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A Novel Neutrosophic Relational Maps Approach for Business Process Maturity Assessment in Logistics and Trade

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Abstract-In today's rapidly evolving logistics and trade environments, assessing the maturity of business processes is crucial for sustaining competitive advantage. However, traditional maturity models often fail to accommodate the inherent uncertainties and indeterminacies in real-world logistics operations. To address this gap, we propose a novel mathematical framework based on Neutrosophic Relational Maps (NRMs) for Business Process Maturity Assessment (BPMA). Unlike existing methods, our approach introduces innovative neutrosophic weighting functions and a new dynamic updating mechanism to handle the simultaneous presence of truth, falsehood, and indeterminacy in logistical relationships. We formulate the mathematical foundations of this approach through new matrix operations and neutrosophic thresholding equations, supported by a fully calculated step-by-step example illustrating the BPMA in a hypothetical logistics scenario. Finally, we provide an in-depth analysis and discussion of the implications, advantages, and limitations of the proposed model, articulated in clear and concise English to ensure accessibility and reproducibility.

Keywords: Neutrosophic sets; Neutrosophic Relational Maps (NRMs); Business Process Maturity Assessment; Logistics and Trade

1. Introduction

In today's fast-paced world, businesses, especially in logistics and trade, are under constant pressure to improve their processes to stay competitive. Continuous improvement has become a cornerstone of operational excellence, ensuring that organizations run smoothly, adapt to challenges, and meet customer demands efficiently [1]. One powerful tool for achieving this is Business Process Maturity Assessment (BPMA), which provides a structured way to evaluate how well-organized and effective business processes are [2]. By assessing maturity, companies can pinpoint weaknesses, streamline operations, and set a clear path for growth. Traditional maturity models, such as the Capability Maturity Model Integration (CMMI), have been widely adopted across industries to guide this process [3]. These models offer a standardized framework to measure process maturity, helping organizations move from chaotic, ad-hoc practices to disciplined, optimized systems.

However, logistics and trade operations present unique challenges that classical maturity models often struggle to address. These models typically assume that relationships between process elements such as tasks, resources, or stakeholders—are clear and predictable [4]. In reality, logistics systems are far more complex. They involve multiple stakeholders, from suppliers to shipping companies to customers, all interacting in dynamic and sometimes unpredictable ways. Add to that volatile market demands, regulatory changes, and external disruptions like weather or geopolitical events, and you get a system filled with uncertainty and ambiguity [5]. This complexity makes it difficult to evaluate processes accurately using traditional models, as they often fail to capture the indeterminate or unclear relationships that define real-world logistics.

This is where Neutrosophic Relational Maps (NRMs) come in. NRMs are an advanced extension of Fuzzy Relational Maps, designed to handle the uncertainty and complexity that traditional models overlook [6]. Unlike binary logic (which sees relationships as simply "yes" or "no") or fuzzy logic (which allows for degrees of truth), NRMs take a more nuanced approach. They evaluate relationships using three independent components: truth (T), indeterminacy (I), and falsehood (F) [7]. This triadic structure allows NRMs to represent not just whether a connection exists, but also how certain or uncertain it is. For example, in a logistics process, NRMs can show that a supplier's reliability is partially true, partially uncertain, and partially false, giving a much richer picture of the system. This makes NRMs uniquely suited for analyzing the intricate, interconnected, and often ambiguous relationships in logistics and trade processes [8].

While NRMs have been used in other fields, such as decision-making, social sciences, and system modeling, their applications have typically been limited [9]. Most studies use NRMs for static modeling creating a snapshot of a system based on expert knowledge—or for mapping straightforward relationships. These approaches, while useful, don't fully tap into the potential of NRMs to handle dynamic, evolving systems like those in logistics [10]. Our research breaks new ground by going beyond these conventional uses. We propose a novel mathematical formulation and analytical framework that leverages the full power of NRMs to create a dynamic, robust, and highly tailored BPMA system for logistics and trade.

Our approach introduces several innovations. First, we develop new matrix formulations that allow NRMs to model complex process relationships more effectively. Second, we design dynamic updating algorithms that enable the system to adapt as conditions change, such as shifts in demand or disruptions in supply chains. Third, we implement robust thresholding mechanisms to ensure that the model focuses on the most significant relationships, making it practical for real-world use [11]. These advancements address the specific challenges of BPMA in logistics, where uncertainty and complexity are the norm, not the exception.

The goal of this research is to create a BPMA system that is not only theoretically sound but also practically valuable for logistics and trade organizations. By integrating NRMs in a new and innovative way, we aim to provide a tool that helps businesses make better decisions, optimize their processes, and achieve operational excellence. To make our work accessible, we use simple language to explain complex ideas and provide clear, calculated examples to illustrate how our framework operates.

The rest of this paper is organized as follows:

Section 1: Definitions and Mathematical Formulations We lay out the mathematical foundation of our approach, including new definitions and properties of NRMs tailored for BPMA.

Section 2: Proposed Methodology We present our step-by-step framework, complete with detailed examples to show how it works in practice.

Section 3: Application We apply our methodology to a hypothetical logistics and trade scenario, demonstrating its effectiveness and real-world relevance.

Section 4: Analysis and Discussion We analyze the results, discuss their implications, and explain their significance in clear, straightforward terms.

Section 5: Conclusion We summarize the strengths and limitations of our approach and suggest directions for future research.

2. Definitions and Mathematical Formulations

1.1 Neutrosophic Relational Maps

Let us consider two disjoint sets of concepts:

$$C = \{C_1, C_2, \dots, C_m\} \text{ and } D = \{D_1, D_2, \dots, D_n\}$$

where C represents causal (input) concepts (e.g., process steps, performance drivers) and D represents resultant (output) concepts (e.g., business process maturity indicators).

The neutrosophic relational matrix N is defined as:

$$N = \left[\left(T_{ij}, I_{ij}, F_{ij}\right)\right]_{m\times n}$$

where each entry (T_{ij}, I_{ij}, F_{ij}) represents the degree of:

Truth (T)

Indeterminacy (I)

Falsehood (F) of the influence from C_i to D_j.

1.2 New Neutrosophic Weighting Function

We introduce an innovative weighting function that dynamically balances these components:

$$W_{ij} = \alpha T_{ij} + \beta I_{ij} + \gamma F_{ij} \quad (1)$$

where:

$\alpha, \beta, \gamma \in [0,1]$ are tunable weights reflecting the relative importance of truth, indeterminacy, and falsehood in the maturity assessment context.

$\alpha + \beta + \gamma = 1.$

This allows domain experts to adapt the system to different levels of maturity risk tolerance (e.g., if indeterminacy is particularly important, set β higher).

1.3 Dynamic Updating Mechanism

Let the input vector (state vector) at time t be:

$$X(t) = [x_1(t), x_2(t), \ldots, x_m(t)] \quad (2)$$

The output vector at time $t + 1$ is:

$$Y(t+1) = X(t) \cdot W \quad (3)$$

where W is the neutrosophic weighted relational matrix:

$$W = \left[W_{ij}\right]_{m \times n} \quad (4)$$

1.4 Neutrosophic Thresholding Function

To ensure the maturity output vector reflects actionable insights, we introduce a neutrosophic thresholding operator:

$$Y_j(t+1) = \begin{cases} 1 & \text{if } Y_j(t+1) \geq \delta \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where δ is a dynamic threshold parameter (can be domain-specific or data-driven).

1.5 Iterative Convergence: Hidden Patterns

To detect the hidden patterns (maturity stability) of the system, we iterate:

$$X(t+1) = Y(t+1) \quad (6)$$
$$Y(t+2) = X(t+1) \cdot W \quad (7)$$

This continues until convergence:

$$Y(t+k) = Y(t+k+1) \quad (8)$$

The converged vector Y^* represents the steady maturity state of the business process.
1.6 Neutrosophic Maturity Index (Novel Contribution)

We introduce a new maturity index to quantify the overall maturity level:

$$\text{NMI} = \frac{\sum_{j=1}^{n} Y_j^*}{n} \quad (9)$$

where:

Y_j^* is the final state (0 or 1) for output concept D_j.

NMI ranges from 0 (immature) to 1 (fully mature).

1.8 Neutrosophic Influence Strength (NIS)

To capture the combined influence of each causal concept C_i on the overall system, we can define:

$$\text{NIS}(C_i) = \sum_{j=1}^{n} W_{ij} \quad (10)$$

This represents how strongly each input concept contributes to the system's maturity.

1.9 Neutrosophic Receptiveness Index (NRI)

To measure how receptive or sensitive each output maturity indicator D_j is to changes in the input layer:

$$\text{NRI}(D_j) = \sum_{i=1}^{m} W_{ij} \quad (11)$$

This will help us identify key bottlenecks or vulnerable indicators in the logistics maturity framework.

1.10 Dynamic Adjustment of Threshold (δ)

Rather than a static threshold, we can introduce a data-adaptive dynamic threshold:

$$\delta = \mu_Y + \lambda\sigma_Y \quad (12)$$

μ_Y is the mean of the current output vector $Y(t+1)$.

σ_Y is the standard deviation of $Y(t+1)$.

λ is a sensitivity parameter that adjusts how strict the threshold is.

This ensures that the threshold adapts to the spread of maturity levels in the system.

1.11 Neutrosophic Impact Matrix (NIM)

A powerful enhancement for later analysis:

$$\mathrm{NIM} = W \cdot W^{\top} \quad (13)$$

This self-multiplication of the weighted matrix W measures influence propagation and feedback loops across the system.

NIM_{ii} shows self-reinforcement of each input.

NIM_{ij} shows cumulative propagation of influences from C_i to C_j.

1.12 Stability Condition

A novel stability condition can be defined by checking the eigenvalues of NIM :

$$\rho(NIM) < 1 \quad (14)$$

where ϱ(·) is the spectral radius (largest absolute eigenvalue).
This ensures that the maturity system converges to a stable state and does not oscillate wildly.

3. Application of the Novel Neutrosophic Relational Maps Framework

Imagine a logistics company wants to assess the maturity of its Order Fulfillment Process (OFP) in the context of international trade.
We define:

Input concepts (Causal drivers, C):
C_1 : Supplier Reliability
C_2 : Warehouse Efficiency
C_3 : IT System Robustness

Output maturity indicators (D) :
D_1 : On-Time Delivery
D_2 : Order Accuracy
D_3 : Customer Satisfaction
2.1 Initial Neutrosophic Relational Matrix

Based on expert opinions, the neutrosophic relational matrix N is:

$$N=\begin{bmatrix}(0.8,0.1,0.1) & (0.7,0.2,0.1) & (0.9,0.05,0.05)\\(0.6,0.2,0.2) & (0.8,0.1,0.1) & (0.7,0.2,0.1)\\(0.7,0.1,0.2) & (0.6,0.2,0.2) & (0.8,0.1,0.1)\end{bmatrix}$$

Weighting Parameters

Let:

$$\alpha=0.5,\beta=0.3,\gamma=0.2$$

which means truth is most important, followed by indeterminacy and falsehood.

Weighted Matrix Calculation

Using Equation (1) for Calculating each entry:

$$W_{11}=0.5\cdot 0.8+0.3\cdot 0.1+0.2\cdot 0.1=0.4+0.03+0.02=0.45$$
$$W_{12}=0.5\cdot 0.7+0.3\cdot 0.2+0.2\cdot 0.1=0.35+0.06+0.02=0.43$$
$$W_{13}=0.5\cdot 0.9+0.3\cdot 0.05+0.2\cdot 0.05=0.45+0.015+0.01=0.475$$
$$W_{21}=0.5\cdot 0.6+0.3\cdot 0.2+0.2\cdot 0.2=0.3+0.06+0.04=0.4$$
$$W_{22}=0.5\cdot 0.8+0.3\cdot 0.1+0.2\cdot 0.1=0.4+0.03+0.02=0.45$$
$$W_{23}=0.5\cdot 0.7+0.3\cdot 0.2+0.2\cdot 0.1=0.35+0.06+0.02=0.43$$
$$W_{31}=0.5\cdot 0.7+0.3\cdot 0.1+0.2\cdot 0.2=0.35+0.03+0.04=0.42$$
$$W_{32}=0.5\cdot 0.6+0.3\cdot 0.2+0.2\cdot 0.2=0.3+0.06+0.04=0.4$$
$$W_{33}=0.5\cdot 0.8+0.3\cdot 0.1+0.2\cdot 0.1=0.4+0.03+0.02=0.45$$

Thus, the weighted matrix W :

$$W=\begin{bmatrix}0.45 & 0.43 & 0.475\\0.4 & 0.45 & 0.43\\0.42 & 0.4 & 0.45\end{bmatrix} \qquad (4)$$

Initial Input Vector

Assume at $t=0$, the initial driver states are:

$$X(0)=[1,0,1]$$

First Output Vector Calculation

Using Equation (3) to compute:

$$Y_1(1)=1\cdot 0.45+0\cdot 0.4+1\cdot 0.42=0.45+0+0.42=0.87$$
$$Y_2(1)=1\cdot 0.43+0\cdot 0.45+1\cdot 0.4=0.43+0+0.4=0.83$$
$$Y_3(1)=1\cdot 0.475+0\cdot 0.43+1\cdot 0.45=0.475+0+0.45=0.925$$
$$Y(1)=[0.87,0.83,0.925]$$

Dynamic Threshold Calculation by Using Equation (12)

$$\mu_Y = \frac{0.87 + 0.83 + 0.925}{3} = \frac{2.625}{3} = 0.875$$

$$\sigma_Y = \sqrt{\frac{(0.87 - 0.875)^2 + (0.83 - 0.875)^2 + (0.925 - 0.875)^2}{3}}$$

$$= \sqrt{\frac{(0.000025 + 0.002025 + 0.0025)}{3}} = \sqrt{0.001516} \approx 0.039$$

Let $\lambda = 1$:

$$\delta = 0.875 + 1 \cdot 0.039 = 0.914$$

Thresholded Output Vector by Using Equation (5):

$$Y(1) = [0,0,1]$$

Next Iteration

Using Equations (6)-(7):

$$X(1) = Y(1) = [0,0,1]$$

$$Y(2) = X(1) \cdot W = [0 \cdot 0.45 + 0 \cdot 0.4 + 1 \cdot 0.42, 0 \cdot 0.43 + 0 \cdot 0.45 + 1 \cdot 0.4, 0 \cdot 0.475 + 0 \cdot 0.43 + 1 \cdot 0.45]$$

$$Y(2) = [0.42,0.4,0.45]$$

Recompute Threshold

$$\mu_Y = \frac{0.42 + 0.4 + 0.45}{3} = \frac{1.27}{3} = 0.423$$

$$\sigma_Y = \sqrt{\frac{(0.42 - 0.423)^2 + (0.4 - 0.423)^2 + (0.45 - 0.423)^2}{3}} = \sqrt{0.00045} \approx 0.021$$

$$\delta = 0.423 + 0.021 = 0.444$$

Thresholding:

$$Y(2) = [0,0,1]$$

Convergence to Fixed Point

The system has converged:

$$Y^* = [0,0,1]$$

Maturity Index Calculation

Using Equation (9):

$$\text{NMI} = \frac{1}{3} = 0.33$$

This example shows how the maturity drivers flow through the system and how the model uses dynamic thresholds to adapt to changing data. It also highlights how the system can find a steady, reliable pattern, showing what parts of the process are consistently working well. Finally, it measures the overall maturity using the Neutrosophic Maturity Index, giving a clear, single score that shows how mature the process is.

4. Analysis and Discussion

In this section, we see how the initial drivers influenced the final outcome in a real-world setting. While Warehouse Efficiency was not active, Supplier Reliability and IT System Robustness were key drivers. Applying the neutrosophic model showed that even with some weak areas, maintaining strong performance in suppliers and IT systems was enough to achieve Customer Satisfaction. This final steady state shows what parts of the system are most reliable and important for business success.

The Neutrosophic Maturity Index shows that the process maturity is low, around 33%, and highlights the need for better performance in areas like Warehouse Efficiency. Unlike older models that only use yes/no logic, this approach includes uncertain factors, making it more flexible and realistic. It adjusts as new data comes in and quickly identifies strong and weak areas in the process. This makes it easier for managers to see what needs attention.

It depends on accurate expert opinions, but with good data, it can be a very useful tool. In the future, it could be even better with real-time data and machine learning to fine-tune the model. This method provides a simple and clear way to see how different parts of the process work together and how mature the overall system is.

5. Conclusion

This work presents a fresh and adaptable way to check how well business processes work in logistics and trade. Using Neutrosophic Relational Maps, we can see not just clear links but also the uncertain parts of the process. We showed every calculation step, leading to a final maturity score of 0.33. This means there's a lot of room for improvement, especially in areas like warehouse operations. The method is good at handling uncertainty and can be updated as new information becomes available. While it does rely on accurate data and expert input, it can still be a very useful tool. Going forward, adding real-time data and smart updates from machine learning will help businesses act more quickly and make smarter choices.

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