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Pythagorean Neutrosophic Fuzzy EM-SWARA-TOPSIS Approach for Green Supply Selection

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Abstract: Green supplier selection has emerged as a vital aspect in supply chain management to assimilate economical advantage with ecological and sustainability goals. To address this, a multi-criteria decision-making method (MCDM) using Pythagorean neutrosophic fuzzy set (PNFS) has been introduced, which can handle uncertain data more effectively. Novel score and accuracy functions have been presented for ranking the Pythagorean neutrosophic numbers (PNNs). The MCDM algorithm integrates novel distance and entropy measure, using, Entropy for objective weights and SWARA for subjective weights. The new distance measure cares the TOPSIS framework for evaluating alternatives. The approach's consistency and robustness in ranking green suppliers is validated through comparative analysis and sensitivity testing.

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Keywords: Pythagorean neutrosophic fuzzy set, Entropy measure, SWARA, TOPSIS, GSS

1. Introduction

Zadeh [1] introduced fuzzy sets (FSs) in 1965, which revolutionized real life decision-making under uncertainty by allowing partial membership in sets instead of relying on binary inclusion. Due to some limitations, FSs have been extended to intuitionistic FSs (IFSs) [2], type-2 FSs (T2FSs) [3, 4], pythagorean fuzzy sets (PyFSs) [5], picture FSs (PFSs) [6], and interval-valued PFSs (IVPFSs) [7], many more. Smarandache [8] presented a new concept known as neutrosophic set (NS), which expands upon FSs and IFSs, among other ideas, as IFSs were inadequate in managing the ambiguous and contradictory information present in belief systems. In 2019, the novel idea of a Pythagorean neutrosophic fuzzy set (PNFS) was investigated by R. Jansi et al [9]. A PNFS consists of three components such as truth membership, indeterminacy membership, and falsity membership in which indeterminacy is an independent component and membership and non-membership degrees are dependent components and the sum of the square of each component must be smaller than two. A collection of algebraic procedures that can be used with PNS was proposed by Jamiatun Nadwa Ismail et al. [10] in 2023. These operations include power, scalar multiplication, addition, and multiplication which make it easier to manipulate and combine PNS effectively, and improve decision-making in situations when there is ambiguity and uncertainty.

The entropy and distance measures are useful tool to create and develop a MCDM. Our main focus in constructing this MCDM technique is based on three key principles:

1. The distance measure is used in the development of the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [11–14] model.

2. The Entropy Measure (EM) [15–19] is employed to determine the objective weights of the criteria.

3. The Stepwise Weight Assessment Ratio Analysis (SWARA) [20–23] methodology is applied to obtain the subjective weights of the criteria.

SWARA is a time-saving and relatively straightforward technique for measuring weights. It assesses the precision of experts and assigns weight to each criterion, with the criteria that have the best rank being considered the most important. Research has shown the significance of the SWARA methodology in MCDM approaches [24–27]. Decision-makers can prioritize issues, conduct analyses, make comparisons, and rank alternatives using the TOPSIS method. TOPSIS is utilized to evaluate the alternatives within the MCDM process.

Due to expanding regulations and greater public awareness of pollution, environmental sustainability is becoming a critical factor in supply chain management. Green supply chains [28] (GSC) are a proactive approach that integrates environmental considerations into every aspect of operations, enhancing sustainability and improving company performance [29]. This approach starts with raw material acquisition and continues through product disposal or recycling. Suppliers play a crucial role in meeting environmental targets, leading to green supplier selection [30]. A sustainable supply chain involves optimizing procurement quality, implementing pollution control measures, adopting environmentally friendly practices, managing costs, and executing effective end-of-life management [31, 32].

Considering the foregoing, the primary driving forces for this study are as follows.

1. In MCDM models, distance and entropy measurements are commonly employed to determine criterion weights and assess alternatives. These metrics may also be leveraged to build methodologies such as TOPSIS, VIKOR, and EDAS.
2. Current approaches frequently calculate objective or subjective weights without taking preference or expert judgment into account, which many lead to information loss.
3. As we know, there is no research work available so far on the GSC selection problem [33, 34] with an integrated EM, distance measure, and SM using SWARA-TOPSIS in PNF environment.

The present study makes the following major contributions:

1. The study introduces an EM for PNFSs and develops a novel weighted model to obtain objective weights for criterion.
2. A novel hybrid MCDM approach, PNF-EM-SWARA-TOPSIS, employs the recommended distance measure and EM to address the complex MCDM problem of ambiguous and hazy information.
3. The article includes a practical investigation on GSS selection, proving the efficacy and utility of PNF-EM-SWARA-TOPSIS. It investigates the sensitivity of criterion weights to measure stability and consistency, and compares the technique to other existing approaches that are already in use.

2. Preliminaries

We will go over the PNFSs and provide some basic definitions.

Definition 1. [35] Let $\mathfrak{E}$ be a universal set and $\mathcal{F}_\Lambda = \{(x, \mathfrak{m}_\Lambda(x)), x \in \mathfrak{E}\}$ be a fuzzy set. Then, a Pythagorean fuzzy set $\wp_\Lambda$, which is a set of ordered pairs over $\wp_\Lambda$, is defined by the following: $\wp_\Lambda = \{(x, \mathfrak{m}_\Lambda(x), \mathfrak{n}_\Lambda(x)), x \in \mathfrak{E}\}$ where the functions $\mathfrak{m}_\Lambda(\mathrm{x}), \mathfrak{n}_\Lambda(x) \in [0,1]$ define the degree of membership and the degree of non-membership, respectively, of the element to $\wp_\Lambda$, which is a subset of $\mathfrak{E}$, and for every $x \in \mathfrak{E}$: $0 \le \mathfrak{m}_\Lambda^2(x) + \mathfrak{n}_\Lambda^2(x) \le 1$.

Supposing $0 \le \mathfrak{m}_\Lambda^2(x) + \mathfrak{n}_\Lambda^2(x) \le 1, \forall\, x \in \mathfrak{E}$, there is a degree of indeterminacy of $x \in \mathfrak{E}$ to $\wp_\Lambda$ defined by $\pi_\Lambda(\mathrm{x}) = \sqrt{1 - \mathfrak{m}_\Lambda^2(x) + \mathfrak{n}_\Lambda^2(x)}$ and $\pi_\Lambda(\mathrm{x}) \in [0,1]$.

Definition 2. [36] Let $\mathfrak{E}$ be a set of objects. Then a NFS $\mathcal{N}_\Lambda$ in $\mathfrak{E}$ is defined by $\mathcal{N}_\Lambda = \{(x, \mathfrak{m}_\Lambda(x), \mathfrak{v}_\Lambda(x), \mathfrak{n}_\Lambda(x)), x \in \mathfrak{E}\}$; where each membership value is expressed by a membership, indeterminacy, and non-membership function which are respectively denoted as $\mathfrak{m}_\Lambda(x), \mathfrak{v}_\Lambda(x), \mathfrak{n}_\Lambda(x)$. Moreover $\mathfrak{m}_\Lambda(x), \mathfrak{v}_\Lambda(x), \mathfrak{n}_\Lambda(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ with the condition $0^- \leq \mathfrak{m}_\Lambda(x) + \mathfrak{n}_\Lambda(x) \leq 1^+, \forall\, x \in \mathfrak{E}$.

Definition 3. [37] The definition of the Pythagorean neutrosophic fuzzy set $\mathcal{P}_\Lambda$ over a non-null set $\mathfrak{E}$ is given by

$$\mathcal{P}_\Lambda = \{(x, \mathfrak{m}_\Lambda(x), \mathfrak{v}_\Lambda(x), \mathfrak{n}_\Lambda(x)), x \in \mathfrak{E}\}$$

where $\mathfrak{m}_\Lambda(x), \mathfrak{v}_\Lambda(x), \mathfrak{n}_\Lambda(x) \in]0^-, 1^+[$ denote respectively, membership, indeterminacy, and non-membership functions, subsequently satisfying the conditions:

$0 \leq \mathfrak{m}_\Lambda(x) + \mathfrak{n}_\Lambda(x) \leq 1, \forall\, x \in \mathfrak{E}$,

$0^- \leq \mathfrak{m}_\Lambda^2(x) + \mathfrak{n}_\Lambda^2(x) \leq 1^+, \forall\, x \in \mathfrak{E}$,

$0^- \leq \mathfrak{m}_\Lambda^2(x) + \mathfrak{v}_\Lambda^2(x) + \mathfrak{n}_\Lambda^2(x) \leq 2, \forall\, x \in \mathfrak{E}$.

For a fixed $x \in \{\mathfrak{m}_\Lambda(x), \mathfrak{v}_\Lambda(x), \mathfrak{n}_\Lambda(x)\}$; i.e., in simply, $\wp = \{\mathfrak{m}, \mathfrak{v}, \mathfrak{n}\}$ is called Pythagorean neutrosophic fuzzy number (PNFN).

Example 3.1. Let $\mathcal{P}_\Lambda \in PFNS(\mathfrak{E})$. Suppose that $\mathfrak{m}_\Lambda(x) = 0.82$, $\mathfrak{v}_\Lambda(x) = 0.61$ and $\mathfrak{n}_\Lambda(x) = 0.15$ for $\mathcal{P}_\Lambda = \{x\}$. Clearly, $0.82 + 0.15 < 1$ and $0.82^2 + 0.15^2 < 1$ and $0.82^2 + 0.61^2 + 0.15^2 \leq 2$.

Then $\mathcal{P}_\Lambda$is a Neutrosophic Pythagorean fuzzy set.

2.1. The Score and Accuracy Functions

Within this section, we have introduced score and accuracy function for PNFN $\wp = \{\mathfrak{m}, \mathfrak{v}, \mathfrak{n}\}$ on .

Definition 4. The Score function is defined as

$$\mathfrak{C}(x) = \frac{(2 + \mathfrak{m}^2 - \mathfrak{v}^2 - \mathfrak{n}^2)}{3} \tag{1}$$

where $\mathfrak{C}(x) \in [0, 1]$.

Definition 5. The accuracy function for a PNFN on $\mathfrak{E}$, is given by

$$\mathfrak{A}(x) = \frac{\mathfrak{m}^2 + \mathfrak{v}^2 + \mathfrak{n}^2}{2} \tag{2}$$

where $\mathfrak{A}(x) \in [0, 1]$.

Let $\wp_1 = (\mathfrak{m}_1, \mathfrak{v}_1, \mathfrak{n}_1)$ and $\wp_2 = (\mathfrak{m}_2, \mathfrak{v}_2, \mathfrak{n}_2)$ are two PNFNs. Considering the aforementioned score function $\mathfrak{C}$ and the accuracy function $\mathfrak{A}$ the relation between two PNFNs is stated as:

If $\mathfrak{C}(\wp_1) < \mathfrak{C}(\wp_2)$, then $\wp_1$ is smaller than $\wp_2$ represented as $\wp_1 < \wp_2$.

If $\mathfrak{C}(\wp_1) = \mathfrak{C}(\wp_2)$, then

If $\mathfrak{A}(\wp_1) < \mathfrak{A}(\wp_2)$, then $\wp_1 < \wp_2$.

If $\mathfrak{A}(\wp_1) = \mathfrak{A}(\wp_2)$, then $\wp_1$ and $\wp_2$ reflect similar information.

Definition 6. Let $\mathfrak{E} = \{x_i : i = 1, 2, \dots, n\}$ be the universal set and $A_i = (\mathfrak{m}_{A_i}, \mathfrak{v}_{A_i}, \mathfrak{n}_{A_i}), i = 1, 2, 3, \dots, n$ be the PNFNs in A. Let $w_i \in \mathfrak{W}$ be the weights corresponding to the element $A_i \in A$ where $0 \leq w_i \leq 1$, $\sum_{i=1}^{n} w_i = 1$ for $i = 1, 2, \dots, n$. Then PNF weighted aggregation operator (PNFWAO) is given by the following expression

$$PNFWAO_w(A_1, A_2, \dots, A_n) = \bigoplus_{i=1}^{n} w_i A_i$$

$$= \left(\left\{ 1 - \prod_{i=1}^{n} \left(1 - \mathfrak{m}_{ij}^2\right)^{w_i} \right\}^{\frac{1}{2}}, \prod_{i=1}^{n} \left(\mathfrak{v}_{ij}^2\right)^{w_i}, \prod_{i=1}^{n} \left(\mathfrak{n}_{ij}^2\right)^{w_i} \right) \tag{3}$$

2.2. Novel entropy and distance measures

In the current segment, we proposed a novel entropy and distance measure on the basis of PNFN and correspondingly some propositions are presented.

Entropy: For a PNFN, the entropy measure is stated as follows

$$\mathfrak{U}(\varsigma_j) = \mathfrak{a}\sum_{i=1}^{n}\left(2 + 2e^2 - e^{1+\mathfrak{m}^2-\mathfrak{n}^2} - e^{1-\mathfrak{m}^2+\mathfrak{n}^2} - e^{1+\mathfrak{v}^2-\mathfrak{v}_c^2} - e^{1-\mathfrak{v}^2+\mathfrak{v}_c^2}\right) \tag{4}$$

where $\mathfrak{v}_c = 1 - \mathfrak{v}$ and $\mathfrak{a} = \frac{1}{2n(e-1)^2}$.

Proposition 1. Let $A = (\mathfrak{m}_A, \mathfrak{v}_A, \mathfrak{n}_A)$ and $B = (\mathfrak{m}_B, \mathfrak{v}_B, \mathfrak{n}_B)$ are two PNFNs. $\mathfrak{U}(A)$ is regarded as an EM. Then the following axioms satisfy:

i. $0 \le \mathfrak{U}(A) \le 1$.
ii. $\mathfrak{U}(A) = 0$ iff A is a crisp set.
iii. $\mathfrak{U}(A) = 1$ iff $\mathfrak{m} = \mathfrak{n}$ and $\mathfrak{v} = 0.5$.
iv. $\mathfrak{U}(A) = \mathfrak{U}(A^c)$.
v. $\mathfrak{U}(A) \le \mathfrak{U}(B)$ if A is more crisper than B, i.e.,$A \le B$.

Proof: For any $x, y \in [-1, 1]$ we have, $e^{1+x}, e^{1-x}, e^{1+y}, e^{1-y} \in [1, e^2]$.

Taking a function $g(x,y) = e^{1+x} + e^{1-x} + e^{1+y} + e^{1-y}$ which has minimum at $x = 0, y = 0$ which is $4e$ that is $g(x,y) \ge 4e$.

Now choosing $x = \mathfrak{m}^2 - \mathfrak{n}^2, y = \mathfrak{v}^2 - \mathfrak{v}_c^2 \in [-1,1]$, we have, $g(x,y) = e^{1+x} + e^{1-x} + e^{1+y} + e^{1-y}$, which implies $2 + e^2 - f(x,y) \le 2 + 2e^2 - 4e = 2(e-1)^2$. Hence, $0 \le \mathfrak{E}(A) \le 1$.

ii. If A is a crisp set, then $\mathfrak{E}(A) = 0$. Conversely, if $\mathfrak{E}(A) = 0$, then $2 + 2e^2 - e^{1+\mathfrak{m}^2-\mathfrak{n}^2} - e^{1-\mathfrak{m}^2+\mathfrak{n}^2} - e^{1+\mathfrak{v}^2-\mathfrak{v}_c^2} - e^{1-\mathfrak{v}^2+\mathfrak{v}_c^2} = 0$, which implies $e^{1+\mathfrak{m}^2-\mathfrak{n}^2} + e^{1-\mathfrak{m}^2+\mathfrak{n}^2} + e^{1+\mathfrak{v}^2-\mathfrak{v}_c^2} + e^{1-\mathfrak{v}^2+\mathfrak{v}_c^2} = 2 + 2e^2$, which hold if any one of $\mathfrak{m} = 1, \mathfrak{v} = 0, \mathfrak{n} = 0$ or $\mathfrak{m} = 0, \mathfrak{v} = 0, \mathfrak{n} = 1$.

So, A is a crisp set.

iii. If $\mathfrak{m} = \mathfrak{n}$ and $\mathfrak{v} = 0.5$, then $\mathfrak{E}(A) = 1$.

Conversely, $\mathfrak{E}(A) = 1$ holds only when $e^{1+\mathfrak{m}^2-\mathfrak{n}^2} + e^{1-\mathfrak{m}^2+\mathfrak{n}^2} + e^{1+\mathfrak{v}^2-\mathfrak{v}_c^2} + e^{1-\mathfrak{v}^2+\mathfrak{v}_c^2} = 4e$.

It is possible only when is, $\mathfrak{m} = \mathfrak{n}$ and $\mathfrak{v} = 0.5$.

iv. Obvious from the definition.

v. Let us consider $h(x,y) = 1 + e^2 - e^{1+x-y} - e^{1-x+y}$

Then, $\frac{\partial h}{\partial x} = -e^{1+x-y} + e^{1-x+y}$ and $\frac{\partial h}{\partial y} = e^{1+x-y} - e^{1-x+y}$

Now, $\frac{\partial h}{\partial x} = 0$ and $\frac{\partial h}{\partial y} = 0$ implies $e^{1+x-y} = e^{1-x+y}$ that is $x = y$.

When $x \ge y$, then h is decreasing (increasing) with variable x (or y).

For two PNFNs A and B such that $\mathfrak{m}_A \le \mathfrak{m}_B$ and $\mathfrak{n}_A \ge \mathfrak{n}_B$.

By monotonicity property of h we get $\mathfrak{E}(A) \le \mathfrak{E}(B)$.

Similarly, for $x \le y$, it can also be proved that, $\mathfrak{E}(A) \le \mathfrak{E}(B)$.

Thus, defined entropy measure is valid.

Distance measure: Let $A = (\mathfrak{m}_A, \mathfrak{v}_A, \mathfrak{n}_A)$ and $B = (\mathfrak{m}_B, \mathfrak{v}_B, \mathfrak{n}_B)$ are two PNFNs, then $\mathfrak{D}(A,B): A \times B \to [0, 1]$ is said to be the distance measure between A and B and is given by

$$\mathfrak{D}(A,B) = \sqrt{\frac{1}{3n}\sum\left(|\mathfrak{m}_A^3 - \mathfrak{m}_B^3|^2 + |\mathfrak{v}_A^3 - \mathfrak{v}_B^3|^2 + |\mathfrak{n}_A^3 - \mathfrak{n}_B^3|^2\right)} \tag{5}$$

Proposition 2. Let $A = (\mathfrak{m}_A, \mathfrak{v}_A, \mathfrak{n}_A), B = (\mathfrak{m}_B, \mathfrak{v}_B, \mathfrak{n}_B)$ and $C = (\mathfrak{m}_C, \mathfrak{v}_C, \mathfrak{n}_C)$ be three PNFNs. Then, the following axioms are satisfied.

i. $0 \le \mathfrak{D}(A,B) \le 1$,
ii. $\mathfrak{D}(A,B) = \mathfrak{D}(A,B)$,
iii. $\mathfrak{D}(A,B) = 0$ iff $A = B$,
iv. If $A \subseteq B \subseteq C$, then $\mathfrak{D}(A,B) \le \mathfrak{D}(A,C)$ and $\mathfrak{D}(B,C) \le \mathfrak{D}(A,C)$.

Weighted distance measure: Let $A = (\mathfrak{m}_A, \mathfrak{v}_A, \mathfrak{n}_A)$ and $B = (\mathfrak{m}_B, \mathfrak{v}_B, \mathfrak{n}_B)$are two PNFNs. Let $w_i \in \mathfrak{W}$ be the weights vectors, with $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. Then a weighted distance between A and B, $\mathfrak{D}_W(A,B)$, defined by

$$\mathfrak{D}_W(A,B) = \sqrt{\frac{1}{3n}\sum w_i(|\mathfrak{m}_A^3 - \mathfrak{m}_B^3|^2 + |\mathfrak{v}_A^3 - \mathfrak{v}_B^3|^2 + |\mathfrak{n}_A^3 - \mathfrak{n}_B^3|^2)} \tag{6}$$

Proposition 3. Let $A = (\mathfrak{m}_A, \mathfrak{v}_A, \mathfrak{n}_A)$ and $B = (\mathfrak{m}_B, \mathfrak{v}_B, \mathfrak{n}_B)$ and $C = (\mathfrak{m}_C, \mathfrak{v}_C, \mathfrak{n}_C)$ be three PNFNs. Then, $\mathfrak{D}_W(A,B)$ satisfies the following axioms:

i. $0 \leq \mathfrak{D}_W(A,B) \leq 1$,
ii. $\mathfrak{D}_W(A,B) = \mathfrak{D}_W(B,A)$,
iii. $\mathfrak{D}_W(A,B) = 0$ iff $A = B$,
iv. If $A \subseteq B \subseteq C$, then $\mathfrak{D}_W(A,B) \leq \mathfrak{D}_W(A,C)$ and $\mathfrak{D}_W(B,C) \leq \mathfrak{D}_W(A,C)$.

3. PNF-EM-SWARA-TOPSIS system for MCDM:

Here, we introduce a novel MCDM method called PNF-EM-SWARA-TOPSIS, by suggested entropy and distance measure, in which the weights of each criterion are obtained by the entropy and SWARA approach. A step-wise procedure is presented now.

Step 1: F*ormulation of the problem.*

Let $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_m\}$ be a collection of all existing alternatives and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \dots, \mathfrak{q}_n\}$ be the criteria set. Let the group of experts $\mathcal{O} = \{\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_k\}$ express their views on all alternatives in linguistic terms, analyzing the given criteria, which are transformed into PNFNs. Let $\mathfrak{D}^r = [d_{ij}^r]_{m\times n}$, $i = 1,2,3,\dots,m$; $j = 1,2,3,\dots,n$; $r = 1,2,3,\dots,k$ be a linguistic decision matrix and $d_{ij}^r = (\mathfrak{m}_{ij}^r, \mathfrak{v}_{ij}^r, \mathfrak{n}_{ij}^r)$ denotes the alternative's $\mathcal{A}_i$'s assessment corresponding to criteria $\mathfrak{q}_j$ given by the experts $\mathcal{O}_r$.

Step 2: *Calculation of experts' weights.*

Let $W = \{w_1, w_2, \dots, w_k\}$ be the set of weights provided by k experts and $w_r = (\mathfrak{m}_r, \mathfrak{v}_r, \mathfrak{n}_r)$ represents PNFN for the r^{th}expert. Then w_r is obtained using the formula

$$w_r = \frac{\mathfrak{v}_r^2 + \left(\frac{|\mathfrak{m}_r^2 - \mathfrak{n}_r^2|}{\mathfrak{m}_r^2 + \mathfrak{v}_r^2 + \mathfrak{n}_r^2}\right)}{\sum_{r=1}^{k}\left(\mathfrak{v}_r^2 + \left(\frac{|\mathfrak{m}_r^2 - \mathfrak{n}_r^2|}{\mathfrak{m}_r^2 + \mathfrak{v}_r^2 + \mathfrak{n}_r^2}\right)\right)} \tag{7}$$

Step 3: *Aggregated decision matrix (ADM) formation.*

The PNFN-ADM is of the form $\mathfrak{D} = [d_{ij}]_{m\times n}$ where d_{ij} represents $(\mathfrak{m}_{ij}, \mathfrak{v}_{ij}, \mathfrak{n}_{ij})$ and calculated by the equation (3).

Step 4: *Assessment of weights for each criteria.*

The Criteria's weight is an important variable in MCDM approaches, and they may not always be the same. Here, we have used both weights, objective and subjective to evaluate the combined weight for each criterion. The entropy measure of PNFNs is accustomed to determine objective and the subjective weights are determined using SWARA technique.

The following equation determines the objective weights

$$\omega_j = \frac{1 - \mathfrak{U}(\varsigma_j)}{n - \sum_{j=1}^{n}\mathfrak{U}(\varsigma_j)} \tag{8}$$

where $\mathfrak{U}(\varsigma_j)$ is the EM provided in Eq. (4).

The SWARA method is employed in calculating the subjective weights. It establishes the weight for addressing MCDM issues by comparing all the criteria. The SWARA methodology involves the following steps:

Step i: Utilizing PNFWAO, get the aggregated PNFNs for the specified criteria. Next, use Eq. (1) to determine the score value $\mathfrak{s}_j$ for each PNFN of the criteria.

Step ii: Initially, the relative relevance scores of each criterion are determined and ranked in descending order. Subsequently, the relative significances (comparative importance) of criterion j and $j-1$ are computed and denoted by $\mathfrak{p}_j$. As a result of this ranking, the comparative importance values of geometric mean are determined by applying the subsequent formula.

$$\mathfrak{p}_j = \mathfrak{s}_j - \mathfrak{s}_{j-1}, \quad j = 2, \dots n. \tag{9}$$

Step iii: Compute the comparative significance for the score value obtained. For all criteria, $\mathfrak{z}_j$ values are obtained from

$$\mathfrak{z}_j = \begin{cases} 1, & j = 1 \\ \mathfrak{p}_j + 1, & j = 2, \dots n. \end{cases} \tag{10}$$

Step iv: The relative weight is determined from

$$\mathfrak{k}_j = \begin{cases} 1, & j = 1 \\ \dfrac{\mathfrak{k}_{j-1}}{\mathfrak{z}_j}, & j = 2, \dots n. \end{cases} \tag{11}$$

Step v: Subjective weights ($\mathfrak{w}_j$) for each criteria are computed as follows:

$$\mathfrak{w}_j = \frac{\mathfrak{k}_j}{\sum_{j=1}^{n} \mathfrak{k}_j} \tag{12}$$

The following equation gives the integrated weights $\mathfrak{w}_j$ for each criteria:

$$w_j = \frac{\mathfrak{w}_j \times \omega_j}{\sum_{j=1}^{n} (\mathfrak{w}_j \times \omega_j)} \tag{13}$$

Step 5: *Computation of PNF-PIS and PNF-NIS.*

The given PNF-ADM of attributes and accessible alternatives, the PNF-PIS Υ^+ and PNF-NIS Υ^- are calculated. The computation formulas Υ^+ and Υ^-: $(\mathfrak{m}_{ij}, \mathfrak{v}_{ij}, \mathfrak{n}_{ij})$

$$\Upsilon^+ = \begin{cases} (\max(\mathfrak{m}_{ij}), \min(\mathfrak{v}_{ij}), \min(\mathfrak{n}_{ij})) & Q_j \in \Omega_b \\ (\min(\mathfrak{m}_{ij}), \max(\mathfrak{v}_{ij}), \max(\mathfrak{n}_{ij})) & Q_j \in \Omega_c \end{cases} \tag{14}$$

$$\Upsilon^- = \begin{cases} (\min(\mathfrak{m}_{ij}), \max(\mathfrak{v}_{ij}), \max(\mathfrak{n}_{ij})) & Q_j \in \Omega_b \\ (\max(\mathfrak{m}_{ij}), \min(\mathfrak{v}_{ij}), \min(\mathfrak{n}_{ij})) & Q_j \in \Omega_c \end{cases} \tag{15}$$

where the specified cost and benefit criteria are denoted, respectively, by Ω_c and Ω_b.

Step 6: *Distance estimation for every alternative using PNF-PIS and PNF-NIS.*

For every $i = 1, 2, \dots, m$, calculate the distance between each alternative $\mathcal{A}_i$ and Υ^+ & $\mathcal{A}_i$ and Υ^- by the suggested weighted distance measure provided by Equation (6).

Step 7: *Determination of the closeness coefficient (CC).*

Using the following formula, the CC is computed

$$\Theta(\mathcal{A}_i) = \frac{\mathfrak{D}_W(\mathcal{A}_i, \mathcal{T}^-)}{\mathfrak{D}_W(\mathcal{A}_i, \mathcal{T}^+) + \mathfrak{D}_W(\mathcal{A}_i, \mathcal{T}^-)} \tag{16}$$

Step 8: *Choosing the optimal alternative.*

The most suitable option is the one with the higher CC value. The preference of the alternatives is obtained by arranging in descending order of the CC values.

Table 1. The LVs for criteria based PNFNs.

Linguistic Terms	Abb.	PNFN
Absolutely High Important	AHI	(0.96, 0.15, 0.12)
Very High Important	VHI	(0.90, 0.20, 0.30)
High Important	HI	(0.85, 0.33, 0.41)
Slightly High Important	SHI	(0.72, 0.45, 0.53)
Moderate Important	MI	(0.96, 0.15, 0.12)
Slightly Low Important	SLI	(0.90, 0.20, 0.30)
Low Important	LI	(0.85, 0.33, 0.41)

Very Low Important	VLI	(0.72, 0.45, 0.53)
Absolutely Low Important	ALI	(0.10, 0.20, 0.90)

Table 2. The LVs for significance of experts.

Linguistic Terms	Abb.	PNFN	Weights (w_r)
Very High significant	VHS	(0.96, 0.15, 0.12)	0.318855472
High significant	HS	(0.90, 0.20, 0.30)	0.283137394
Medium High significant	MHS	(0.85, 0.33, 0.41)	0.23963914
Low significant	LS	(0.72, 0.45, 0.53)	0.158367994

Table 3. PNF decision matrix in form of LVs.

Alternatives	Experts	Criteria						Alternatives	Experts	Criteria					
		$\mathfrak{Q}_1$	$\mathfrak{Q}_2$	$\mathfrak{Q}_3$	$\mathfrak{Q}_4$	$\mathfrak{Q}_5$	$\mathfrak{Q}_6$			$\mathfrak{Q}_1$	$\mathfrak{Q}_2$	$\mathfrak{Q}_3$	$\mathfrak{Q}_4$	$\mathfrak{Q}_5$	$\mathfrak{Q}_6$
$\mathcal{A}_1$	$\mathcal{O}_1$	VHI	MI	SHI	LI	VLI	AHI	$\mathcal{A}_4$	$\mathcal{O}_1$	AHI	MI	VHI	SLI	HI	MI
	$\mathcal{O}_2$	SLI	HI	AHI	VHI	MI	SHI		$\mathcal{O}_2$	SLI	AHI	SHI	SHI	VLI	HI
	$\mathcal{O}_3$	AHI	SLI	MI	VLI	SHI	HI		$\mathcal{O}_3$	SHI	AHI	LI	VHI	MI	AHI
	$\mathcal{O}_4$	VHI	AHI	HI	SLI	SHI	SLI		$\mathcal{O}_4$	VHI	VLI	HI	MI	AHI	SLI
$\mathcal{A}_2$	$\mathcal{O}_1$	VLI	SHI	VHI	HI	AHI	SLI	$\mathcal{A}_5$	$\mathcal{O}_1$	VLI	MI	HI	AHI	SHI	VHI
	$\mathcal{O}_2$	MI	SHI	VHI	HI	SHI	AHI		$\mathcal{O}_2$	SHI	MI	SLI	HI	AHI	SHI
	$\mathcal{O}_3$	VLI	HI	LI	SHI	SLI	MI		$\mathcal{O}_3$	AHI	VLI	SHI	VHI	MI	HI
	$\mathcal{O}_4$	MI	HI	AHI	VHI	HI	AHI		$\mathcal{O}_4$	VHI	MI	SHI	HI	AHI	SLI
$\mathcal{A}_3$	$\mathcal{O}_1$	VHI	MI	AHI	SLI	VLI	HI	Weights	$\mathcal{O}_1$	HS	LS	MHS	HS	VHS	VHS
	$\mathcal{O}_2$	AHI	LI	VHI	SHI	MI	HI		$\mathcal{O}_2$	HS	VHS	VHS	LS	HS	HS
	$\mathcal{O}_3$	VLI	SLI	MI	SHI	AHI	SHI		$\mathcal{O}_3$	VHS	LS	HS	HS	VHS	LS
	$\mathcal{O}_4$	VLI	AHI	HI	SHI	MI	VHI		$\mathcal{O}_4$	HS	HS	VHS	LS	HS	VHS

Table 4. Aggregated PNF decision matrix.

Criteria	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$
$\mathfrak{Q}_1$	{0.8516, 0.0418, 0	{0.392, 0.3968, 0.1	{0.8408, 0.0447, 0	{0.8608, 0.0286, 0	{0.7559, 0.0962, 0
$\mathfrak{Q}_2$	{0.8076, 0.0484, 0	{0.7188, 0.0951, 0	{0.7356, 0.0934, 0	{0.79, 0.0595, 0.0	{0.4796, 0.2764, 0
$\mathfrak{Q}_3$	{0.8286, 0.0361, 0	{0.9036, 0.0166, 0	{0.8908, 0.0178, 0	{0.7981, 0.0665, 0	{0.6684, 0.1382, 0
$\mathfrak{Q}_4$	{0.6371, 0.2237, 0	{0.8281, 0.0499, 0	{0.5779, 0.1868, 0	{0.5954, 0.1947, 0	{0.8848, 0.0189, 0
$\mathfrak{Q}_5$	{0.5029, 0.2743, 0	{0.8445, 0.0302, 0	{0.5658, 0.2323, 0	{0.8031, 0.0573, 0	{0.885, 0.0146, 0.0
$\mathfrak{Q}_6$	{0.813, 0.0433, 0.0	{0.885, 0.0146, 0.0	{0.8281, 0.0499, 0	{0.6889, 0.1268, 0	{0.758, 0.0915, 0.0

Table 5. Computation of PNF-PIS and PNF-NIS

Criteria	PNF-PIS (Υ^+)	PNF-NIS (Υ^-)
$\mathfrak{Q}_1$	{0.8608, 0.0286, 0.0001}	{0.392, 0.3968, 0.1121}
$\mathfrak{Q}_2$	{0.8076, 0.0484, 0.0003}	{0.4796, 0.2764, 0.0466}
$\mathfrak{Q}_3$	{0.9036, 0.0166, 0.0001}	{0.6684, 0.1382, 0.0045}
$\mathfrak{Q}_4$	{0.8848, 0.0189, 0}	{0.5779, 0.2237, 0.0203}
$\mathfrak{Q}_5$	{0.885, 0.0146, 0.0001}	{0.5029, 0.2743, 0.0212}
$\mathfrak{Q}_6$	{0.885, 0.0146, 0.0001}	{0.6889, 0.1268, 0.0012}

Table 6. Importance of criteria by different experts.

Criteria	Aggregated PNFN	$\mathfrak{s}_j$
$\mathfrak{Q}_1$	{0.7317,0.0526,0.0001}	0.8442
$\mathfrak{Q}_2$	{0.7096,0.0856,0.0016}	0.8321
$\mathfrak{Q}_3$	{0.8167,0.0331,0.0000}	0.8886
$\mathfrak{Q}_4$	{0.5411,0.1839,0.0098}	0.7529
$\mathfrak{Q}_5$	{0.8174,0.0281,0.0000}	0.8891
$\mathfrak{Q}_6$	{0.8419,0.0239,0.0001}	0.9028

Table 7. Subjective weights calculation by SWARA approach.

Criteria	Crisp Value	Comparative significance $(\mathfrak{p}_j)$	Comparative coefficient $(\mathfrak{Z}_j)$	Relative weight $(\mathfrak{k}_j)$	Subjective weight $(\mathfrak{w}_j)$	Objective weight (ω_j)	Integrated weight (w_j)
$\mathfrak{Q}_6$	0.9028	-	1	1	0.17501	0.1897	0.19811
$\mathfrak{Q}_5$	0.8891	0.0136	1.0136	0.9865567488	0.17266	0.1548	0.15951
$\mathfrak{Q}_3$	0.8886	0.0005	1.0005	0.9860554637	0.17257	0.2062	0.21236
$\mathfrak{Q}_1$	0.8442	0.0444	1.0444	0.9441394348	0.16523	0.1669	0.16456
$\mathfrak{Q}_2$	0.8321	0.0122	1.0122	0.9327842393	0.16325	0.1451	0.14142
$\mathfrak{Q}_4$	0.7529	0.0791	1.0791	0.8644049253	0.15128	0.1374	0.12405

Table 8. Computation of the integrated weights.

Criteria	Subjective weight (ϖ)	Objective weight (φ)	Integrated weight (w)
$\mathfrak{Q}_1$	0.16523	0.1669	0.16456
$\mathfrak{Q}_2$	0.16325	0.1451	0.14142
$\mathfrak{Q}_3$	0.17257	0.17257	0.21236
$\mathfrak{Q}_4$	0.15128	0.1374	0.12405
$\mathfrak{Q}_5$	0.17266	0.1548	0.15951
$\mathfrak{Q}_6$	0.17501	0.1897	0.19811

Table 9. The CC values and the alternatives' ranking

Alternatives	$\mathfrak{D}_{W_i}^{+}$	$\mathfrak{D}_{W_i}^{-}$	CC	Ranking
$\mathcal{A}_1$	0.0689	0.075	0.5212	5
$\mathcal{A}_2$	0.0588	0.0852	0.5917	1
$\mathcal{A}_3$	0.0662	0.077	0.5377	3
$\mathcal{A}_4$	0.0631	0.0782	0.5534	2
$\mathcal{A}_5$	0.069	0.0774	0.5287	4

Table 10. Different weight sets of criteria.

Criteria	S_1	S_2	S_3	S_4	S_5	S_6
$\mathfrak{Q}_1$	0.16456	0.14142	0.21236	0.12405	0.15951	0.19811
$\mathfrak{Q}_2$	0.14142	0.21236	0.12405	0.15951	0.19811	0.16456
$\mathfrak{Q}_3$	0.21236	0.12405	0.15951	0.19811	0.16456	0.14142
$\mathfrak{Q}_4$	0.12405	0.15951	0.19811	0.16456	0.14142	0.21236
$\mathfrak{Q}_5$	0.15951	0.19811	0.16456	0.14142	0.21236	0.12405
$\mathfrak{Q}_6$	0.19811	0.16456	0.14142	0.21236	0.12405	0.15951

Table 11. The CC values and the alternatives' ranking of GSSs for different weight sets.

Alternatives	S_1	S_2	S_3	S_4	S_5	S_6
$\mathcal{A}_1$	0.5212	0.4963	0.5106	0.5057	0.5	0.5274
$\mathcal{A}_2$	0.5917	0.5996	0.5553	0.6211	0.5903	0.556
$\mathcal{A}_3$	0.5377	0.4882	0.5116	0.5142	0.5014	0.5166
$\mathcal{A}_4$	0.5534	0.558	0.5549	0.52	0.5792	0.5436
$\mathcal{A}_5$	0.5287	0.5597	0.5792	0.5278	0.5561	0.5615

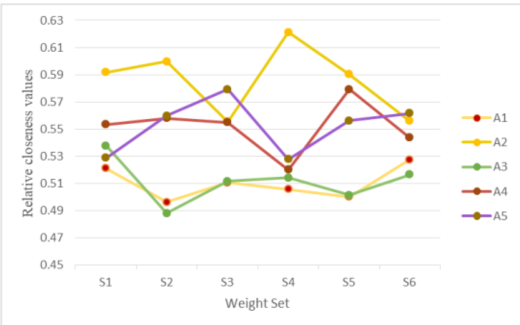

Figure 1. Alternative's rank based on closeness coefficient for different

4. Numerical Example

Advancements in engineering systems and information technology have enabled businesses to redefine core competencies and develop new models. Agricultural Implement Company (AIC) in India uses the GSS approach to assess supplier performance. AIC aims to expand its product range while maintaining leadership in innovation and quality. To align with industry standards, they plan to implement an environmental management system that collaborates with suppliers across the supply chain. GSS plays a key role in AIC's decision-making process.

AIC Corporation is striving to apply industry standards to identify the most suitable green supplier. Face-to-face interviews were conducted to assess suppliers by a panel of four specialists $\mathcal{O} = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4\}$. Four executives from different departments inside the company offered their opinions on the proposed approach. $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5\}$ specifies the five providers from whom the required part can be acquired. The six most vital eco-friendly criteria are 'delivery performance', 'pollution control', 'manufacturing', 'quality service', 'environmental representation', and 'technological capabilities' represented by $\mathfrak{Q} = \{\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3, \mathfrak{Q}_4, \mathfrak{Q}_5, \mathfrak{Q}_6\}$. 'Delivery performance', 'pollution control', 'production, quality service', 'environmental representation', and 'technological capabilities' are among the selection criteria used to identify green suppliers.

4.1. PNF-EM-SWARA-TOPSIS step by step procedure

To rank the GSSs, the suggested PNF-EM-SWARA-TOPSIS algorithm is now used. An explanation of the entire implementation process is given below.

Step 1: Expert opinions for each alternative are compiled into a PNF decision matrix (PNF-DM) in terms of LVs using the specified criteria shown in Table 3.

Step 2: Each expert's weight is presented in Table 2 obtained from Equation (7).

Step 3: The LV are used to transformed the decision matrix (Table 3) is into a PNF valued matrix. Next, using Eq. (3), a PNF-ADM (Table 4) is generated.

Step 4: This model combines objective and subjective weights to create integrated criteria weights, which are important for solving the MCDM problem. The opinion of experts is crucial for determining subjective weight but insignificant for determining objective weight. The suggested EM (Eq. (8)), is used to calculate objective weights, and the SWARA approach is applied to evaluate subjective weights, presented in Table 6 and Table 7. The combined weight set that obtained by Eq. (13) is given by $\mathfrak{W} = \{0.16456, 0.14142, 0.21236, 0.12405, 0.15951, 0.19811\}$.

Step 5: The best choice for GSS has been identified using the TOPSIS method. Equations (14) and (15) are used to calculate the PNF-PIS and PNF-NIS, which are shown in Table 5.

Step 6: The presented weighted distance measured provided in Eq. (6) is used to compute the distance between each option and PNF-PIS and PNF-NIS.

Step 7: The CC of every alternative are calculated using equation (16) and displayed in Table 9.

Step 8: The order preference of GSS is arranged as $\mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_1$ as presented in Table 9, which shows that the best suited alternative for GSS is $\mathcal{A}_2$ whereas $\mathcal{A}_1$ is the worst one.

5. Sensitivity analysis for weights

The sensitivity analysis is performed to explore the performance of the proposed method under different sets of criterion weights. The analysis is carried out using six different sets of criterion weights. As illustrated in Table 10, sets are constructed by altering the integrated weights generated by the proposed method. Figure 1 illustrates how the CC values vary according to the different criterion weights and the results are shown in Table 11. Interestingly, almost across all weight sets, alternative $\mathcal{A}_2$ emerges as the optimal or second optimal choice. Alternative $\mathcal{A}_1$ is the worst preferred alternative in sets for S_1, S_3, S_4 and S_5 while $\mathcal{A}_3$ is the least preferred for S_2 and S_6 weight sets. The analysis clearly demonstrates that the GSS evaluation is highly sensitive for criterion weights, with alternative $\mathcal{A}_2$ consistently performing well, while alternatives $\mathcal{A}_1$ and $\mathcal{A}_3$ are often the least preferred in certain weight sets.

6. The comparative study

We have contrasted our method with the prevailing methods in this study. The technique employed the process for determining the criteria weight, alternative assessment data, and the type of information on the criteria are the main factors that need to be compared. The following are the benefits of the suggested PNF-EM-SWARA-TOPSIS approach over the other popular GSS approaches.

Table 12. The suggested model's comparison with other methods.

Ref.	Methodology	Criteria assessment	Assessment information	Preference order
[38]	Picture fuzzy TOPSIS	OWA (objective)	PFS	$\mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3$
[39]	Spherical fuzzy TOPSIS	SWAM (objective and subjective)	SFS	$\mathcal{A}_2 \succ \mathcal{A}_4 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_3$
[40]	q-rung picture fuzzy VIKOR	Entropy (objective)	q-RPFS	$\mathcal{A}_2 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_1$
[41]	SLDFS TOPSIS	OWA (subjective)	SLDFS	$\mathcal{A}_2 \succ \mathcal{A}_5 \succ \mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_3$
Proposed method	**PNF-EM-SWARA-TOPSIS**	**Entropy and SWARA (objective and subjective)**	**PNFS**	$\boldsymbol{\mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_1}$

The proposed method operates within a PNF environment, to minimize information loss effectively over picture fuzzy set, q-rung picture fuzzy set, soft fuzzy set, and spherical linear Diophantine fuzzy set. Unlike other methods that focus solely on objective weights ([38, 40, 41]) or assume equal expert weightings ([39]), our approach integrates both objective and subjective criteria using the EM and SWARA techniques. This dual-weight strategy addresses limitations in previous studies and improves the reliability of results. As shown in Table 12, our method yields an identical preference ranking for optimal alternative, though it aligns closely with existing methods when only one type of weight is considered.

7. Conclusions

This study presents PNF, aimed to address uncertainty in MCDM challenges effectively by introducing novel distance and entropy measures. Also introduce new score and accuracy functions to rank PNFNs. The method integrates objective weights, derived through entropy, and subjective weights, calculated via the SWARA method, to determine comprehensive criteria weights. The application of TOPSIS validates the method’s effectiveness in selecting optimal alternatives, with sensitivity analysis demonstrating its impact on criterion weights. While the proposed approach provides a robust decision-making framework, future work could explore its extension to more complex scenarios, such as Circular Intuitionistic Fuzzy Set (CIFS), m-neutrosophic fuzzy set (m-NFS), and Fermatean neutrosophic fuzzy set (FNFS), and its application in fields like sustainable resource allocation, climate change mitigation, electric vehicles, and healthcare management etc.

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