,\ \mathfrak{y}_2)\ \in\ \mathfrak{V}\ \times\ \mathfrak{F}$

Now, $\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}\{((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))\}\ \geq\ \text{rmin}\{\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}$

Put $\varpi_1\ =\ \varpi_2\ = 0$, then

$\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}\{((0,\ \mathfrak{y}_1) * (0,\ \mathfrak{y}_2))\}\ \geq\ \text{rmin}\{\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_2)\}$

$\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}\{(0\ * 0), (\mathfrak{y}_1\ \ *\mathfrak{y}_2)\}\ \geq\ \text{rmin}\{\bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_2)\}$

$\bar{\mu}_{\bar{T}_{\xi_2}}(\mathfrak{y}_1\ \ *\mathfrak{y}_2)\ \geq\ \text{rmin}\{\bar{\mu}_{\bar{T}_{\xi_2}}(\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{T}_{\xi_2}}(\ \mathfrak{y}_2)\}$

Similarly, $\bar{\mu}_{\bar{I}_{\xi_2}}(\mathfrak{y}_1\ \ *\mathfrak{y}_2)\ \geq\ \text{rmin}\{\bar{\mu}_{\bar{I}_{\xi_2}}(\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{I}_{\xi_2}}(\ \mathfrak{y}_2)\}$

$\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}\{((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))\}\ \leq\ \text{rmax}\{\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}$,

Put $\varpi_1\ =\ \varpi_2\ = 0$, then

$\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}\{((0,\ \mathfrak{y}_1) * (0,\ \mathfrak{y}_2))\}\ \leq\ \text{rmax}\{\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_2)\}$

$\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}\{(0\ * 0), (\mathfrak{y}_1\ \ *\mathfrak{y}_2)\}\ \leq\ \text{rmax}\{\bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{F}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_2)\}$

$\bar{\mu}_{\bar{F}\ \xi_2}(\mathfrak{y}_1\ \ *\mathfrak{y}_2)\ \leq\ \text{rmax}\{\bar{\mu}_{\bar{F}\ \xi_2}(\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{F}\ \xi_2}(\ \mathfrak{y}_2)\}$

Also prove for, $\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}\{((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))\}\ \leq\ \text{rmax}\{\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}$

Put $\varpi_1\ =\ \varpi_2\ = 0$, then

$\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}\{((0,\ \mathfrak{y}_1) * (0,\ \mathfrak{y}_2))\}\ \leq\ \text{rmax}\{\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_2)\}$

$\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}\{(0\ * 0), (\mathfrak{y}_1\ \ *\mathfrak{y}_2)\}\ \leq\ \text{rmax}\{\bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{T}_{\xi_1\times\xi_2}}(0,\ \mathfrak{y}_2)\}$

$\bar{\vartheta}_{\bar{T}\ \xi_2}(\mathfrak{y}_1\ \ *\mathfrak{y}_2)\ \leq\ \text{rmax}\{\bar{\vartheta}_{\bar{T}\ \xi_2}(\ \mathfrak{y}_1),\ \bar{\mu}_{\bar{T}\ \xi_2}(\ \mathfrak{y}_2)\}$

Similarly, $\bar{\vartheta}_{\bar{I}\ \xi_2}(\mathfrak{y}_1\ \ *\mathfrak{y}_2)\ \geq\ \text{rmin}\{\bar{\vartheta}_{\bar{I}\ \xi_2}(\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{I}\ \xi_2}(\mathfrak{y}_2)\}$

$\bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}\{((\varpi_1,\ \mathfrak{y}_1) * (\varpi_2,\ \mathfrak{y}_2))\}\ \geq\ \text{rmin}\{\bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_1,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{F}_{\xi_1\times\xi_2}}(\varpi_2,\ \mathfrak{y}_2)\}$

Put $\varpi_1 = \varpi_2 = 0$, then

$$\bar{\vartheta}_{\bar{F}_{\xi_1 \times \xi_2}}\{((0,\ \mathfrak{y}_1) * (0,\ \mathfrak{y}_2))\} \geq \mathrm{rmin}\{\bar{\vartheta}_{\bar{F}_{\xi_1 \times \xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{F}_{\xi_1 \times \xi_2}}(0,\ \mathfrak{y}_2)\}$$

$$\bar{\vartheta}_{\bar{F}_{\xi_1 \times \xi_2}}\{(0 * 0), (\mathfrak{y}_1 * \mathfrak{y}_2)\} \geq \mathrm{rmin}\{\bar{\vartheta}_{\bar{F}_{\xi_1 \times \xi_2}}(0,\ \mathfrak{y}_1),\ \bar{\vartheta}_{\bar{F}_{\xi_1 \times \xi_2}}(0,\ \mathfrak{y}_2)\}$$

$$\bar{\vartheta}_{\bar{F}_{\xi_2}}(\mathfrak{y}_1 * \mathfrak{y}_2) \geq \mathrm{rmin}\{\bar{\vartheta}_{\bar{F}_{\xi_2}}(\mathfrak{y}_1),\ \bar{\vartheta}_{\bar{F}_{\xi_2}}(\mathfrak{y}_2)\}.$$

Conclusion

The preceding discussion introduces the concept of interval-valued intuitionistic neutrosophic Ẑ-subalgebra and provides a full overview of the underlying theories supporting this topic. It describes the essential principles of fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, and their algebraic structures, which serve as the theoretical foundation for this research paper. A careful survey of the available literature reveals a diverse set of algebraic frameworks and generalised structures developed and used in a variety of disciplines. This research aims to expand the concept of Ẑ-subalgebra by embracing sophisticated extensions of fuzzy set theories, including interval-valued and neutrosophic environments. This paper proposes a more generalised and flexible algebraic structure that can manage larger degrees of uncertainty, indeterminacy, and imprecision, with the goal of contributing to the increasing body of knowledge in the field.

Future Work

Future study could investigate interesting paths based on the proposed interval-valued intuitionistic neutrosophic Ẑ-subalgebra. The alternative path is to create new sorts of algebraic structures by combining previous generalised fuzzy set extensions, such as image fuzzy sets, Pythagorean fuzzy sets, and spherical fuzzy sets. Future research may focus on incorporating these algebraic structures in real-world decision-making settings, such as fuzzy MCDM problems, pattern recognition, or medical diagnosis, where ambiguity and reluctance are important factors. Integration of computational techniques and software tools to facilitate practical applications and simulation is also an option. Furthermore, studying the categorical and topological characteristics of these structures may lead to new theoretical possibilities and increase the field's mathematical depth.

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Conflict of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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